Hopf Fibration, Non-Homotopy and Mass

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Abstract

Unable to determine a particle's specific mass the Standard Model leaves rest mass a free parameter. For an alternative theory we consider a 3-sphere intersecting three dimensional space. The intersection is a Hopf fibration with non Euclidean topology. Resistance to a force is due to non-homotopy. The intersection signature appears across mass splitting formulae that treat lighter hyperons as functions of the proton, neutron and in some cases electron. In MeV the derived values are: $\Sigma^+ \approx 1189.371$, $\Sigma^0 \approx 1192.655$, $\Sigma^- \approx 1197.580$, $\Xi^0 \approx 1314.810$, $\Xi^- \approx 1321.711$, $\Omega^- \approx 1672.482$. The electron, Σ, Ξ masses provide a solution to the neutron-proton mass difference problem. This is a pure mathematical relationship ensuring the derived Σ, Ξ mass values are not ad hoc. The mathematical status of Ω^- is less secure. To make the case, a scaling factor S_M is introduced using Ω^- volume. Rest mass in any system of units is scaled to a dimensionless number proportional to MeV. For precise results less than one fifth of an electron-volt is shaved from the 2022 CODATA neutron adjustment (our value 939.565 421 76). To justify S_M we consider the difference between Gaussian and SI units. In the final count nine free parameters reduce to two.

Keywords— Hopf fibration, Hopf map, Euclidean space, non-homotopy, particle mass, symmetry breaking, 3-sphere, theory of mass, scaling factor.

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The Higgs field imparts mass to fundamental particles. In the crowd analogy the field acts like a throng impeding a celebrity.[1] The stronger the interaction the slower the progress, the heavier the particle. To dig a little bit deeper, particles that exhibit internal Lie group symmetry gain mass when spontaneous symmetry breaking couples with the Higgs field at higher energy states.[2, 3] The caveat is that quarks, leptons and some bosons interact with the field, but not photons; whilst the bulk of a Hadron's mass is due to the energy that goes into quark confinement. The problem with this picture is the Standard Model is unable to predict why a particle has the precise mass that it does. By radically rethinking how a particle resists a force, a very different answer emerges to the one just sketched. A particle does not break its symmetry. It is the force that breaks away from the symmetry of Euclidean space. Non-homotopy accounts for the entirety of a particle's mass. We lay out what is needed for this alternative picture.

In lieu of the Higgs scalar field we consider a vector field. Ordinary space is the set of three dimensional points $\mathbb{R}^3$. 3-space is ordinary space filled with forces constrained by Euclidean topology. For instance, a force has the connected topology of a point when it makes contact with an object.

The theory also considers an S^3 Hopf fibration.[4] S^3 is the set of four dimensional coordinates that form a 3-sphere. A 2-sphere is described by the set of three dimensional points S^2 (a subset of $\mathbb{R}^3$). A Hopf fibration continuously maps the 3-sphere to the 2-sphere. This is done with Hopf maps. A Hopf map ($h : S^3 \to S^2$) is a surjective function mapping a subset of S^3 elements to a point in S^2. An individual Hopf map describes an S^1 circle (Hopf circle) embedded in S^3. There is one unique Hopf circle for each point in S^2. This means a single point on the 2-sphere surface is the image (shadow) of a circle embedded in higher dimensions. Spatial intuition is lost, but the fibration makes it mathematically possible to return to the circle. Continuous mapping also entails an infinite number of maps for each four dimensional point ensuring S^3 space is transitive.

We consider a 3-sphere intersecting 3-space. The intersection generates a 2-sphere image of the 3-sphere; a situation described by a Hopf fibration. A force making contact with the 2-sphere raises the question of homotopic non-equivalence. The non-homotopy may be pictured with cone mapping. A point at the apex of a cone is unable to pass to the base circle unless its connected topology is punctured. As S^3 is transitive, a force able to make the jump is dispersed throughout the 3-sphere. The extent of the dispersal is the size of the resistance. An ordinary object with Euclidean topology is part of 3-space and unable to resist a force. The object is massless. Resistance (due to non-homotopy) entails mass is relative to 3-space (so long as there is a force to resist). Therefore, mass is not an intrinsic property.

Five equations characterise particle mass. Eq. (1) tells us rest mass is determined by the size of the 3-sphere.

$$M = 2\pi^2 r^3. \tag{1}$$

The Hopf mass occupies the volume of a standard ball in 3-space, as Eq. (2).

$$V = \frac{4\pi}{3} r^3. \tag{2}$$

If $M_p \approx 938.272$ then $V_p \approx 199.108$. The mass / volume disparity means the density (ρ) of the 2-sphere interior, Eq. (3), is a pure number > 1.

$$\rho = \frac{M}{V} = \frac{3\pi}{2}. \tag{3}$$

Hypermass (H) is the difference between mass and volume, as Eq. (4).

$$H = M - V. \tag{4}$$

Eq. (5) is the hypermass signature (h-signature).

$$M = H\left(\frac{\rho}{\rho - 1}\right). \tag{5}$$

Σ rest masses are h-signatures, as Eqs. (6, 7, 8). ($M_p = 938.272\,089\,43$, $M_n = 939.565\,421\,94$).[5]

$$M_{\Sigma^+} = (2M_p - M_n)\left(\frac{\rho}{\rho - 1}\right) \approx 1189.3712. \tag{6}$$

$$M_{\Sigma^0} = M_n\left(\frac{\rho}{\rho - 1}\right) \approx 1192.6546. \tag{7}$$

$$M_{\Sigma^-} = (4M_n - 3M_p)\left(\frac{\rho}{\rho - 1}\right) \approx 1197.5797. \tag{8}$$

Eq. (6) is a match for the Particle Data Group (PDG) fit for M_{Σ^+} (1189.37 $\pm$0.07).[6] The PDG fit for M_{Σ^0} is 1192.642 $\pm$0.024. Eq. (7) is particularly close to Wang 1192.65 $\pm$0.020.[7] However, Eq. (8) is over four standard deviations adrift of the current PDG fit (1197.449 $\pm$0.030). This value draws on three results. Schmidt (1197.43) and Gurev (1197.417) are too low to be the number derived here.[8, 9] Schmidt is an old paper from 1965, and Gurev is a proof of method. M_{Σ^-} is within one standard deviation of Gall (1197.532 $\pm$0.057).[10]

We consider what it means if Eqs. (9, 10) are Ξ rest masses. (Subtracting a volume indicates a mass is also a hypermass).

$$M_{\Xi^0} = M_{\Sigma^0}\left(\frac{\rho}{\rho - 1}\right) - V_p \approx 1314.8104. \tag{9}$$

$$M_{\Xi^-} = M_{\Sigma^-}\left(\frac{\rho}{\rho - 1}\right) - V_p \approx 1321.0622. \tag{10}$$

M_{Σ^0} is within one standard deviation of the PDG fit (1314.86 ± 0.20) and close to Fanti (1314.82 $\pm$0.06)[11]. Eqs. (7, 8, 9, 10) resolve (11, 12, 13).

$$\frac{M_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}} = \frac{\rho}{\rho - 1}. \tag{11}$$

$$M_{\Sigma^0}\left(\frac{M_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}}\right) - M_{\Xi^0} - V_p = 0. \tag{12}$$

$$M_{\Sigma^-}\left(\frac{M_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^0} - M_{\Sigma^-}}\right) - M_{\Xi^-} - V_p = 0. \tag{13}$$

We are about to see why Eq. (11) is unsustainable. The present PDG fit for M_{Ξ^-} (1321.71 ± 0.07) draws on a 2006 study of 4.8k events.[12] Faced with an unlikely nine standard deviation adjustment, Eq. (14) introduces the electron rest mass energy ($M_e = 0.510\ 998\ 950\ 69$ MeV) as a *fudge* factor. (Hint: it is not a fudge).

$$M^*_{\Xi^-} = (M_{\Sigma^-} + M_e)\left(\frac{\rho}{\rho - 1}\right) - V_p \approx 1321.7109. \tag{14}$$

Eqs. (7, 8, 9, 14) resolve (15). This mathematical solution to the neutron - proton mass difference problem ensures Σ, Ξ values are not ad hoc.

$$\frac{M_e}{3\left(\frac{M^*_{\Xi^-}-M_{\Xi^0}}{M_{\Sigma^-}-M_{\Sigma^0}}-\frac{\rho}{\rho-1}\right)} = M_n - M_p. \tag{15}$$

The $M^*_{\Sigma^-}$ adjustment averts the threat of infinity due to (11). If the M_e adjustment is applied to the Ξ_0 mass, we get Eq. (16), and a $\frac{3}{4}$ adjustment gives (17).

$$M^*_{\Xi^0} = (M_{\Sigma^0} + M_e)\left(\frac{\rho}{\rho-1}\right) - V_p \approx 1315.4591. \tag{16}$$

$$M^\diamond_{\Xi^0} = \left(M_{\Sigma^0} + \frac{3}{4}M_e\right)\left(\frac{\rho}{\rho-1}\right) - V_p \approx 1315.2969. \tag{17}$$

Shifting Ξ adjustments generates alternative formulations for M_{Σ^+}, M_{Σ^0}, M_{Σ^-}.

$$M_{\Sigma^+} = \left(M_p + \frac{M_e}{3\left(\frac{M_{\Xi^-}-M^*_{\Xi^0}}{M_{\Sigma^-}-M_{\Sigma^0}}-\frac{\rho}{\rho-1}\right)}\right)\left(\frac{\rho}{\rho-1}\right). \tag{18}$$

$$M_{\Sigma^0} = \left(M_p + \frac{M_e}{3\left(\frac{M^*_{\Xi^-}-M_{\Xi^0}}{M_{\Sigma^-}-M_{\Sigma^0}}-\frac{\rho}{\rho-1}\right)}\right)\left(\frac{\rho}{\rho-1}\right). \tag{19}$$

$$M_{\Sigma^-} = \left(M_p + \frac{M_e}{3\left(\frac{M^*_{\Xi^-}-M^\diamond_{\Xi^0}}{M_{\Sigma^-}-M_{\Sigma^0}}-\frac{\rho}{\rho-1}\right)}\right)\left(\frac{\rho}{\rho-1}\right). \tag{20}$$

Eqs. (18, 19, 20) affirm Σ, Ξ particles belong to a single family but overlooks Ω^-. For Ω^- mass Eq. (22) follows the template provided by (21).

$$M_{\Sigma^0} = \left(\frac{3H_{\Sigma^+} + 2H_{\Sigma^-}}{5}\right)\left(\frac{\rho}{\rho-1}\right) \approx 1192.6546 \tag{21}$$

$$M_{\Omega^-} = \left(\frac{3M_{\Xi^0} + 2M^*_{\Xi^-}}{5}\right)\left(\frac{\rho}{\rho - 1}\right) \approx 1672.4824. \tag{22}$$

The PDG fit for the Ω^- mass is 1672.45 ± 0.29 MeV. Eq. (21) is the second clue suggesting Ξ particles are the hypermasses of two heavier particles; a neutral particle 1668.9787 MeV and a negative particle 1677.7380 MeV. These masses fall within the uncertainties of a number of potential candidates, i.e. N(1675), N(1680), Λ(1670), Σ(1660), Σ(1670),[6] but the uncertainties are presently too wide to be definitive. The Ξ(1318) resonance is also a plausible candidate for $H_{\Omega^-} \approx 1317.5706$ with Ω^- the middle mass of a triple {1668.9787, 1672.4824, 1677.7380} and their hypermasses the Ξ triple {1314.8104, 1317.5706, 1321.7109}.

At Eq. (23) the $M^*_{\Xi^-}$ adjustment necessitated by (15) replaces volume V_p at (12, 13) with V_{Ω^-}. However, the approximate equivalence is in MeV only.

$$M_{\Sigma^0}\left(\frac{M^*_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}}\right) - M_{\Xi^0} - V_{\Omega^-} \approx \frac{\rho}{\rho - 1}. \tag{23}$$

The alignment provides the scaling factor introduced at Eqs. (24, 25).

$$S_M = \left(\frac{\{M_x\}}{M_{\Sigma^0}\left(\frac{M^*_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}}\right) - M_{\Xi^0} - V_{\Omega^-}}\right)\left(\frac{\rho}{\rho - 1}\right). \tag{24}$$

$$S_M = \left(\frac{\{M_x\}}{M_{\Sigma^-}\left(\frac{M^*_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}}\right) - M^*_{\Xi^-} - V_{\Omega^-}}\right)\left(\frac{\rho}{\rho - 1}\right). \tag{25}$$

S_M scales to a number proportional to MeV. Eq. (26) ≈ 0.511, for example.

$$\left(\frac{M_e}{M_{\Sigma^0}\left(\frac{M^*_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}}\right) - M_{\Xi^0} - V_{\Omega^-}}\right)\left(\frac{\rho}{\rho - 1}\right) \approx 0.511. \tag{26}$$

Eq. (26) = 0.511 007 366 716 with 2022 CODATA adjustments in u, The kg adjustments give 0.511 007 405 8334. MeV is 0.511 007 615 2234. The values are marginally high for the electron but S_M is sensitive to its inputs. Using the proton and electron to dial-in the more uncertain neutron mass in u, Eq. (26) = 0.510 998 950 69 when $M'_n = 1.008\,664\,915\,876\,394\,072$. The ultra precision is overdone but M'_n represents a downward adjustment within one standard deviation or less than one fifth of an electron-volt. Given the uncertainties, a neutron mass energy 939.565 421 76 $\pm\, 0.000\,000\,06$ MeV resolves Eqs. (23, 26).

The obvious question is why does S_M favour millions of electron-volts? The clue is in the millions. A 10^6 factor points to the difference in scale between SI and Gaussian units. Gaussian fields are part of a single mechanical framework sharing the same set of cgs dimensions. If mass is non-homotopic, a dimension independent of field dimensions is needed. This is not available in the Gaussian framework. As field units the joule and kilogramme are also not suitable. Though independent the arbitrary carbon 12 benchmark is reason to dismiss u. It is the ampere that offers a non-arbitrary independent number. If mass is measured as *no of elementary charge* $\times$ *electrical potential*, where a current produces a magnetic force, then **B**, **E** field densities are implicit. If we ask what does nature prefer? Imbalanced field dimensions seems the wrong answer. Whilst not an SI unit the electron-volt is an adjunct to the SI system (due to the ampere and volt). For balance, the Gaussian statVolt is converted into volts, as Eq. (27).

$$1\ statVolt = \frac{299792458}{10^6}\ Volts. \tag{27}$$

Reference to SI values is eliminable. (28) walks through each step.

$$\begin{array}{ll}
\dfrac{299792458}{c}\ Volts & 1 \\
= 1\ V \cdot m^{-1} \cdot s & 2 \\
\times\ \ 10^{-6}\ N^{-1} \cdot s^{-1} & 3 \\
= 10^{-6}\ V \cdot N^{-1} \cdot m^{-1} & 4 \\
\times\ \ 1.60219 \cdot 10^{-19} A \cdot s & 5 \\
= 1.60219 \cdot 10^{-25}\ eV \cdot N^{-1} \cdot m^{-1} & 6 \\
\times\, 6.24151 \cdot 10^{18}\ eV & 7 \\
= 10^{-6}\ eV \cdot N^{-1} \cdot m^{-1} & 8 \\
\times\, 510998.95069\ eV & 9 \\
= 0.510\,998\,950\,69\ eV \cdot N^{-1} \cdot m^{-1} & 10 \\
= 0.510\,998\,950\,69 & 11
\end{array} \tag{28}$$

The conversion factor 299792458 at line 1 is already dimensionless but directly references the speed of light in SI units. Dividing by c excises this number, though it introduces the additional dimensions $m^{-1} \cdot s$, which have to be cancelled later. This explains the statVolt to volt conversion factor 10^{-6} introduced as an impulse at line 3. At line 4, line 2 dimensions are divided by $newton \cdot seconds$ from line 3. Line 5 introduces elementary charge in SI units, which converts volts to electron-volts at line 6. Line 7 leaves $A \cdot s = 1$ which ensures all trace of the SI value is erased when dimensions cancel. Line 8 is the conversion factor for mass energy that gives a number in reduced electron-volts. Line 9 introduces the electron mass energy but this could be any mass energy in electron-volts. The reduced number at line 10 is parsed in its base dimensions as follows.

$$\begin{pmatrix} \begin{matrix} \textit{electrical potential} \\ \textit{difference} \\ \approx 0.511 \\ m^2 \cdot kg \cdot s^{-3} \cdot A^{-1} \end{matrix} & \times & \begin{matrix} \textit{no of charges} \\ \textit{moved for a second} \\ = 1 \\ A \cdot s \end{matrix} \end{pmatrix} m^{-2} \cdot kg^{-1} \cdot s^2. \tag{29}$$

(29) quantifies the work needed per ampere to move an electron with an ampere current, per mass energy. At line 11 dimensions are allowed to cancel. If this line of reasoning is correct S_M scales rest mass and rest mass energies to a dimensionless magnitude for a generic system with balanced fields and independent elementary charge. The magnitude quantifies the size of a particle's reluctance to break its own symmetry when interacting with a force. Whilst this definition of particle rest mass along with its roundabout explanation provides the beginnings of a conceptual framework, we are still missing the math needed to be able to say Eq. (23) is exact. Until a proof becomes available Ω^- is a tentative member of the Σ, Ξ mathematical family. Future CODATA/PDG adjustments converging on S_M would also give support to this thesis.

We consider in passing the last member of the baryon eightfold way. A mathematical solution for Λ^0 mass has proved elusive. Eq. (30) is a simple relationship that gets close and accords with a hierarchy of neutral particles already evident at Eqs. (7, 9).

$$M_{\Sigma^0}\left(\frac{\rho}{\rho - 1}\right) - 2V_p = 1115.7029. \tag{30}$$

The octet might still be neatly tied together if Eq. (30) were an exact match but the present PDG fit is 1115.683 MeV ± 0.006, a precise measurement based on 38k events.[13] With such a large data set another four standard deviation adjustment is unlikely. However, we get within one standard deviation using M_n as the base.

$$M_{\Lambda^0} = M_n + \frac{M_{\Omega^-} - (V_{\Xi^0} + H^*_{\Xi^-})}{2} \approx 1115.68334. \tag{31}$$

As we lack an additional feature like a scaling factor to motivate Eq. (31) it remains ad hoc, which leaves us to close this limited survey of hyperon mass on a disappointing note.

In conclusion: evidence for non-homotopic mass is provided by Σ, Ξ, Ω rest mass energies derived from h-signatures. The derived values are exceptionally close to observation and a number of predictions are made. The Σ and Ξ mass values provide an explanation for the long standing problem of neutron - proton mass difference; this, and the possibility of reducing nine free parameters to two is further evidence for non-homotopy. The S_M scaling factor is a novel development that stakes its claim as the dimensionless and natural scale for particle rest mass.

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