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"I know of nothing more beautiful in arithmetic than these numbers, called planetary by some and magical by others."
Pierre de Fermat.

Numerical symmetry of magic squares and cubes

Annotation. Formulas are defined for calculating the sum of a series of numbers – the so–called constant - that make up a square and a cube of a certain order. The central symmetry of magic squares from the 2nd to the 8th order, as well as magic cubes from the 3rd to the 5th order, is analyzed. It is revealed that the magic squares of the 2nd, 3rd, 4th, 5th, 7th, and 8th order have a "homogeneous" central symmetry (relative to the diagonals), and the magic square of the 6th order has a "mixed" central symmetry. The character of symmetry is determined in the same way for magic cubes of the 3rd, 4th, and 5th orders. "Homogeneous" symmetry is characteristic of the magic cube of the 3rd order and the 5th order, and "heterogeneous" - for the magic cube of the 4th order. Based on the logic of constructing magic squares and cubes, two similar magical objects are constructed – a cube in a cube and cubes in a cube. The first one is based on a magic square of the 4th order (Albrecht Dürer, 1514), and the second one is based on a magic square of the 8th order. These magical figures have a "mixed" symmetry.

Keywords: central symmetry, magic squares, magic cubes, the order of the magic square and cube, the constant of the magic square and cube.

Introduction. It is known that a magic square in which any two numbers located symmetrically relative to its center add up to the same number is called symmetrical. Even and odd numbers are arranged symmetrically both relative to the center of the square and relative to each of its axes of symmetry. The sums of the pairs of numbers occupying the centrally symmetrical cells are the same and twice as large as the number in the center. The number located in the center of the square (for squares of odd orders) will be equal to the ordinal number of the cell if all the cells are numbered line by line from top to bottom. For magic cubes, if the numbers on each diagonal of the cross section are also added to the magic number of the cube, then the cube is called a perfect magic cube. If the sums of the numbers on the broken spatial diagonals of the magic cube are also equal to the magic number of the cube, the cube is called a pandiagonal magic cube. It seems very relevant to consider the numerical symmetry of magic squares and cubes.

The main part. Formulas are defined for calculating the sum of a series of numbers – the so–called constant ($\sum_S$) - that make up a square (1) and a cube (2) of a certain order (n). Based on them, tables of basic mathematical indicators and their values for magic squares (Table 1) and cubes (Table 2) from the 2nd to the 8th order have been compiled:

$$\sum_S = n\times(1 + n^2)/2 \quad (1)$$

$$\sum_S = n\times(1 + n^3)/2 \quad (2)$$

Formulas are also defined for calculating the sum of all the numbers (=S) that make up a magic square (3) and a magic cube (4) of a certain order (n):

$$\sum_S = n\times(1 + n^2)\ /2\times n \quad (3)$$

$$\sum_S = n\times(1 + n^3)/2\times n^2 \quad (4)$$

Table 1 – Basic mathematical indicators and their values for magic squares from the 2nd to the 8th order

The order of the magic square, n	The sum of a symmetric pair (the square of the order of the pair + 1), ∑	The sum of a series of numbers, ∑	The part of a symmetric pair from the sum of a series, a fraction	The sum of all the numbers of the figure, ∑
2	5	–	1/1	10
3	10	15	1/1,5	45
4	17	34	1/2	136
5	26	65	1/2,5	325
6	37	111	1/3	666
7	50	175	1/3,5	1225
8	65	260	1/4	2080

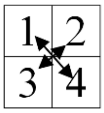
Table 2 – Basic mathematical indicators and their values for magic cubes from the 2nd to the 8th order

The order of the magic Cube, n	Sum of a symmetric pair (cube of the order of a pair + 1), ∑	The sum of a series of numbers, ∑	The part of a symmetric pair from the sum of a series, a fraction	The sum of all the numbers of the figure, ∑
2	9	9	1/1	
3	28	42	1/1,5	378
4	65	130	1/2	2080
5	126	315	1/2,5	7875
6	217	651	1/3	23436
7	344	1204	1/3,5	58996
8	513	2052	1/4	131328

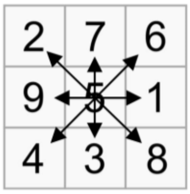
The central symmetry of magic squares from the 2nd to the 8th order (Figure 1), as well as magic cubes from the 3rd to the 5th order (Figure 2), is analyzed. It is revealed that the magic squares of the 2nd, 3rd, 4th, 5th, 7th, and 8th order have a "homogeneous" central symmetry (relative to the diagonals), and the magic square of the 6th order has a "mixed" central symmetry. The character of symmetry is determined in the same way for magic cubes of the 3rd, 4th, and 5th orders.

"Homogeneous" symmetry is characteristic of the magic cube of the 3rd order and the 5th order, and "heterogeneous" - for the magic cube of the 4th order (Figure 2).

Based on the logic of constructing magic squares and cubes, two similar magical objects are constructed – a cube in a cube and cubes in a cube. The first (Figure 3) is built [3] on the basis of a magic square of the 4th order (Albrecht Dürer, 1514), and the second (Figure 4) is based on a magic square of the 8th order (Figure 1). These magical figures have a "mixed" symmetry.



A



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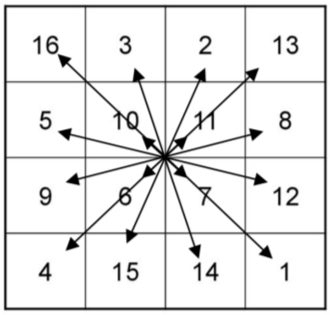

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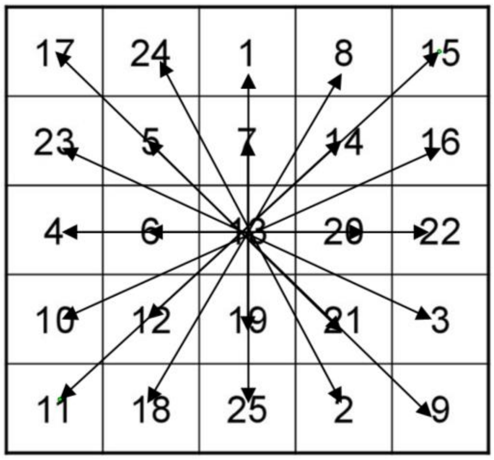

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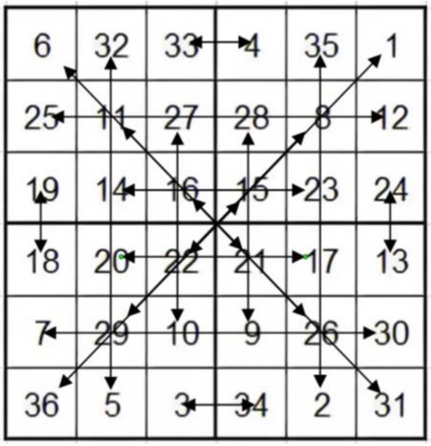

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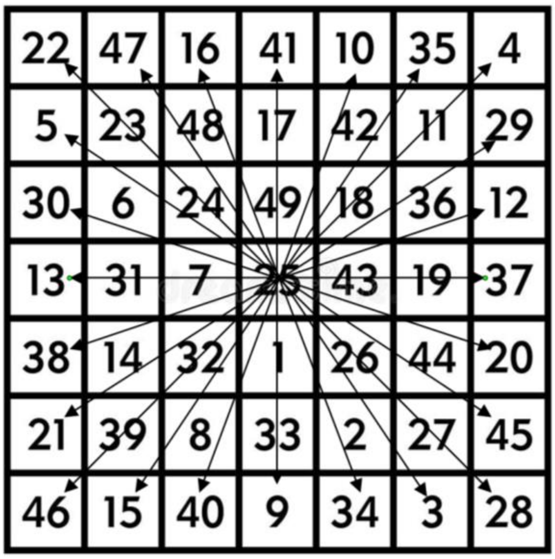
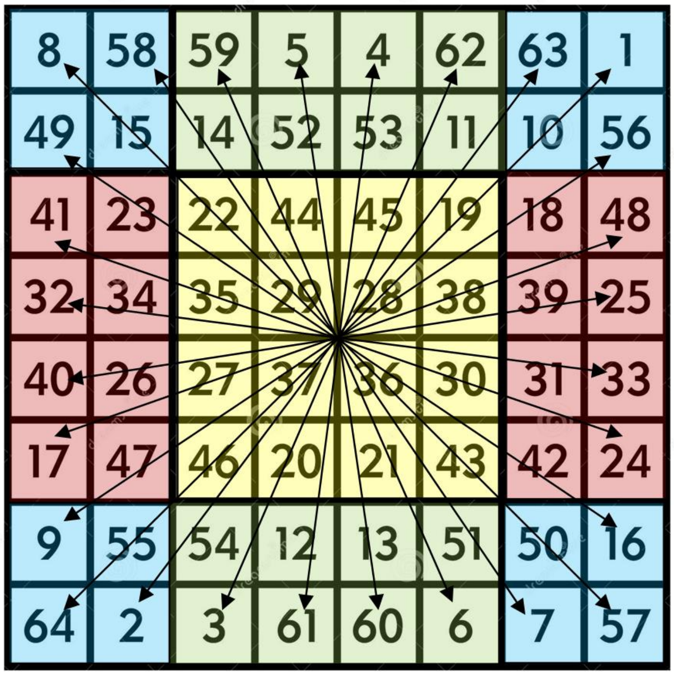


F



G

Figure 1 – The central symmetry of the magic squares:
A – 2nd order; B – 3rd order; C - 4th order (Albrecht Dürer, 1514); D – 5th order;
E – 6th order; F – 7th order; G - 8th order

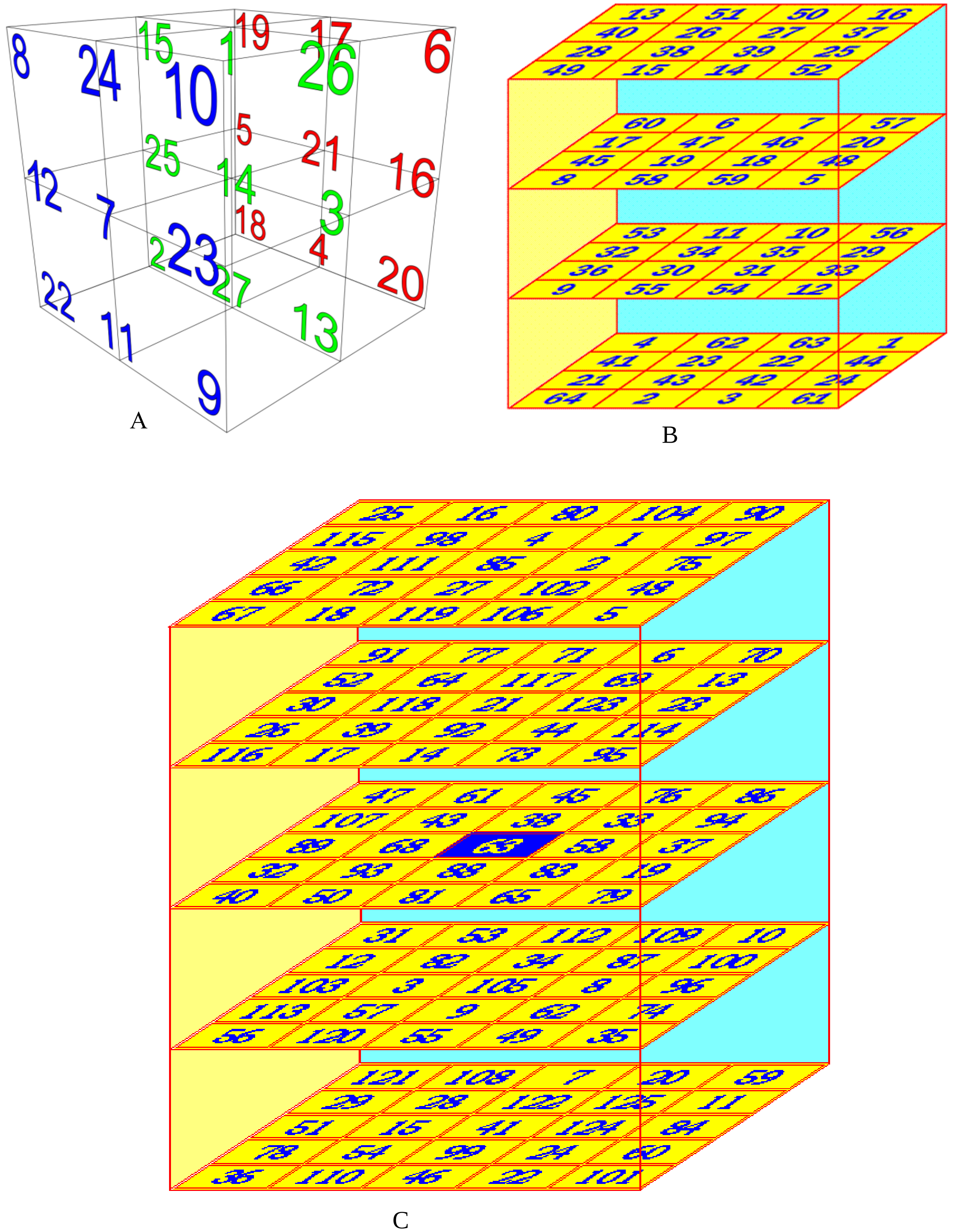


Figure 2 – Magic cubes: A – 3rd order, B – 4th order (Pierre de Fermat, 1640 [1]), C – 5th order (Christian Boyer, Walter Trump, 2003 [2])

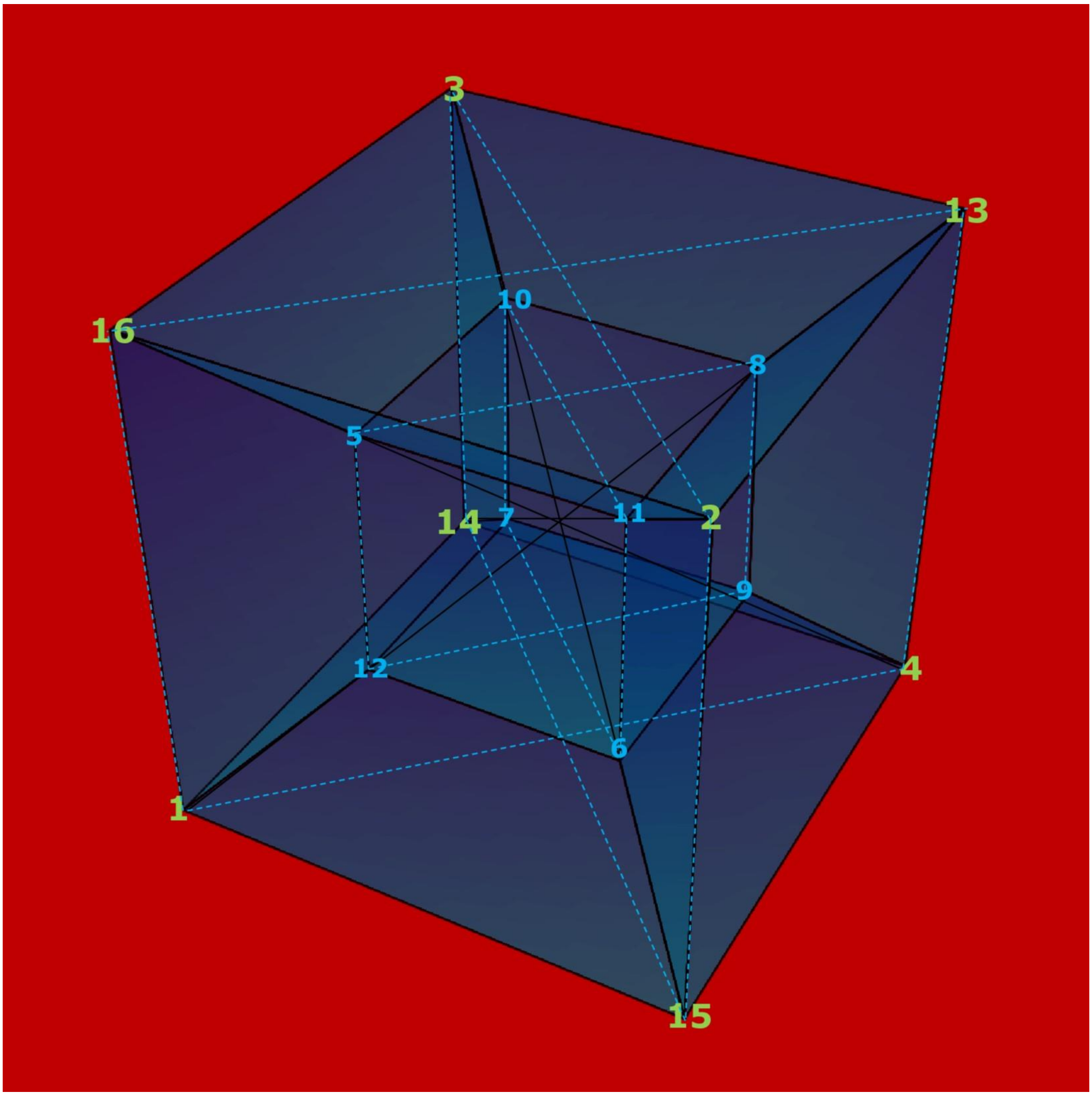


Figure 3 – A magic cube in a cube (Andrey V. Voron, 2018 [3]), built on the basis of a magic square of the 4th order (Albrecht Dürer, 1514)

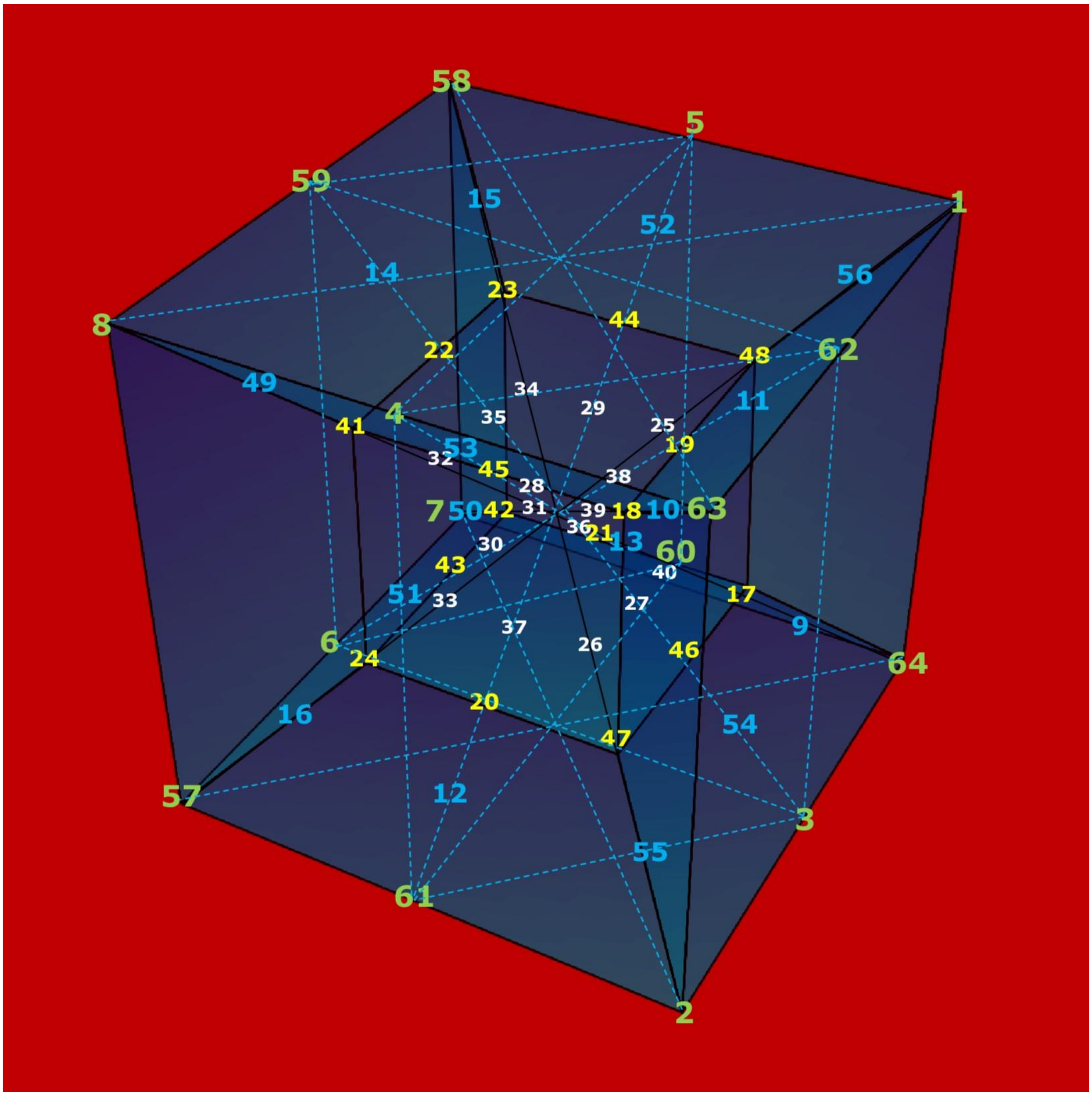


Figure 4 – Magic cubes in a cube (Andrey V. Voron, 2025), built on the basis of a magic square of the 8th order

Conclusion.

1. Formulas are defined for calculating the sum of a series of numbers – the so-called constant – that make up a square and a cube of a certain order.

2. The central symmetry of magic squares from the 2nd to the 8th order, as well as magic cubes from the 3rd to the 5th order, is analyzed. It is revealed that the magic squares of the 2nd, 3rd, 4th, 5th, 7th, and 8th order have a "homogeneous" central symmetry (relative to the diagonals), and the magic square of the 6th order has a "mixed" central symmetry. The character of symmetry is determined in the same way for magic cubes of the 3rd, 4th, and 5th orders. "Homogeneous" symmetry is characteristic of the magic cube of the 3rd order and the 5th order, and "heterogeneous" - for the magic cube of the 4th order.

3. Based on the logic of constructing magic squares and cubes, two similar magical objects are constructed – a cube in a cube and cubes in a cube. The first one

is based on a magic square of the 4th order (Albrecht Dürer, 1514), and the second one is based on a magic square of the 8th order. These magical figures have a "mixed" symmetry.

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