New mathematical rules and methods for the strong conjecture of Goldbach to be verified

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Abstract

This article emphasizes the most fundamental rules to verify Goldbach's strong conjecture that an even number is the sum of two primes. One rule states that for an even number E to split into two primes there must be two equidistant prime numbers p and p' such that E/2 - p = p' - E/2. The strong conjecture also applies to biprime numbers that are $x^2 - y^2$. Two prime numbers equidistant with respect to an integer n have a specific property of Modulo when divided by the gap that separates them from n. The paper further proposes methods to convert even and odd numbers into sums of two and three prime numbers by the equation M ± 1 such that M is prime or multiple of primes except 2 and 3 knowing that there are two types of prime numbers 6x - 1 and 6x + 1. The data also show a strong correlation coefficient between close equidisant primes indicating they are likely to happen in a regular fashion. Finally, the paper describes new rules that explain how a prime numbers gives another one and this is where the truth of Goldbach's conjecture lies and show congruence rules between the two additive primes. These rules allow to demonstrate how an even ends up to be a sum of two primes and proves Goldbach's strong conjecture. This article can have new applications in computing and sheds new lights on the Goldbach's strong and weak conjectures.

Key words : Goldbach. Strong Conjecture. Weak Conjecture. Primes. Addition. Equidistant primes. Euclidean division. Remainders. Prime factor. Congruence modulo. Gap. Correlation coefficient.

1. Introduction

There are two conjectures of Christian Goldbach (1690-1764) that have been the focus of mathematical research for a very long time, they are called the weak and the strong one (Goldbach, 1742). The strong conjecture states that every even natural number greater than 4 is the sum of two prime numbers whereas the weak one says that every odd integer greater than 8 is the sum of three primes. Today websites such like https://www.dcode.fr/conjecture-goldbach or https://wims.univ-cotedazur.fr propose to put Goldbach's conjectures into practice to convert an even number into a sum of prime numbers. In addition, the Goldbach partition function is the function that associates to each even integer the number of ways it can be decomposed into a sum of two primes. Its graph looks like a comet and is therefore called Goldbach's comet (Fliegel and Robertson, 1989). Goldbach's weak conjecture has been verified for all integers up to $8{,}875.10^{30}$ (Helfgott and Platt, 2013). But what exactly do these conjectures mean in the strict mathematical sense? They postulate that by combining the prime numbers by adding them is enough to regenerate any even or odd number. In mathematics, the fundamental theorem of arithmetic, also called the unique factorization theorem states that every integer greater than 1 can be represented uniquely as a product of prime numbers and algorithms are today available to facotrize integers (Pollard, 1974). The difference between the Goldbach's conjectures and the unique factorization theorem is that the conjectures suggest that a number might be converted to many different sums of prime numbers while the latter says that there is only one product of prime numbers for an integer. Many attemps have been underken since then to provide a proof for their truthfulness (Helfgott and Platt, 2013; Estermann et al, 1938; Markakis, 2013).

These conjectures suggest that there are enough prime numbers to generate the entire set N of integers from 5 to infinity. However, the prime number theorem which describes the asymptotic distribution of prime numbers and allows us to calculate the density of prime numbers in a predefined area of numbers (Chaudhuri, 2017; Liu, 2013), rather show that prime numbers become rarer as we tend to infinity so that these conjecture might not hold true to infinity. We still cannot predict where and when a prime number appears by a unitary equation although many mathematicians still believe those conjectures hold true (Guiasu, 2019; Markakis et al, 2013). There have been many empirical verifications of it, up to astronomic numbers, but it has remained unproven since 1742 and that what is still believed today. Therefore, Goldbach's conjecture remains one of the best-known unsolved problems in mathematics. Otherwise some think they might be viewed as an axiom because if they are unrpoven then they must be true (https://www.irishtimes.com/news/science/goldbach-s-conjecture-if-it-s-unprovable-it-must-be-true-1.4492890). Markakis et al (2013) presented a detailed study on the classification of even numbers by the equation 6x + n (n= 0, n= 2 and n= 4) and a method for their conversion into sums of prime numbers. Armed with three theorems Markakis et al (2013) lean in favor of the truth of Goldbach's strong conjecture and discusses the distribution of prime numbers claiming that it is not random but rather predetermined. Guiasu (2019) has shown that for every positive composite number n, strictly larger than 3, there are two primes equidistant with respect to n. The paper contains a proof of this prime symmetry property and, implicitly, of Goldbach's conjecture for 2n as well.

The present article aims to define new rules of calculation as well as a method to put into practice the two Goldbach conjectures and discuss their mathematical meaning by resorting to deductive reasoning (if A then B). It focuses on the rules of calculation of addition between prime numbers. Second, it proposes a simpler and elementary accessible method based on the equations M + 1 and M + 5 (M is either prime or a multiple of primes except 2 and 3) to convert an even or odd number into a sum of primes. It states specific rules based on the fact that there are two types of prime numbers 6x + 1 and 6x - 1. This new method is not only programmable but can be exploited on a large scale to verify Goldbach's conjectures. Finally, this article explains how a prime number leads to another by explaning the gaps that separate them and show new congruence rules that determine if two primes can add together to form an even. Globally, this paper provides a basic demonstration of Goldbach's strong conjecture and draws the limits of its truthfulness.

2. Results

In the first section of the article (**2A**), GSC will be assumed to be true and then the initial conditions required for it to be verified by computational rules will be defined. In the second section, we'll also look at the addition rules obeyed by GSC (**2B**). For the rest and until the end, we'll look at how to find prime numbers that satisfy this GSC for any even number.

2A- Calculation rules for the strong and weak conjectures of Goldbach

1. If $E = p_1 + p_2$ and $p_2 > p_1 \rightarrow p_1 < E/2$ and $p_2 > E/2 \rightarrow E/2 - p_1 = p_2 - E/2$. E/2 is any integer ≥ 4 and E any even ≥ 8 (this is true for this entire article). The prime numbers p_1 and p_2 are said to be equidistant relatively to E/2. For the Goldbach's strong conjecture (GSC) to be true, there must exist at least two equidistant primes.
2. Two prime numbers p_1 and p_2 which are both < E/2 or both > E/2 will not verify the Goldbach's conjecture $E = p_1 + p_2$.
3. If two prime numbers p and p' are equidistant with respect to any integer n then $2n = p + p'$. Example, 37 and 29 are equidistant relatively to 33 and then 37 + 29 = 2 x 33 = 66. For any even number $E \geq 8$ its half E/2 is surrounded by two equidistant prime numbers including one before (p_1) and one after (p_2) such that $p_1 + p_2 = 2 \times E/2 = E$. That starts with 8 = 5 + 3 with E/2 = 4. This article will discuss in details this rule that determines if GSC is true.
4. The GSC therefore means that an even E is constructed with two prime numbers p and p' that are located at the same distance of E/2. These two primes are said to be equidistant relatively to E/2. It is under this condition that the GSC stating that an even E = p + p' is verified correctly. For example 100 = 3 + 97 such that 50 - 3 = 97 – 50 or 18 = 5 + 13 then 9 - 5 = 13 – 9. Or 190 = 17 + 173 such that 95 – 17 = 173 – 95.
5. Suppose we have an even number that we want to convert to the sum of two prime numbers. For example, let's take 1256 and divide it by 2 = 628. We will look for the prime numbers that surround 628 and find those that are at the same distance from 628. We have the two prime numbers 613 = 628 - 15 and 643 = 628 + 15. And so 613 + 643 = 1256.
   Here is another example. The number randomly chosen 14896 the half of which is 7448. We have two prime numbers 7349 and 7547 such that 7448 – 7349 = 99 and 7547 – 7448 = 99. Hence 7349 + 7547 = 14896. See table 1 below for more examples of calculation.

Table 1. For the Goldbach's strong conjecture (GSC) to be verified and if an even $E = p_1 + p_2$ then $E/2 - p_1 = p_2 - E/2$. The table shows examples of verification of this rule with chosen numbers. Primes p_1 and p_2 shown are equidistant because $E/2 - p_1 = p_2 - E/2$.

E	$p_1 + p_2$	E/2	E/2 - p_1	p_2 . E/2
66	29 + 37	33	4	4
1780	557+1223	890	333	333
37674	18191+19483	18837	646	646
1173850	174989 + 998861	586925	411936	411936
2460650	880069 + 1580581	1230325	350256	350256
690116436	678955259 + 11161177	345058218	333897041	333897041
9077236708	331582187 + 8745654521	4538618354	4207036167	4207036167
1574407869450	699845716519 + 874562152931	787203934725	87358218206	87358218206

6. If we already know its prime factors we can frame any biprime number by two perfect squares as follows: be a biprime number $N_b= xy$ such $x < y$; we calculate $(x + y)/2 = z$ and then $y – z = t$; then $N_b = z^2 – t^2$. For example let's take the biprime number 13 289 = 97 x 137. Let's calculate (97 + 137)/2 = 117 and then 137 – 117 = 20 then $13\,289 = (117)^2– (20)^2 = (117 – 20)\ (117 + 20) = 97 \times 137$.
   There is a link between GSC and the remarkable identity $x^2 - y^2$ which is used to factor biprime integers.
7. Let E be an even number and let E = 2pq (p and q are any prime factors > 2).
   $E/2 = p \times q$ such that $q > p$ and therefore $E/2 = x^2 – y^2$. First let calculate $(p + q)/2 = M$ and $q – M = z \rightarrow E/2 = M^2 – z^2 = (M – z)(M + z) \rightarrow E = 2\,(M – z)\,(M + z)$. Hence $p = M – z$ and $q = M + z$. Therefore, there always exist two equidistant prime numbers such that $p + z = M$ and $q – z = M$ to form $E = 2pq$ or $Nb = pq$. Because p and q might be any prime number (except 2) then all prime numbers are equidistant relatively to an integer value M such that $p + z = M$ and $q – z = M$. Given that M might be any integer then 2M might be any even which is therefore a sum of the two primes p and q. In fact $p + z = M$ and $q – z = M \rightarrow 2M = (p + z) + (q – z) = p + q$. The GSC also applies for biprime numbers. This is a demonstration going from the multiplicative structure of integers to the additive one. This means that prime numbers are equidistant in addition or multiplication when combined by two.

- Following the demonstration cited above we can substitute z by t and M by n kowing that E is any even = 2pq (q > p ; q > 2 ; p > 2) and n = (p + q)/2. So we have **$E/2 = pq = (n – t)(n + t) = n^2 - t^2$**. Using the principle of equivalence we can say that ***the*** *factorization of a biprime number implies that an even is the sum of two prime numbers because it implies the existence of two prime numbers equidistant to n*. Therefore **$E/2 = pq = (n – t)(n + t) \leftrightarrow p = n – t$ and $q = n + t \leftrightarrow 2n = p + q$**. This means that if all biprime numbers are written $x^2 - y^2$ it is because all even numbers > 4 are sums of two equidistant primes.
- Let us note in passing that prime numbers can be written as sums of squares. If a prime number is then written as $x^2 + y^2$, it will not have a prime equidistant from a specific mean. for example 89 = 64 + 25 does not have a symmetric prime at position 64 – 25 = 39 = 3 x 13. Here the mean value between 89 and 39 is 64. This applies even for contiguous primes example $101 = 10^2 + 1^2$ will not have a twin with respect to 100 because 100 – 1 = 99 = 9 x 11. Here 100 is the mean value between 99 and 101.
- $E/2 = pq$ and because $q > p$ and $q = E/2 + t$ and $p = E/2 – t \rightarrow q – p = (E/2 + t) – (E/2 – t) \rightarrow q = p + 2t \rightarrow E/2 = p(p + 2t) \rightarrow E/2 = p^2 + 2tp \rightarrow$ *$t = (E/2 – p^2)/p$*. Or $E/2 = q(q – 2t) \rightarrow E/2 = q^2 – 2tq \rightarrow$ *$t = (q^2 – E/2)/q \rightarrow (E/2 – p^2)/p = (q^2 – E/2)/q$*.

8. As an important reminder, equidistant prime numbers introduced in this article are not to be confused with twin prime numbers. The difference between two twin prime numbers that = 2 is visible because it separates two numbers that follow each other in the set of integers. But the symmetry between two equidistant prime numbers is only visible between them when they are prime factors of a biprime number in product or when they add up to form an even number.
9. A prime number p has an infinity of equidistant primes numbers. There is no prime number that does not have an equidistant prime number (except 2) and therefore GSC is true. A counterexample cannot be found to contradict this fact.

10. This rule works with twin prime numbers because they are equidistant relative to the even number between them and their addition is in agreement with GSC. Twin prime numbers are not the only ones to be equidistant. But all prime numbers are equidistant relatively to a mean whey they are in a sum or in a biprime product. Two given primes are equidistant to one single value.
11. It is known that between E and 2E, there is always a prime number (Bertrand's postulate that for every $n > 1$ there is a prime p with $n < p < 2n$). Between 0 and E/2 on one hand, and E/2 and E on the other hand, there would exist two equidistant primes satifsying the GSC and therefore Bertrand's postulate is not enough to prove GSC is true.

12. Be E any even ≥ 8 (note E/2 is thus any integer $n \geq 4$). Because there are always two integers such that $(E/2 - x) \in [0 - E/2]$ and $(E/2 + x) \in [E/2 - E]$ that are both primes (noted p and p' respectively) then any even $2E = 2n = (E/2 - x) + (E/2 + x) = p + p'$. Hence GSC is true. A counterexample cannot be found to contradict this fact.
13. P and P' are two equidistant prime numbers relatively to E/2 such that $t = E/2 - P = P' - E/2$. E/2 is any integer ≥ 4 and E any even ≥ 8 . Then, $E/2 \equiv P \equiv P'$ modulo (t). Demonstration is below with r the remainder of the euclidean division.
    $P \rightarrow E/2 \leftarrow P'$. $E/2 - P = t$ and $P' - E/2 = t$
    $E/2 = at + r \rightarrow P = E/2 - t = at + r - t \rightarrow$ $\mathbf{P = t(a-1) + r}$
    $E/2 = at + r \rightarrow P' = E/2 + t = at + r + t \rightarrow$ $\mathbf{P' = t(a+1) + r}$ $\rightarrow E/2 \equiv P \equiv P'$ modulo (t).

Here are some examples below :

- $666 = 2 \times 3^2 \times 37$ and $89 + 577 = 666$ (89 and 577 are primes)

666/2 = 333. 333 – 89 = 244. 577 – 333 = 244
333/244 = 1.3647540983606557377049180327868852459016393 (r = 89)
89/244 = 0.3647540983606557377049180327868852459016393 (r = 89)
577/244 = 2.3647540983606557377049180327868852459016393 (r = 89)

- $1764 = 2^2 \times 3^2 \times 7^2$

613 + 1151 = 1764 (613 and 1151 are primes)
1764/2 = 882
882 – 613 = 269
1151 – 882 = 269
882/269 = 3.2788104089219330855018587360594795539033457 (r = 75)
613/269 = 2.2788104089219330855018587360594795539033457 (r = 75)
1151/269 = 4.2788104089219330855018587360594795539033457 (r = 75)

- If $E/2 - P = t$ and $P' - E/2 = t$, Tables 2A-C show that t is either prime or composite for even numbers. Any integer is surrounded by two equidistant primes and any prime has a equidistant prime relatively to an integer. Equidistant primes give the same remainder when divided by t.
- Any integer increased or decreased gives either a composite or a prime number but both are likely to happen because if not there would be no prime numbers or much lesser in the set of integers. GSC means that for any integer N there is an integer t ($t < N$) such that $N - t$ and $N + t$ are equidistant prime numbers the sum of which gives any even and therefore any even is sum of two primes. However, these equidistant primes cannot be predicted with an established equation which explain why this conjecture remains unsolved. We can however prove it by following the calculation rules described here. Otherwise, search for a counter-example to reject this rule.

Tables 2: Remainders (r) of the euclidean divisions and the difference (t) between an even or an odd number and the equidistant prime numbers that surround them. Euclidean divisions are calculated with X, Y and the number shown. Note t has specific values either prime or 3n in an increasing order. Equidistant primes X and Y give the same remainder. Equidistant primes are highlighted. Note that sum of the two equidistant primes = 2 x the number shown (60 for 30; 58 for 29; 100 for 50; 98 for 49; 96 for 48, and 94 for 47).

Table 2A- Numbers 30 and 29.

X	r (30 : X)	r (Y : 30)	Y	t	X	r (29 : X)	r (Y : 29)	Y	t
29	1	1	31	1	28	1	1	30	
28	2	2	32		27	2	2	31	
27	3	3	33		26	3	3	32	
26	4	4	34		25	4	4	33	
25	5	5	35		24	5	5	34	
24	6	6	36		23	6	6	35	
23	7	7	37	7	22	7	7	36	
22	8	8	38		21	8	8	37	
21	9	9	39		20	9	9	38	
20	10	10	40		19	10	10	39	
19	11	11	41	11	18	11	11	40	
18	12	12	42		17	12	12	41	2x 6
17	13	13	43	13	16	13	13	42	
16	14	14	44		15	14	14	43	
15	15	15	45		14	15	15	44	
14	16	16	46		16	16	16	45	
13	17	17	47	17	12	17	17	46	
12	18	18	48		11	18	18	47	2 x 9
11	19	19	49		10	19	19	48	
10	20	20	50		9	20	20	49	
9	21	21	51		8	21	21	50	
8	22	22	52		7	22	22	51	
7	23	23	53	23	6	23	23	52	
6	24	24	54		5	24	24	53	2 x 12
5	25	25	55		4	25	25	54	
4	26	26	56		3	26	26	55	
3	27	27	57		2	27	27	56	
2	28	28	58		1	28	28	57	
1	29	29	59						

Table 2B- Numbers 50 and 49.

X	r (50 : X)	r (Y : 50)	Y	t	X	r (49 : X)	r (Y : 49)	Y	t
49	1	1	51		48	1	1	50	
48	2	2	52		47	2	2	51	2
47	3	3	53	3	46	3	3	52	
46	4	4	54		45	4	4	53	
45	5	5	55		44	5	5	54	
44	6	6	56		43	6	6	55	
43	7	7	57		42	7	7	56	
42	8	8	58		41	8	8	57	
41	9	9	59	9 = 3 x 3	40	9	9	58	
40	10	10	60		39	10	10	59	
39	11	11	61	11	38	11	11	60	
38	12	12	62		37	12	12	61	2 x 6
37	13	13	63		36	13	13	62	
36	14	14	64		35	14	14	63	
35	15	15	65		34	15	15	64	
34	16	16	66		33	16	16	65	
33	17	17	67		32	17	17	66	
32	18	18	68		31	18	18	67	2 x 9
31	19	19	69		30	19	19	68	
30	20	20	70		29	20	20	69	
29	21	21	71	21 = 3 x 7	28	21	21	70	
28	22	22	72		27	22	22	71	
27	23	23	73		26	23	23	72	
26	24	24	74		25	24	24	73	
25	25	25	75		24	25	25	74	
24	26	26	76		23	26	26	75	
23	27	27	77		22	27	27	76	
22	28	28	78		21	28	28	77	
21	29	29	79		20	29	29	78	
20	30	30	80		19	30	30	79	2 x 15
19	31	31	81		18	31	31	80	
18	32	32	82		17	32	32	81	
17	33	33	83	33 = 3 x 11	16	33	33	82	
16	34	34	84		15	34	34	83	
15	35	35	85		14	35	35	84	
14	36	36	86		13	36	36	85	
13	37	37	87		12	37	37	86	
12	38	38	88		11	38	38	87	
11	39	39	89	39 = 3 x 13	10	39	39	88	
10	40	40	90		9	40	40	89	
9	41	41	91		8	41	41	90	
8	42	42	92		7	42	42	91	
7	43	43	93		6	43	43	92	
6	44	44	94		5	44	44	93	
5	45	45	95		4	45	45	94	
4	46	46	96		3	46	46	95	
3	47	47	97	47	2	47	47	96	
2	48	48	98		1	48	48	97	
1	49	49	99						

Table 2C- Numbers 48 and 47.

X	r (48 : X)	R (Y : 48)	Y	t	X	r (47 : X)	R (Y : 47)	Y	t
47	1	1	49		46	1	1	48	
46	2	2	50		45	2	2	49	
45	3	3	51		44	3	3	50	
44	4	4	52		43	4	4	51	
43	5	5	53	5	42	5	5	52	
42	6	6	54		41	6	6	53	2 x 3
41	7	7	55		40	7	7	54	
40	8	8	56		39	8	8	55	
39	9	9	57		38	9	9	56	
38	10	10	58		37	10	10	57	
37	11	11	59	11	36	11	11	58	
36	12	12	60		35	12	12	59	
35	13	13	61		34	13	13	60	
34	14	14	62		33	14	14	61	
33	15	15	63		32	15	15	62	
32	16	16	64		31	16	16	63	
31	17	17	65		30	17	17	64	
30	18	18	66		29	18	18	65	
29	19	19	67	19	28	19	19	66	
28	20	20	68		27	20	20	67	
27	21	21	69		26	21	21	68	
26	22	22	70		25	22	22	69	
25	23	23	71		24	23	23	70	
24	24	24	72		23	24	24	71	2 x 12
23	25	25	73	23	22	25	25	72	
22	26	26	74		21	26	26	73	
21	27	27	75		20	27	27	74	
20	28	28	76		19	28	28	75	
19	29	29	77		18	29	29	76	
18	30	30	78		17	30	30	77	
17	31	31	79	31	16	31	31	78	
16	32	32	80		15	32	32	79	
15	33	33	81		14	33	33	80	
14	34	34	82		13	34	34	81	
13	35	35	83	35	12	35	35	82	
12	36	36	84		11	36	36	83	2 x 18
11	37	37	85		10	37	37	84	
10	38	38	86		9	38	38	85	
9	39	39	87		8	39	39	86	
8	40	40	88		7	40	40	87	
7	41	41	89	41	6	41	41	88	
6	42	42	90		5	42	42	89	2 x 21
5	43	43	91		4	43	43	90	
4	44	44	92		3	44	44	91	
3	45	45	93		2	45	45	92	
2	46	46	94		1	46	46	93	
1	47	47	95						

- What do the weak and strong Goldbachs conjectures signify? They signifiy that whenever there is an even or an odd number, there will be a prime number (prime number theorem allows to count prime numbers before an integer). Let N be any integer, then N ± t such that t < N and t is any non-zero integer would give any other number, prime or not. But there might always be a value t such that N – t and N + t are equidistant primes (Table 3).
- Since 2N = (N - t) + (N + t) with t < N and since N ± t produces prime numbers equidistant or not (Table 3), then an even can be the sum of two primes.Therefore, prime numbers do happen equidistanty at all levels of divisibility of integers. An infinitely larger number will produce by the N ± t equation an infinite number of prime numbers, equidistant or not. We understand why the equations of Fermat $2^x + 1$ ($x = 2^n$ and n is an integer > 0) and that of Mersenne $2^n – 1$(n must be prime for the Mersenne's number to be prime and so the equation is rather $2^p – 1$) were able to produce very long prime numbers. For instance, one of the Mersenne's numbers has 24 862 048 digits. Altough both formula are not always giving prime numbers, they show that a very long number tending to $+\infty$ and whatever the number of its prime factors can become prime when increased or decreased by one unit. This is why the simpler equations N ± t were used here to produce prime numbers some of which are equidistant with respect to the value obtained, by just adding or removing two units in series.

Table 3. Formation of prime numbers and couples of equidistant numbers by the equation N ± t such that N and t are integers and t < N. Two numbers N are chosen, N = 20 and N= 37 while t is the sequence of evens or odd numbers < N. The equidistant prime numbers are highlighted. All other individual prime numbers are underlined. Note that the sum of the two equidistant primes = 2N (or 40 for 20 and 74 for 37).

	20		
-3	17	+3	23
-5	15	+5	25
-7	13	+7	27
-9	11	+9	29
-11	9	+11	31
-13	7	+13	33
-17	3	+17	37
-19	1	+19	39

	37		
-2	35	+2	39
-4	33	+4	41
-6	31	+6	43
-8	29	+8	45
-10	27	+10	47
-12	25	+12	49
-14	23	+14	51
-16	21	+16	53
-18	19	+18	55
-20	17	+20	57
-22	15	+22	59
-24	13	+24	61
-26	11	+26	63
-28	9	+28	65
-30	7	+30	67
-32	5	+32	69
-34	3	+34	71
-36	1	+36	73

14. Because all primes numbers are equidistant from each other relatively to any integer then any even can be sum of two primes. Thus we can deduce that GSC is true because there exists between 1 and E/2 a prime number p, and another prime number p' between E/2 and E such that E/2 – p = p' – E/2.

- Therefore, Goldbach's conjectures are related to distribution of prime numbers around integers, and if these conjecture are true, this means that prime numbers are not randomly distributed because they would implie that there is at least two prime numbers that fulfill the rules stated above.
- Goldbach's conjectures implies that if we take a very large integer N and divide it by all primers p < N so as to obtain N/2, N/3, we would have prime numbers before and after each fraction. Goldbach restricted himself to the two fractions of 1/2 and 1/3. The strong conjecture is based on their distribution around N/2, the weak conjecture around N/3. In other words, prime numbers are present at all levels of divisibility of a natural integer, especially the fraction 1/2 and 1/3. We can round the decimal or irrational numbers obtained with these fractions to one unit, this will be recovered in the choice of prime numbers and their addition. Here is a simple example, 100/2 = 50, 111/3 = 37. We have therefore to take 50 as the first lever to distribute 100 as a sum of two prime numbers and 33 to distribute 111 as a sum of three prime numbers. Then 100 = 41 + 59 and 101 = 37 + 31 + 43.
- Here we touch on the theorem of unique factorization which teaches us that prime numbers are the factors of any integer and are consequently its divisors and this is how they can by themselves reconstitute any integer by adding together by 2 (≥ 4) or by 3 (≥ 8) and by much more. There is a relationship between divisibility and addition. As shown above with the $x^2 – y^2$ equation, biprime numbers are formed of two equidistant primes.

15. A similar rule applies to the weak conjecture which states that if the odd number O does not have a prime number > its third 1/3, or if all prime numbers < O are also less than its 1/3, then the weak conjecture is inapplicable. In the conjecture $O = p_1 + p_2 + p_3$, the three prime numbers p_1, p_2 and p_3 cancel each other out to form the number O.
16. For an odd number $O = p_1 + p_2 + p_3$ the sum of $p_1/O + p_2/O + p_3/O = 1$. We also have $(O - p_1) + (O - p_2) + (O - p_3) = 2O$. Taking $(1/3 \text{ x } O) – p_1$, $(1/3 \text{ x } O) – p_2$ and $(1/3 \text{ x } O) – p_3$ and if we have $p_3 > p_2 > p_1$ then $(1/3 \text{ x } O) – p_3 = (1/3 \text{ x } O) – p_1) + (1/3 \text{ x } O) – p_2)$ in absolute value. This means that for two prime numbers p_1 and p_2 there is only one prime number that will add to them to form O. Since it is unusual to find three primes that are close to one-third of an odd number unless there are twin primes around or in a prime-dense region, the weak conjecture holds true only if at least 1 of the three primes > 1/3 x O.
17. If the gap between two consecutive primes is > E/2 (half of an even number) or 1/3 of an odd number which are at the end of the gap, these numbers cannot be formed by the weak nor by strong Goldbach conjectures. However, to date the largest published gaps (wikipedia) do separate giant primes and therefore they remain very negligible compared to E/2 or 1/3 of the even and odd numbers placed at the ends of the gap.

2B. Calculation rules to verify the strong and the weak conjectures of Goldbach

2B1. Primes numbers and their multiples (except those of 2 and 3) are all $6x \pm 1$

- If we separate the even numbers and multiples of 3 from the rest of the natural numbers, we realize that the prime numbers and their multiples all line up in two separate lines that we call here the "P/M lines" (Table 4).

Table 4. Arranging the natural numbers in 6 categories shows that the prime numbers (P) and multiples of prime numbers (M) are $6x \pm 1$ or $3x \pm 2$. They form two lines called the P/M lines (P is prime and M is multiple of primes). Multiples of 2 (even numbers) and 3 are excluded from the P/M lines. There is a difference of 6 units between two consecutive P or M and this is also true for a P and M that follow each other. The data are shown for up to 100 but this is true to infinity.

$6x + 1$ or $3x – 2$ (P/M line)	1	7	13	19	25	31	37	43	49	55	61	67	73	79	85	91	97
Evens 2n	2	8	14	20	26	32	38	44	50	56	62	68	74	80	86	92	98
Odds 3n	3	9	15	21	27	33	39	45	51	57	63	69	75	81	87	93	99
Evens 2n	4	10	16	22	28	34	40	46	52	58	64	70	76	82	88	94	100
$6x – 1$ or $3x + 2$ (P/M line)	5	11	17	23	29	35	41	47	53	59	65	71	77	83	89	95	101
Evens 2 x 3n	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102

- We already know that prime numbers are all $6x \pm 1$ except 2 and 3. Those from the top P/M starting with 7 are $6x + 1$ whereas those on the one below starting with 5 are $6x – 1$ (Table 4). All multiples of prime numbers are also $6x \pm 1$ except those of 2 and 3 (unpublished data not shown). These rules can help to transform an even number into the sum of two prime numbers.

If GSC is true then any even number denoted E is :
$E = p + p' = (6x \pm 1) + (6x' \pm 1)$. This signifies that E is $6x$; $6x – 2$; or $6x + 2$.

- If we take any odd number $\neq 3n$ and reduce or increase it by 6 units in a sequential manner, at some point we have a prime number. It is therefore possible to transform an even number denoted by E into the sum of two prime numbers p and p' such that $p < E/2$ and $p' > E/2$ and p # p'.
- Unlike factorization where a number has unique prime factors, GSC might hold true with many combinations of prime numbers.

2B2 – An elementary Method for Converting an even number into an addition of two prime numbers

- In this paper we pose the GSC as follows Even = p + p' and p#p'.
- First we set any even number E = Odd_1 + Odd_2 such that both Odd_1 and Odd_2 are not 3n. Odd_1 and Odd_2 are either primes or mutliples of prime numbers other than 2 and 3. Then E = $\downarrow Odd_1 \downarrow$ + $\uparrow Odd_2 \uparrow$ which amounts to decreasing Odd_1 by 6 and increasing Odd_2 by 6 by scanning the P/M lines of table 4 from top to bottom or vice versa at a rate of 6. At some point or another, we might have two prime numbers that will add up.
- Because prime numbers are also 3x ± 2, Odd_1 and Odd_2 should not be multiples of 3 and if they are, they will have to be modified at the begining of the conversion. This article gives detailed examples of calculation. Not only must rules be stated, but it must also be shown how to verify them by calculation. We will start gradually with examples of calculation and step by step the method will become clearer.

Examples :

- 378 = 189 + 189 because 189 is 3n, we will reduce it by 2 and increase the other by 2 so 378 = 187 + 191. We can now apply the method of addition and subtraction of 6.
  378 = 187 + 191 → 378 = (187 – 6) + (191 + 6) → 378 = 181 + 197 (both primes). Or,
  378 = 1 + 377 = 7 + 371 = 13 + 365 → 378 = 19 + 359 (both primes).
- 1000 = 500 + 500. First we have to put ourselves on a P/M line and therefore we have to put 1000 in the form of a sum of two odd numbers which are not multiples of 3.
  1000 = 497 + 503 but 497 is not prime.
  1000 = (497 – 6) + (503 + 6) → 1000 = 491 + 509. Both 491 and 509 are primes.
  1000 = 1 + 999 We cannot pose this equality because 999 is a multiple of 3, so we drop 1 and start with 5.
  1000 = 5 + 995 = 11 + 989 → 1000 = 17 + 983.
- Let's take an even number that is one unit more than Fermat's number known as the 6th Fermat number 4294967297, which is composed of 10 digits.
  4294967298 = 1 + 4294967297 (note 1 is not prime) → 4294967298 = 7 + 4294967291.
  Let us take an even number which is one unit more than the 37th Mersenne number M(37) 137438953471, which is composed of 12 digits.
  137438953472 = 1 + 137438953471 = 7 + 137438953465 = 13 + 137438953459 = 19 + 137438953453 = 25 + 137438953447 → 137438953472 = 31 + 137438953441.

2B3. Rules of the conversion of evens in sum of primes by GSC

As shown by table 4, odd numbers are either 3n, multiples of prime numbers, or primes.
Prime numbers are odd numbers which have one unit more or less to be 3n therefore they are either 3x – 1 or 3x + 1. The 3x – 1 are also 3x + 2 and 3x + 1 are 3x – 2. These are the cases of all odd numbers which are not multiple of 3. On the other hand, 3x – 2 are 6x + 1 and 3x + 2 are 6x – 1. For example 19 is 3x – 2 because it needs 2 units to be 3n (21), 19 is therefore 6x + 1. While 17 is 3x + 2 and needs only one unit to be 3x and is 6x – 1. This interplay between multiples of 3 and the 6x ± 1 equations is important for putting Goldbach's conjecture into practice.

→ The first rule. There are two types of prime numbers: those that are 6x + 1 and those 6x – 1. Note that 6x – 1 equation will be used as 6x + 5 because the two are the same given that 6x – 1 = 6x – 6 + 5 = 6(x – 1) + 5 = 6X + 5 (x ou X both are any non-zero integer). If we start with 1 and add 6 consecutively, we will have 6x + 1 prime numbers. If we start with 5 and add 6 consecutively, we will have primes which are 6x – 1. If we start with a 6x – 1 prime we will have 6x – 1 primes, and 6x + 1 primes lead to 6x + 1 ones. For example:
92 = $\underline{1}$ + 91 = $\underline{7}$ + 85 = $\underline{13}$ + 79 (all 6x + 1 primes).
92 = 5 + 87 would not work because 87 is 3n. (see the second rule below).
96 = $\underline{1}$ + 95 = $\underline{7}$ + 89
96 = $\underline{5}$ + 91 = $\underline{11}$ + 85 = $\underline{17}$ + 79 (all 6x – 1 primes).

→The second rule. A 3n number will never lead to primes by the addition of 6. It only leads to 3n because 3n ± 6 is always 3n.
92 = 5 + $\underline{87}$ = 11 + $\underline{81}$ = 17 + $\underline{75}$ = 23 + $\underline{69}$ = 29 + $\underline{63}$ = 35 + $\underline{57}$....= 89 + $\underline{3}$.

When we have a multiple of 3 we will first add or remove one or two units from it so that we can obtain prime numbers by successive additions or subtractions of 6.

→ The third rule. « An even number ≥ 6 is either 6x, 6x + 2 or 6x + 4 ». An even number that is 6x will be in the form of a sum = (6x + 1) + (6x – 1) or (6x – 1) + (6x + 1). An even number that is 6x + 2 makes a sum of 6x + 1 and 6x + 1 prime numbers. Finally, an even 6x + 4 is a sum of two 6x – 1 primers which make 6x – 2. Indeed 6x – 2 is the same as 6x + 4 because 6x – 2 = 6x – 6 + 4 = 6(x – 1) + 4 = 6X + 4 so 6x + 4 given that X or x are any non-zero integer.
Examples.

- 36 is 6x and 36 = 7 + 29 with 7 a 6x + 1 prime and 29 a 6x – 1 prime the sum of which make 6x.
- 38 is 6x + 2 and 38 = 7 + 31 with 7 a 6x + 1 prime and 31 a 6x + 1 prime the sum of which make 6x + 2.
- 40 is 6x + 4 or 6x - 2 and 40 = 11 + 29 with 11 a 6x – 1 and 29 a 6x – 1 the sum of which make 6x – 2 or 6x + 4.
- Care must be taken when applying these rules. For example, 6x – 1 is also 6x + 5 and 6x – 2 is also 6x + 4. For example, 11 + 89 = 100. We know that 11 is 6x – 1; 89 is 6x – 1 but 100 is 6x + 4. In fact, 100 is 6x – 2, which is the same as 6x + 4. Let's take another example: the number 124 = 23 + 101 with 23 being 6x – 1 and 101 being 6x – 1. In fact 124 will be 6x – 2. But 124 is also 6x + 4. In other words, 23 is 6x – 1 and therefore 6x **+ 5** and 101 is 6x **– 1** or vice versa and therefore 124 is 6x + 4 or 6x – 2. In fact, you have to put the prime primes that sum to 6x ± y (y < 6) and add the y's. The rule of 6x ± 1 sums always applies when we apply Goldbach's strong conjecture.

2B4. Perform the conversion from an even to addition of prime numbers in a table

We will apply GSC to some even numbers using Tables 5 in accordance with the three rules stated above.

- The even number to be converted must be set at the very beginning as M + 1 or M + 5 such that M is a multiple of prime numbers except 2 and 3 (M might be prime). M is therefore the leftt-hand term of the addition and 1 or 5 are the right-hand terms.
- The method is to transfer 6 by 6 from the left-hand side of the addition (M) to the right-hand side (1 or 5). According to Table 4, M – 6n and 1 + 6n or 5 + 6n either give a prime number or another M' number that is < M and that is a multiple of prime numbers. There is then a chance that two prime numbers will appear to the right and left of the addition.

- As soon as two prime numbers meet and sum, we mark them as an exact verification of GSC. Each sum of two prime numbers will be designated by the letter S followed by a number which indicates the order of its appearance.
- Here the number itself is converted directly in sum of two primes (not by searching for equidistant primes as above but by finding out additive primes).

Tables 5 : Conversion of evens in sum of primes by the three rules stated. The sums are denoted S followed by a number. Note a same sum can appear twice for a same number and in this case it is denoted the same way. The number 136 is posed as M + 5 (131 + 5 or 5 + 131), 218 as M + 1 (217 + 1 or 1 + 217) and 282 as M + 5 (277 + 5 or 5 + 277). The number 2042 in separate tables is posed M + 1 (2041 + 1 or 1 + 2041).

Table 5-1. Numbers 136, 218 and 282.

Sum	136		Sum	218		Sum	282		
S1	5	131		1	217	S1	5	277	
	11	125	S1	7	211	S2	11	271	
	17	119		13	205		17	265	
S2	23	113	S2	19	199		23	259	
S3	29	107		25	193		29	253	
	35	101		31	187		35	247	
	41	95	S3	37	181	S3	41	241	
S4	47	89		43	175		47	235	
S5	53	83		49	169	S4	53	229	
	59	77		55	163	S5	59	223	
	65	71	S4	61	157		65	217	
	71	65	S5	67	151		71	211	
	77	59		73	145		77	205	
S5	83	53	S6	79	139	S6	83	199	
S4	89	47		85	133	S7	89	193	
	95	41		91	127		95	187	
	101	35		97	121	S8	101	181	
S3	107	29		103	115		107	175	
S2	113	23		109	109		113	169	
	119	17		115	103		119	163	
	125	11		121	97		125	157	
S1	131	5		127	91	S9	131	151	
				133	85		137	145	
			S6	139	79		143	139	
				145	73		149	133	
			S5	151	67		155	127	
			S4	157	61		161	121	
				163	55		167	115	
				169	49	S10	173	109	
				175	43	S11	179	103	
			S3	181	37		185	97	
				187	31		191	91	
				193	25		197	85	
			S2	199	19		203	79	
				205	13		209	73	
			S1	211	7		215	67	
				217	1		221	61	
							227	55	
							233	49	
						S12	239	43	
							245	37	
						S13	251	31	
							257	25	
						S14	263	19	
						S15	269	13	
							275	7	
							281	1	

Table 5-2-1. Number 2042.

	2042			2042			2042			2042	
	1	2041		127	1915		247	1795		367	1675
	7	2035		133	1909		253	1789	S11	373	1669
S1	13	2029		139	1903		259	1783	S12	379	1663
	19	2023		145	1897		265	1777		385	1657
	25	2017		151	1891		271	1771		391	1651
S2	31	2011		157	1885		277	1765		397	1645
	37	2005	S5	163	1879	S9	283	1759		403	1639
S3	43	1999		169	1873		289	1753		409	1633
	49	1993		175	1867		295	1747		415	1627
	55	1987	S6	181	1861		301	1741	S13	421	1621
	61	1981		187	1855		307	1735		427	1615
	67	1975		193	1849		313	1729		433	1609
	73	1969		199	1843		319	1723	S14	439	1603
	79	1963		205	1837		325	1717		445	1597
	85	1957	S7	211	1831		331	1711		451	1591
	91	1951		217	1825		337	1705		457	1585
	97	1945		223	1819		343	1699	S15	463	1579
	103	1939		229	1813	S10	349	1693		469	1573
S4	109	1933		235	1807		355	1687		475	1567
	115	1927	S8	241	1801		361	1681			
	121	1921									

Table 5-2-2. Number 2042.

	2042			2042			2042			2042	
	481	1561		595	1447		709	1333		823	1219
	487	1555		601	1441		715	1327	S25	829	1213
	493	1549		607	1435		721	1321		835	1207
S16	499	1543	S18	613	1429		727	1315		841	1201
	505	1537	S19	619	1423		733	1309		847	1195
	511	1531		625	1417	S22	739	1303		853	1189
	517	1525		631	1411		745	1297		859	1183
	523	1519		637	1405	S23	751	1291		865	1177
	529	1513	S20	643	1399		757	1285		871	1171
	535	1507		649	1393		763	1279		877	1165
	541	1501		655	1387		769	1273		883	1159
	547	1495	S21	661	1381		775	1267		889	1153
	553	1489		667	1375		781	1261		895	1147
	559	1483		673	1369		787	1255		901	1141
	565	1477		679	1363		793	1249		907	1135
S17	571	1471		685	1357		799	1243		913	1129
	577	1465		691	1351		805	1237	S26	919	1123
	583	1459		697	1345	S24	811	1231		925	1117
	589	1453		703	1339		817	1225		931	1111

Table 5-2-3. Number 2042.

	2042			2042			2042			2042	
	937	1105	S27	1051	991		1165	877		1279	763
	943	1099		1057	985		1171	871		1285	757
	949	1093		1063	979		1177	865	S23	1291	751
	955	1087		1069	973		1183	859		1297	745
	961	1081		1075	967		1189	853	S22	1303	739
	967	1075		1081	961		1195	847		1309	733
	973	1069		1087	955		1201	841		1315	727
	979	1063		1093	949		1207	835		1321	721
	985	1057		1099	943	S25	1213	829		1327	715
S27	991	1051		1105	937		1219	823		1333	709
	997	1045		1111	931		1225	817		1339	703
	1003	1039		1117	925	S24	1231	811		1345	697
S28	1009	1033	S26	1123	919		1237	805		1351	691
	1015	1027		1129	913		1243	799		1357	685
S29	1021	1021		1135	907		1249	793		1363	679
	1027	1015		1141	901		1255	787		1369	673
S28	1033	1009		1147	895		1261	781		1375	667
	1039	1003		1153	889		1267	775	S21	1381	661
	1045	997		1159	883		1273	769		1387	655

Table 5-2-4. Number 2042.

	2042			2042			2042			2042			2042	
	1393	649		1507	535	S13	1621	421		1735	307		1849	193
S20	1399	643		1513	529		1627	415		1741	301		1855	187
	1405	637		1519	523		1633	409		1747	295	S6	1861	181
	1411	631		1525	517		1639	403		1753	289		1867	175
	1417	625		1531	511		1645	397	S9	1759	283		1873	169
S19	1423	619		1537	505		1651	391		1765	277	S5	1879	163
S18	1429	613	S16	1543	499		1657	385		1771	271		1885	157
	1435	607		1549	493	S12	1663	379		1777	265		1891	151
	1441	601		1555	487	S11	1669	373		1783	259		1897	145
	1447	595		1561	481		1675	367		1789	253		1903	139
	1453	589		1567	475		1681	361		1795	247		1909	133
	1459	583		1573	469		1687	355	S8	1801	241		1915	127
	1465	577	S15	1579	463	S10	1693	349		1807	235		1921	121
S17	1471	571		1585	457		1699	343		1813	229		1927	115
	1477	565		1591	451		1705	337		1819	223	S4	1933	109
	1483	559		1597	445		1711	331		1825	217		1939	103
	1489	553	S14	1603	439		1717	325	S7	1831	211		1945	97
	1595	547		1609	433		1723	319		1837	205		1951	91
	1501	541		1615	427		1729	313		1843	199		1957	85

Table 5-2-5. Number 2042.

	1042	
	1963	79
	1969	73
	1975	67
	1981	61
	1987	55
	1993	49
S3	1999	43
	2005	37
S2	2011	31
	2017	25
	2023	19
S1	2029	13
	2035	7
	2041	1

- The tables 5 show that when we start with the equation E = M + 1, we have a center of symmetry beyond which we fall back on the same series of addition operations as is the case with the number 2042 (Table 5-2-1 to 5-2-5) after the sum S29 = 2042 = 1021 + 1021.
- The two terms of the sums always have the same unit digits and therefore for a given prime number we only have one kind of prime numbers with a precise unit digit which is suitable for constructing the sum. If you look at the unit digits of the prime numbers participating in the sums you will see that they are periodically the same.

- We see that with this method, we can verify the SGC on any length of the P/M lines and thus list many sums corresponding to the tested even. This article proposes this method for the first time.

2B5. The so-called weak Goldbach conjecture or Odd = p + p'+ p'' (p, p', p'' are prime numbers)

- Suppose a non-prime odd number O = p x q then O = (p - 1)q + q. Since p and q are primes then p - 1 is even which we denote by E and therefore O = E + q. In other words, a non-prime odd number can be the sum of an even and a prime number.
- If the odd number is prime we denote it by p' > 5. We know that if we have any prime number p' > p then p' - p = 2n and so p' = 2n + p. Therefore an odd number whether prime or not is the sum of an even number and a prime number.
- Whether it is weak or strong Goldbach, it is verified with several sums, we can therefore apply the formula O = E + p starting with any prime number p removed from O and not only with p being a prime factor of O (in case it is composite) or p being the prime number preceding O (in case it is prime). Afterwards, it remains to convert E into the sum of two prime numbers.
- We deduce that if GSC is true then an $O = E + p_3 = p_1 + p_2 + p_3$ such that $E = p_1 + p_2$ and with p_1, p_2, and p_3 being prime numbers. Therefore the weak conjecture depends on the truth of the strong conjecture. We will then set any odd number as Odd = E + p_3 and then convert E to the sum of p_1 and p_2. Thus Odd = $p_1 + p_2 + p_3$ with p1#p2#p3.
- Odd = p + p ' + p'' = Even + p'' with E = $\downarrow Odd_1 \downarrow + \uparrow Odd_2 \uparrow$. We apply the method described above based on the M + 1 and M + 5 equations with the even thus chosen to convert it into the sum of two prime numbers.

For example:

131 = <u>100</u> + 31 = <u>5 + 95</u> + 31 = <u>11 + 89</u> + 31 → 131 = 11 + 89 + 31

131 = <u>90</u> + 41 = <u>1 + 89</u> + 41 = 7 + 83 + 41 → 131 = 7 + 83 + 41

18 971 523 157 = 53+ 18 971 523 104 = 1 + 53 + 18 971 523 103 = 7 + 53 + 18 971 523 097 →

18 971 523 157 = 7 + 53 + 18 971 523 097.

Table 6 shows additional examples and how to apply the method by three steps.

Table 6: Conversion of an odd number into a sum of 3 prime numbers. The method involves three steps: first put the odd number in the form of E + p_3 then convert E into $p_1 + p_2$. As a third step, O = $p_1 + p_2 + p_3$. The weak Goldbach' conjecture is therefore here deduced from the strong one. In the table, all letters p indicate prime numbers. E is any even > 4 and O any odd number >8.

Odd number (O)	O = E + p_3 (E = 2n)	E = $p_1 + p_2$	O = $p_1 + p_2 + p_3$
Step	Remove a prime number from O such that E can be divided into two prime numbers.	Convert E into a sum of two prime numbers using M + 1 and M + 5 equations method	Final verification of weak Goldbach's conjecture
2053	1362 + 691	1362 = 293 + 1069	293 + 1069 + 691
20995	10988 + 10007	10988 = 4909 + 6079	4909 + 6079 + 10007
3506641	173310 + 3333331	173310 = 61559 + 111751	61559 + 111751 + 3333331
1025894774731	92589477472 + 100000000003	92589477472 = 147895132739 + 777999641989	147895132739 + 777999641989 + 100000000003

2C. Expalining the gap between prime numbers and the truth of the strong Goldbach's conjecture

There are three types of even numbers 6x, 6x + 2 and 6x – 2.
There are three tyes of odd numbers : 3n, multiple of pirme numbers except 3 and 2 (M) and prime numbers (P).
There are two types of primes numbers 6x + 1 and 6x – 1.
We can therefore understand the gaps between primes numbers and and anticipate them or calculate the probability of their co-occurrence when it comes to equidistant prime numbers.
Here are examples of representative cases.

1. If we have an even E = 6x, we must progress by regular intervals of 6x - 1 or 6x + 1 to find either a P or a M. We progress in the same way either from E/2 to 0 or E/2 to E. The process is symmetrical.
2. If we have an even 6x – 2, we add one unit and then advance by 6x therefore we span 6x + 1 intervals to get to the P/M line (see Table 4).
3. If we have an even E = 6x + 2, we must subtract 1 to get to the P/M line (see Table 4) and then advance by intervals of 6x and therefore we advance by 6x - 1. We will then have P or M and we do the same either from E/2 to 0 or from E/2 to E.

Examples:

- *The number E = 60 (E/2 = 30) is an even 6x*. And therefore 30 will be away from prime numbers by 6x + 1 or 6x – 1 gaps. Therefore we add values of 6x ± 1 to 30 to get new primes. Here is the case when we add 6x + 1 primes: 30 + 7 = **37**; 30 + 13 = **43**; 30 + 19 = 49; 30 + 31 = **61**. Or adding 6x – 1 primes : 30 + 5 = 35; 30 + 11 = **41**; 30 + 17 = **47**; 30 + 23 = **53**.
  On the other hand, we must do the same to go down : 30 – 7 = **23**; 30 – 13 = 1**7**; 30 – 17 = **13**; 30 – 23 = **7**. Or 30 – 5 = 25; 30 – 11 = **19**; 30 – 17 = **13**; and 30 – 23 = **7**.

π(30)

3 5 7 11 13 17 19 23 29

- *The number E = 80 and E/2 = 40 is 6x – 2 (or 6x + 4)*. Therefore we add 1 and get to the 6x – 1 number 41 then we add 6 to go up to 47. 40 + 7 = **47**; 40 + 13 = **53**; 40 + 19 = **59**; 40 + 25 = 65; 40 + 31 = **71**; 40 + 37 = 77. Or reduce the number by 4 and we get 36 then advance by 6x – 1 or 6x + 1 intervals. Then we have 40 – 4 = 36 + 5 = **41** + 6 = **47** + 6 = **53** + 6 = **59** + 6 = 65 + 6 = **71** + 6 = 77. Or 40 – 4 + 7 = **43** + 6 = 49 + 6 = 55 + 6 = **61** + 6 = **67** + 6 = **73**. We go down the same: 40 – 4 = 36 – 5 = **31** – 6 = 25 – 6 = **19** – 6 = **13** – 6 = 7. Or 40 – 4 = 36 – 7 = **29** – 6 = **23** – 6 = **17** – 6 = **11** – 6 = **5**.

π(40)

3 5 7 11 13 17 19 23 29
31 37

- *The number E = 100 and E/2 = 50 is 6x + 2*. We reduce it by one and then go up or down by 6x intervals. Then 50 – 1 = 49 + 6 = 55 + 6 = **61** + 6 = **67** + 6 = **73** + 6 = **79** + 6 = 85 + 6 = 91 + 6 = **97**. Or 50 – 1 = 49 – 6 = **43** – 6 = **37** – 6 = **31** – 6 = 25 – 6 = **19** – 6 = **13** – 6 = **7**.
  We can also substract 2 to get 6x and add 5 or 7 and then advance by 6x intervals. 50 – 2 = 48 + 5 = **53** + 6 = **59** and so on or 50 – 2 = 48 + 7 = 55 + 6 = **61** + 6 = **67** and so on. We do the same to go down.

π(50)

	3	5	7	11	13	17	19	23	29
31	37	41	43	47					

Here are some other examples number E = 120, E/2 = 60; E = 140, E/ 2 = 70; and E = 180, E/2 = 90 to show that there are always more prime numbers between [0 – E/2] than [E/2 – E] because there are always more primes close to 0 (2; 3; 5; 7; 11;...) (2 is excluded here).

- We see that there is a *limiting prime number (LPN)* for every even number from which we cannot obtain it even if the LPN > E/2 (highlighted). For example, we cannot obtain 60 with prime numbers P < 31, nor 70 with P < 41 nor 90 with P < 47. Limiting prime numbers are highlighted also in the case of E = 60, E/2 = 30; E = 80 and E/ 2 = 40; and E = 100, E/2 = 50. The LPN is close to E/2 but > E/2. The LPN for the above numbers 30; 40; and 50 are also highlighted.

π(60)

3	5	7	11	13	17	19	23	29
31	37	41	43	47	53	59		

π(70)

3	5	7	11	13	17	19	23	29
31	37	41	43	47	53	59	61	67

π(90)

3	5	7	11	13	17	19	23	29	
31	37	41	43	47	53	59	61	67	71
73	79	83	89						

- Using the rules stated in this article, let us explain the gaps between prime numbers (from 40 to 97 as examples). First let us mark them (bold)

40 **41** 42 **43** 44 48 46 **47** 48 49 50 51 52 **53** 54 55 56 57 58 **59** 60 **61** 62 63 64 65 66 **67** 68
69 70 **71** 72 **73** 74 75 76 77 78 **79** 80 81 82 **83** 84 85 86 87 88 **89** 90 91 92 96 94 95 96 **97**

In the first place, we must separate the prime numbers $6x - 1$ and $6x + 1$ and identify their sequences. We have on one hand ***41 (6x – 1)*** → 47 → 53 → 59 → 65 (M) → 71 → 77 (M) → 83 → 89. On the other hand, we have ***43 (6x + 1)*** → 49 (M) → 55 (M)→ 61 → 67 → 73 → 79 → 85 (M) → 91(M) → 97. So there are gaps of 6n between prime numbers P of the same writing in equation $6x \pm 1$. However, there are other gaps like between 41 and 43; 67 and 71; and 89 and 97.

40 **41** 42 **43** 44 48 46 **47** 48 49 50 51 52 **53** 54 55 56 57 58 **59** 60 **61** 62 63 64 65 66 **67** 68
69 70 **71** 72 **73** 74 75 76 77 78 **79** 80 81 82 **83** 84 85 86 87 88 **89** 90 91 92 96 94 95 96 **97**

- The number P 41 is two units from 43. How to explain this? In fact 41 is $6x - 1$ and 43 is $6x + 1$ and therefore $(6x + 1) - (6x - 1) = \mathbf{2}$ (here we assume that x is any integer > 0). This is also the case of 59 ($6x - 1$) and 61 ($6x + 1$) and of all the twin prime numbers.

- Between 67 and 71 we have four units. In fact 67 is 6x – 1 or 6x + 5 and 71 is 6x + 1 and thus (6x + 5) – (6x + 1) = **4**. We have to make [*(6x + 1) – (6x – 1)*] but [*(6x + 5) – (6x + 1)*] when calculating the gaps to avoid negative values (to stand in the N set of integers).
- Let us explain the difference between 97 – 89 = 8. Because 97 is 6x + 1 and 89 = 6x – 1, they do not progress by 6 intervals. The last prime number 6x + 1 before 97 is 79 and *97 – 79 = 18*. And because *89 – 79 = 10* therefore the gap between 79 and 97 is **18 – 10 = 8**.
- Let us take this sequence of prime numbers and explain the gap between 181 and 191.

157	163	167	173						
179	**181**	**191**	193	197	199	211	223	227	229

- Again 191 is 6x – 1 (and thus 6x + 5) and 181 is 6x + 1. An so 191 is preceded by numbers 191 – 6 = 185 – 6 = 179 while 181 – 6 = 175 – 6 = 169 – 6 = 163. The last prime number before 191 is 179 and 191 – 179 = 12. But 181 (6x + 1) – 179 (6x – 1) = 2. Therefore the gap between 181 and 191 = **12 – 2 = 10**.
- By those rules combined we explain any gap occurring between primes. First 6x – 1 and 6x + 1 progress in two different overlaping series ; either a prime number P or a multiple of primes (M) occupies a position corresponding to 6x – 1 or 6x + 1. The gap between primes is 6n between the 6x – 1 primes on the one hand, and between 6x + 1 primes on the other hand. But the gap is 2, 4, 8, 10 or 2n between prime numbers 6x – 1 and 6x + 1 and it depends on how many times a number M occupies the 6x ± 1 positions of the lines P/M (see table 4).
- Be an Even = E and E/2. We have four possibilities

1. **M** → E/2 ← **M**. Two numbers M occupy the equidistant positions.
2. **M** → E/2 ← **P**. There is only one prime without an equidistant one because there is instead a M number.
3. **P** → E/2 ← **M**. There is only one prime without an equidistant one because there is instead a M number.
4. **P** → E/2 ← **P**. There are two-equidistant primes.

Let us assume that these 4 possibilities are equiprobable because we cannot anticipate or predict where a prime number P ou M will appear. In this case, there is a 25% chance or a probability of 0.25 that Goldbach's conjecture holds true. Hence it is true.Note that this should be assessed for every prime of π(n) or π(E/2) (n or E/2 any integer ≥ 4) to determine if its equidistant number at E/2 is P or M.

- We also see that the prime numbers are formed symmetrically from E/2 to 0 and from E/2 to E. On one side subtraction and on the other side addition. This also supports Goldbach's conjecture because without this symmetry there would be no equidistant prime numbers and the even number E cannot be converted into the sum of two prime numbers. Prime numbers are always formed in the same way even if we cannot translate it into an equation. This equation must give all equidistant primes numbers produced by any integer n ≥ 4.
- *We all know that π(E) (E any even* ≥ 8) *contains equidistant primes to E/2 but what we are missing is to directly deduce equidistant prime numbers from an integer by a formula or a theorem. Otherwise, pose an axiom that states that any integer n or E/2* ≥ 4 *is surrounded by at least one couple of equidistant primes and assume it is true unless one counterexample is found.*
- Even if a gap comes after E/2, prime numbers > E/2 and close to E will combine with increasingly smaller prime numbers < E/2 and close to 0 and since the latter are more numerous, they will increase the chances that two equidistant prime numbers appear.

2D- Examples of applying the rules described to convert an integer ≥ 4 into the sum of two prime numbers

2D-1. Posing the mathematical problem of Goldbach's strong conjecture

Here we will consider Goldbach's strong conjecture as being for an even number ≥ 8. E = p + p' such that p' > p and p'#p. So E ≥ 8 and E/2 ≥ 4 recall that E/2 is any integer ≥ 4. To prove the GSC, we need to predict at least one pair of two prime numbers equidistant at E/2. If we set p = E/2 - **t** and p' = E/2 **+ t**, in other words, we have to predict the value of **t**. For a number E that tends to infinity, t can also tend to infinity.
There are well-known prime number postulates that have become theorems, but which unfortunately can't help to solve Goldbach's strong conjecture. For example, the prime number theorem : « *The number of primes less than x tends asymptotically towards x/log x: n/ln(n)*
We have improved the approximation by taking: π(n) ~ n /(ln(n) – 1) » gives just an approximation to the number of primes before a natural number, but in no way predicts the position of the equidistant primes. Similarly, Bertrand's postulate : « Between n and 2n, there is always a prime. In other words, the gap between a prime number p and its successor is smaller than p » indicates the presence of a prime number between n and 2n, but does not predict its position. Also, the theorem « Between n and 2n and n > 6, there is at least one prime in 4k - 1 and at least one in 4k + 1 - Proven by Erdös. Example between 7 and 14: 7 = 4x2 - 1; 11 = 4x3 - 1; 13 = 4x3 + 1 » doesn't predict the position of all equidistant primes either.
We can't use the laws of probability calculation, because the positions of numbers are not events that happen in a dependent or independent way. The formula nln(n), which approximates the nth prime number, is of no help, as variations of a few or several units will distort the calculation, since exact values of t are required.
The GCS problem can be posed as follows: we have an even number E ≥ 8 (E/2 ≥ 4) and a prime number p, we have p + 2t = p' and we need p + t = E/2 and E/2 + t = p' so that E = p + p'.
We know that by adding 2n to a number P1, we'll get another prime number P2 at some point, and we know that there's always an even or odd natural number at equal distance between P1 and P2. For example, between 11 and 31, there's the number 22 at equal distance. Or 31 + 47 = 78 and therefore 39 in the middle between 31 et 47. But the real problem here is that we have a prime number p, and we have to add a certain value of 2n = 2t to it, so as to predict in advance that it is indeed E/2 that is at equal distance between p and p'. This article will show that the only safe approach is to analyze the remainders of Euclidean divisions of p and p'.
This approach will be discussed in this article (see below). It can be used to predict whether adding 2t to p will produce an equidistant p' or not. It's all about analyzing successive Euclidean divisions. Furthermore, this article will also define which values of t added to or substracted of E produce prime numbers.

2D2. The gap between equidistant primes has specific values depending on whether the even sum of two prime numbers is a multiple of 3 or not

∀x, x∈N and x≥*8*, ∃t t∈N and t < x such that x – t and x + t are primes → ∀x, x ∈N and x ≥*8*, 2x = (x – t) + (x + t) = p + p' (p and p' are primes). Goldbach's conjecture holds true.

- E is any even ≥ 8 and E = (P1 – t) + (P2 – t) and thus P1 and P2 are equidistant primes. In tables 7-9, t values are going to be determined for four numbers (E = 200, E = 400, E = 600 and E = 2000). Therefore, equidistant primes before and after E/2 are located and then **t** calculated and shown in the tables. The data show that **t** has specific values depending on E if it is a multiple of 3 or not.

∀x, x∈N and x≥*8*, ∃t t∈N and t < x such that x – t and x + t are equidistant primes at x/2 → t is prime or composite. If x/2 is even, t is odd. If x /2 is odd t is even
First case: x/2 is even
1)- If x = 3n; t is either prime or composite the prime factors of which are in an ascending order ***but not 3n***.
2)- If x ≠ 3n; t is odd 3n, prime or composite but ***3n values are the most frequent***.
Second case: x/2 is odd
3)- If x = 3n; t is even composite the prime factors of which are in an ascending order ***but not 3n***.
4)- If x ≠ 3n; ***t is even 3n***.

We see that t represents the gap that separates each of the two equidistant prime numbers from E/2 with E being any even ≥ 8 and E/2 is any integer ≥ 4. P → E/2 ← P'. E/2 – P = ***t*** *and P' – E/2 =* ***t****. Table 7 shows the values of t before and after two numbers chosen as examples 100 and 200. Note that both 100 and 200 are ≠ 3n.* ***Table 7 shows that t has values of 3n with both numbers****. In Table 8 only the t-values are represented of two numbers that are not 3n (E = 200 and E/2 =100; E = 400 and E/2 = 200) and of a number that is 3n (E = 1200 and E/2 = 600). It is clear that the values of t are not identical.* ***When the number is 3n such the case of 1200, t values are either prime or composite but not 3n****. These data show that the gap between E/2 and equidistant primes has different values depending on the number E if it is 3n or not. At the bottom of each column of Table 8, the gaps between equidistants prime numbers and E/ 2 are represented depending on their order of appearance and we see that there is a good linear correlation (R^2 = 0.97-0.99).*

Table 9 shows data consistent with those in Table 8. ***The t-values between equidistant primes and E/2 are almost all the time 3n for a number that is not itself a multiple of 3 (E = 2000, E/2 = 1000)****. Although the gaps between equidistant primes and E/2 = 1000 in the case of E = 2000 show a good correlation of 0.97-0.98, randomly chosen primes between 1009 and 1213 show a similar correlation (see graphics below table 9). But the larger the number (600, 2000) we notice a shift and a curve which winds (snake-like) but the correlation coefficient remains almost the same.*

The gaps between equidistant prime numbers of an even E and its half E/2 obey a same linear distribution compared to that of natural prime numbers in an increasing order. These data were confirmed with larger numbers including two numbers that are not multiples of 3, 100000 and 10000 (**see the supplementary data on pages 51-54 below at the end of this article**). The number 100000 has more than 500 equidistant primes and the t-values separating them from E/2=50000 are all 3n (additional data, Table S1). The number 10000 has 145 equidistant primes and the t-values separating them from E/2=5000 are all 3n (Table S2). The number 3n 9000, on the other hand, has 242 equidistant primes and the t-values separating them from E/2=4500 are either primes or composites , but in no case 3n (additional data, Table S3). However the associated equations cannot be used for integers because the linearity is not absolute.

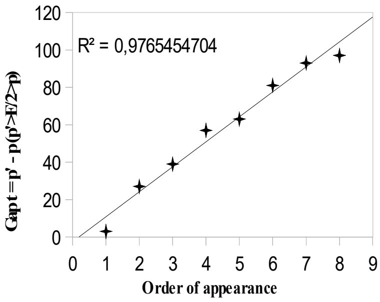
Strong linear correlation coefficients means that equidistant primes appear after relatively close or fairly regular intervals, whereas if this were not the case, the correlation would have been very weak. This indicates that equidistant primes of an integer value are very likely to occur and argues in favor of the authenticity of the strong Goldbach conjecture. This also indicates that primes do not appear randomly but follow pre-established rules depending on whether the number is a multiple of 3 or not. ***The larger the number, the greater the number of equidistant primes so that the linear correlation increases to = 1 (this is due to the very large number of t values, as opposed to a smaller number). This shows that GSC touches on a fundamental rule that governs the appearance of primes after precise gaps.***

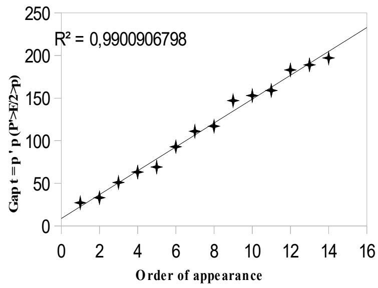
Table 7 : The gap **t** has values of 3n when E/2≠3n. Note that **t** is the gap between p and p' of E/2 if E = P1 + P2 → P1 and P2 are equidistant primes and p' > p such that ***E/2 – t = p and E/2 + t = p'***. Arrow on the left and right indicates t values of corresponding equidistant primes (for example in case of 100; t = 3 corresponds to 103 + 97 = 200; t = 27 for 127 + 73 = 200 and so on). Two examples are shown : E = 200 with E/2 = 100 ; and E = 400 with E/2 = 200. The arrow → alone means from one prime number to another.

100 → 200	← 100 →		0 → 100	200 → 400	← 200 →		0 → 200
101	1	97	3	211	11	197	3
103	3	95	5	223	23	195	5
107	7	93	7	227	27	193	7
109	9	89	11	229	29	189	11
113	13	87	13	233	33	187	13
127	27	83	17	239	39	183	17
131	31	81	19	241	41	181	19
137	37	77	23	251	51	177	23
139	39	71	29	257	57	171	29
149	49	69	31	263	63	169	31
151	51	63	37	269	69	163	37
157	57	59	41	271	71	159	41
163	63	57	43	277	77	157	43
167	67	53	47	281	81	153	47
173	73	47	53	283	83	147	53
179	79	41	59	293	93	141	59
181	81	39	61	307	107	139	61
191	91	33	67	311	111	133	67
193	93	29	71	313	113	129	71
197	97	27	73	317	117	127	73
199	99	21	79	331	131	121	79
		17	83	337	137	117	83
		11	89	347	147	111	89
		3	97	349	149	103	97
				353	153	99	101
				359	159	97	103
				367	167	93	107
				373	173	91	109
				379	179	87	113
				383	183	73	127
				389	189	69	131
				397	197	63	137
						61	139
						51	149
						49	151
						43	157
						37	163
						33	167
						27	173
						21	179
						19	181
						9	191
						7	193
						3	197
						1	199

Table 8: The gap **t** has values of 3n when E/2≠3n. Note that **t** is the gap between p and p' of E/2 if E = P1 + P2 → P1 and P2 are equidistant primes and p' > p such that ***E/2 – t = p and E/2 + t = p'***. t-values for numbers that are not 3n (200 and 400) and a 3n number (600). t-values that are multiples of 3 are marked with an asterisk. Unmarked numbers are either prime or composite (bold) with prime factors > 3 in increasing order. Below each column the graphic showing correlation between t values and their order of appearance.

100±t → 200	**200±t → 400**	**300±t → 600**
97	197	293
93*	189*	**287**
81*	183*	277
63*	159*	271
57*	153*	269
39*	147*	263
27*	117*	257
3*	111*	**247**
	93*	**221**
	69*	**203**
	63*	199
	51*	**187**
	33*	163
	27*	**161**
		149
		143
		133
		121
		119
		109
		101
		89
		73
		67
		49
		37
		31
		17
		7





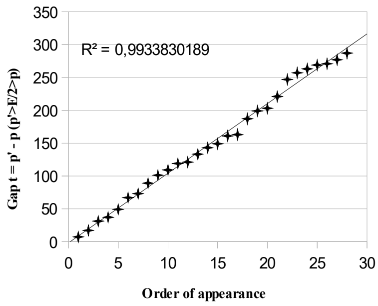


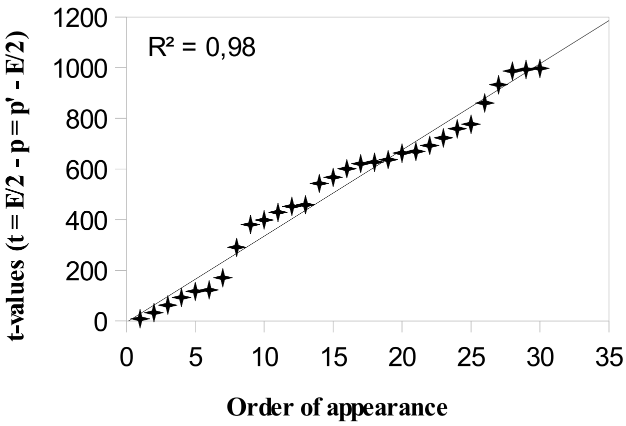
*Table 9: t-values for a number that is not 3n (E = 2000, E/2 = 1000). The t-values are mostly multiple of 3 (3n marked with *) except in three cases (bold underlined). Below is the correlation between the t-values and their order of appearance. As a control, correlation between a same number of Prime numbers from 1009 to 1213 is shown for comparison. The t values or gaps between equidistant primes and E/2 show similar linear corelation than natural prime numbers in their increasing order.

← 1000 →									
9*	33*	63*	93*	117*	123*	171*	291*	381*	399*
429*	453*	459*	543*	567*	**601**	621*	627*	**637**	663*
669*	693*	723*	759*	777*	861*	933*	987*	993*	**997**

Underlined numbers 601 and 997 are primes while $637 = 7^2 \times 13$.

↓

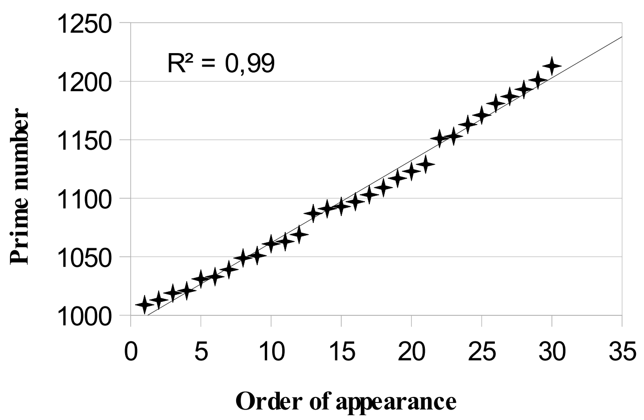
E = 2000 and E/2 = 1000.



Natural prime numbers from 1009 to 1213 (30 primes).

↓

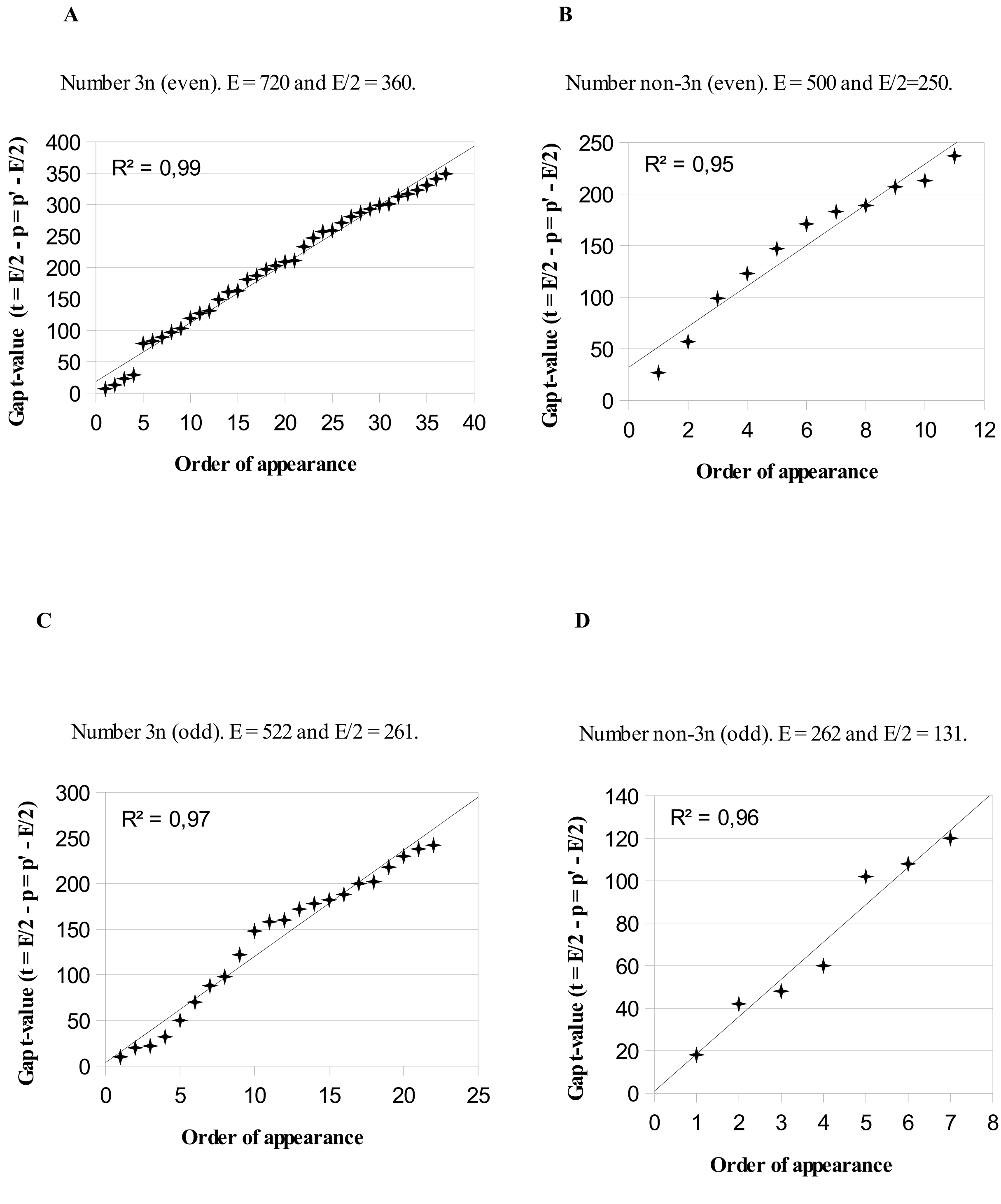
Primes numbers in their natural order



2D3. Linear correlation between the gaps separating equidistant primes and E/2 in all cases of even numbers

Below in **Figure 1**, four graphics which represent the four cases of E/2 numbers to take into account for the conversions of evens E in sum of two primes. E/2 is either 3n even (Figure 1A) or non-3n even (1B). On the other hand, E/2 is either 3n odd or non-3n odd. In all these graphics, E = p + p' (p' > p and both primes) and **t = E/2 – p = p' – E/2**. The graphics show distribution of t-values relatively to their order of appearance. In all graphics, the t-values are strongly correlated for any of the four cases. Each dot represents a pair of equidistant primes. Equidistant primes appear regularly as any other prime number in the four cases of evens (Fig 1A-D) which shows that any even can split into sum of two primes. The evens differ by the density of equidistant primes and the more larger the number is, the higher their densities. In all cases, density of equidistant primes is always < π(E) ~ E/log(E), where π(E) is the prime-counting function (the number of primes less than or equal to E) and log(E) is the natural logarithm of E (the prime number theorem). Another point is that equidistant primes are found between 0 and E/2 on one hand, and E/2 and E on the other hand while total count of primes might differ between 0-E/2 and E/2-E. For the strong conjecture of Goldbach to hold true, there must be at least one couple of equidistant primes p and p' among π(E) such that t= E/2 – p = p' – E/2. If one prime results from E/2 – t, then it is very likely tat E + t is prime and this probability is never zero therefore proving GSC.

Figure1. The four cases of evens to take into account for the conversion of evens in sums of two primes (p and p' such that p' > p). Each graphic shows the distribution of t-values with t = E/2 – p = p' – E/2. Linear correlation coefficients are shown. Each type of E/2 number is indicated on the top of each graphic.

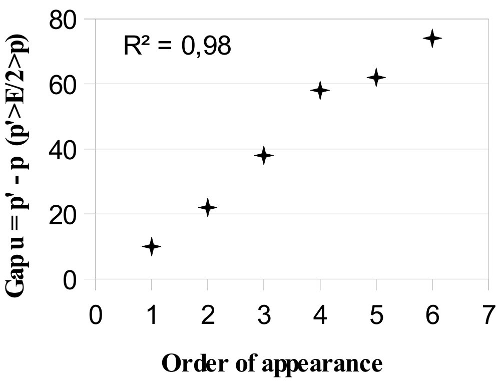


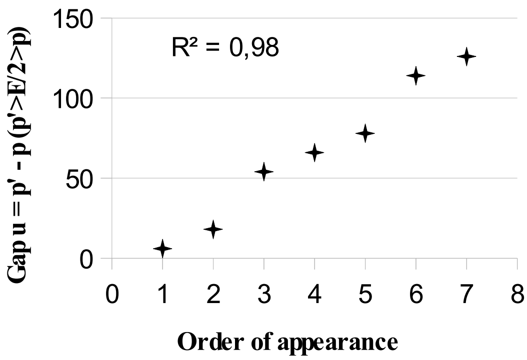
If E = P1 + P2 with P2 > P1 let pose u = P2 – P1. In table 10, u obtained with a 3n number (E = 84, E/2 = 42) is compared to that obtained with a non 3n number (E = 140, E/2 = 70).
The gap u = P2 – P1 is 3n or 6n when the even E is not 3n. By contrast, u is 2n when the number E is 3n. The GSC is linked to the formation of prime numbers from the integers which precede them. The data show that for the goldbach's conjecture to be true, there must be a value t such that for any integer n, n – t and n + t are primes and equidistant to n. The value of t will depend on whether the integer n is 3n or not.

Table 10. The gap between two equidistant primes noted **u** is not the same depending on E/2 of the even number E. If E/2 is 3n (42), u values are 2n in increasing order. If E/2 is non-3n (70), u values are 3n or 6n in increasing order. The u values obtained in both cases show a good linear correlation of 0.98; shown by graphics below (left, u values of E/2 = 42; right, u values of E/2 = 70).

u (E/2 = 42)	Factors ≠ 3n	u (E/2 = 70)	Factors = 3n
22	2 x 11	6	6 x 1
38	2 x 19	18	6 x 3
58	2 x 29	54	6 x 9
62	2 x 31	66	6 x 11
74	2 x 37	78	6 x 13
		114	6 x 19
		126	6 x 21

↓ ↓





The strong correlation observed with u values also indicates that equidistant primes appear after a regular interval of the same order. And even if the number tends to infinity, there will always be a strong correlation between equidistant primes close to each other.

2E. Two rules that explain the equidistance of prime numbers and which are at the origin of the strong conjecture of Goldbach

The main question is to determine how an integer gives a prime number by increasing or decreasing in a symmetrical way. For GSC to be proven, we have to demonstrate that there are two equidistant primes around E/2 with E being an even. In this section, two rules are given that explain how an integer produces a prime number. Let E be any even number and let us calculate E ± T such that T is an integer < E. We will apply the same rules as seen previously, if E is an even which is not 3n, then T is odd 3n values. There is a rule for E – T and another for E + T and both of them are going to give equidistant primes around E/2. This is different from what described above since we start now with the even E and then fall back on the equidistant primes around E/2. Another method of obtaining equidistant primes is also included here which consists of euclidean divisions of E by prime factors q out of π(E) of which are > E/2. Let us pose E = aq + r with a the quotient, q any prime factor out of π(E) < E and r the remainder and this is the classic equation of the Euclidean division.

Be E any even ≥ 8 and T any integer < E. For E – T if T = r + nq then E – T is not prime (n is any integer ≥ 0). For E + T if T = nq – r then E + T is not prime. Only if T ≠ r + nq in the first case and T ≠ nq – r in the second case can we have equidistant primes. Both T values are symmetrical. These two rules are required to understand the GSC.

Demonstration:

- E – T and T = r + nq. Knowing that E = aq + r → E – T = aq + r – (r + nq) = (a + n) q → E – T not prime. For each T value, this must be true for all q out of π(E) < E.
- E + T and T = nq – r. Knowing that E = aq + r → E + T = aq + r + (nq – r) = (a + n) q → E + T not prime. For one T value, this must be true for all q out of π(E) < E.

2E.1 First rule: In order to have prime numbers by subtracting T from an even number E : if E = aq + r then E – T is prime if T ≠ r; or T ≠ r + q; or T ≠ r + nq (n is any integer and q all primes < E).

E is any even ≥ 8. E = P1 + P2 with P2 > P1 and P1 and P2 are equidistant primes. The method is as follows.
1)- Take T-values as odd 3n (for an even number that is not 3n). Calculate E – T.
2) - Determine π(E) the primes of which are named q and divide E –T by prime factors **q < E/2** to apply the rule T ≠ r and T ≠ r + q or T ≠ r + nq (n is any integer > 0). Primes are numbers E – T with T satisfying the rule for each euclidean division of E by **q out of π(E) < E/2**. This leads to equidistant primes to E/2 that sum up to form E. Therefore, this rule allows us to find out equidistant primes around E/2.
3) Meanwhile, when we divide E by **q out of π(E) > E/2**, the remainder = P1 and the divisor = q = P2. This time we have at once two equidistant primes if the remainder is prime. This is another method to find out equidistant primes around E/2. The data obtained with q < E/2 and q > E/2 are shown in tables 11A+B and 13A+B. In table 12, the specific case of q > E/2 is further discussed separately by puttig emphasis on other rules.

Example number E = 112 and E/2 = 56 which is not 3n and then T is mostly 3n (Table 11A). On the first column of Table 11A, we have prime factors q of π(E) < E and the second column the remainders r of euclidean division of E with each q. T values (odd 3n) are shown in the first line which have to be substracted from E = 112 (only T values are shown).
The colored columns indicate prime numbers while non-colored columns correspond to non-primes and have a color spot that indicate which remainder is concerned. Note equidistant primes are E – T (Table 11A) and E + T (Table 11B) that are both primes. All equidistant primes are underlined and highlighted in bold in the first line.
On the other hand, equidistant primes directly obtained by Euclidean divisions of E by q > E/2 are shown on the first two columns and they are also underlined and highlighted in bold.

- Here are some examples for $q < E/2$ (Table 11A).

112 – 21 is not prime because 112 : 13 (13 is q) has a remainder (r) of 8 and at the same time 21 – 8 = 13 → 21 = 8 + 13 (r + q). If we substract 21 of 112, we take off the remainder 8 and one factor 13 and what remains is therefore multiple of 13 → 112 – 21 = 91 = 7 x 13.

112 – 27 is not prime because 112 : 5 (q) has a r = 2 and thus 27 – 2 = 25 → 27 = 2 + 25 = 2 + 5 x 5 (r + nq).

112 – 57 is not prime because 112 : 5 has a r = 2 and 57 = 2 + 55 = 2 + 11 x 5 (r + nq). Furthermore, 112 : 11 (q) has a r = 2 and 57 = 2 + 55 = 2 + 5 x 11 (r + nq).

112 – 63 is not prime because 63 is a multiple of 7 and 112 : 7 has r = 0.

112 – 87 is not prime because 112 : 5 has r = 2 and 87 = 2 + 85 = 2 + 17 x 5. (r + nq)

- But when $q > E/2$ or 112/2 = 56 the remainder r is either prime or not. For $q > E/2$ the strong conjecture (E = p + p') itself becomes Euclidean division in the form E = aq + r with **q = P2** and **r = P1** and the quotient a = 1 → E = P2 + P1 = P1 + P2 such that P2 > E/2 > P1. And in this case T = q = P2 and E – T = E – q = P1 = r. Note that r = P1 may be prime or not. This brings new equidistant primes (See Table 12 with the comments that follow) . In this case, we also have the rule stated above. For instance 100 = 53 + 47. Here we have for example 100 : 11 has a **r' = 1** while 53 : 11 has a **r = 9** and 47 : 11 has a **r = 3** and we see that r'≠r in both cases. However if we have 100 = 67 + 33 we have 67 : 11 has **r = 1** and we see that 33 is a composite relatively to q = 11 = 3 x 11 which has a r = 0 → r' = r and 33 = n'q = 3 x 11. Here is another example. 100 = 61 + 39. We have 100 : 13 has a **r' = 9** while 61 : 13 has a **r = 9** and r' = r → 39 is composite relatively to q = 13 ad 39 = 3 x 13. If for one q, r'=r then P + X → X is composite = n'q except if n' = 1 (see Table 12 and what follows).

Table 11A. Primality test of (E – T) numbers by looking at the remainders of euclidean divisions E:q and (E – T):q. Be E any even E ≥ 8 such that p and p' are equidistant primes (p' > p) to E/2 and so p = E/2 – t and p' = E/2 + t and E = p + p'. In the table, E = aq + r (euclidean division) with **a** the quotient (not shown) and **r** the remainder (shown). The divisor q or prime divisors < E are shown in the first column and remainders r on the second one. E – T (E = 112) numbers are calculated with T values shown in the first line (odd 3n). Columns colored are those corresponding to E – T being prime numbers and columns with an isolated colored spot indicate non-prime numbers and the remainders they are related to. If T = r + nq (n any integer including 0) then E – T is not prime. Underlined numbers in bold on the first line correspond to equidistant primes in Tables 11A+B. The highlighted and underlined numbers in the two first columns are equidistant primes obtained with E : q such that q > E/2. The prime factor q > E/2 is indicated by a colored line.

π(E)	E:q	T values to substract from E = 112 and divide by q (E – T : q) to determine remainders (r)																	
q	r	3	9	**15**	21	27	33	**39**	**45**	**51**	57	63	69	75	**81**	87	93	**99**	105
3	1																		
5	2																		
7	0																		
11	2																		
13	8																		
17	10																		
19	17																		
23	20																		
29	25																		
31	19																		
37	1																		
41	30																		
43	26																		
47	18																		
53	6																		
59 >E/2	**53**																		
61	51																		
67	45																		
71	**41**																		
73	39																		
79	33																		
83	**29**																		
89	**23**																		
97	15																		
101	**11**																		
103	9																		
107	**5**																		
109	**3**																		

2E.2 Second rule: In order to have prime numbers by adding T to E or E + T : T ≠ q – r or T≠ nq – r.

Only some of E + T that are not prime are going to be explained (Table 11B).

1. 112 + 9 is not prime because 112 : 11 has a remainder r = 2 and 9 = 11 – 2 (q – r).
2. 112 + 33 is not prime because 112 : 5 has r = 2 and 33 = 35 – 2 = 7 x 5 – 2 (nq – r). Furthermore, 112 : 29 has r = 25 and 33 = 58 – 25 = 2 x 29 – 25 (nq – r).
3. A last example. 112 – 75 is not prime because 112 : 11 has a r = 2 and 75 = 77 – 2 = 7 x 11 – 2 (nq – r). In addition, 112 : 17 has r = 10 and 75 = 85 – 10 = 5 x 17 – 10 (nq – r). In tables 11A and 11B corresponding equidistant primes are underlined in the first line, and two first coloumns (q > E/2).

Table 11B. Primality test of (E + T) numbers by looking at the remainders of euclidean divisions E:q and (E + T):q. Be E any even ≥ 8 such that p and p' are equidistant primes (p' > p) to E/2 and so p = E/2 – t and p' = E/2 + t and E = p + p'. In the table, E = aq + r (euclidean division) with a the quotient (not shown) and r the remainder (shown). The divisor q or prime factors < E are shown in the first column from left and remainders r on the second one. E + T (E = 112) numbers are calculated with T values shown in the first line. Columns colored are those corresponding to E + T being prime numbers and columns with an isolated colored spot indicate non-prime numbers and the remainders they are related to. If T = nq – r (n any integer including 0) then E + T is not prime. Underlined numbers in bold on the first line correspond to equidistant primes in Tables 11A+B. The highlighted and underlined numbers in the two first columns are equidistant primes obtained with E : q such that q > E/2. The prime number q > E/2 is indicated by a colored line.

π(E)	E:q	T values to add to E = 112 and divide by q to determine remainders (E + T : q)																	
q	r	3	9	**15**	21	27	33	**39**	**45**	**51**	57	63	69	75	**81**	87	93	**99**	105
3	1																		
5	2																		
7	0																		
11	2																		
13	8																		
17	10																		
19	17																		
23	20																		
29	25																		
31	19																		
37	1																		
41	30																		
43	26																		
47	18																		
53	6																		
59 (>E/2)	**53**																		
61	51																		
67	45																		
71	**41**																		
73	39																		
79	33																		
83	**29**																		
89	**23**																		
97	15																		
101	**11**																		
103	**9**																		
107	**5**																		
109	**3**																		

- *2E3. The specific case of q > E/2 where we have E : P2 = 1 with the remainder r = P1. The congruence rules for this case.*

Table 12. Congruence rules that determine whether the strong Goldbach conjecture holds in the case of q = P2 > E/2. Let E be an even number ≥ 8, q any prime number < E in $\pi(E)$, and P2 a prime > E/2. To convert E to the sum of two primes P2 and P1 (E = P2 + P1) such that P1 < E/2 we perform the Euclidean divisor E : P2 which has a quotient = 1 and a remainder = P1 or C (C is any composite number). If P2 ≡ E modulo (q) (for example in the table q3) then E = P2 + C unless C = q . If E≢P2 on all the remainders of E: q (r1 to rn) then P1 is prime and E = P2 + P1. We see that a prime number is a solution to a problem: that of finding a number which has no congruence with the number of which it is an addition term. In the table ≢means no congruence. If there is a congruence (for example modulo q3) P1 is composite (C) except if C = q.

q < E	P2 > E/2 and P1 or C < E/2 E : P2 = P1 or E : P2 = C → E = P2 + P1 or E = P2 + C		
Prime factor (q) of $\pi(E)$	Remainder E : q	P2 : q P1 Composite (C) Except if C = q	P2 : q P1 Prime
q1	r1	≢	≢
q2	r2	≢	≢
q3	r3	≡	P1 = C not prime except if C = q.
q4	r4	≢	≢
q5	r5	≢	≢
q6	r6	≢	≢
q7	r7	≢	≢
...	...	≢	≢
qn	rn	≢	≢

Demonstrations in the case of q > E/2 (also the case of the equidistant primes of the two first colums in Tables 11A+B above and 12A+B below)**.**

1)- Be E = aq + r and P2 a prime number > E/2.
Be P2 = a'q+ r
then E – P2 = X = (a – a')q → *X is not prime* except if a – a' = 1. Only if a' - a = 1 is the GSC verified E = P2 + P1 with P1 < E/2.

2)- Be E = aq + r
Be P2 = a'q+ r'
then E – P2 = X = (a – a')q + (r – r') → *X is prime* if r ≠ r' for any q < E. Only under this condition is the GSC verified E = P2 + P1 with P1 < E/2.

3)- If the GSC is verified E = P2 + P1 with P2 > E/2 and P1 < E/2 → E ≡ P2 moduloP1.
E = aP1 + r → P2 = aP1 + r – P1 **→ *P2 = (a – 1)P1 + r*** → E ≡ P2 moduloP1.

4)- If E = aq + r and P2 = (a – 1)q + r then E – P2 = P1 is prime. E – P2 = (aq + r) - ((a – 1)q + r)) = (a – a + 1)q + (r – r) = q knowing that q is any prime < E.

5)- E : P2 = X (note P2 is prime > E/2). Let E = aq +r ; P2 = a'q + r' and X = a''q + r''. In all cases we have r' + r'' = r or r' + r'' = nq + r (n ≥ 0). If this is true for all q < E or any q of $\pi(E)$ then E = P2 + P1 which are both primes and the GSC is verified. If for one q of $\pi(E)$, r'' = 0 and r = r' then X is composite except if X = q.

Examples :
100 = 67 + X knowing that 67≡100 modulo11 then X is composite (except if X = q = 11) but X = 33 = 3 x 11.
1000 = 571 + X knowing that 571≡100 modulo11 X is composite X = 429 = 3 x 11 x 13.
100 = 89 + 11 Even if 89≡100 modulo11 X is prime because X = 11 (the case in the table when X = q).
2000 = 1303 + X knowing that 1303≡2000 modulo 41 X is compostite X = 697 = 17 x 41.
2000 = 1873 + X Even if 1873≡2000 modulo127 X is prime because X = 127 (the case in the table whe X = q).
2000 = 15 x 127 + 95 and 1873 = 14 x 127 + 95 (according to demostration 3 above).
200 = 149 + 51. For all q of $\pi(200)$ the remainders r' of (149 : q) + r'' of (51 : q) = nq + (r of 200 : q) except for 3 and 17 for which r'' = r and 51 is composite = 3 x 17.
200 = 139 + 61 For all q of $\pi(200)$ the remainders r' of (139 : q) + r'' of (61 : q) = nq + (r of 200 : q) therefore 61 is prime and therefore Goldbach conjecture is verified.

What then do these demonstrations mean in the case where E : P2 = X knowing that E is any even number ≥ 8 and P2 > E/2 and denoting any prime number < E or $\pi(E)$ as q ? For any even number E, there are three possible numbers P2 > E/2: composite (C), prime numbers P2≡E modulo at least one factor q, and prime numbers P2≢ E for any factor q. The congruent P2 will always add to a composite number C to form E except if C is a unit prime factor. Whereas the non-congruent P2 will necessarily add to a prime number P1 to form E. Why? By using the same demonstrations cited above. In fact, if there is congruence between E and P2 and if we write E = P2 + X this means that the remainders of E : q and P2 : q are identical and that necessarily X is a multiple of q except in the case where X is itself the prime factor q, which could happen sometimes but not always. On the contrary, if there is never any congruence between E and P2 ; and if we write E = P2 + X and we note E = aq + r ; P2 = a'q + r' ; and X = a''q + r'' we therefore have r' + r'' = r or r' + r'' = nq + r. In this case, r'' cannot be zero because we contradict ourselves since there will be congruence between E and P2. Therefore, r'' is always non-zero in this case for any factor q. In other words, the non-congruence of E and P2 entails that of E and P1 whatever the factor q of $\pi(E)$. Consequently in this case X = P1 which is prime and E = P1 + P2. This is a demonstration of Goldbach strong conjecture because there will always be at least one probability chance that a non-cogruent prime number will appear after E/2, this probability is never zero. *All prime numbers after E/2 cannot all be congruent because this is incompatible with the progression of natural numbers unit by unit*. This is why Goldbach's conjecture is always true if we admit that there always exist enough prime numbers between E/2 and E whatever the value of E. For instance 100 = 73 + 27 means 100≡73 modulo(3) while 100 = 59 + 41 means 100≢59 and 100≢41 for any factor q < E.

We can conclude that the progression of natural numbers always produces two types of prime numbers. Among the latter we have those which are never congruent with an even number E ; E/2 is located at an equal distance between a non-congruent prime number P2 > E/2 and another prime number P1 < E2.

The strong Goldbach's conjecture E = P1 + P2 ↔ E≢P1 for any prime q < P1 and E≢P2 for any prime q < P2 with q any prime of π(E) and such that P2 > E/2 et P1 < E/2 . Howevere, E≡P2 if P1 = q.

- *2E4. Another example : a 3n number E = 240 and E/2 = 120.*

Table 13A. Equidistant primes around 120 the sum of which make 240. Because 240 is 3n, T takes values of primes (or composites but primes are used here). The same legends as in tables 11. Here E/2 – T. Empty Columns are those corresponding to E – T being prime numbers and an isolated colored spot indicate non-prime numbers and the remainders they are re lated to. *If T = r + nq (n any integer) then E – T is not prime*. Underlined numbers in bold on the first line correspond to equidistant primes in Tables 12A+B. Both equidistant primes are shown on the two left-columns if q > E/2 or q > 60 for 120 number. Note equidistant primes are E – T and E + T that are both primes.

π(E)	E: q	T values to substract from E = 120 and divide by q to determine remainders (E – T : q)																												
q	r	3	5	**7**	**11**	**13**	**17**	**19**	**23**	29	**31**	**37**	**41**	43	**47**	**53**	57	**61**	**67**	71	**73**	**79**	83	89	97	101	**103**	**107**	**109**	**113**
3	0																													
5	0																													
7	1																													
11	10																													
13	3																													
17	1																													
19	6																													
23	5																													
29	4																													
31	27																													
37	9																													
41	38																													
43	34																													
47	26																													
53	14																													
59	2																													
61 >E/2	**59**																													
67	**53**																													
71	49																													
73	**47**																													
79	**41**																													
83	**37**																													
89	**31**																													
97	**23**																													
101	**19**																													
103	**17**																													
107	**13**																													
109	**11**																													
113	**7**																													

Table 13B. Equidistant primes around 120 the sum of which make 240. Because 240 is 3n, t takes values of primes (or composites but primes are used here). The same legends as in tables 11. Here E + T. Empty Columns are those corresponding to E/2 + T being prime numbers and an isolated colored spot indicate non-prime numbers and the remainders they are related to. *If T = nq – r (n any integer) then E + T is not prime*. Underlined numbers in bold on the first line correspond to equidistant primes in Tables 12A+B. Both equidistant primes are shown on the two left-columns if q > E/2 or q > 60 for 120 number. Note equidistant primes are E – T and E + T that are both primes.

π(E)	E:q	T values to add to E = 120 and divide by q to determine remainders (E + T) : q																												
q	r	3	5	**7**	**11**	**13**	**17**	**19**	**23**	29	**31**	**37**	**41**	43	**47**	**53**	57	**61**	**67**	71	**73**	**79**	83	89	97	101	**103**	**107**	**109**	**113**
3	0																													
5	0																													
7	1																													
11	10																													
13	3																													
17	1																													
19	6																													
23	5																													
29	4																													
31	27																													
37	9																													
41	38																													
43	34																													
47	26																													
53	14																													
59	2																													
61	**59**																													
67	**53**																													
71	49																													
73	**47**																													
79	**41**																													
83	**37**																													
89	**31**																													
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101	**19**																													
103	**17**																													
107	**13**																													
109	**11**																													
113	**7**																													

2F. The GSC explains how a prime number gives the prime number that follows it and this progression obeys the two rules decribed above in relation to the remainders of the Euclidean divisions

Be any two prime numbers p and p' such that $p' > p \rightarrow p' - p = 2n$. Let us suppose a number noted $X - p = 2n$ and let us see if X is prime nor not. X will be prime if $2n \neq mq - r$ with q any prime factor $< X$ and m any integer ≥ 1. When $n = 1$ and this rule verified then we have twin prime numbers. But if we have $n = 1$ and the rule not verified (meaning $2n = mq - r$) then we do not have twin prime numbers. For instance, let us take 17 and 17 : 3 has a remainder r = 2 ; 17 : 5 has r = 2 ; 17 : 7 has r = 3, 17 : 11 has r = 6, and 17 : 13 has r = 4. Therefore if we add 2 to 17 we have $2 \neq mq - r$ in all those euclidean divisions and so 17 + 2 = 19 is prime. By contrast if we take a number like 31 we have 31 : 11 = 2 and r = 9 and so $2 = 11 - 9 = mq - r \rightarrow 31 + 2 = 33$ is not prime because it is a multiple of 11. In a similar way 31 : 3 = 10 and r = 1 and $2 = 3 - 1 = mq - r$ and so if we add 2n to 31, it is not prime because it is a multiple of 3.
This rule determines if p + 2n is prime or not and can therefore explain how equidistant primes are produced. Let us take some examples. 11 + 12 knowing that 11 : 12 = 0 and r = 11.
In this case $12 \neq mq - r = m11 - 11$ for instance $12 \neq 22 - 11$ or $12 \neq 33 - 11$ and so on. Therefore 11 + 12 = 23 is prime. We have two primes 11 and 23 and 11 + 23 = 34 : 2 = 17 and therefore 11 and 23 are equidistant to 17. In this specific case 2 x 17 = 34 = 11 + 23.
If we take 11 + 10 = 21 not prime because 11 : 3 = 3 and r = 2 and $10 = 12 - 2 = 4 \times 3 - 2 = mq - r$. Or 11 : 7 = 1 and r = 4 and we have $10 = 14 - 4 = 2 \times 7 - 4 = mq - r$. Therefore 21 is a multiple of 3 and 7.
Let take another number like 31 + 12 and whatever prime factor < 31; $12 \neq mq - r$. For instance if q = 7, then 31 : 7 = 4 and r = 3. Hence $12 \neq m \times 7 - 3$ whatever m value; if m = 1, $12 \neq 4$; if m = 2, $12 \neq 11$; and m = 3, $12 \neq 18$ and so on. Hence 31 + 12 = 43 is prime $\rightarrow$ 31 + 43 = 74 : 2 = 37 $\rightarrow$ 31 and 43 are equidistant to 37 and 37 x 2 = 74 = 31 + 43. We can argue differently 31 + 12 = 43 the mean value is either 31 + 6 or 43 – 6 which also means that 2 x 6 is the distance between 31 and 43 and therefore 37 x 2 = 31 + 43.
Because an even value has to be added to a prime number p to get the next one p' ($p' = p + 2n$) therefore there is always a mean value M located at the same distance from the two such that $M = p + n = p' - n$ and therefore $2M = p + p'$. However if q is any prime factor < p', the rule $2n \neq mq - r$ has to be verified to get the next prime number p'.
The most important element is that the rule $2n \neq mq - r$ is always verified because there is an infinity of n values of 2n to get the next prime number. For example, if we take any prime number like 73 we can get 79 (73 + 2 x 3); 89 (73 + 2 x 8); 97 (73 + 2 x 12) and so on. In other words, we will never find a prime number that will not give another prime number by adding to it 2n with n being any integer > 0. When we say prime numbers are infinite this means that any prime p increased by 2n would give another prime p' and therefore $p + n = p - n = N \rightarrow 2N = p + p'$. This proves that GSC is always true as long as a prime number p increased by 2n gives another one noted p'.
Reciprocally, if we have one prime number p' and want to go down to p such that p < p' then $p' - 2n = p$. This time we divide p' by all prime factors noted q < p' and $2n \neq r$ or $2n \neq mq + r$ (m any integer including 0). For instace 97 – 6 = 91 is not prime because 97 : 7 = 13 and r = 6 so 6 is the remainder (2n = r). Therefore $6 = r \rightarrow 6 = mq + r$ with m = 0.
Let us take another example 443 – 234 = 209. We have 443 : 11 has a remainder r = 3. However $234 = 231 + 3 = 11 \times 21 + 3 = mq + r$. Therefore 209 not prime because multiple of 11. Or 443 : 19 has a remainder r = 6. And $234 = 228 + 6 = 122 \times 19 + 6 = mq + r$.

- Demonstration:
- $p + 2n = p'$. If $p = aq + r$ and $2n = mq$ **– r** then $p + 2n = aq + r + mq – r = (a + m)q$ thus not prime.
- $p' – 2n = p$. If $p' = a'q + r$ and $2n = mq$ **+ r** then $p' - 2n = a'q + r – (mq + r) = (a' + m)q$ thus not prime. The same if $2n = r$ then $p' - 2n = a'q + r – r = a'q$ thus not prime.
- If p is prime and if $p + 2n = p'$ then p' is prime only if $2n \neq mq – r$ with q being any prime factor < p and r the rremainder of the Euclidean division of p by q. Let determine $\pi(p)$ and then divide p by all prime factors of $\pi(p)$ and calculate the remainder r for each euclidean division then apply this rule.
- If p is prime and if $p' - 2n = p$ then p is prime only if $2n \neq r$ and $2n \neq mq + r$ with q being any prime factor < p' and r the remainder of the Euclidean division of p' by q. Let determine $\pi(p')$ and then divide p' by all prime factors of $\pi(p')$ and calculate the remainder r for each euclidean division then apply this rule.

For $p + 2n = p'$ or $p' – 2n = p$ and knowing that $n \to + \infty$, there must exist at least one value of n such that p' and p are primes. GSC means that one or more values of n always exist such that p and p' are primes. Given that there exists a limitless possibilities that one value of n exists such that p and p' are primes then $p + 2n = p' \to p + n = p' – n \to$ be N any integer such that $N = p + n = p' – n$ $\to 2N = (p + n) + (p' – n)= p + p'$. Goldbach conjecture is therefore verified to be true. In other words, this conjecture means that whatever values of any prime numbers p and p' such that p' > p and whatever $\pi(p)$ or $\pi(p')$, there always exist a value n such that $p + n = p' – n$. Because prime numbers are limitless, then their additions would produce all possible even numbers ↔ any even is a sum of at least two primes. If the strong conjecture is true ↔ the weak one is also true. Even if prime numbers might be less frequent beyond E/2, this is compensated by their much higher frequence below E/2 leading to at least one verification of Goldbach's conjectures.

- The rules stated above indicate that GSC is linked to the progression of prime numbers one after another. If we take any prime number and divide it by all prime factors lesser than it, we will get remainders. These latters will determine the next prime number and so on. For instance, if we take a prime number like 31, we have 3, 5, 7, 11, 13, 17, 19, 23, and 29 prime numbers that are < 31 and therefore we have 9 remainders of 9 euclidean divisions between 31 and each of them. On the other hand, we have too many possibilities not to complete these remainders and not to get non-prime numbers. For instance, if we add 8 to 31, that is no prime, but if we add 6 to it, that is prime. Whatever the size of a prime number, there will always be too many possibilities to bypass all the remainders and get a new prime number, which is in accordance with the fact that prime numbers are limitless. Any time a prime number p gives another one p' that follows it, Goldbach conjecture is verified because $p + 2n = p'$ ↔ Even = p + p' as demonstrated above. In addition, any prime number combines with a limitless prime numbers to form an even number so that each even number is a sum of two prime numbers. The other property of prime numbers is that if we take any two prime numbers and whatever the distance between them, we will find the same rules that explain how a prime numbers gives a new one. This is where the truth of Goldbach's conjecture lies.

2G. Calculation examples

2G1. Direct calculation with small numbers using the rules described

Let E/2 being any integer ≥ 4 and E any even ≥ 8 be the sum of two prime numbers P1 and P2 such that P2 > P1. If E = P1 + P2 then P1 = E/2 - t and P2 = E/2 + t with t being any non-zero integer. We say that P1 and P2 are two equidistant prime numbers. In this case E/2 mod t = P1 mod t = P2 mod t. This rule allows us to find the equidistant prime numbers around E/2 and thus convert the even number E into the sum of two prime numbers P1 and P2 according to Goldbach's conjecture. Given that E/2 can be any integer ≥ 4 we can deduce that all natural integers ≥ 4 are in the middle of two equidistant prime numbers whether they are evens or odds, primes or composite. The only parameter to take into consideration as demonstrated above is to see if the number is a multiple of 3 or not. Other rules are described above which are based on 6x ±1 equations.

Here are detailed calculation examples to prove the authenticity of these rules to verify GSC.
Let's take for example the number E = 84 (E/2 = 42). Because 42 is 3n then equidistant primes are located after gaps = prime numbers or multiple of prime numbers. Let focus on gaps = prime numbers only.

- 42 – 5 = 37 → 42 + 5 = 47 → 37 and 47 are equidistant primes and 37 + 47 = 84.
- 42 – 7 = 35 → 42 + 7 = 49
- 42 – 11 = 31 → 42 + 11 = 53 → 31 and 53 are equidistant primes and 31 + 53 = 84.
- 42 – 13 = 29 → 42 + 13 = 55
- 42 – 17 = 25 → 42 + 17 = 59
- 42 – 19 = 23 → 42 + 19 = 61 → 23 and 61 are equidistant primes and 23 + 61 = 84.
- 42 – 23 = 19 → 42 + 23 = 65
- 42 – 29 = 13 → 42 + 29 = 71 → 13 and 71 are equidistant primes and 13 + 71 = 84.
- 42 – 31 = 11 → 42 + 31 = 73 → 11 and 73 are equidistant primes and 11 + 73 = 84.
- 42 – 37 = 5 → 42 + 37= 79 → 5 and 79 are equidistant primes and 5 + 79 = 84.
- 42 – 41 = 1 → 42 + 41 = 83

The mod rule applies as follows:

- 37 and 47 are equidistant to 42 and the gap = 5. Then the remainders of the euclidean divisions 37 : 5 ; 47 : 5 ; and 42 : 5 are the same = 2.
- 31 and 53 are equidistant to 42 and the gap = 11. Then the remainders of the euclidean divisions 31 : 11 ; 53 : 11 ; and 42 : 11 are the same = 9.
- 23 and 61 are equidistant to 42 and the gap = 19. Then the remainders of the euclidean divisions 31 : 19 ; 53 : 19 ; and 42 : 19 are the same = 4.
- 13 and 71 are equidistant to 42 and the gap = 29. Then the remainders of the euclidean divisions 13 : 29 ; 71 : 29 ; and 42 : 29 are the same = 13.
- 11 and 73 are equidistant to 42 and the gap = 31. Then the remainders of the euclidean divisions 11 : 31 ; 73 : 31 ; and 42 : 31 are the same = 11.
- 5 and 79 are equidistant to 42 and the gap = 37. Then the remainders of the euclidean divisions 5 : 37 ; 79 : 37 ; and 42 : 37 are the same = 5.

If the number is not 3n such like 140, we then substract or add 3n values to 140/2 = 70.

70 - 3 = 67	→	70 + 3 = 73	→	67 + 73 = 140
70 - 9 = 61	→	70 + 9 = 79	→	61 + 79 = 140
70 – 21= 49	→	70 + 21 = 91		
70 – 27= 43	→	70 + 27 = 97	→	43 + 97 = 140
70 – 33 = 37	→	70 + 33 = 103	→	37 + 103 = 140
70 – 39 = 31	→	70 + 39 = 109	→	31 + 109 = 140
70 – 51 = 19	→	70 + 51 = 121		
70 – 57 = 13	→	70 + 57 = 127	→	13 + 127 = 140
70 – 63 = 7	→	70 + 63 = 133	→	7 + 133 = 140

Be E = P1 + P2 such that P2 > P1 and P1 = E/2 – t and P2 = E/2 + t. Hence t is the gap between E/2 and the equidistant primes P1 and P2.
Be E/2 = at + r with a the quotient and r the remainder of the euclidean equation or division of E : t.

$$E/2 = at + r \rightarrow P1 + t = at + r \rightarrow P1 = (a - 1)t + r.$$
$$E/2 = at + r \rightarrow P2 - t = at + r \rightarrow P2 = (a + 1)t + r.$$

These equations can be useful to convert an even number into a sum of two prime numbers. Examples of these are given below.

E/2 = 42. 42 : 5 = 8 and r = 2. Then P 1 = (8 – 1) x 5 + 2 = 7 x 5 + 2 = **37**. P2 = (8 + 1) x 5 + 2 = 9 x 5 + 2 = **47**.
E/2 = 42. 42 : 7 = 6 and r = 0. Then P 1 = (6 – 1) x 7 + 0 = 5 x 7 + 0 = **35**. P2 = (6 + 1) x 7 + 0 = 7 x 7 + 0 = **49**. However neither P1 nor P2 is prime.
E/2 = 42. 42 : 11 = 3 and r = 9. Then P 1 = (3 – 1) x 11 + 9 = 2 x 11 + 9 = **31**. P2 = (3 + 1) x 11 + 9 = 4 x 11 + 9 = **53**.
E/2 = 42. 42 : 19 = 2 and r = 4. Then P 1 = (2 – 1) x 19 + 4 = 1 x 19 + 4 = **23**. P2 = (2 + 1) x 19 + 4 = 3 x 19 + 4 = **61**.
E/2 = 42. 42 : 23 = 1 and r = 19. Then P 1 = (1 – 1) x 23 + 19 = 0 x 23 + 19 = **19**. P2 = (1 + 1) x 23 + 19 = 2 x 23 + 19 = **65**. However P2 = 65 is not prime.
E/2 = 42. 42 : 29 = 1 and r = 13. Then P 1 = (1 – 1) x 29 + 13 = 0 x 29 + 13 = **13**. P2 = (1 + 1) x 29 + 13 = 3 x 19 + 4 = **71**.
E/2 = 42. 42 : 37 = 1 and r = 5. Then P 1 = (1 – 1) x 37 + 5 = 0 x 37 + 5 = **5**. P2 = (1 + 1) x 37 + 5 = 3 x 19 + 4 = **79**.

Let E = P1 + P2 such that P2 > P1 → P1 < E/2 and P2 > E/2. Therefore, E/2 : P2 = 1 and r = P1. In fact Goldbach's conjecture E = P1 + P2 can be posed as an euclidean equation E = a x P2 + P1 with a (quotient) = 1 and the remainder r = P1 and P2 > E/2. Then there is a third prime number P3 = 2P2 + P1 such that P3 + P1 = 2P2 + 2P1 = 2E. Here is the demonstration.
The equation results from the Mod rule. If we divide E/2 by P2 which is >E/2 the quotient is = 1 and the remainder is necessarily P1 because E = P1 + P2. And since P1 = (a – 1)t + r and P2 = (a + 1)t + r ; P1 remains unchanged while a new prime number P3 will appear and which is equal to 2P2 + P1. In fact P1 = (1 – 1)t + r knowing that r = P1 thus P1 = P1. While P2 = (a + 1)t + r = (1 + 1) x P2 + P1 (note t = P2 the divisor) and because it is impossible that P2 = 2P2 + P1 we rather set a new prime number P3 = 2P2 + P1 → P3 + P1 = 2P2 + 2P1 = 2E. To convert an even 2E (E is also even) in sum of two primes, we start with its half E. Note that this equation cannot give a prime any time but rather gives equidistant primes after one or more operations. This equation can be used to convert an even in sum of two primes as follows.

Let's take the number 180 as an example. Then we start with 90 = 180/2 and 90/2 = 45. Let us take a prime P2 > 45 and < 90 such that the remainder r = 1.
P2 = 47. Then 90 : 47 = 1 and r = 43 → P1 = **43** and P2 = 47. Therefore, P3 = 2 x 47 + 43 = **137**. Therefore, P3 + P1 = 137 + 43 = 180.
P2 = 59. Then 90 : 59 = 1 and r = **31**. P3 = 2 x 59 + 31 = **149** and 149 + 31 = 180.
P2 = 83. Then 90 : 83 = 1 and r = **7**. P3 = 2 x 83 + 7 = **173** and 173 + 7 = 180. Therefore 173 and 7 are equidistant to 90.
The equation 2P2 + P1 gives the gap separating the two equidistant primes which the P2 value. In the case above of P2 = 83. P1 = **7**. P3 = **173**. We have 83 separating 7 and 173 from 90. And in the latter P1 = **13** and P3 = 2 x 77 + 13 = **167**, we have 77 separating 167 and 13 from 90. Another example 90 : 59 = 1 and r = 31. Therefore, P3 = 2 x 59 + 31 = 149 and thus 149 + 31 = 180. The primes 31 and 149 are both 59 away from 90, the P2 value.
These calculations will apply to any even number ≥ 8 to convert it to the sum of two prime numbers.

2G2. Calculation with 6x ± 1 equations using tables with numbers relatively larger in value

Note that the rules explained here apply to any number. However, the direct calculation shown above is easier with relatively small numbers but with larger numbers, a table is essential to be able to proceed. Here are two examples of conversion of evens into the sum of two prime numbers.

As aforementioned, there are two types of even numbers 2n with even or odd n. We have seen examples of 2n with even n, here is one example of even with odd n and another 2n with even n is added.

1. *Even 2n with odd n*

Let's first take a small number to explain the rules of calculation.
The number 66 : 2 = 33 and thus E = 66 and E/2 = 33. This times E/2 is divided by evens and not by odds to get prime numbers. For instance 33 : 10 = 3 and r = 3. P1 = 10 x 2 + 3 = 23. P2 = 10 x 4 + 3 = 43. P1 + P2 = 23 + 43 = 66. Or 33 : 20 = 1 and r = 13. Hence P1 = 13. P2 = 2 x 20 + 13 = 53. P1 + P2 = 13 + 53 = 66.
Here is another example E = 206. E/2 = 103 → 103 : 16 = 6 and r = 7. But P1 = 5 x 16 + 7 = 87 which is not prime. We see that we have to set the calculation so that we have one prime at first. 103 : 20 = 5 and r = 3. P1 = 4 x 20 + 3 = 83. P2 = 6 x 20 + 3 = 123 which is not prime. 103 : 24 = 4 and r = 7. P1 = 3 x 24 + 7 = 79. P2 = 5 x 24 + 7 = 127 → P1 + P2 = 79 + 127 = 206.
Let's take now a larger number E = 2380106 = 2 x 1190053. We are going to apply the mod rule by dividing the number by any even number < E/2 such as 895020. We are going to convert E in sum of two primes P1 and P3 such that P1 < P3 using mod rules stated above with P3 = 2P1 + P2.

Note that P3 + P1 = E if we divide E by a divisor < E/2; but P3 + P1 = 2E if we divide it by a divisor > E/2. This is always the case whether the divisor is even or odd. But the result is the same: either we start with 2E, find equidistant primes around E and then convert 2E. Otherwise, start with E, find out equidistant primes around E/2 and convert E. All depends on which divisor we choose in comparison to E/2. The two cases are detailed here with this example with a divisor < E/2 and the next one involving a divisor > E/2.

We have E = 2380106 and E/2 = 1190053. Let us take any even divisor such 895020.

Therefore, 1190053 : 895020 = 1 + r and r = 295033. Hence P1 = (1 – 1) + r = 0 + 295033 = 295033.

P3 = (1 + 1) x 895020 + 295033 = 2085073 + 295033 = 2085073. However P3 is not prime. We will have to apply the rule of 6x ± 1 equations to find out two équidistant primes. A table is thus needed (table 14).

However there are two major rules already discussed above.

1. Prime numbers or odd multiples of prime numbers that are not multiples of 3 are all written as 6x ± 1. So the first step is to determine whether an odd number is 6x + 1 or 6x – 1.
2. It should be noted that prime numbers or their multiples which have the same writing in equation 6x ± 1 follow each other by gaps of 6n. But the numbers 6x + 1 and 6x – 1 are separated by variable gaps having any possible value of 2n. It is therefore necessary to separate the numbers 6x – 1 from the 6x + 1 to facilitate the calculation. In table 14 only prime numbers that follow or precede the investigated numbers by 6n gaps are shown.

P1 = 295033; P3 = 2085073.

295033 + 2085073 = 2380106 : 2 = 1190053 but P3 = 2085073 is not prime.

The number 295033 = 6 x 49172 + 1 → 6x + 1.

The number 2085073 = 6 x 347512 + 1→ 6x + 1.

Table 14: Conversion of a larger even number = 2n with odd n into the sum of two prime numbers using the 6x ± 1 equation method. The calculated equidistant primes are highlighted.

295033	**+ 6n**	2085073	**- 6n**
295039	6	2085049	24
295081	48	2085037	36
295111	78	2085007	66
295123	90	2084989	84
295129	96	2084983	90

According to table 14 we have two equidistant primes relatively to E/2 = 1190053. Therefore, (295033 **+ 90**) + (2085073 **– 90**) = 295123 + 2084983 = 2380106 : 2 = 1190053. Note that both 295123 and 2084983 are both primes and therefore 2380106 = 2 x 1190053 was converted in sum of two primes.

2. *Even 2n with even n*

Let convert 2^{38} in sum of two primes.
E = 2^{37} = 137438953472
E/2 = 137438953472 : 2 = 68719476736
Let choose any prime number > E/2, such 68719479749.
137438953472 : 68719479749 = 1 and the remainder r = 68719473723 = P1.
Then we calculate P3 = 2 x 68719479749 + 68719473723 = 206158433221 = 6 x 34359738870 + 1.
While 68719473723 is 3n = 3 x 22906491241. Because we cannot get 6x ± 1 equation with the latter we have to make a change: remove two units from P3 = 206158433221 (- 2) = 206158433219 = 6 x 4359738869 + 5 (6x – 1). Add them to P1 → P1 + (2) = 68719473723 + (2) = 68719473725 = 6 x 11453245620 + 5 (6x – 1).
Neither 206158433219 nor 68719473725 is prime. We therefore have to set a table (table 15).

Table 15: Conversion of a larger even number = 2^{38} with even n into the sum of two prime numbers using the 6x ± 1 equation method.

68719473725	+ 6n	206158433219	- 6n
68719473839	114	206158433213	6
68719473917	192	206158433189	30
		206158433177	42
		206158433111	108
		206158433099	120
		206158433083	138
		206158433051	168
		206158433027	192

Therefore (68719473725 **+ 192**) + (206158433219 **– 192**) = 68719473917 + 206158433027 = 274877906944 = 2^{38}
274877906944 : 2 = 137438953472 = 2^{37}.
Note as said above if the initial divisor is > E/2 then we get 2E because the two additive primes are equidistant to E.
Both 68719473917 and 206158433027 are both equidistant primes and therefore 274877906944 = 2 x 2^{37} was converted in sum of two primes.

3. Discussion

This article discusses the major rules that Goldbach's conjecture must obey because in mathematics everything obeys rules or theorems. However, with this conjecture one is forced to reason in terms of probabilities since the prime numbers are almost impossible to put into an equation. One sees that Goldbach's conjecture is very closely linked to the distribution of prime numbers but also to their progression, that is to say how a prime number produces the other one that follows it or the one that precedes it. First, this article shows that the conversion of an even number into the sum of two prime numbers obeys the equation $6x \pm 1$. Then, it shows that two equidistant prime numbers obey a new modulo rule with respect to the gap that separates them from half of the even number. On the other hand, the article gives methods for identifying equidistant prime numbers or additive equidistant prime numbers that reconstitute an even number. Finally, the article also draws its originality by stating two major rules relating to the remainders of Euclidean divisions which allow us to understand the progression of prime numbers and thus know how one prime number leads to another.

Overall, the article clarifies some aspects of prime numbers such as the gaps between them and their progression. This article argues for the truth of the strong Goldbach conjecture as well as the weak one. Examples of calculations based on the stated rules are given, but despite all possible efforts, no counterexample could be found to reject these conjectures. They derive their truth from the very progression of natural numbers which produces an infinity of equidistant prime numbers producing in turn all the even numbers (two primes) and all the odd numbers (three primes). Biprimes are all products of two equidistant prime factors (excluding 2) which proves that all primes are equidistant and therefore their average will produce an even number. Suppose we take all the even numbers at infinity, and see all their partitions of sums, the article says that there would be at least one sum of two primes. If we follow the prime numbers, we realize that there is a perfect summetry from 0 to infinity and vice versa from infinity to 0.

A prime number is a solution to an equation or a problem that results from the progression of numbers; it represents the number that will bypass all the remainders of the Euclidean divisions of the numbers that follow or precede it. This is shown in the article with two major rules relating the primality of a number and the remainders of Euclidean divisions of the number from which it comes divided by all the prime factors that are less than it or those enumerated by the prime counting function of a number. Suppose a prime number p (or any other number), however giant it may be, and consider all the prime numbers preceding it, which we call q, the Euclidean division of p by each q will produce a remainder. Since p will produce another larger prime number only by adding to 2n, this article suggests that there is always a value of n that will circumvent all the remainders of p : q according to the two major rules stated in this article, and gives a larger prime number called p'. This is also true in the opposite direction, i.e. starting from $p' - 2n = p$. This is also true for any integer $n \geq 4$ to which we subtract or add a certain quantity. Since the process is symmetric, it generates equidistant prime numbers at key positions, which explains Goldbach's conjecture. Therefore, the prime number is the one that makes the natural numbers progress to infinity because if the equation $N + T$ or $N - T$ ($T < N$, N and T two integers ≥ 4) does no longer produce prime numbers, this means that the numbers more graduated to infinity are only multiples of the preceding prime numbers, but this is not the case. Goldbach's conjecture means a continuous progression of integers and therefore a continuous production of natural numbers with newer prime factors.

It is true that for any integer $n = a + b$ ($a < n/2$ and $b > n/2$) there exists a value $x < n$ such that $n = (a + x) + (b - x)$. This value x can be calculated by the mean (M) of $n \rightarrow M = (a + b)/2$ and $b - M = x$. However, when n is any even noted E sum of two primes p and p', this means that p and p' are equidistant to E/2 such that $p + p' = 2 \times E/2 = E$. And reciprocally $E = p + p'$ only if p and p' are equidistant with respect to E/2 such that $E/2 - p = p' - E/2$. This is also true for any even $E = 2pq$ (p and q are any prime factors except 2) so that $E/2 = p \times q$ such that $q > p$. Because E/2 can be in the form of $x^2 - y^2$ and therefore $E/2 = (M - z)(M + z) \rightarrow E = 2 (M - z) (M + z)$.

By resorting to deductive reasoning, one can argue that since all prime numbers are in advance equidistant with respect to any integer value, then it is logical to admit that their addition will give any possible even and therefore any even ≥ 4 is the sum of two primes because if p and p' are equidistant relatively to E/2 then 2 x E/2 = p + p'. The results of this paper confirm that GSC is true. And because the weak one depends on the strong one, then both of them are true.
With all the prime numbers known to date, the largest of which can have millions of digits, the results of this paper can be verified by calculation: take any even number, divide it by 2 and look for prime numbers equidistant to this fraction, you will see the conjectures are verified. However, a theorem that directly gives us the values of the two equidistant prime numbers is still missing. Hence the fact that these conjectures are always considered unproven. We can therefore say that for any integer there exists at least one pair of equidistant prime numbers that obey Mod's rule such that E/2 mod t = P mod t = P' mod t (E is any even ≥ 8 and E/2 is any integer ≥ 4).
The article published in 2019 by Guiasu contains the proof that every positive composite integer n strictly larger than 3, is located at the middle of the distance between two primes, which implicitly proves Goldbach's Conjecture for 2n as well. *However, the present article shows that every integer ≥ 4 (prime or compostite) is surrounded by equidistant primes indicating that the rule is true all the time. Furthermore, the present paper is designed differently by targeting the basic rules of calculation and from there deriving prerequisites for these conjectures to be true or verified. It also provides easy and reliable method to verify them by calculation*.
The best known equidistant primes are the twins but their density would seem not to be sufficient to reproduce all the even integers of the set N (not to mention odd ones). They only form the even which is the double of the even which is between them, for example 17 + 19 = 36 = 2 x 18. Therefore, Goldbach's conjecture makes a prediction on prime numbers and imposes a certain equidistant distribution with respect to integers. If an even number E does not have at least one prime number > E/2 then the strong conjecture can no longer remain true in its initial version ($2n = p_1 + p_2$). However, it is indeed known that any interval [x-2x] $x \geq 2$ contains at least one prime number but there must be two equidistant primes so that E can form by their addition. Till now, the amount of prime numbers < n is $\pi(n) \approx n/\ln(n)$ with n an integer and that means the prime numbers become very rare when $n \to +\infty$. This also means that evens $E \to +\infty$ might not have that primer > E/2 for the strong conjecture of Goldbach to be true. Nevertheless, the gap between E and E/2 is several times greater than ln(E) which represents the average gap with the nearby prime number (gap $\approx \ln(n)$ with n being an integer). This means that between E and E/2 it is very likely that one or many prime numbers p > E/2 satisfies GSC. Mathematics seeks absolute theorems which are true at infinity and this is undoubtedly the real problem with Goldbach's conjectures: to what extent are they true? But what is paradoxical is what we call infinity is a relative notion because its limits recede as computers become more powerful. We can reason differently and say that these conjectures are true as long as we cannot demonstrate that they are false by finding an even number which does not have a prime number equidistant between E/2 and E. Each integer ≥ 4 has its own pattern of equidistant primes. and the larger is the number the more complex it is.
On the other hand, this article proposes a method to convert an even or odd numbers in sums of primes numbers which is based on the equations M + 1 and M + 5 with M being a multiple of prime numbers except 2 and 3 or M is prime. This method shows that there are two types of prime numbers 6x - 1 and 6x + 1 and that there are three types of even numbers 6x, 6x + 2 and 6x + 4 (also previously reported by Markakis et al (2013)). The method described here based upon M + 1 and M + 5 equations could be programmed in a computer and generate a new algorithm by converting even numbers into the sum of two or three prime numbers. Goldbach's conjectures touch on the foundations of arithmetic, namely the distribution of prime numbers with respect to integers. The truth of these conjectures depends on the presence of prime numbers equidistant from integers.

An even number may have many equidistant prime numbers but their number may decrease to infinity or the gaps may increase but the result of the paper show that for any prime number there exist a equidistant one and therefore Goldbach's conjecture holds true to infinity. A counterexample cannot be found to contradict this rule.
The data of the present paper show a strong correlation between equidistant primes (by measuring their distance from E/2 or the gap between them) even though this seems to decrease as the number is larger, the linear correlation coefficient will be always stronger between close equidisant primes which proves that they are occurring in a regular fashion. This leans in favor of the truthfulness of Goldbach's strong conjetcure because if equidistant primes were not correlated and occur randomly then even numbers not satisfying this conjeture would be easier to find. Furthermore, this article gives for the first time new two rules to determine why a number $N - T$ or $N + T$ ($N \geq 4$) is not prime. These rules relate to the rest of the Euclidean divisions of the even E to be converted into sums of prime numbers with all the prime factors < E. These two rules apply especially for the prime factors < E/2 but beyond the Goldbach conjecture E = P1 + P2 itself becomes an Euclidean division with the remainder = P1, the divisor is P2 and the quotient denoted a = 1 $\rightarrow$ E = aP2 + P1. To express it more simply beyond E/2, the subtraction E - P2 (P2 > E/2) will give P1 which is prime or not. It is likely that other hidden rules also related to remainders would dictate if P1 resulting from such Euclidean divisions are prime or not.

If we take an integer n and all prime numbers < n. Since [0-n/2] and [n/2-n] have the same length and the prime numbers 6x - 1 and 6x + 1 swap after the same intervals of 6n, we can assume that a given position is either occupied by a prime number (P) or multiple of prime numbers (M). Calculating the probability will tell us that P or M have an equal chance of occupying this position either before or after n. For example a P < n/2 and another P' > n/2 may well occupy two equidistant positions, the probability is never zero neither negilgeable and therefore Goldbach's conjecture cannot be refuted, and therefore it can be that admitted as true. Even if we tend to infinity and we take at random an integer n, the largest that we can imagine, this rule of probability would not change and would not be zero. If this is not the case then formal mathematics are not unitary because this means that its rules are not the same when we tend to 0 and when we tend to infinity.

Undoubtedly the major factor in GSC is the fact that the same integer $n \geq 4$ gives two prime numbers in a symmetrical way: n - t and n + t with t < n. The prime number equation, if there is one, must take this fact into consideration and generates the two equidistant prime numbers in a reciprocal way like an equation that has two or more solutions. For instance, if we have all the prime numbers present in [0-n] then at least two of them noted p and p' such that p' > n/2 > p must be equidistant ($n/2 - p = p' - n/2$) so that the GSC be true. Therefore if one equation gives us these prime numbers of one integer n or $\pi(n)$ and if none of them are equidistant then the conjecture is false in the strict sense of mathematics (one exception causes rejection of the rule). Nevertheless, when we perform calculations with the rules described here in this paper, we always find those equidistant primes in a same way and showing a strong linear correlation.

How to set the equation of prime numbers? This article shows that we must start with an integer, any integer, and then extract all possible prime numbers of it. For example, we can define intervals whose largest is [0-2n]. This equation must give symmetric solutions and equidistant prime numbers, otherwise it is inconsistent or Goldbach's conjectures are false. Goldbach's conjecture will weigh heavily in this equation of prime numbers. This would probably be the true indisputable formal mathematical demonstration of the strong Goldbach conjecture that has been awaited for centuries. Without it, and whatever the size of the number and the limit which verifies this conjecture, a shadow of doubt will always hover, and this conjecture will remain mathematically unproven and can only be verified by applying rules of calculation such as those stated in this article.

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Supplementary data including Table S1 ; Table S2 and Table S3
(see page 25 of the article above)

Table S1 : Be an even E = p + p' such that p = E/2 – t and p' = E/2 + t. The values of t = p' – E/2 for E = 100000 and E/2 = 50000 (**non-3n**). The t values are all 3n (see the table).The graphic shows a high correlation coefficient of 1 of the t-values.

19941	33117	39501	45549
20079	33177	39513	45603
20121	33339	39567	45651
20163	33471	39657	45717
20181	33579	39669	45747
20241	33639	39753	45783
20571	33777	39819	45789
20589	33813	39849	45873
20667	33873	39867	45987
20769	33903	39897	46053
20793	33933	39909	46137
20853	33939	39939	46149
20877	34191	39963	46167
20937	34239	40059	46179
20979	34263	40071	46221
20991	34317	40149	46233
21039	34431	40179	46281
21129	34449	40281	46323
21249	34503	40371	46329
21339	34533	40527	46377
21429	34701	40533	46419
21453	34713	40659	46443
21483	34731	40677	46461
21537	34737	40863	46587
21597	34827	40971	46671
21693	34869	40989	46749
21711	34947	41139	46779
21789	35049	41151	46797
21837	35061	41163	46911
21849	35103	41193	46959
21999	35109	41253	46989
22047	35121	41331	47001
22053	35133	41373	47073
22173	35229	41457	47103
22221	35247	41463	47157
22227	35259	41499	47259
22251	35331	41571	47301
22353	35361	41577	47367
22383	35439	41631	47379
22461	35451	41703	47523
22551	35577	41757	47553
22671	35751	41781	47577
22701	35793	41961	47583
22719	35847	42051	47607
22893	35991	42177	47649
22923	36069	42243	47787
23013	36117	42297	47847
23019	36171	42357	47859
23079	36201	42459	47871
23121	36243	42567	47919
23277	36249	42669	47931
23331	36291	42693	47961
23583	36351	42717	47973
23607	36381	42753	48123
23613	36423	42789	48129
23643	36477	42849	48213
23679	36501	42921	48387
23751	36531	42957	48429
23823	36579	42987	48507
23847	36771	43053	48519
23859	36783	43083	48561
24159	36813	43089	48573
24201	36837	43131	48627
24297	36951	43239	48639
24357	36993	43263	48711
24411	37041	43281	48717
24531	37083	43419	48807
24561	37107	43479	48837
24609	37179	43683	48849
24699	37257	43701	48897
24747	37359	43827	48909
24771	37473	43887	48939
24831	37509	43911	48981
24873	37587	43971	49017
24903	37623	44121	49023
25011	37671	44151	49053
25029	37719	44307	49089
25083	37797	44331	49119
25149	37803	44343	49137
25323	37881	44349	49173
25329	37887	44427	49191
25377	37959	44449	49257
25389	38019	44529	49317
25407	38169	44559	49347
25527	38211	44583	49401
25557	38223	44613	49431
25629	38301	44727	49497
25641	38379	44811	49551
25683	38589	44847	49581
25797	38607	44949	49611
25821	38721	44961	49671
26091	38883	45027	49689
26253	38997	45063	49707
26259	39021	45111	49719
26367	39051	45213	49767
26463	39153	45267	49809
26541	39261	45279	49833
26631	39387	45327	49929
26667	39393	45483	49971
	39399	45507	49989

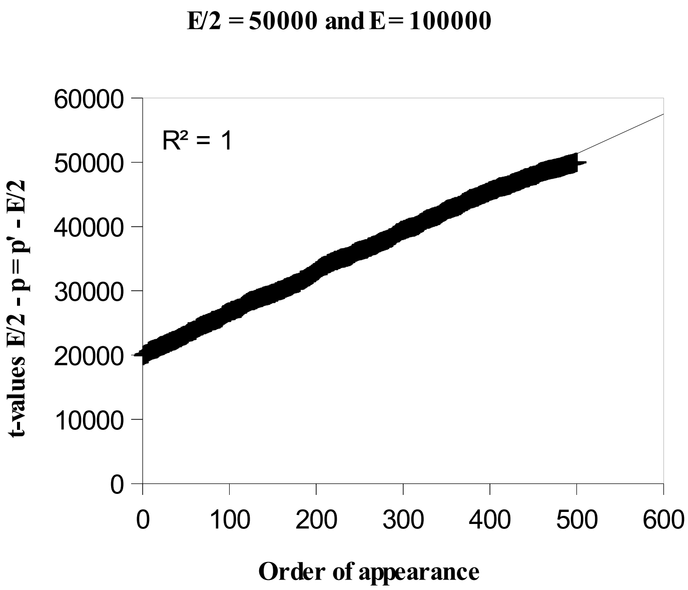

Table S2: Be an even E = p + p' such that p = E/2 – t and p' = E/2 + t. The values of t = p' – E/2 for E = 10000 and E/2 = 5000 (**non-3n**). The t values are all 3n (see the table).The graphic shows a high correlation coefficient of 1 of the t-values.

4941	2703
4929	2691
4887	2649
4851	2643
4833	2607
4803	2589
4767	2583
4749	2577
4743	2559
4719	2541
4689	2523
4551	2457
4539	2451
4533	2307
4521	2247
4497	2211
4491	2121
4479	2103
4437	2043
4431	2001
4413	1977
4341	1959
4323	1917
4281	1911
4257	1863
4239	1833
4227	1791
4203	1779
4173	1701
4161	1653
4137	1551
4059	1473
4029	1329
3969	1323
3951	1299
3849	1221
3837	1203
3819	1197
3807	1089
3783	1053
3741	1011
3699	987
3693	981
3681	927
3627	867
3573	861
3513	843
3501	783
3447	741
3429	717
3387	711
3363	651
3291	591
3123	519
3111	507
3093	483
3087	477
3069	417
2937	351
2919	309
2901	297
2793	279
2757	81
2727	

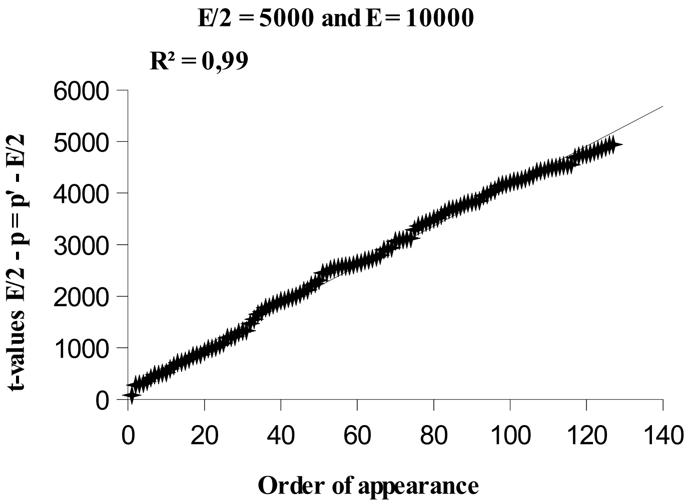

Table S3: Be an even E = p + p' such that p = E/2 – t and p' = E/2 + t. The values of t = p' – E/2 for E = 9000 and E/2 = 4500 (**3n**). The t values are either prime or composite but not 3n (see the table). The graphic shows a high correlation coefficient of 1 of the t-values..

163	1769	3203
173	1771	3217
203	1787	3223
229	1801	3241
259	1811	3329
283	1817	3377
289	1823	3383
299	1829	3409
361	1837	3437
371	1843	3449
389	1853	3451
409	1867	3509
443	1879	3517
451	1921	3553
473	1949	3559
487	1951	3581
493	1969	3589
499	2053	3593
511	2063	3617
577	2077	3623
581	2107	3647
619	2119	3661
647	2153	3671
653	2159	3679
667	2161	3691
679	2189	3731
697	2191	3743
731	2203	3773
733	2219	3791
761	2233	3817
773	2261	3853
781	2263	3869
803	2279	3887
809	2293	3923
823	2357	3929
887	2363	3931
893	2369	3943
907	2371	4001
917	2411	4013
919	2417	4021
941	2447	4037
943	2461	4039
971	2471	4043
983	2483	4081
1001	2497	4099
1031	2501	4127
1139	2513	4141
1149	2527	4147
1153	2569	4163
1157	2621	4169
1169	2627	4189
1193	2677	4193
1201	2679	4207
1241	2711	4219
1243	2713	4231
1249	2747	4237
1279	2753	4261
1283	2803	4303
1291	2831	4307
1313	2833	4319
1379	2893	4321
1381	2917	4337
1439	2933	4349
1481	2951	4361
1529	2957	4363
1537	2977	4387
1543	2989	4393
1547	3007	4429
1573	3017	4433
1591	3029	4441
	3041	4463
	3047	4469
		4471