Hopf Fibration, Non-Homotopy and Mass

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Abstract

The Standard Model is unable to determine a particle's specific rest mass. For an alternative theory we consider a 3-sphere intersecting three dimensional space. The intersection is a Hopf fibration with non Euclidean topology. Mass is due to a force jumping topologies. Mass splitting formulae with intersection signature treat the mass of lighter hyperons as a function of the proton, neutron and in some cases electron. The approach is numerical. In MeV the derived values are: $\Sigma^+ \approx 1189.371$, $\Sigma^0 \approx 1192.655$, $\Sigma^- \approx 1197.580$, $\Xi^0 \approx 1314.810$, $\Xi^- \approx 1321.711$, $\Omega^- \approx 1672.482$. Eqs. (15, 19) bring Σ and Ξ particles together as a single family of massive particles ensuring the individual formulae are not ad hoc. The familial status of Ω^- is less secure. To make the case, a scaling factor SM is introduced using Ω^- volume. SM scales rest masses given in any system of units to a dimensionless number proportional to MeV. For precise results less than one fifth of an electron-volt is shaved from the 2022 CODATA neutron adjustment (our value 939.565 421 76). To justify SM we consider the difference between Gaussian and SI units. In the final count nine free parameters reduce to two.

Keywords— Hopf fibration, non-homotopy, rest mass, scaling factor.

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The Higgs field imparts mass to fundamental particles. In the crowd analogy the field acts like a throng impeding a celebrity as they attempt to cross a room. [1] The stronger the interaction the slower the progress, the heavier the particle. To dig a little bit deeper, particles that exhibit internal Lie group symmetry gain mass when spontaneous symmetry breaking couples with the Higgs field at higher energy states.[2, 3] The caveat is that the field interacts with quarks, leptons and some bosons, but not photons; while the bulk of a Hadron's mass is due to quark confinement and not the Higgs field. Unable to predict why a particle has the precise mass that it does, the Standard Model leaves rest mass an open question. To address this problem we radically rethink how a particle resists a force. Reluctance to interact with a field is due to homotopic non-equivalence. The non-homotopy explains the entirety of a particle's mass. In order to act on the particle it is the force that breaks field symmetry, not the particle. Reworking the analogy: crowd members must work harder and do something weird to elicit a response from an aloof celebrity.

In lieu of the Higgs scalar field we consider a vector field. Ordinary space is the set of three dimensional points $\mathbb{R}^3$. 3-space is ordinary space filled with forces constrained by the Euclidean topology. For instance, a force has the connected topology of a point when it makes contact with an object.

The theory also considers an S^3 Hopf fibration.[4] S^3 is the set of four dimensional coordinates that form a 3-sphere. A 2-sphere is described by the set of three dimensional points S^2 (a subset of $\mathbb{R}^3$). A Hopf fibration continuously maps the 3-sphere to the 2-sphere. This is done with Hopf maps. A Hopf map ($h : S^3 \to S^2$) is a surjective function mapping a subset of S^3 elements to a point in S^2. An individual Hopf map describes an S^1 circle (Hopf circle) embedded in S^3. There is one unique Hopf circle for each point in S^2. This means a single point on the 2-sphere surface is the image (shadow) of a circle embedded in higher dimensions. Spatial intuition is lost, but the fibration makes it mathematically possible to return to the circle. Continuous mapping also entails an infinite number of maps for each four dimensional point ensuring S^3 space is transitive.

We consider a 3-sphere intersecting 3-space. The intersection generates a 2-sphere image of the 3-sphere. A situation described by a Hopf fibration. The location where 3-space force contacts the 2-sphere raises the question of homotopic non-equivalence. The non-homotopy may be pictured with cone mapping. A point force at the apex of a cone is unable to pass to the base circle unless its connected topology is punctured. As S^3 space is transitive a force able to make the jump is dispersed throughout the 3-sphere. The sudden dispersal registers as resistance to the force. To be clear, a 2-sphere with ordinary interior does not resist because it is part of 3-space. But where there is resistance due to non-homotopy a particle has mass relative to 3-space.

Five equations characterise Hopf particle rest mass. Eq. (1) tells us a particle's mass is determined by the size of the 3-sphere.

$$M = 2\pi^2 r^3. \tag{1}$$

The Hopf mass occupies the volume of a standard ball in 3-space, as Eq. (2).

$$V = \frac{4\pi}{3} r^3. \tag{2}$$

If $M_p \approx 938.272$ then $V_p \approx 199.108$. The mass / volume disparity means the density of the interior of the 2-sphere, Eq. (3), is a pure number > 1.

$$\rho = \frac{M}{V} = \frac{3\pi}{2}. \tag{3}$$

Hypermass (H) is the difference between mass and volume, as Eq. (4).

$$H = M - V. \tag{4}$$

Eq. (5) is the Hopf/hypermass signature (h-signature).

$$M = H\left(\frac{\rho}{\rho - 1}\right). \tag{5}$$

Σ rest mass h-signatures are functions of the proton and neutron masses, as Eqs. (6, 7, 8). [The results shown are due to the 2022 CODATA rest mass energies in MeV (ignoring uncertainties) for proton and neutron: $M_p = 938.272\,089\,43$, $M_n = 939.565\,421\,94$].[5]

$$M_{\Sigma^+} = (2M_p - M_n)\left(\frac{\rho}{\rho - 1}\right) \approx 1189.3712. \tag{6}$$

$$M_{\Sigma^0} = M_n\left(\frac{\rho}{\rho - 1}\right) \approx 1192.6546. \tag{7}$$

$$M_{\Sigma^-} = (4M_n - 3M_p)\left(\frac{\rho}{\rho - 1}\right) \approx 1197.5797. \tag{8}$$

Eq. (6) is a match for the Particle Data Group (PDG) current fit for M_{Σ^+} (1189.37 $\pm$0.07).[6] The PDG fit for M_{Σ^0} is 1192.642 $\pm$0.024, Eq. (7) is particularly close to Wang 1192.65 $\pm$0.020.[7] However, Eq. (8) is over four standard deviations adrift of the PDG fit (1197.449 $\pm$0.030). This value draws on three results. Schmidt (1197.43) and Gurev (1197.417) are too low to be the number derived here.[8, 9] Schmidt is an old paper from 1965, and Gurev is a proof of method. M_{Σ^-} is within one standard deviation of Gall (1197.532 $\pm$0.057).[10]

We consider what it means if Eqs. (9, 10) are Ξ rest masses. (Subtracting a volume indicates a mass is also a hypermass; a point we return to later).

$$M_{\Xi^0} = M_{\Sigma^0}\left(\frac{\rho}{\rho - 1}\right) - V_p \approx 1314.8104. \tag{9}$$

$$M_{\Xi^-} = M_{\Sigma^-}\left(\frac{\rho}{\rho - 1}\right) - V_p \approx 1321.0622. \tag{10}$$

M_{Σ^0} is within one standard deviation of the PDG fit (1314.86 $\pm$ 0.20) and close to Fanti (1314.82 $\pm$0.06)[11]. Eqs. (7, 8, 9, 10) resolve (11, 12, 13).

$$\frac{M_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}} = \frac{\rho}{\rho - 1}. \tag{11}$$

$$M_{\Sigma^0}\left(\frac{M_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}}\right) - M_{\Xi^0} - V_p = 0. \tag{12}$$

$$M_{\Sigma^-}\left(\frac{M_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^0} - M_{\Sigma^-}}\right) - M_{\Xi^-} - V_p = 0. \tag{13}$$

Eqs. (12, 13) have import for (22) later. But first, we are about to see why Eq. (11) is unsustainable. The present PDG fit for M_{Ξ^-} (1321.71 ± 0.07) draws on a 2006 study of 4.8k events from 1992-1995 data.[12] Faced with an unlikely nine standard deviation downward adjustment, Eq. (14) introduces the electron rest mass energy ($M_e = 0.510\ 998\ 950\ 69$ MeV) as a *fudge* factor. (Hint: it is not a fudge).

$$M^*_{\Xi^-} = (M_{\Sigma^-} + M_e)\left(\frac{\rho}{\rho - 1}\right) - V_p \approx 1321.7109. \tag{14}$$

Eqs. (7, 8, 9, 14) resolve (15). The equivalence ensures the individual formulae are not ad hoc.

$$\frac{M_e}{3\left(\frac{M^*_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}} - \frac{\rho}{\rho-1}\right)} = M_n - M_p. \tag{15}$$

At Eq. (15) the $M^*_{\Sigma^-}$ adjustment heads off the potential threat of infinity due to (11). Eq. (11) is revised as (16).

$$\frac{M^*_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}} - \frac{M_e}{3M_n - 3M_p} = \frac{\rho}{\rho - 1}. \tag{16}$$

If the M_e adjustment is applied to the Ξ_0 mass, we get Eq. (17).

$$M^*_{\Xi^0} = (M_{\Sigma^0} + M_e)\left(\frac{\rho}{\rho - 1}\right) - V_p \approx 1315.4591. \tag{17}$$

Eq. (17) entails a variation of (16) with $\frac{M_e}{3M_n - 3M_p}$ addended as (18) shows.

$$\frac{M_{\Xi^-} - M^*_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}} + \frac{M_e}{3M_n - 3M_p} = \frac{\rho}{\rho - 1}. \tag{18}$$

Eq. (19) reveals M_{Σ^+} is also a function of the other Σ and Ξ masses.

$$\left(M_p + \frac{M_e}{3\left(\frac{M_{\Xi^-} - M^*_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}} - \frac{\rho}{\rho-1}\right)}\right)\left(\frac{\rho}{\rho - 1}\right) = M_{\Sigma^+}. \tag{19}$$

Eqs. (15, 19) demonstrate Σ, Ξ particles formally belong to a single family of Hopf particles. The question is whether Ω^- also belongs. For Ω^- mass we take a lead from Eq. (20).

$$M_{\Sigma^0} = \left(\frac{3H_{\Sigma^+} + 2H_{\Sigma^-}}{5}\right)\left(\frac{\rho}{\rho - 1}\right) \approx 1192.6546 \tag{20}$$

The most recent PDG fit for the Ω^- mass is 1672.45 ± 0.29 MeV. Eq. (21) follows the template provided by (20).

$$M_{\Omega^-} = \left(\frac{3M_{\Xi^0} + 2M^*_{\Xi^-}}{5}\right)\left(\frac{\rho}{\rho - 1}\right) \approx 1672.4824. \tag{21}$$

The $\frac{3x+2x}{5}$ schema is indicative of two hypermasses resulting in the mass of a third particle. Eq. (21) is the second clue suggesting Ξ particles are the hypermasses of two heavier particles. A neutral particle 1668.9787 MeV and a negative particle 1677.7380 MeV. These masses fall within the uncertainties of a number of potential candidates, i.e. N(1675), N(1680), Λ(1670), Σ(1660), Σ(1670),[6] but the uncertainties are presently too wide to be definitive. The Ξ(1318) resonance is also a plausible candidate for $H_{\Omega^-} \approx 1317.5706$ with Ω^- the middle mass of a triple {1668.9787, 1672.4824, 1677.7380} and their hypermasses the Ξ triple {1314.8104, 1317.5706, 1321.7109}.

The $M^*_{\Xi^-}$ adjustment necessitated by Eq. (15) replaces volume V_p at (12, 13) with V_{Ω^-} at (22). The near equivalence is for MeV only.

$$M_{\Sigma^0}\left(\frac{M^*_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}}\right) - M_{\Xi^0} - V_{\Omega^-} \approx \frac{\rho}{\rho - 1}. \tag{22}$$

Helping Eq. (21) seem less ad hoc the close alignment with $\frac{\rho}{\rho-1}$ in MeV provides a scaling factor introduced at (23, 24).

$$SM = \left(\frac{\{M_x\}}{M_{\Sigma^0}\left(\frac{M^*_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}}\right) - M_{\Xi^0} - V_{\Omega^-}}\right)\left(\frac{\rho}{\rho - 1}\right). \tag{23}$$

$$SM = \left(\frac{\{M_x\}}{M_{\Sigma^-}\left(\frac{M^*_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}}\right) - M^*_{\Xi^-} - V_{\Omega^-}}\right)\left(\frac{\rho}{\rho - 1}\right). \tag{24}$$

SM scales rest mass values in any system of units to a number close to mass energy in MeV. Eq. (25), for example, returns the number ≈ 0.511 for the electron.

$$\left(\frac{M_e}{M_{\Sigma^0} \left(\frac{M^*_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}} \right) - M_{\Xi^0} - V_{\Omega^-}} \right) \left(\frac{\rho}{\rho - 1} \right) \approx 0.511. \tag{25}$$

The nucleon and electron masses are more accurately known in u. Using the 2022 CODATA adjustments (ignoring uncertainties), $M_n = 1.008\,664\,916\,06$, $M_p = 1.007\,276\,466\,5789$, $M_e = 0.000\,548\,579\,909\,0441$, Eq. (25) $= 0.511\,007\,366\,716$. With the kg adjustments we get $0.511\,007\,405\,8334$. The mass energy equivalent in MeV is $0.511\,007\,615\,2234$. These values are marginally high for the electron but SM is sensitive to its inputs. Using the proton and electron to dial-in the more uncertain neutron mass in u, Eq. (25) $= 0.510\,998\,950\,69$ when $M'_n = 1.008\,664\,915\,876\,394\,072$. The ultra precision is overdone but M'_n represents a downward adjustment within one standard deviation or less than one fifth of an electron-volt. Given the uncertainties, a neutron mass energy $939.565\,421\,76 \pm 0.000\,000\,06$ MeV resolves Eqs. (22, 25). However, if not a coincidence, the obvious question is why does SM favour millions of electron-volts? The clue is the million. A 10^6 factor immediately points to the difference in scale between SI and Gaussian systems.

Gaussian fields are part of a single mechanical framework sharing the same set of cgs dimensions. If mass is non-homotopic, a dimension independent of classical fields is needed. This is not available in the Gaussian single framework. The joule and kilogramme are also field units and not suitable. Unified atomic mass units (u) provide an independent option but the choice of the carbon 12 benchmark is arbitrary. It is the ampere that offers a non-arbitrary independent number. Whilst not an SI unit the electron-volt is an adjunct to the SI system due to the ampere and volt. If mass is measured as *no of elementary charge* $\times$ *electrical potential* where the number of charges is proportional to a magnetic force then **B**, **E** field densities are implicit. If we ask what does nature prefer? Unbalanced field dimensions seems the wrong answer. This leaves us to resolve non-homotopic mass with a Gaussian / electron-volt synthesis. To rescale electrical potential with balanced field dimensions Gaussian statV is first converted into volts, as Eq. (26).

$$1\ statV = \frac{299792458}{10^6}\ V. \tag{26}$$

When dimensions are cancelled, reference to SI values are eliminable. (27) walks through each step.

$$\begin{array}{lr}
\dfrac{299792458}{c}\ V & 1 \\
= 1\ V \cdot m^{-1} \cdot s & 2 \\
\times\ \ 10^{-6}\ N^{-1} \cdot s^{-1} & 3 \\
= 10^{-6}\ V \cdot N^{-1} \cdot m^{-1} & 4 \\
\times\ \ 1.60219 \cdot 10^{-19} A \cdot s & 5 \\
= 1.60219 \cdot 10^{-25}\ eV \cdot N^{-1} \cdot m^{-1} & 6 \\
\times\, 6.24151 \cdot 10^{18}\ eV & 7 \\
= 10^{-6}\ eV \cdot N^{-1} \cdot m^{-1} & 8 \\
\times\, 510998.95069\ eV & 9 \\
= 0.510\,998\,950\,69\ eV \cdot N^{-1} \cdot m^{-1} & 10 \\
= 0.510\,998\,950\,69 & 11
\end{array} \tag{27}$$

At line 1 of (27) the conversion factor 299792458 is already dimensionless but one that directly references the speed of light in SI units. Dividing by c excises this number, though it introduces the additional dimensions $m^{-1} \cdot s$, which have to be cancelled later. This explains the statV to volt conversion factor 10^{-6} introduced as an impulse at line 3. At line 4, line 2 dimensions are divided by $newton \cdot seconds$ from line 3. Line 5 introduces elementary charge in SI units, which converts volts to electron-volts at line 6. Line 7 leaves $A \cdot s = 1$ which ensures all trace of the SI value is erased when dimensions cancel. Line 8 is the conversion factor for mass energy that gives a number in reduced electron-volts. Line 9 introduces the electron mass energy but this could be any mass energy. The reduced number at line 10 is parsed in its base dimensions as follows.

$$\begin{pmatrix} no\ of\ charges & & electrical\ potential \\ moved\ for\ 1\ sec & \times & difference \\ = 1 & & \approx 0.511 \\ A \cdot s & & m^2 \cdot kg \cdot s^{-3} \cdot A^{-1} \end{pmatrix} m^{-2} \cdot kg^{-1} \cdot s^2. \tag{28}$$

(28) quantifies the work needed to move the electron per unit of mass energy. At line 11 dimensions are allowed to cancel. If this line of reasoning is correct SM

scales rest mass to a dimensionless magnitude for a generic system with balanced fields and an independent elementary charge.

There is one more particle we should at least mention in passing. The octet n, p, Λ^0, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^- forms the baryon eightfold way. This leaves Λ^0 still to be considered. Unfortunately an h-signature within one standard deviation has proved elusive. Eq. (29) is a simple relationship that gets close, and accords with the intuition Λ^0 is primarily related to other neutral particles.

$$M_{\Xi^0} - V_p = 1115.7029 \tag{29}$$

The octet might still be neatly tied together if Eq. (29) were the Λ^0 mass but the present PDG fit is 1115.683 MeV ± 0.006; a precise measurement based on 38k events.[13] This leaves a future PDG adjustment of 1115.7029 highly unlikely. However, we get within one standard deviation using M_n as the base.

$$M_n + \frac{M_{\Omega^-} - (V_{\Xi 0} + H^*_{\Xi^-})}{2} \approx 1115.68334. \tag{30}$$

As we lack an additional feature like a scaling factor to motivate Eq. (30) it remains ad hoc, leaving Λ^0 still to be convincingly combined with the Σ, Ξ family; so we close our limited survey of hyperon mass on a disappointing note.

In conclusion: evidence for a non-homotopic cause of mass is provided by rest mass energies for Σ, Ξ, Ω particles derived from h-signatures. The derived values are in the majority of cases exceptionally close to observation. It is anticipated future CODATA and PDG adjustments will favour the values derived here. The added possibility of reducing nine free parameters to two is further evidence for non-homotopy. The SM scaling factor is a novel development that stakes its claim as the dimensionless and natural scale for particle rest mass.

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