Four dimensional spacetime will continue in the future?

Tatsuyuki Sugawa*

December 11, 2024

Internal Medicine Devision,
Shinseikai hospital
Kitasakueazuka, Toyonaka, 560-0022, Japan
and
Osaka University
Yamadagaoka, Suita, 565-0701 Japan

Abstract

In this short report we speculate the spacetime dimension in our universe , by using both the Kasner solutions to the Einstein field equations and the Kaluza-Klein concept of the extra dimension. Surprisingly around the past Big-Bang, the spacetime dimension may have been two dimensional (1+1) , and in the future the spacetime dimension may tend to three dimensional one (2+1) , though our observable space-time is four dimensions at the moment. This is a toy model.

*tatsuyukisugawa@hotmail.com

Contents

1 Introduction

We live in the four dimensional space-time at the present stage. My question is : this universe has been four dimensional space-time from the birth and will continue as four dimensional space-time toward the future? In the Kaluza-Klein cosmology , A. Chodos and S. Detweiler [1] discussed the possibility that a simple solution to the vacuum Einstein equations with no matter in $4+1$ space-time dimensions leads to a universe which has $3+1$ observable dimensions, by one dimensional space shrinking[1] .They pointed out that the five-dimensional Kasner solution [2] describes a universe in which one spatial dimension shrinks with time while the other three spatial ones expand. This not only accords with the observed expanding universe , but also can explain why the fifth dimension is so small. In our report we will study the possibility that our universe would be a $2+1$ dimensional space-time plus one spatial extra dimension. namely we consider the Kaluza-Klein theory which is the $2+1$ space-time and 1 extra spatial dimension. In the past we studied the wave function of the universe in three dimensions.[3] If our universe began as three dimensional Big Bang ,the concept of the Big Bang would be changed drastically . In other words, all space times were not born as one Big Bang around the same time, one space had been existed beforehand. After that another two spaces were born as the Big Bang. Now our space-time is the four dimensional space-time.In the past two dimensional space-time had already existed and another 2 spaces were born as Planck scale to become growing up to the present scale. So around the present we live in 3+1 space-time. In the future one space will be shrunk to be very small , maybe the Planck Length. As the result , the future space-time may tend to the three dimensional one(2+1 space-time).

2 Set-Up

We follow the Chodos-Detweiler method. We set time to be a continuous parameter ranging over the real time and take the spatial coordinates x^α to be periodic.

$$0 \leq x^\alpha < L$$

where L is some parameter with the dimension of length. In our model with no matter, the basic equation is the vacuum Einstein field equations (the energy-momentum tensor $T_{\mu\nu} = 0$, cosmological constant $\Lambda = 0$)

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 0 \tag{1}$$

where $R_{\mu\nu}$ is the four- dimensional Ricci tensor. R is the four-dimensional Ricci scalar. Following Chodos-Detweiler, we assume that the universe has evolved according to the Kasner solution [2] to Eq.(1) which is a homogeneous and anisotropic vacuum solution in the case of ($T_{\mu\nu} = 0$, $\Lambda = 0$). In d spatial dimensions, the Kasner solutions to Eq.(1) have the form

$$ds^2 = -dt^2 + \sum_{i=1}^{d} (\frac{t}{t_0})^{2p_i} (dx^i)^2 \tag{2}$$

provided Kasner conditions are satisfied, which are

$$\sum_{i=1}^{d} p_i = \sum_{i=1}^{d} p_i^2 = 1 \tag{3}$$

The above relations require at least one of the p_i to be negative which means contraction in at least one spatial dimension. As Chodos Detweiler pointed out, in four dimensions the vacuum Kasner solution may seem a poor description of the universe since it predicts contraction in at least one dimension. Here we dare consider the four dimensional case, since we concentrate on the three dimensional gravity originally. The Kasner conditions in four dimensions($d = 3$) are

$$p_1 + p_2 + p_3 = p_1^2 + p_2^2 + p_3^2 = 1 \tag{4}$$

We take the choice to satisfy the above relations.

$$p_1 = p_2 = \frac{2}{3},\ p_3 = -\frac{1}{3} \tag{5}$$

Thus the four - dimensional Kasner metric is

$$\begin{aligned} ds^2 &= -dt^2 + (\frac{t}{t_0})^{\frac{4}{3}} dx^2 + (\frac{t}{t_0})^{\frac{4}{3}} dy^2 + (\frac{t}{t_0})^{-\frac{2}{3}} dz^2 && (6) \\ &= g_{\mu\nu} dx^\mu dx^\nu && (7) \end{aligned}$$

where

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & (\frac{t}{t_0})^{\frac{4}{3}} & 0 & 0 \\ 0 & 0 & (\frac{t}{t_0})^{\frac{4}{3}} & 0 \\ 0 & 0 & 0 & (\frac{t}{t_0})^{-\frac{2}{3}} \end{pmatrix} \tag{8}$$

$g^{\mu\nu}$ as the inverse of $g_{\mu\nu}$ is

$$g^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & (\frac{t}{t_0})^{-\frac{4}{3}} & 0 & 0 \\ 0 & 0 & (\frac{t}{t_0})^{-\frac{4}{3}} & 0 \\ 0 & 0 & 0 & (\frac{t}{t_0})^{\frac{2}{3}} \end{pmatrix} \tag{9}$$

3 three dimensional cosmology in four dimensional Kaluza-Klein model

Ordinary Kaluza-Klein theory is considered to be 3+1 space-time plus one extra spatial dimension. However here we consider 2+1 space-time dimension plus one extra spatial dimension. Namely we consider three dimensional cosmology. Anyway we investigate the Kasner metric (6).
If we look at t_0 as our present time,

(a) $t \approx t_0$

The metric (6) is

$$ds^2 \approx -dt^2 + dx^2 + dy^2 + dz^2 \tag{10}$$

This is the almost flatlike and isotropic metric which shows nowadays our (3+1) dimensional universe.
This is a good metric, considering $\Lambda \approx 0$ though observably $\Lambda \gtrapprox 0$
But the distance around any dimension is L. $L \approx 10^{30}$cm.

(b) $t \ll t_0$

$$ds^2 \approx -dt^2 + (\frac{t}{t_0})^{-\frac{2}{3}} dz^2 \tag{11}$$

the universe had only one spatial dimension, whose length approached infinity at the initial singularity $t = 0$.
This shows the 2 dimensional (1+1) universe.
The other spacial dimensions, x, y, will come to be compactified with the scale of the Planck length. This means the new big bang. The compactified two dimensional space is the essencial manifold which is explained by three dimensional quantum gravity (2+1).
Two space (x,y) was born in the (1+1) space-time.

(c) $t \gg t_0$
The metric is

$$ds^2 \approx -dt^2 + (\frac{t}{t_0})^{\frac{4}{3}} dx^2 + (\frac{t}{t_0})^{\frac{4}{3}} dy^2 \tag{12}$$

the distance around the fourth dimension (z) has shrunk to $(\frac{t}{t_0})^{-\frac{1}{3}} L$ to be compactified to the Planck length, on the other hand the other two spatial dimensions(x,y) have grown to $(\frac{t}{t_0})^{\frac{2}{3}} L$.
This shows the 3 dimensional(1+2) space-time universe. In the far future the universe will become three dimensional space-time.

4 conclusion

We spaculated our universe dimension considering the Kasner's metric and the Kaluza-Klein concept. Our universe is at the present $(3 + 1)$ dimensional space-time, but the initial universe may have been $(1 + 1)$ dimensional space-time as a starting point. In addition in the future it will be $(2 + 1)$ dimensional space-time. Of course we have no observational evidence . However our universe (or Multiverse) may be very mysterious for us. If my speculation may be true, the Big-Bang may have started as the birth of 2 dimensional space on the background of $(1 + 1)$ space-time . If so , our next question is how the other two dimensional space was emerged? In order to avoid the two dimensional space-like singularity we may use the (2+1) dimensional space-time quantum gravity. Our toy model may touch the conflict of basical physics principle.

5 Acknowledgements

The author is grateful to Professor. Douglas Smith and the department of Mathematical Sciences, Durham University for giving me an opportunity to stay there in summer 2024 . Besides he thanks many researchers for giving me the useful informations and discussions when he took part in the conferences (domestic and international). He also thanks co-workers of medical hospital for their warm kindness.

References

[1] A. Chodos and S. Detweiler, " Where has the fifth dimension gone?" Phys.Rev.D 21. 2167 (1980)

[2] E. Kasner, Am J. Math 48. 217(1921)

[3] Tatsuyuki Sugawa , " Wave Function of the Universe near Cosmological Singularity in three dimensions I" Google Scholar