Pseudo-trigonometric functions

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Abstract

This paper introduces a new concept concerning periodic functions. These functions entitled: pseudo- trigonometric functions, allow us to draw curves and periodic straight line segments.

I defined the functions : pseudo-sine denoted(spx) and pseudo-cosine denoted (cpx), as well as their reciprocal functions. I defined the hyperbolic pseudo-sine and hyperbolic pseudo-cosine functions.

this new mathematical tool allows me to calculate differential equations and integrals of a new kind.

keywords:

pseudo-sine functions, pseudo-cosine, hyperbolic pseudo-sine, hyperbolic pseudo-cosine. derivatives, Integrals , differential equations

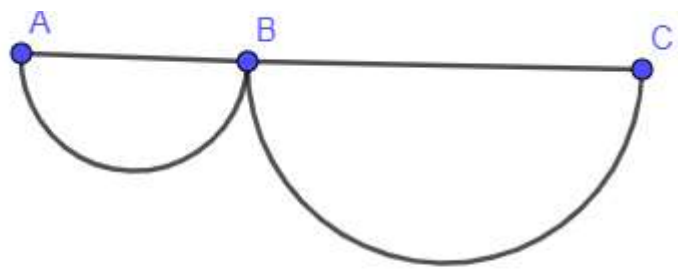
. pseudo-trigonometric functions

we consider the intervals : $[0;5]$, $[5;10]$, $[10;15]$....... etc

1.1 **Definition**

Any function defined and positive continuous by intervals is called a pseudo-trigonometric function, which verifies the following conditions.

let be the interval $[A;C[$ and the point B of abscissa x



1.2 The pseudo-sine function, denoted spx is equal to $|AB|$

1.3 The pseudo-cosine function, denoted cpx is equal to $|BC|$

* These functions are discontinuous at points of abcsissses 5k

Exemple1.4: sp0=0 cp0=5 sp5=0 cp5=5 sp-3 =2

Sp4=4, cp4 =1 sp12=2 sp9=4 sp15=0 cp-3=3

Sp6=1 ; cp6=4 cp12=3 cp9=1 cp15=5

Note1.5: in this example, the intervals of length 5 have been taken. we can choose any length for the intervals. $\forall a \in \mathbb{R}$, $spa + cpa = 5$

Note1.6: in the whole article the interval is equal to 5.

2.1 derived from pseudo-trigonometric functions

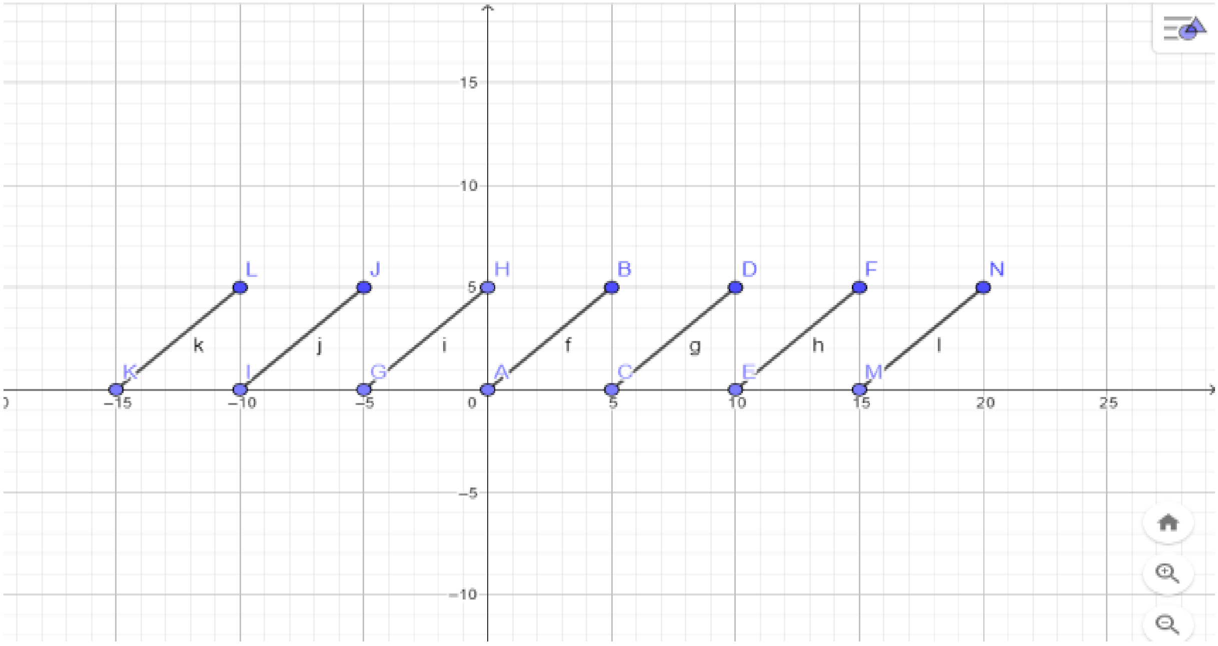
2.2 Derived from the pseudo- sine function: $(spx)'=1$

2.3 Derived from the pseudo-co sine function: $(cpx)'=-1$

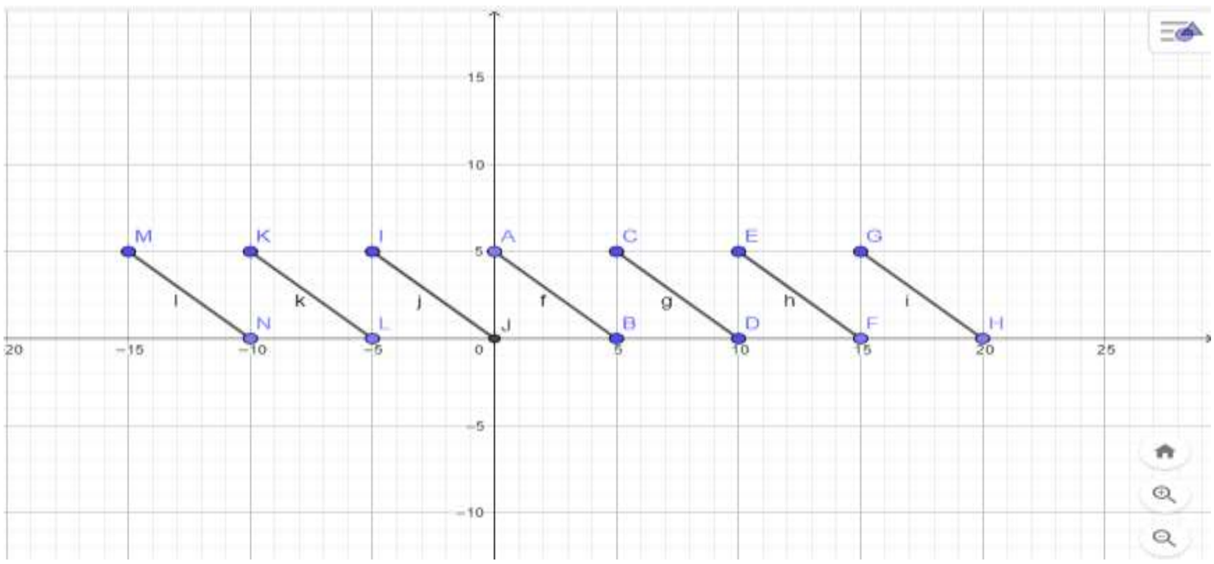
Note 2.4: all formulas on derivatives are valid for pseudo-trigonometric functions.

The plane is provided with a direct orthonormal reference $(O;\vec{OI};\vec{OJ})$

3.1 curve of the function $x \to spx$

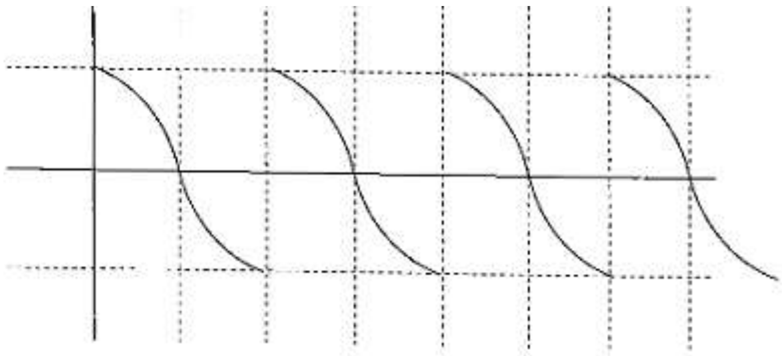


3.2 curve of the function $x \to cpx$

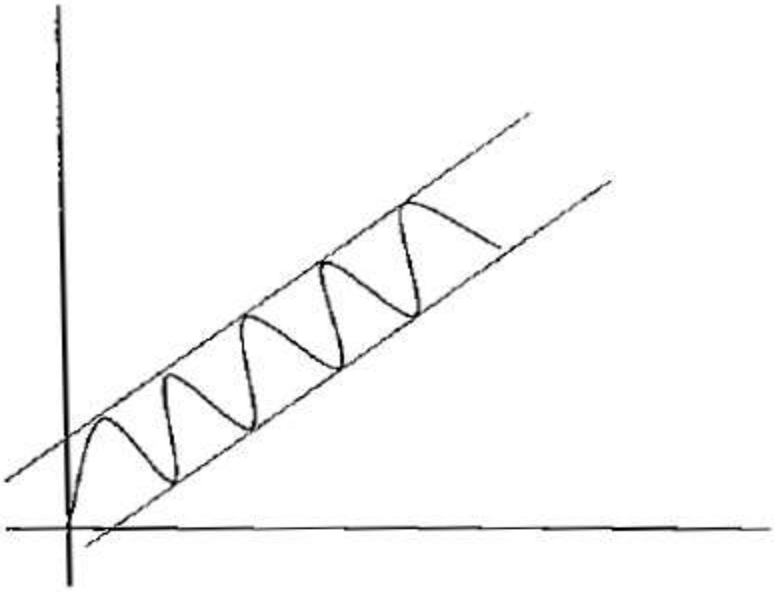


3.3 curves of some pseudo-trigonometric functions

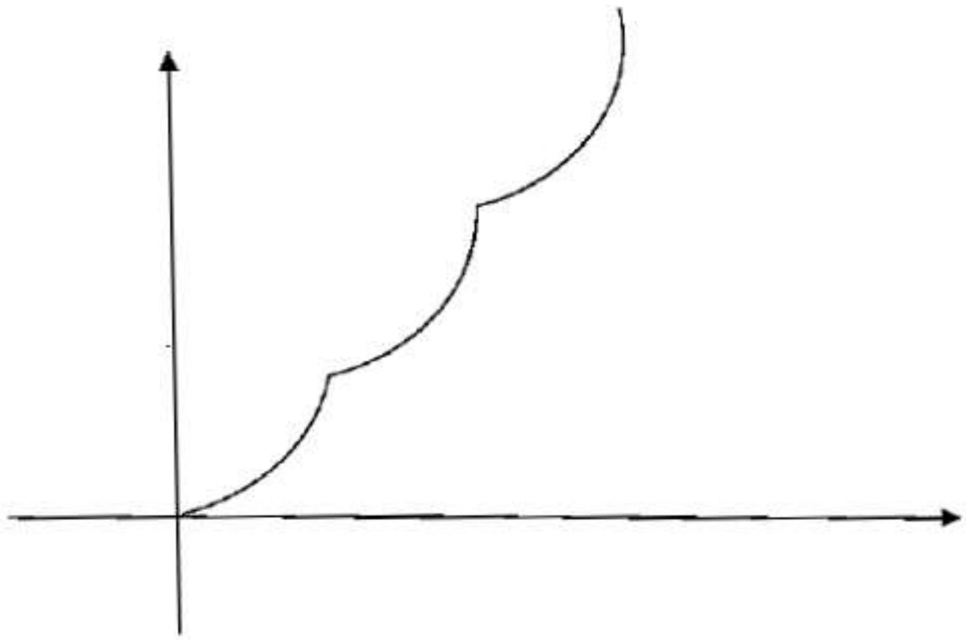
$$x \to \cos\frac{\pi}{6}cpx$$



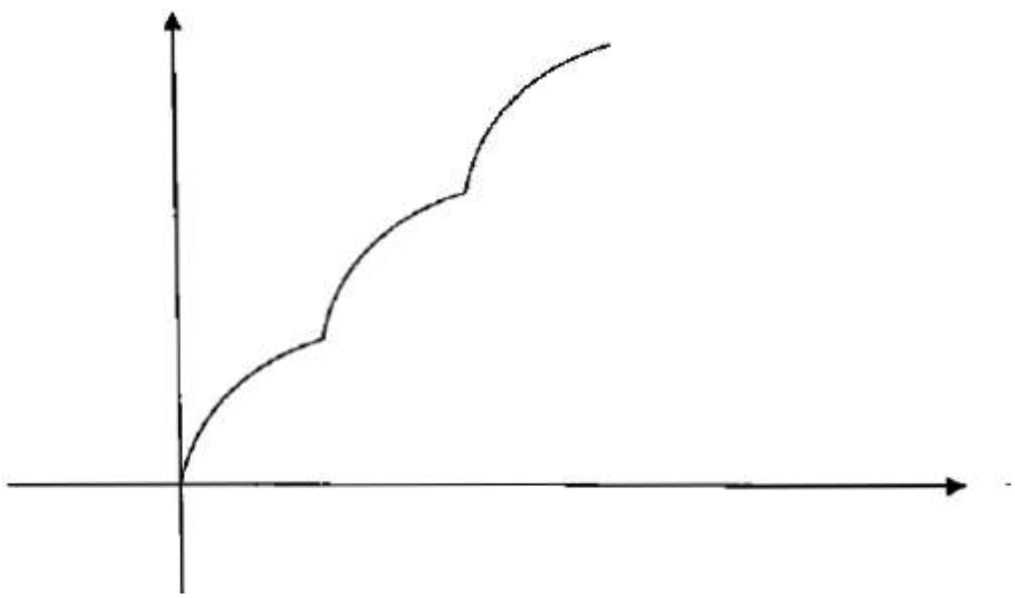
$$x \to 5(x - \frac{1}{5}cpx + 1)(\sin \pi x + cpx)$$



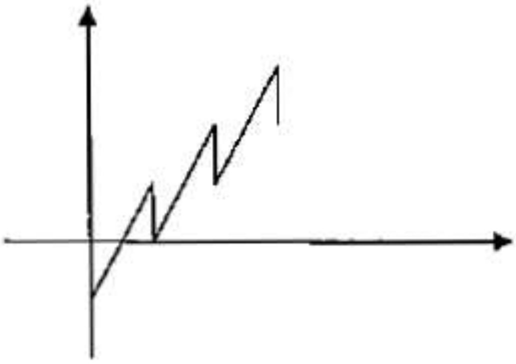
$x \to 3x - spxcpx$



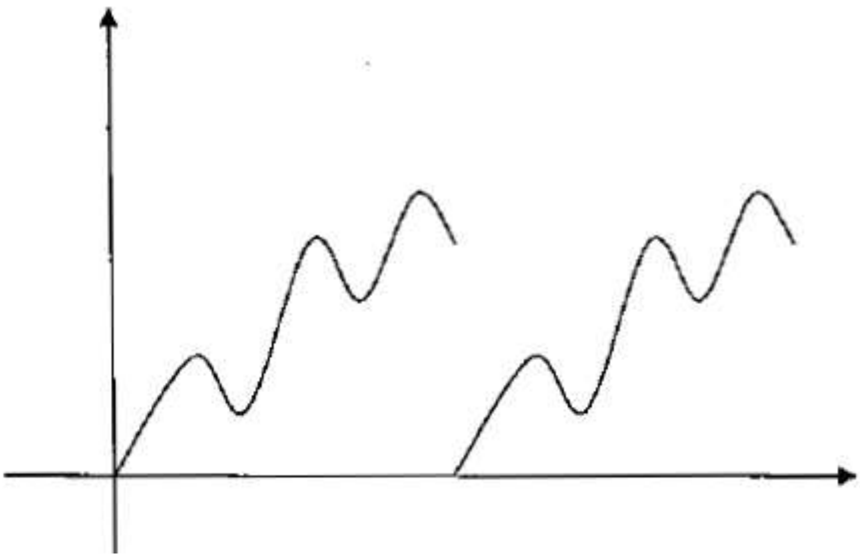
$x \to 3x + spxcpx$



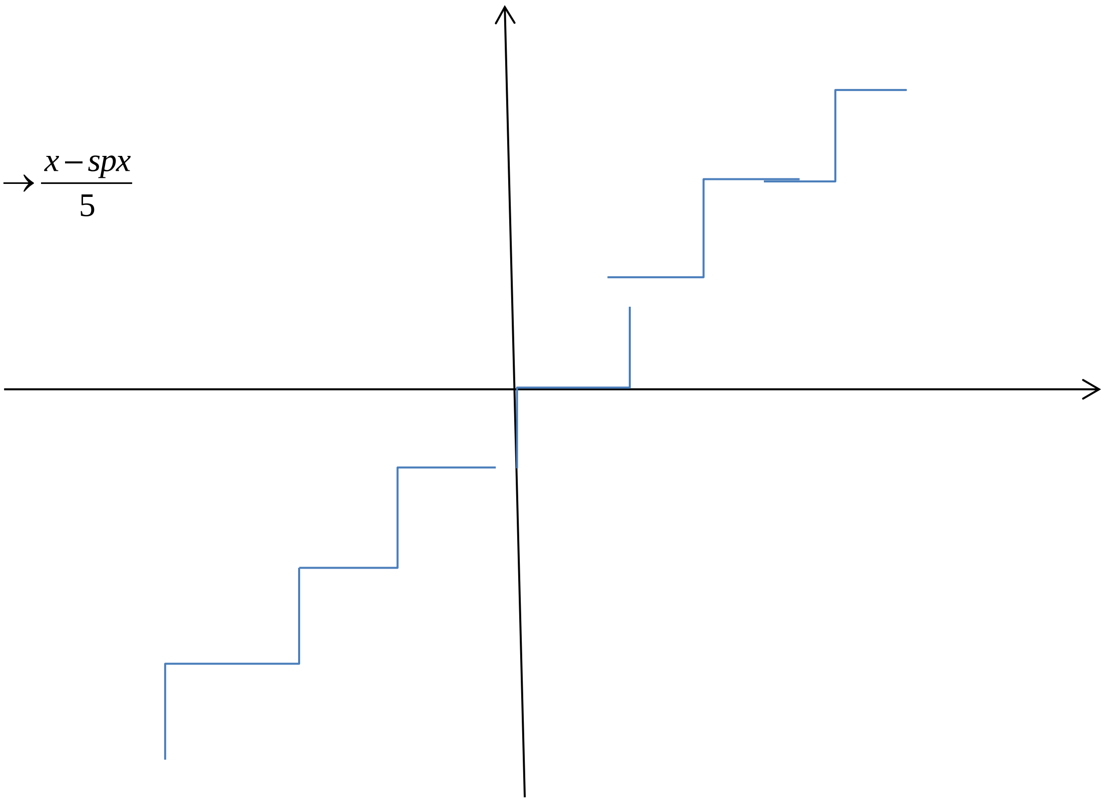
$x \to x - spx$



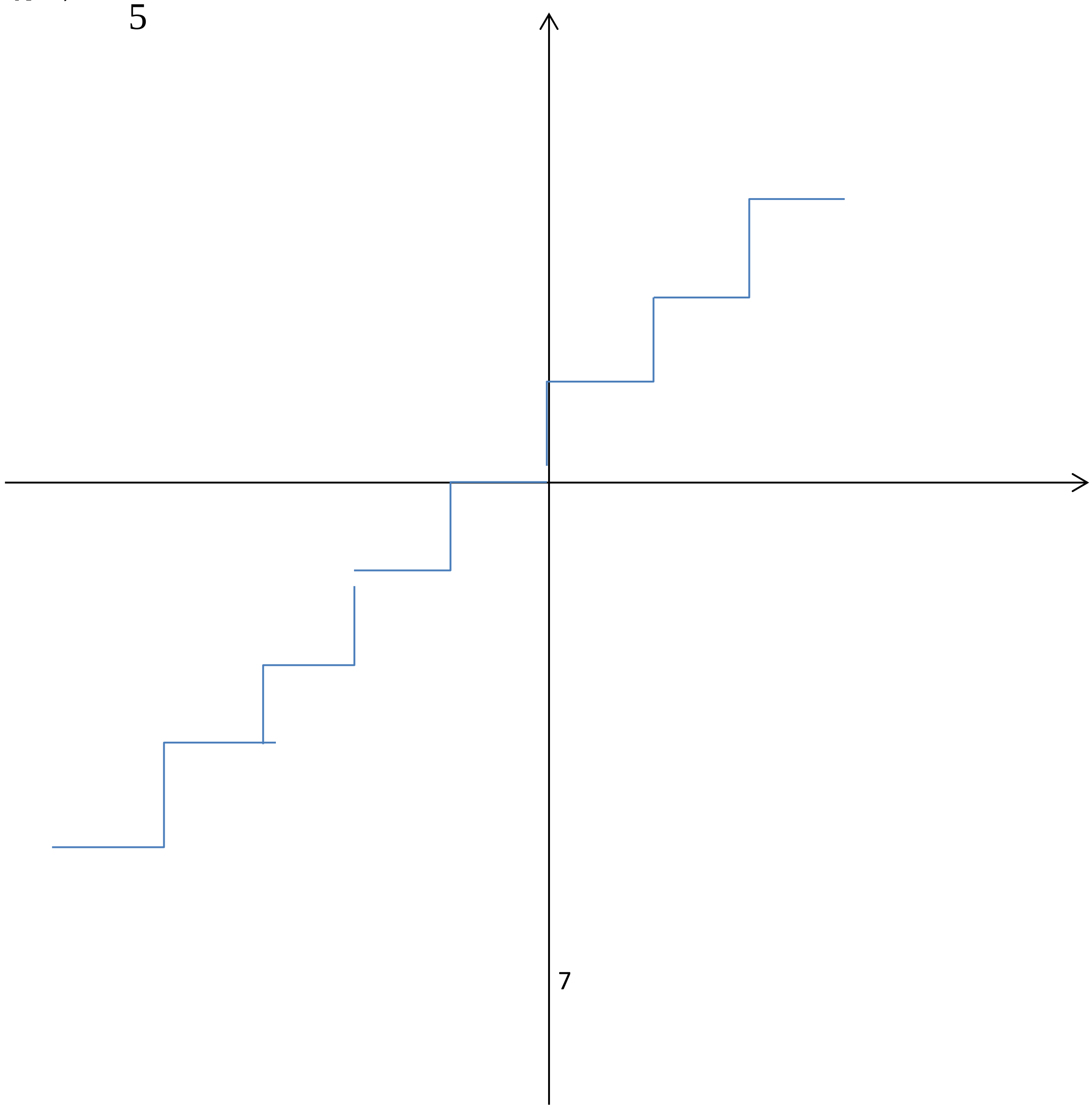
$x \to \sin \pi x + spx$



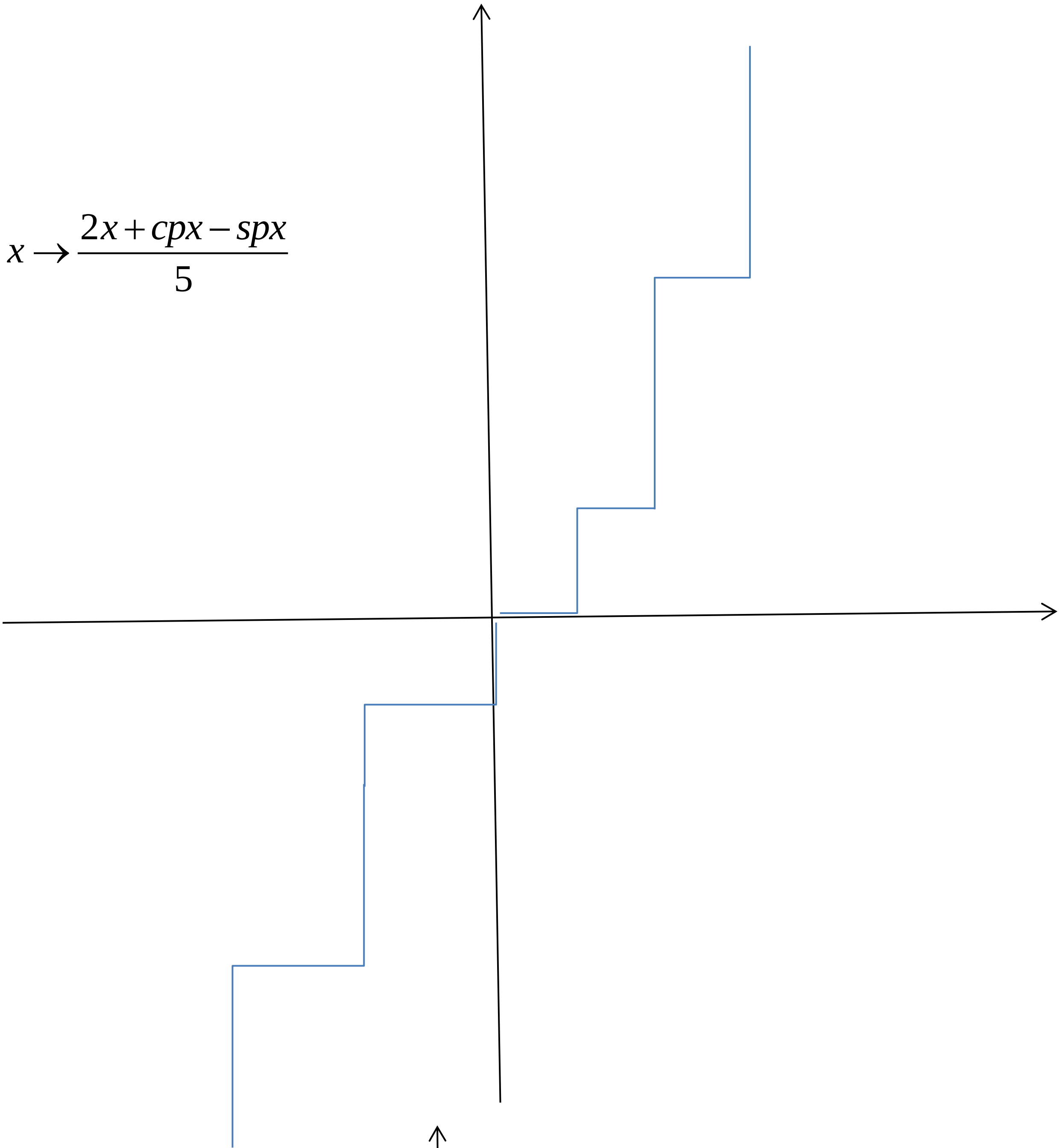
$$x \to \frac{x - spx}{5}$$



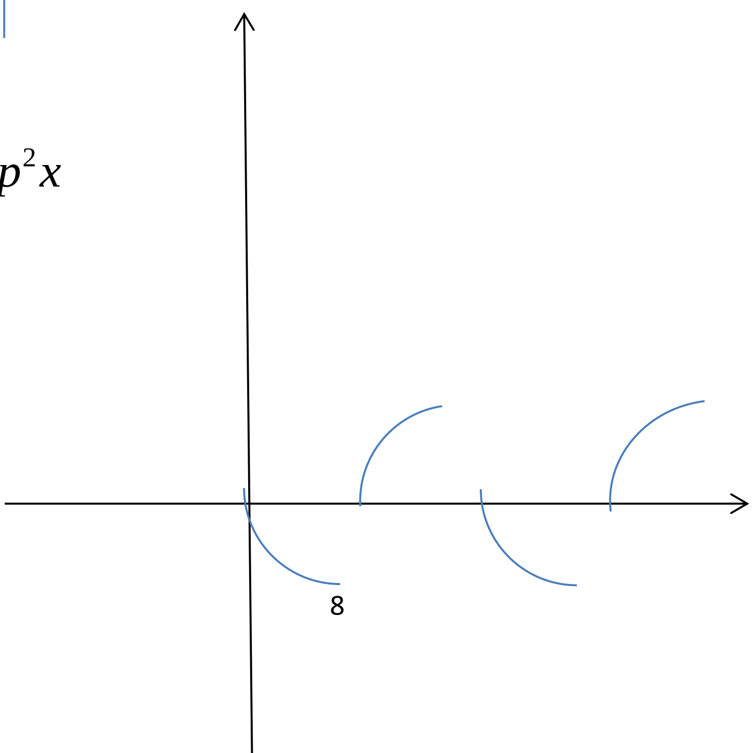
$$x \to \frac{x + cpx}{5}$$



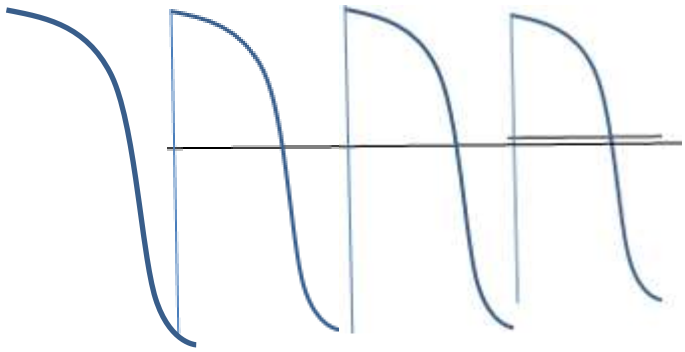
$$x \rightarrow \frac{2x + cpx - spx}{5}$$



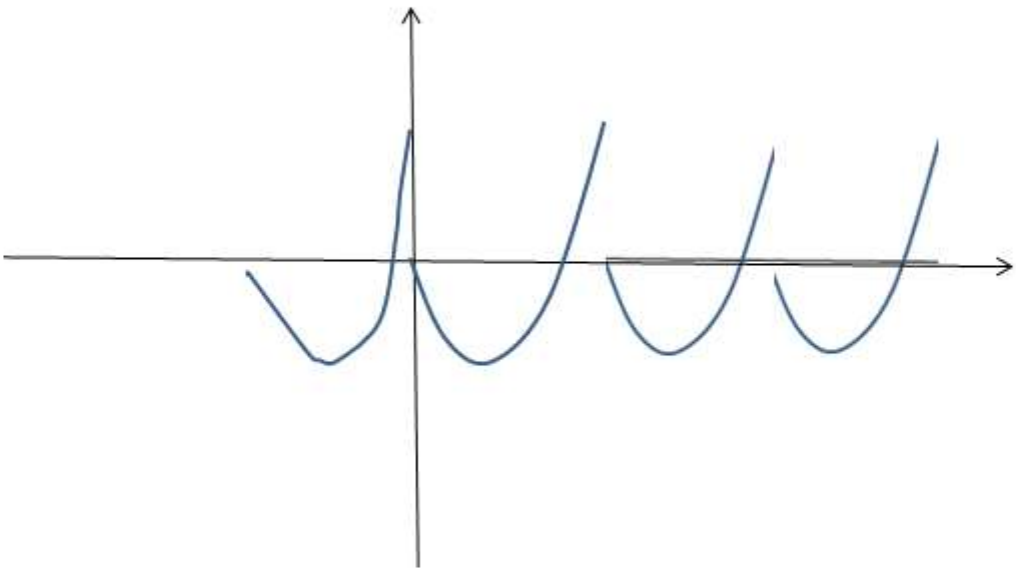
$$x \rightarrow (-1)^{\frac{x+cpx}{5}} sp^2 x$$



$x \to -\cos x cox(x + cpx)$

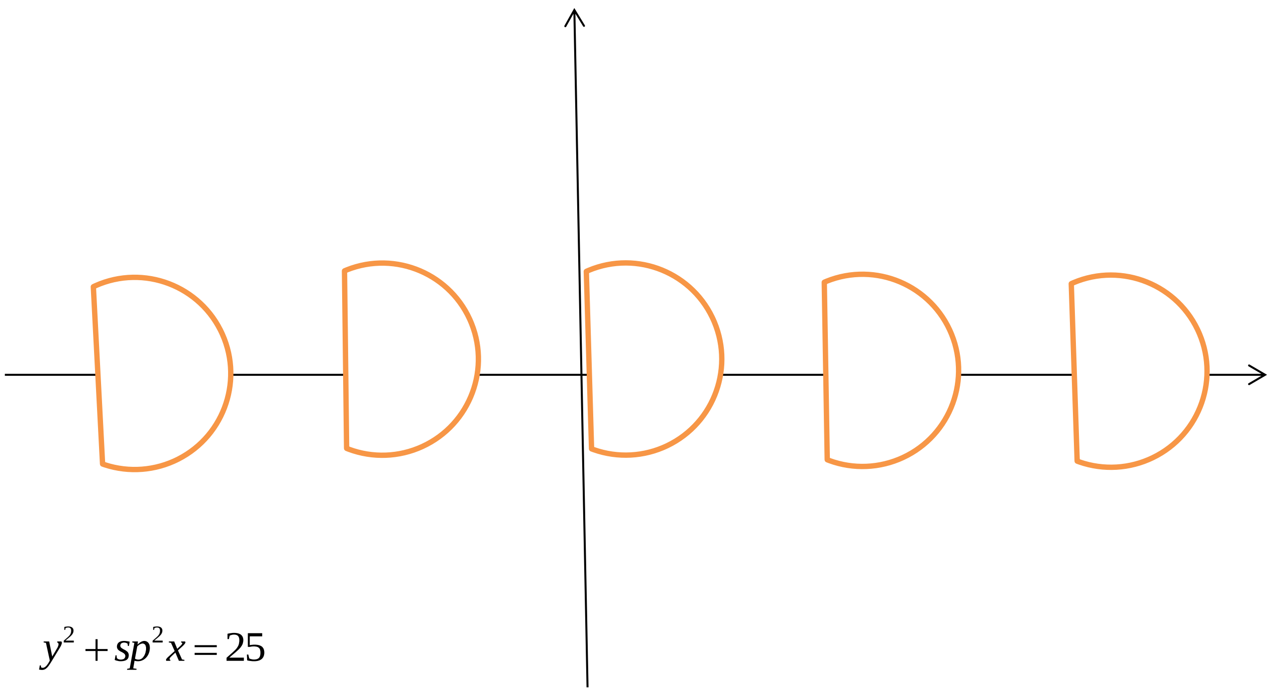


$x \to spx(cpx - 3)$

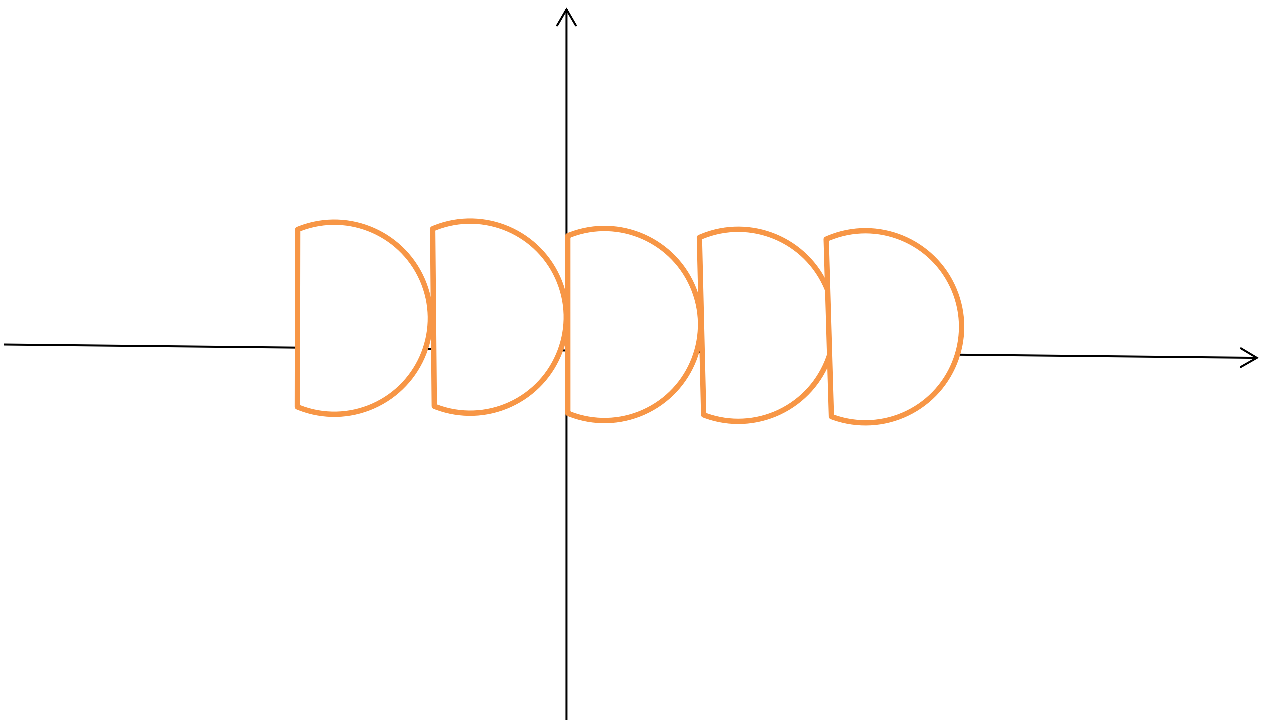


4.1 algebraic curves

$$y^2 + sp^2 x = 1$$



$$y^2 + sp^2 x = 25$$



5.1 properties of spx and cpx functions

for all values of a and b **Integers**

$$sp(a+b)=sp(spa+spb) \qquad spa*b=sp(spa*sp$$

$$cp(a+b)=sp(cpa+cpb) \qquad cpa*b=cp(cpa*cpb)$$

6.1 hyperbolic pseudo-sine and hyperbolic pseudo-cosine functions

$$pshx=\frac{e^{spx}+e^{cpx}}{2e^{\frac{5}{2}}}$$ Hyperbolic pseudo- sine

$$pchx=\frac{e^{spx}-e^{cpx}}{2e^{\frac{5}{2}}}$$ Hyperbolic pseudo- cosine

6.2 $psh^2x-pch^2x=1$

6.3 derived from pseudo-trigonometric functions

the pseudo trigonometric functions are derivable in their set of definition.

6.4 $psh'x=pchx$; $pch'x=pshx$

6.5 Hyperbolic pseudo-tangent function : $pthx=\frac{pshx}{pchx}$

$$(pthx)'=\frac{1}{pch^2x}=1-pth^2x$$

7.1 primitive functions :

$$7.2:\int spxdx=\frac{sp^2x}{2}+c$$

$$7.3:\int xspxdx=\frac{x^2}{2}spx-\frac{x^3}{3}+c$$

$$7.4:\int sp^2xdx=xsp^2x-x^2spx+\frac{x^3}{3}+c$$

$$7.5:\int cpxdx=-\frac{cp^2x}{2}+c$$

$$7.6:\int e^xcpxdx=e^xcpx+e^x+c$$

$$7.7:\int \sin(x)cpxdx=-\cos(x)cpx-\sin x+c$$

$$7.8:\int spxcpxdx=\frac{cpxsp^2x}{2}+\frac{xsp^2x}{2}-\frac{x^2spx}{2}+\frac{x^3}{6}+c$$

$$7.9:\int\frac{spx}{cpx}dx=\int\frac{5-cpx}{cpx}dx=-5\int\frac{-1}{cpx}-\int dx=-5\ln cp(x)-x+c$$

8.1 differential equations

8.2 $y'=y^2\Rightarrow y=\frac{1}{cpx}$ **; 8.3** $y''=2y^3\Rightarrow y=\frac{1}{cpx}$ **; 8.4** $y'''=3y^4\Rightarrow y=\frac{1}{cpx}$ **….etc**

8.5 $y'=-y^2\Rightarrow y=\frac{1}{spx}$ **8.6** $y''=2y^3\Rightarrow y=\frac{1}{spx}$ **8.7** $y'''=-3y^4\Rightarrow y=\frac{1}{spx}$ **…etc**

8.8 $y'-ay=e^{ax}\Rightarrow y=e^{ax}spx$ **8.9** $y''-a^2y=2ae^{ax}\Rightarrow y=e^{ax}spx$

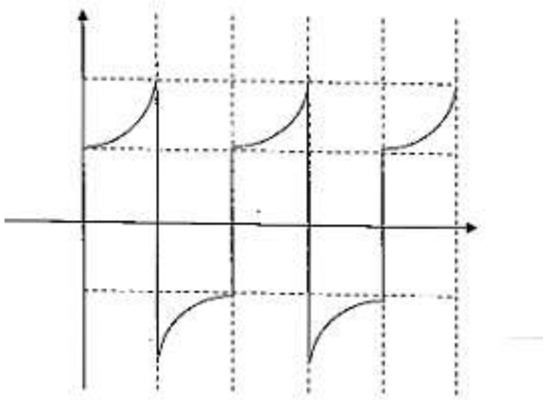
8.10 $y'''-a^3y=3a^2e^{ax}spx\Rightarrow y=e^{ax}spx$

8.11 $y'+ay=e^{axcpx}\Rightarrow y=e^{axcpx}spx$ **a reel**

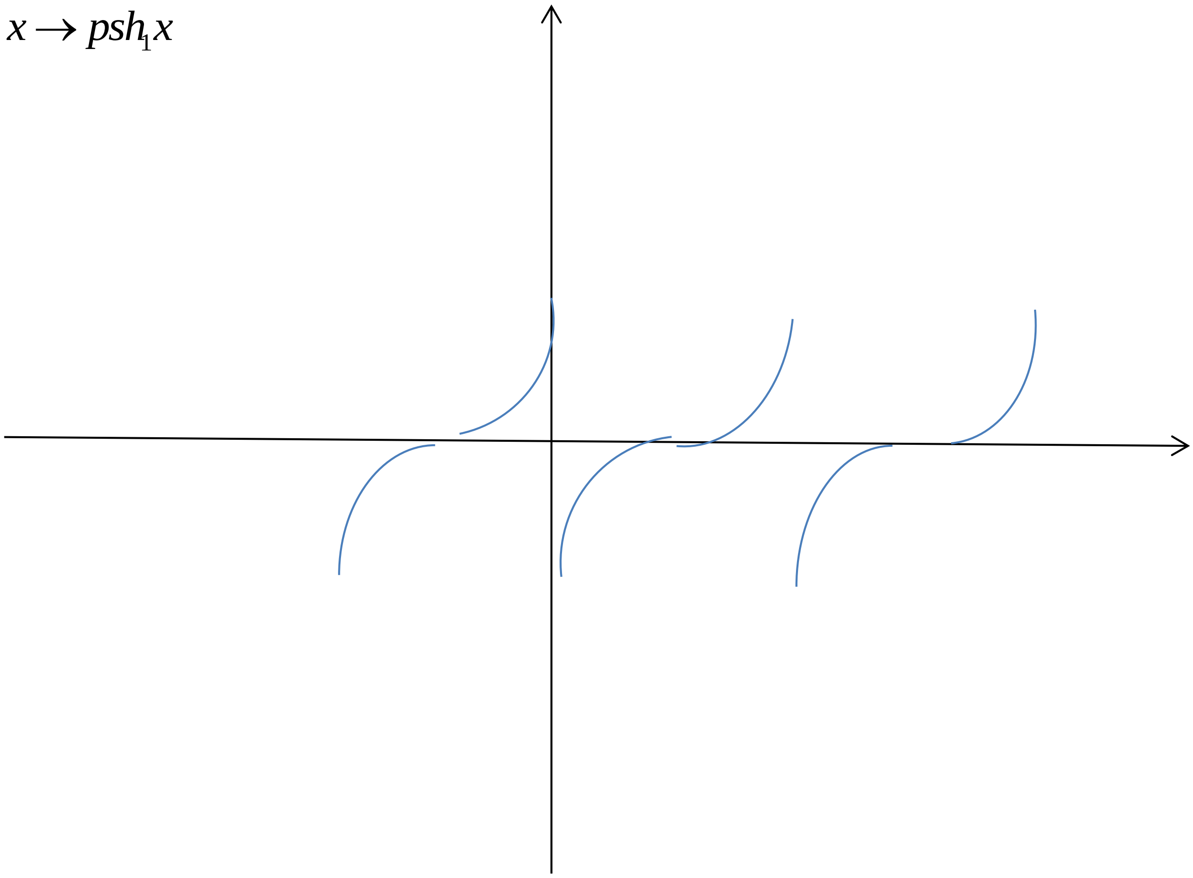
$$y''-a^2y=-2ae^{axcpx}\Rightarrow y=e^{axcpx}spx$$

9.1 pshx and pchx function curves

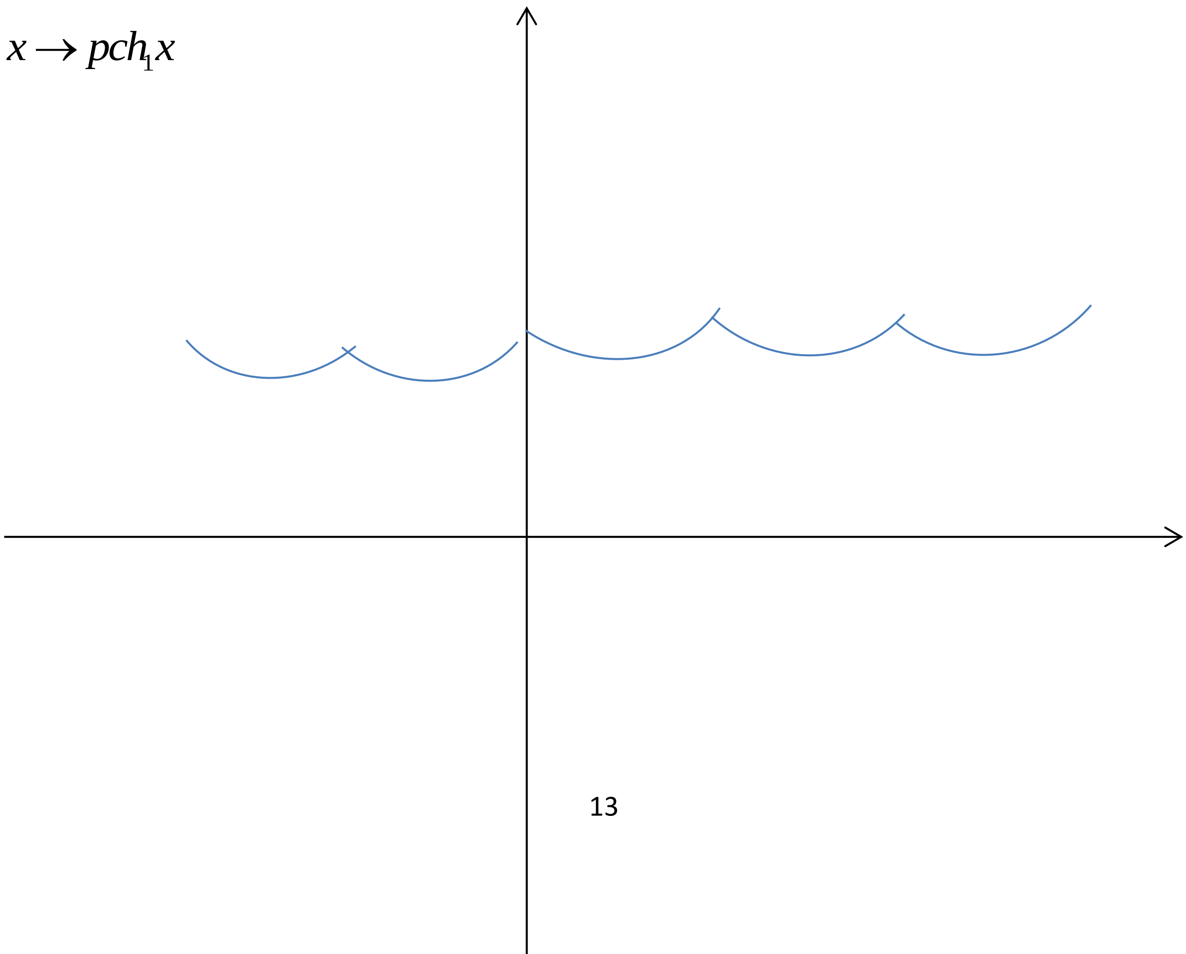
$x\to pchx$



$x \to psh_1 x$



$x \to pch_1 x$



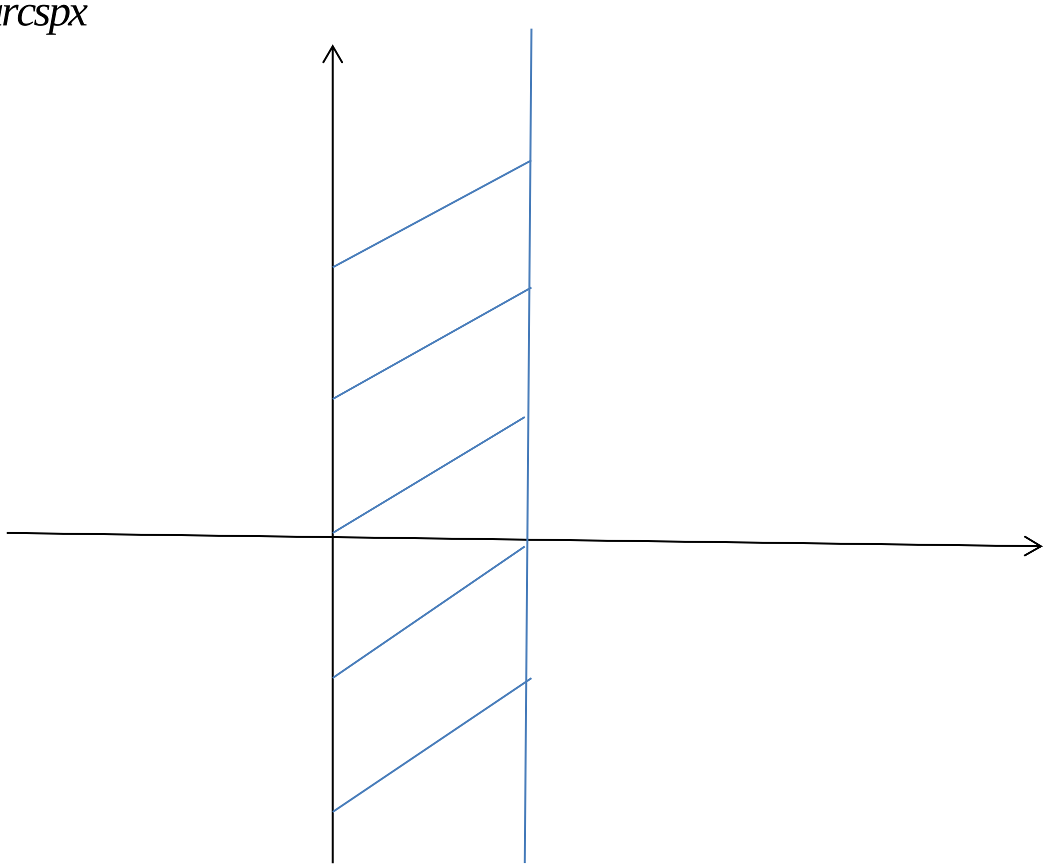
9.2 note: There are other pseudo-hyperbolic functions.

$$psh_1 x = \frac{e^{sp2x-spx} - e^{spx-sp2x}}{2}$$

$$pch_1 x = \frac{e^{sp2x-spx} + e^{spx-sp2x}}{2} \; ; \quad \Rightarrow \quad psh_1^2 x - pch_1^2 x = 1$$

9.3 the reciprocal function of spx is : arcspx

$x \rightarrow arcspx$



9.4 the reciprocal function of spx is : arccpx

$x \rightarrow arccpx$