HOOKE-KEPLER LAW OF GRAVITATION: the rejected law of gravitation, which turned out to be more accurate and perfect than Newton's law.

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Abstract. *The role of Robert Hooke in the discovery of the law of universal gravitation is shown in a new light. New circumstances are revealed that relate to the priority dispute between Hooke and Newton. It is shown that Newton's law of gravitation is not the only law of gravitational interaction. There is another law of gravitation that was outside Newton's field of vision. The existence of this law was indicated by Robert Hooke in his correspondence with Newton. Robert Hooke pointed out that the law of gravitation should take into account the elliptical orbits of the planets and the inverse square law. In 1687, Newton presented the law of gravitation, which includes the inverse square law. But the parameters of the elliptical orbit were not included in Newton's law. Instead of the orbital parameters, Newton introduced mass into his law. As a result, a more perfect law of gravitation than Newton's law was not discovered. Here we present this law of gravitation. It has a beautiful and mathematically perfect form:* $F_{H-K} = mR^3/(T*r)^2$*. I call this physical law the Hooke-Kepler law of gravitation. It includes the Kepler constant. It includes the inverse square law. It does not include the central mass. This is a more accurate and perfect law of gravitation than Newton's law, since distances and periods are known from observations with greater accuracy than mass. Thus, Hooke's path to the law of gravitation was more promising than Newton's. Robert Hooke was very close to discovering an alternative law of gravitation. Obviously, Hooke knew something about gravitation that Newton did not.*

Keywords: *Robert Hooke; Isaac Newton; astronomy; Newton's law of gravitation; Kepler's 3rd law.*

1. Introduction

It is believed that with the advent of Newton's law of gravity, the desired law of gravitational interaction, to which Hooke and Newton were moving, was discovered. At the same time, it is believed that the law discovered by Newton is the only possible law of gravitational interaction. The possibility of the existence of an alternative law of gravity is not considered.

There is currently a major re-evaluation of Hooke's contribution to mechanics. Researchers of the work of Hooke and Newton did not pay attention to the fact that Hooke and Newton understood the future law of gravity differently. Researchers passed by the fact that Hooke and Newton were moving towards the future law of gravity in different ways. These different approaches and different paths must have led them to two different laws of gravity. Newton's approach ended with the discovery of only one of the two laws of gravity existing in Nature. The second law of gravity, to which Hooke's path objectively led, remained undiscovered.

Newton presented the law of universal gravitation in a verbal formulation in 1687. At the same time, as of 1687, there were not one, but two verbal formulas for the future law of gravitation. They differed significantly from each other. One verbal formulation was given by Newton. The second verbal formulation of the law of gravitation was given by Hooke. In essence, these were verbal formulas for two different laws of gravitation. Newton's verbal formulation became the law of universal gravitation. Hooke failed to transform his knowledge into a law of gravitation. His verbal formulation was unreasonably forgotten after the appearance of Newton's law of universal gravitation. Newton and Hooke, without suspecting it, spoke of two different laws of gravitation.

Hooke and Newton took different paths to the law of gravitation. These different paths led them to different laws of gravitation. Newton discovered only one of the two laws of gravitation. Newton used masses as parameters in the law of gravitation. Hooke was very close to discovering the second alternative law of gravitation. Hooke did not rely on masses, but persistently investigated elliptical trajectories. This can be seen both in his works and in his experiments with a pendulum [1, 2].

Hooke hoped that Newton would help him explain the law of gravity. Robert Hooke, in his correspondence with Newton, indicated that the law of gravity should take into account the elliptical orbits of the planets and the inverse square law [1]. Newton was more interested in masses as parameters of the law of gravity and the relationship of elliptical orbits with the inverse square law. Newton did not see the advantage of using the parameters of an elliptical orbit in the law of gravity and overestimated the role of mass as a parameter in the law of gravity. Newton, in his letter to Hooke dated December 13, 1679, stated regarding elliptical orbits: "...it being of no great moment..." [1].

As a result, the law of gravity that Hooke was working towards was not discovered. It was a completely different law, not similar to Newton's law. We show that the law of gravitation that Hooke was working towards is an exact and mathematically perfect law of gravitation, more perfect than Newton's law. We present this law of gravitation and give it a complete explanation. We show that Hooke's path to the law of gravitation was more promising than Newton's. Hooke's path led him to yet another law of gravitation, more perfect than Newton's law. Obviously, Hooke knew something about gravitation that Newton did not.

2. About Newton's original law of gravity $F_N \propto mM/r^2$.

Newton formulated his law of gravity in verbal form. In his formulation, the gravitational force is proportional to the masses and inversely proportional to the square of the distance. When trying to present Newton's law of gravity in symbolic form, a mistake is often made. It is mistakenly presented as: $\mathbf{F_N = mM/r^2}$. The formula for force uses the equal sign. This symbolic representation of Newton's law in the form of equality is a fairly common mistake. Newton actually said that the force of attraction is proportional to the masses. Therefore, the equal sign in the symbolic representation of the law proposed by Newton is unacceptable. The correct form of presentation of the original law proposed by Newton is as follows:

$$F_N \propto \frac{mM}{r^2} \qquad (1)$$

Where: F_N is the force, m, M are the masses of bodies, r is the distance, ∝ is the proportionality sign.

As we can see, Newton presented the world with an unfinished law of gravity. Formula (1) is not an exact formula for the law of gravitation. Using masses as parameters did not allow Newton to obtain an exact formula for the law of gravitation. In this unfinished form, Newton's law existed for almost 200 years. It turned into the exact equality $\mathbf{F_N = GmM/r^2}$ only after the gravitational constant G was introduced into it [3 - 6]. It took almost 200 years before the proportionality law $\mathbf{F_N \propto mM/r^2}$ proposed by Newton turned into the exact law of gravitation $\mathbf{F_N = GmM/r^2}$. In fact, Newton's proportionality law $\mathbf{F_N \propto mM/r^2}$ with the appearance of the constant G was "refined" to the exact law of gravitation $\mathbf{F_N = GmM/r^2}$. The appearance of the constant G in Newton's law of gravitation was a revolutionary step. The constant G saved Newton's unfinished law and turned it into an exact law of gravitational interaction of two bodies.

But even in its unfinished form, Newton's law became a real breakthrough in science. Gravitational interaction became one of the 4 fundamental interactions in Nature. At the same time, the illusion appeared and took root that Newton's law gives a complete description of gravitational interaction. It is still believed that this is the only formula for the force of gravitational interaction. Below we will show that there is another law of gravity that was outside Newton's field of vision and we will give its formula. The existence of this law of gravity was indicated by Robert Hooke in his correspondence with Newton long before the discovery of Newton's law. Robert Hooke pointed out that the law of gravity should take into account the elliptical orbits of the planets and the inverse square law. Hooke did not mention mass as a parameter in the law of gravity.

3. Hooke's hint that Newton did not use

It is known that in 1679-1680 Robert Hooke entered into correspondence with Newton. Hooke in his correspondence with Newton gave him two hints for the future law of gravity. The first hint: the law of gravity should include the inverse square law. The second hint: the law of gravity should take into account the elliptical orbits of the planets, as in Kepler's law. In practice, Hooke gave an approximate verbal description of the future law of gravity. This verbal description differs from the verbal description of the law of gravity (1) given by Newton. Elliptical orbits were known at that time from Kepler's laws. The inverse square law was proposed by Bullialdus [3]. [7]. But the possibility of their application in the future law of gravity was not obvious. Hooke was the first to guess that the law of gravity could be derived from Kepler's empirical laws. He persistently studied motion along elliptical trajectories and studied the inverse square law. To do this, he conducted experiments with a pendulum and used a graphical method to describe an elliptical orbit [1]. In his letters to Newton, he emphasized elliptical orbits. Newton confirmed that he had received Hooke's suggestion that motion along an ellipse should be taken into account: "...*as if he had found the motion in the Ellipsis, which inclined me to try it* ..." [8].

The law of gravitation, published in 1687, shows that Newton rejected one of Hooke's hints. Of Hooke's two hints, only the inverse square law is included in Newton's law of gravitation. As a result, the law of gravitation that included the parameters of the elliptical orbit hinted at by Hooke remained undiscovered. Perhaps the situation can be clarified by Newton's letter to Hooke dated December 13, 1679. In it, Newton writes [1]: *«Your acute Letter having put me upon considering thus far ye species of this curve, I might add something about its description by points quam proximè. But the thing being of no great moment I rather beg your pardon for having troubled you thus far with this second scribble….».*

What was of great importance to Hooke meaning, for Newton "... *being of no great moment* ...". Newton did not see the advantage of using the parameters of an elliptical orbit and overestimated the role of mass as a parameter in the law of gravity. Newton used only the first hint - the inverse square law. Obviously, Hooke's first hint confirmed his own opinion. Newton's manuscripts show that Newton came to the understanding of the need to use the inverse square law in the law of gravity before 1669 [9, 10]. It was this hint from Hooke that caused the dispute about priority. Newton rejected the second hint. He did not use Hooke's second hint about elliptical orbits, like Kepler's. Instead of using the parameters of an elliptical orbit, he introduced masses into the law of gravity. It is known that Kepler's laws do not include masses. The use of masses in the law of gravity led to the fact that instead of an exact law of gravity, Newton proposed only a proportional dependence of force on masses.

There is a known entry in his notebook from 15 years before his correspondence with Hooke in 1679: "*if a body moves in an ellipse, then the force at each point... can be found*" [11]. Newton found a formula for the gravitational force using mass, but did not find a more perfect formula for the same force using the parameters of an ellipse. And this is against the background of the fact that Kepler's laws had been known since 1619. And this is against the background of the fact that Hooke persistently pointed out the need to take into account elliptical motion. Why was Newton satisfied with an inaccurate law of gravity $\mathbf{F_N \propto mM/r^2}$, using mass as parameters? Why did he reject Hooke's hint and not use the parameters of an ellipse? This remains a mystery.

4. Two different approaches should have led to two different laws of gravity.

Newton introduced masses as parameters in his law of gravity. Hooke did not talk about masses in the context of the law of gravity. Hooke believed that the law of gravity should be represented by the inverse square law and elliptical orbits, as in Kepler. Kepler's laws do not include masses. Hooke and Newton had different ideas about the future law of gravity.

Robert Hooke's 1685 manuscript shows that he was working hard to create a quantitative theory of motion using the geometric method of constructing an ellipse [12]. He studied elliptical orbits to determine the gravitational force. Could this have led Hooke to discover a law of gravity different from Newton's? We show that Keplerian dynamics could indeed have led Hooke to discover a law of gravity better than Newton's.

5. Hooke-Kepler law of gravitation.

Let us consider in more detail the second hint that Hooke so persistently suggested to be taken into account in the law of gravitation. Maybe Hooke was wrong? After all, Newton rejected his hint and used masses instead of elliptical orbit parameters? Is it possible to propose a formula for the law of gravitation in which the parameters of the orbit are used instead of mass? To answer these questions, let us consider what the law of gravitation would look like in symbolic form, adequate to Hooke's verbal formulations.

According to Hooke's first hint, the law of gravitation should include the inverse square law. According to Hooke's second hint, the law of gravitation should include the parameters of an elliptical orbit. The central mass should not be included in the law of gravitation, since masses are not included in Kepler's laws. Direct use of Kepler's laws does not lead to the formula for gravitational force. The problem is solved by Kepler's constant [13], which directly follows from Kepler's 3rd law.

Using Hooke's verbal description of the future law of gravity, it is not difficult to obtain a symbolic formula for the alternative law of gravity. Based on Hooke's verbal formulations, the alternative law of gravity can be represented by the following symbolic equivalent:

$$F_{H-K} = \frac{mR^3}{T^2 r^2}$$

Fig. 1. Hooke-Kepler law of gravity. Where: m is the mass of the body, R and T are orbit parameters, r is the distance.

As we can see, Hooke was not mistaken with the second hint. The fundamental law of gravity that Hooke hinted at does exist. The parameters of the elliptical orbit together with the inverse square law do lead to a new law of gravity. Robert Hooke was very close to discovering an alternative law of gravity.

The law of gravity (Fig. 1) includes Kepler's constant. It includes the inverse square law. It does not include the central mass. This precise and beautiful law of gravity was not discovered by Newton, despite Hooke's hint. The reason is that Newton used masses as parameters and did not use Hooke's second hint about elliptical orbits.

Paying tribute to the genius of Robert Hooke, I call this mathematically perfect law of gravity the *Hooke-Kepler law*. This is a more precise and perfect law of gravity than Newton's law, since distances and periods are known from observations with greater accuracy than mass. If Newton had heeded both of Hooke's hints, he could have offered the world two laws of gravity. One approximate law of gravity $\mathbf{F_N \propto mM/r^2}$, and the second exact law of gravity $\mathbf{F_{H\text{-}K} = mR^3/T^2r^2}$.

As we can see, Newton had the opportunity to give the world an exact law of gravity back in 1687. To do this, it was enough to use Kepler's law. Robert Hooke clearly indicated this in his letter. Historians have paid more attention to the first hint: the inverse square law. Newton took this hint from Hooke into account. The inverse square law is equally compatible with both masses and orbital parameters. It was because of this hint that a dispute about priority arose between Newton and Hooke. The second hint about the need to take into account motion along an elliptical orbit in the law of gravity was ignored. Newton rejected this hint. And in vain! As a result of Newton rejecting Hooke's second hint, the more perfect law of gravity was not discovered. Hooke knew something more profound about gravity, but he failed to transform his knowledge into the law of gravity. One can only guess what a great impetus to the development of science would have been received in the second half of the 17th century if Newton had accepted both of Hooke's hints.

6. Comparison of two laws of gravity.

Hooke and Newton had different approaches to the law of gravity. Newton and Hooke, without even realizing it, were talking about two different laws of gravity. Hooke relied on Kepler's laws. Newton used masses as parameters. Newton managed to discover the law of gravity. But it was not the exact law of gravity. Hooke was very close to discovering the exact law of gravity. But he failed to do so. Hooke turned to Newton for help. But Newton also failed to discover the exact law of gravity, the signs of which Hooke suggested to him.

From a comparison of the two laws it follows that the law of gravity $\mathbf{F_N \propto mM/r^2}$ proposed by Newton is clearly inferior to the Hooke-Kepler law of gravity $\mathbf{F_{H\text{-}K} = mR^3/T^2r^2}$ (Fig. 2).

$$F_{H-K} = \frac{mR^3}{T^2 r^2} \qquad F_N \propto \frac{mM}{r^2}$$

Fig. 2. Hooke-Kepler law of gravitation and Newton's law of gravitation.

The law of gravitation (1), proposed by Newton, was not a quantitative physical law. It was presented as an approximate verbal formula. Law (1), proposed by Newton, is an incomplete law of gravitational interaction of two bodies. Hooke-Kepler law of gravitation (Fig. 1) is a quantitative physical law. Hooke-Kepler law is an exact and complete law of gravitational interaction of two bodies. Newton's law includes masses as parameters. Hooke-Kepler law of gravitation does not include central mass. Hooke-Kepler law includes Kepler constant as a parameter [13].

Hooke-Kepler law of gravitation (Fig. 1) and Newton's law of gravitation (1) are not equivalent physical laws. These are different laws of gravitation. There is no equal sign between them:

$$\frac{mM}{r^2} \neq \frac{mR^3}{T^2 r^2} \qquad (2)$$

The common feature of the two laws of gravity is the inverse square law. This feature became the cause of the priority dispute between Hooke and Newton.

7. The gravitational constant G includes the Kepler constant R^3/T^2.

The constant G is called the Newtonian constant of gravitation [14]. But Newton did not even mention it. The constant G appeared in the formula for the law of gravitation much later than the discovery of the law of universal gravitation. Henry Cavendish in 1798 also did not know about the existence of the constant G and did not measure it. He measured the density of the Earth. The constant G appeared in the formula $F = GmM/r^2$ almost 200 years after the discovery of the law of universal gravitation and 75 years after the Cavendish experiment.

J. A. M. Pereira in [15] showed the history of the appearance of the constant G. In 1803, S. D. Poisson presented the formula for the law of universal gravitation $F = f\,mM/r^2$ with the coefficient f, which later became the constant G [15, 16].

In 1873, A. Cornu and J. B. Baille also used the symbol f for the coupling constant in Newton's law of gravitation (4 - 6, 15).

The law of universal gravitation in the form familiar to us $F = GmM/r^2$ was presented by J. H. Poynting in 1913 [15, 17].

The constant G is now considered a fundamental constant [14]. Limits on the change of G have been confirmed. Analysis of observations shows that the gravitational constant G has changed by less than one part in ten billion per year over the past nine billion years [18].

There is an ongoing debate about the fundamental status of the constant G [19 - 21]. In [22], it was shown that the constant G can be represented using the fundamental physical constants of the electron by the following formula:

$$\mathbf{G = r_e^3/t_e^2 m_e D_o} \qquad (3)$$

Where: r_e is the radius of the electron, $t_e = r_e/c$, me is the mass of the electron, Do is the large Weyl number (D_0 = 4.16561... x 10^42).

The constant G is presented in formula (3) by electromagnetic constants. The formula for calculating G includes the Kepler constant for the electron r_e^3/t_e^2.

In [13] it is shown that the constant G can be presented using the parameters of the Universe by the following formula:

$$\mathbf{G = R_u^3/T_u^2 M_u} \qquad (4)$$

Where: R_u is the radius of the Universe, T_u is the time of the Universe, M_u is the mass of the Universe.

Formula (4) includes the Kepler constant for the Universe R_u^3/T_u^2. Formulas (3) and (4) show that G is a composite constant. A similar equation is valid for the solar system:

$$\mathbf{G = R^3/T^2 M_\odot} \qquad (5)$$

Where: R, T are the parameters of the planet's orbit, $M_\odot$ is the mass of the Sun.

The universality of the constant G leads to an interesting connection between the parameters of the electron, the parameters of the solar system and the parameters of the Universe:

$$\mathbf{r_e^3/t_e^2 m_e D_o = R^3/T^2 M_\odot = R_u^3/T_u^2 M_u} \quad (6)$$

Equation (6) lacks a formula that includes the parameters of the galaxy. The universality of the constant G indicates that this equation must include a formula that includes the parameters of the galaxy, represented by the Kepler constant. This equation is valid for all structural objects in the Universe:

$$\mathbf{r_e^3/t_e^2 m_e D_o = R^3/T^2 M_\odot = R^3_{galaxy}/T^2_{galaxy} M_{galaxy} = \ ...= R_u^3/T_u^2 M_u} \qquad (7)$$

What is surprising is the presence in the equation of the electron parameters in the same row with the parameters of objects in the Universe.

8. Conclusion.

Hooke offered Newton cooperation. Newton chose competition. Hooke gave two hints for the future law of gravity: the law of gravity should include the inverse square law, like Bullialdus, and elliptical orbits, like Kepler's. Newton included the inverse square law in his law of gravity, but did not include the parameters of the elliptical orbit. Instead of the parameters of the elliptical orbit, he used mass. Newton did not see the decisive role of the parameters of the elliptical orbit and overestimated the role of mass as a parameter in the law of gravity. As a result, the world received an incomplete law of gravity in the form: $\mathbf{F_N \propto mM/r^2}$. This law of Newton's gravity existed in an incomplete form for about 200 years until the constant G appeared in it.

In those years, nothing prevented Newton from giving the world an exact law of gravity in the form: $\mathbf{F_{H\text{-}K} = mR^3/T^2 r^2}$. At that time, both Kepler's laws and inverse square law were well known. These laws opened the way to a more perfect law of gravity. But to do this, Newton would have to accept both hints that Hooke gave him. In this case, there would be very little Newtonian in the law of gravity of the form: $\mathbf{F_{H\text{-}K} = mR^3/T^2r^2}$. This would be Hooke's law.

Perhaps Newton's rivalry played its fatal role and prevented him from giving the world the exact and mathematically perfect law of gravity that Hooke pointed him to. Hooke argued with Newton over the inverse square law in Newton's inaccurate and incomplete law. Hooke did not know how close he was to the exact and more valuable law of gravity.

In 1853, H. Garcet presented Newton's gravitational force as: $\boldsymbol{F} = \mathbf{4}\boldsymbol{\pi^2}(\boldsymbol{a^3/T^2})(\boldsymbol{m/r^2})$ [15, 23]. Garcet presented the force formula in the context of Newton's verbal formula. He did not notice that this formula for gravitational force is not adequate to Newton's verbal formula. He also did not notice that this formula for gravitational force is very close to Robert Hooke's verbal formula. Again, an opportunity was missed to see in this formula an alternative law of gravitation that Hooke had hinted at.

After Newton's law was refined 200 years later and presented as $\mathbf{F_N = GMm/R^2}$, it became clear how closely the law of gravity and Kepler's laws are connected. Now a student or even a schoolchild can show the connection between Newton's law and Kepler's laws. It is easy to derive both Newton's law from Kepler's laws and Kepler's laws from Newton's law. When deriving such a connection, it is easy to see Kepler's constant $\mathbf{R^3/T^2}$. When deriving, this constant $\mathbf{R^3/T^2}$ appears in formulas every time and reminds us that its rightful place is in the new law of gravitation, which Newton was unable to discover. Hooke saw and predicted the close connection between Kepler's laws and the future law of gravitation long before Newton's law of gravitation appeared. Hooke directly pointed out to Newton that the law of gravitation should take into account the elliptical orbits of the planets and the law of inverse squares.

9. Conclusions

1. There were two different verbal formulations of the law of gravity. One formulation belonged to Newton: the force is proportional to the masses and inversely proportional to the square of the distance. The other verbal formulation belonged to Hooke. Robert Hooke pointed out that the law of gravity should take into account the elliptical orbits of the planets and the inverse square law.

2. Different verbal formulas could not refer to the same law of gravitation. They referred to two different laws of gravitation.

3. Using masses as parameters led Newton to the law of gravity of the form: $\mathbf{F_N \propto mM/r^2}$. As a result, the world received the law of universal gravitation. To turn this proportional formula into an exact equation, the gravitational constant **G** was needed. Newton's law existed in an unfinished form for about 200 years until the constant **G** appeared in it.

4. Hooke's approach should have led to an exact law of gravity of the form: $\mathbf{F_{H\text{-}K} = mR^3/T^2r^2}$. Robert Hooke was very close to discovering this exact alternative law of gravity.

5. The law of gravity $\mathbf{F_{H\text{-}K} = mR^3/T^2r^2}$ surpasses in its perfection the law proposed by Newton: $\mathbf{F_N \propto mM/r^2}$. The Hooke-Kepler law surpasses in its perfection the law of gravity containing the constant **G** ($\mathbf{F_N = GMm/r^2}$), since distances and periods are known from observations with greater accuracy than mass and the constant **G**.

6. If Newton had accepted both of Hooke's hints, the world would have received the exact and perfect law of gravity $\mathbf{F_{H\text{-}K} = mR^3/T^2r^2}$ back **in 1687**. Nothing stood in the way. At that time, both

Kepler's laws and the inverse square law of Bullialdus were well known. Hooke's verbal formulations gave the necessary hint. But Newton used mass in the law of gravity instead of the parameters of an elliptical orbit.

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