


The Multi-fold Least Action Principle, a Quasi Theory Of Everything

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Abstract:

The principle of least action is known to apply to many domains of Physics. Yet its justifications and derivations usually come from archaic classical principles like the Fermat or Maupertuis principles, or mathematical analytical frameworks used for classical mechanics like the Lagrange, Hamilton and D'Alembert formalisms. As is well known, it is widely applicable to other domains, without necessarily a clear justification for it, especially when applied to Physics where Lagrangians or Hamiltonians may not be defined or do not exist. In addition, the Feynman Path Integral formulation of Quantum Mechanics gave it a second life, and another way to justify its validity not only in classical, but also in quantum Physics. However, the conventional logic to reach these conclusions is at time circular, and the principle is applied without proof that it is applicable: it just works and everybody does it. It is this applicability that led us in the past to treat the Action path integral as God's equation.

The paper justifies the principle of least action and Feynman's path integrals through different physical reasoning instead of the traditional mathematical formalism, and on that basis we confirm its validity in most situations, even if Hamiltonians or Lagrangians may be shaky to define, or the system may be dissipative. We also explain when, and why, sometimes, it may not conventionally apply. The derivation uncovers different links between Entropy and Action, which also opens a different way to look at the origin of the probability distribution of even just a single lone particle; especially when complemented with the W-type multi-fold hypothesis, and makes some considerations on Feynman diagrams, instantons, renormalons and resurgence. Also discontinuities in the paths of the path integrals imply that spacetime is discrete, and non-commutative, and supersymmetry in flat spacetime is unphysical. Surfaceology, as optimized computation of the Feynman diagram is shown to trivially predict that all gauge theories double copy dual to gravity share the same forbidden scatterings.

Relying on a recently encountered paper, we show how the results of the multi-fold theory, in particular the multi-fold space time matter induction and scattering, built explicitly on the path integral formalism, its underlying principles, and the principle of least action, allows recovering almost everything in Today's physics. It leaves out essentially only some of the coupling constants and mixing angles/parameters. This goes a long way towards the ambition of a Theory Of Everything (TOE), even if not presented as usually expected. It maybe the best that can be achieved considering Gödel's incompleteness theorems.

Our justification of the principle of least action, and path integrals, and our work on multi-fold theory encounters relationships between Entropy and Action. We review these encounters, including how our early prediction that entanglement might be irreversible match recent published results that argue against the existence of a (second) Law of entanglement analogous to the second Law of Thermodynamics.

Also, from our approach, we obtain a formulation of the evolution of entropy, including for irreversible systems, which matches Onsager and Prigogine's famous models and principles.

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1. Introduction

To level the plain field, we do not want to assume that the reader is familiar or interested in the multi-fold theory. But we need to provide a minimum of background so that it will be possible to follow the reasoning when multi-fold considerations appear. Note that our derivation of the principle of least action does not require any such considerations.

An overview of the multi-fold theory [1,8-10,22,209,229,234-236] is presented in Appendix A. *Note added on December 12, 2024: In this paper references in italic were added on December 12, 2024.*

We are now ready to discuss the structure of this paper. We start with our justification of the classical principle least action, with a new derivation. Then, we derive in Quantum Physics why the uncertainty principle implies path integral, and the “quantum” version of the principle of least action. Interestingly, it relies on the introduction of Wick rotations [67-69], and related insights of the relationship between Thermodynamic entropy and (quantum) Action. Even if there is nothing really new here, we believe that the derivation is original, and introduces a different way to look at the relationship between entropy and action, which can also lead to a different take on the notion of probability distribution associated to wave functions. To the best of knowledge, we have never encountered such an analysis. We think that it motivates more fundamentally why all of Physics seems often correctly modeled by Actions and path integrals; unless when it can't, for other reasons as we will explain.

Note added on December 12, 2024: We discussed how path integrals amount to 2D random walks, something established early on by Feynman [1,215,243-245]. If the paths are discontinuous, it implies that spacetime is non-commutative, discrete [240,246], and supersymmetry is unphysical in a universe with a positive (dS) or null (flat) cosmological constant [190,240]. We have also shown that our universe must have a strictly positive cosmological constant [190].

Feynman evolves its path integral formalism to provide a perturbative QFT approach to the SM, starting from QED, with his Feynman diagrams [247-251]. We justify why the models remains interesting when the approach diverge as in non-perturbative phenomenon like the need to renormalize them, tunneling with instantons, renormalons, topological effects and strong coupling/interactions as for example in QCD with confinement as in [252-255] and references therein. This is in particular directly related to the theory of resurgence and the future for Feynman diagrams [256-258]. In particular we position asymptotic series, non-perturbative asymptotic series, amplituhedron, and surfaceology. We will show that with multi-folds, all gauge theories double copy dual to gravity trivially share the same forbidden scatterings.

After, we provide additional considerations on entropy versus action, including recovering the evolution of the entropy of systems that are irreversible, and/or out of equilibrium, as well as considerations on entanglement vs. the second law of Thermodynamics, by showing that equivalent versions of entropy may not always exist for entanglement, and that entanglement seems to include some irreversibility, as we already hinted in [1]. It includes a new insight on the evolution of the entropy of irreversible systems, and another justification for the arrow of time.

Armed with confidence in the least action principle, we refer to, and criticize where needed, paper [70], which derives most of Today's Physics from the principle of least action, and a few additional principles, for a total of nine principles, and we review some select topics. *Note added on December 12, 2024: On the other hand, we will repeat an early but controversial argument from us that, based on Gödel's incompleteness theorems [241], no full Theory of Everything is actually possible. See [242,264] and references therein. In the present case, formulating the complete TOE action may be an example of unprovable statement, or justifying the Action principle, if one couldn't rely on the derivation that we present here, could be another example. This paper strive to minimize the unproved aspects of former, and prove the Action principle, so that is no more an unprovable statement.*

Then, we discuss how the results and principles of the multi-fold theory allow us to reduce and evolve the number of basic principles proposed in [70]. In particular, it include relating the multi-fold no supra luminosity principle to discrete spacetime and quantum physics (and positioning for example with respects to instantons and tunneling), the multi-fold mechanisms to GR, dark matter and dark energy effects, and the multi-fold 7D space time matter induction and scattering to the SM symmetries [23,31,54,56,72,73], interactions, particles, and their masses as well as addressing many open issues with the SM and the Standard Cosmological model (Λ CDM) [1,4-40,43-66,136]. It qualifies it as a TOE as already proposed in [70], but, in our view, significantly upgraded in a multi-fold universe, (*Note added on December 12, 2024: while aligned with [242]*). And per [6], there are significant possibilities that our real universe is multi-fold.

We conclude with a discussion of some of the predictions provided in [70], especially in terms of New Physics and quantum gravity. *Note added on December 12, 2024: Based on [224], we know that gravity is quantum.*

2. Revisiting the Principle of Least Action

Since Newton proposed the foundations of modern Physics, it underwent several evolutions with more powerful mathematical and analytic frameworks to characterize the dynamics of systems. In particular, this led to the (classical) introduction of D'Alembertian, Lagrangian and Hamiltonian formalisms [74-76].

Underlying these different models appeared what is known as the principle of least action (originally proposed by Fermat in Optics then the Maupertuis principles in classical mechanics). The classical mechanics principle can be formulated in Lagrange mechanics, or as Hamiltonian principle [75], with the challenges that Lagrangian or Hamiltonians may not always exist [74-76]. A good historical overview is presented in [75]. Also, note that the action is more generically extremized, rather than minimized as is often presented, e.g. it can be a maximum, minimum or saddle point. In this paper we continue to use the terminology of least action, but it is with the understanding that we mean extremizing, including saddle points.

The principle is usually introduced mathematically as a result of the different Lagrangian and Hamiltonian formalisms, and applications of variational methods [74-76]. But other derivations from different (first) principles exist, freeing it to some extent from the not always guarantee of the existence of an Hamiltonian [1,74-76,77,78,79]. However, as discussed in these references, it does not mean that the formalism is always suitable: no Hamiltonian may exist, or no, or multiple, Lagrangians may exist or may not be known [1,80]. In particular, it is known that dissipative systems and systems with constraints which depend on the velocity, especially if associated to non-fundamental forces that depend on the velocity are challenging and sometimes not suitable, unless if it is possible to properly treat them as in [75-77]. As an example where it does not create problems, the Lorentz force of a charged particle in a magnetic field, also a force with a velocity dependency, is not an issue because it is a fundamental and non-dissipative interaction, as discussed for example in [75]. Examples where no Hamiltonian can be defined can be found in [74]. Generalized actions for dissipative systems that can be expressed with action-dependent Lagrangians ², see [78,79] and references therein. We will consider at this stage that any fundamental interaction has a Lagrangian. When it is not the case it is because of limitation of the model or because we are dealing with an EFT (Effective Field Theory) approach.

In the mathematical analysis of Lagrange and Hamilton, while quite aesthetic, one may lose the physical justification for the principle. And without the same formalism beyond classical mechanics, one could question if the principle is well motivated. Here, in this section, we try to provide a generic and more physical justification of

² With the caveat that the energy and momentum that results are not always accepted as physically meaningful [79].

the principle of least action (extremized action), without resorting to Lagrangian or Hamiltonian mechanics, and the knowledge / existence of a formulation of the Lagrangian or Hamiltonian.

Let us formally define the Lagrangian as the difference between kinetic energy³ (K) and other energies (U) of the system, all these being covered under the notion of potential energy, up to some constant; which is ok as Lagrangians are not unique anyway up to a total time derivative[75].

$$L = T - U \tag{1}$$

If a system is fully described and isolated (i.e., with conserved energy, or $\frac{d(K+U)}{dt} = 0$), any evolution of the system will be such that the path in phase space, followed by the system, exchanges back and forth as needed, kinetic energy for potential energy. Nothing really new here.

As a result, for a small variation of the system by varying a parameter $\delta\alpha$.

$$\delta \tag{2}$$

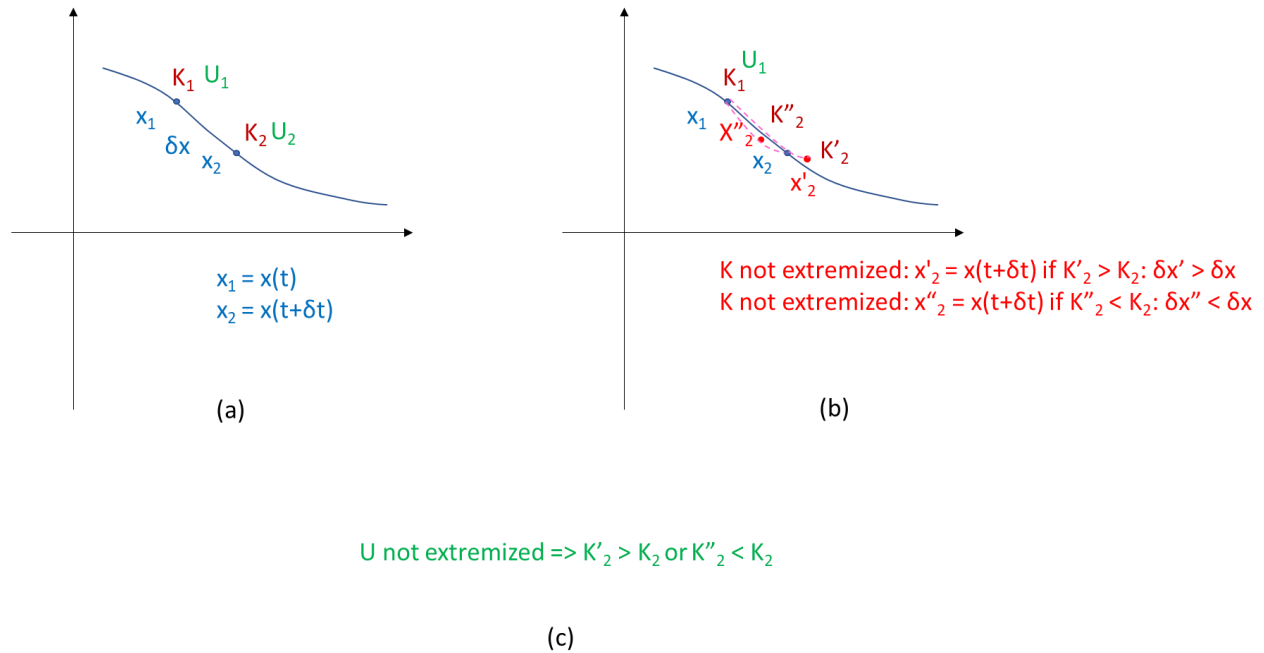


Figure 1: (a) shows a x to $x+\delta x$ path over δt . x is a 3D (or another dimension) vector. All paths between $(t,x1)$ and $(t+\delta t,x2)$ are considered under the variations δ . They all have same end points. (b) illustrates how two paths from varying a parameter $\delta\alpha$, dashed red if K is not extremized, $\delta K \neq 0$, and as a result, the path is not realizable. $x+\delta x$ and δt are small enough to ensure that the path is unambiguous. So being too short or too long is not compatible with the path. (c) uses (3) to show that $\delta K \neq 0$ puts us in the (b) case (for a system that conserves energy). Alternatively that would be different potential energy in the same position and system configuration.

If we now consider δL , we have:

$$\delta L = \delta K - \delta U = 2 \delta K = -2 \delta U \tag{3}$$

³ Kinetic energy in the context of Lagrangians and Hamiltonians is often denoted as T. We prefer to avoid this notation in this paper because of the potential confusions with temperature or Energy momentum (stress) tensor.

This can be seen in the phase space, or in the configuration space, as a path. Along a classical path,

$$\delta L = 0 \tag{4}$$

Indeed, as shown on Figure 1, if $\delta K \neq 0$, then with extra (not enough) kinetic energy, the systems can not evolve from x to $x+\delta x$ over δt . It would overshoot (undershoot). Equivalently, if $\delta U \neq 0$ on the infinitesimal path, we would revert to the previous case or, said differently, the potential energy would depend on the path between the end points, and one would have different potential energy options at a same location / for a same spatial configuration.

As a result, any completely described and isolated system follow the/a principle of least action.

If the system is dissipative, and therefore typically irreversible, not isolated, or with problematic constraints, the reasoning does not automatically hold. It can still be recovered with models accounting for the dissipation [74-79] or by modeling also the external world (e.g., all the particle involved in the dissipation like friction or irreversibility in heat exchanges). But the approach does not depend any more on necessarily holding a well-defined or formulated Hamiltonian or (single) Lagrangian. Also, if the model is effective or phenomenological, or involving phase transitions (e.g., Ising model) ⁴, the applicability depend on the suitability of the model [77,80,84].

Because only (classically) involving particles and classical fields, the Lorentz force example from above follows the least action.

However, as we defined it, i.e., just as the time integral over the path of $K - U$:

$$\delta \mathcal{S} = \delta \int_{t_1}^{t_2} (K - U) dt = 0 = \delta \int_{t_1}^{t_2} L dt \tag{5}$$

\mathcal{S} is the action. These quantities always exist classically.

With this simple derivation, we find a unique insight and intuition behind actions, Lagrangians and the principle of least action. When it exists, the Hamiltonian is a Legendre transformation away ⁵, and can be seen as a change of variable, that now considers the energy of the system, i.e., $K + U$, and leads to interesting geometrical modeling of mechanics, and anticipates quantum mechanics, where the Hamiltonian operator matches the energy of the system.

There is nothing new, or subtle, in our approach, yet we have not seen such arguments explicitly presented in the literature, without having to infer it by reading between the lines⁶. It seems missing to add intuition to such a fundamental principle.

With the caveats⁷ and limitations mentioned, the principle of least action derived as we propose, applies to all classical physics (if modeled down to its fundamental constituting particles), or with correct handling of special cases. It includes for example relativistic theory, curved spacetime and General Relativity (GR) for which we like to

⁴ High energy/ strong coupling interactions are also cases with no, or multiple, Lagrangians, but we are now talking of quantum systems.

⁵ While not needed for this paper, the Legendre transform is often not that well intuitively understood either. To remediate this, we would like to refer the reader to [81,82].

⁶ And even there, we have rather in mind Schwinger's variational principle, which is no more a classical discussion [83].

⁷ Especially in terms of weeding out the non-physical solutions that can occur for example in electromagnetism. *Note added on December 12, 2024: These are mathematical solutions that satisfy the variational equations but do not correspond to real, physically observable electromagnetic field configurations; these solutions might involve unrealistic field behaviors like infinite energy densities or singularities in specific regions of space, superluminal particle motion, or solutions with negative energy densities [1,217,259-263].*

refer to [75,85-88], even if conservation of energy is a bit trickier in GR as discussed in [1,89], and references therein.

With our definition of the Lagrangian, it, and the Action, can always be defined and computed, physically, even if not in a closed form, and the principle will hold for dissipative system if we also include the full system, or the heat reservoir / full external system, or if the dissipative system or constraints can be suitably modeled (e.g., mirror system of [74,75]).

3. Feynman Principle, Action Path Integrals, Quantum Physics and God's Equation

3.1 Quantum Systems

The fathers of quantum physics, like Heisenberg, Schrödinger (non-relativistic), and Dirac (relativistic), provided recipes for “quantizing” a theory, ultimately and typically starting from the Hamiltonian formalism. For QFT, with the second quantization, the Lagrangian then turned out to be even more convenient if it exists. Again, that may not be the case for strongly coupled, effective, phenomenological or phase transition models, where no or multiple Lagrangians may exist [1,80]. For QFT, see for example [144]. Note also the alignment of the principle of least action with Gauge symmetry, key to QFT and the SM [75,144].

The conventional derivation of the path integration and quantum action principle is especially well described in [74,75,90], and it is self-explanatory. Early in [1], we also summarize the idea.

However, we would like to continue the derivation, the way that we proposed in the previous section, and show how it can also lead the path integrals and the Feynman principle. Otherwise we feel that the description provided above would not be consistent, if contradicted as soon as the system is quantum, which every system is at some small enough spacetime scales. Of course, our ambition is not to produce a new treatise of Quantum Physics, so we will have some circularities, even if we try to minimize them. The conventional proof is found in [74,90].

So what happens when a system is quantum⁸?

Let us assume that the system is quantum because it must respect the Heisenberg uncertainty relations between conjugate variables or non-commuting operators [83,91]. These uncertainties, considered between time and energy, but one could also consider momentum (related to K) and position (impacting U), mean that, in the reasoning on section 2, even for a conserved system, additional energy $\delta\epsilon$ (positive or negative) can appear added to $\delta U + \delta K$, consistent with the Heisenberg uncertainty for δt . Therefore all possible ways to introduce $\delta\epsilon$ can be encountered and be valid, exploding what can be possible paths⁹, as illustrated in figure 2. In a probability distribution model, each path is possible with a certain probability. That probability specifies the contribution to the change to the probability distribution, and therefore the wave function.

⁸ Even if we acknowledge the incestuous circular arguments, we want to stick to the claims of [70], that we will discuss later, that the principle of least action, with a minimum action $\hbar/2$, contains Quantum Physics, something that [70] frankly does not concretely explain. Therefore we do not want to derive the principle following the approach [24,90], which acknowledges already too much of the quantum formalism.

⁹ Of course other considerations can filter out some path as proposed in [1], and at the difference of what others proposed, as also discussed with references in [1].

So, a priori, we do not have a principle of least action anymore, but a wide range of physically possible/allowed paths for any given δt . It seems that our reasoning does not work anymore: on most of the physically allowed and considered paths, the physical action is no longer expected to be extremized. Instead we have options for different and multiple paths with probabilities for each path.

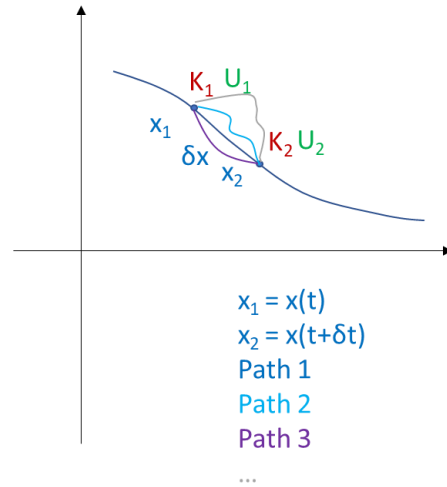


Figure 2: For quantum systems, multiple paths are allowed and contributing thanks to the uncertainty principle.

In a probability framework, it makes sense that the outcome is the sum of all the options weighted by the probabilities associated to each path (it is in fact equiprobable, which makes sense in the absence of more knowledge, but it implies correct renormalization¹⁰) times the amplitude evolution along that path.

If one neglects the concerns with the expansion of the universe, or the question of unitarity vs. hermiticity of the wave function (see [72,92,93]), we can easily accept that the evolution of the wave function is to be unitary. Following [83], it results that, by considering equations (1.8) and (2.14) in [83], with W in [83] being the action \mathcal{S} , and considering a δt infinitesimal time evolution as the different representations:

$$\delta \langle \zeta_1, \sigma_1 | \zeta_2'', \sigma_2 \rangle = \frac{i}{\hbar c} \langle \zeta_1, \sigma_1 | \delta \int_{\sigma_1}^{\sigma_2} L dt | \zeta_2'', \sigma_2 \rangle = \frac{i}{\hbar c} \langle \zeta_1, \sigma_1 | \delta \mathcal{S} | \zeta_2'', \sigma_2 \rangle \quad (6)$$

With the definitions of [83], this can be understood as the evolution of the amplitude of a state as we consider an infinitesimal time evolution δt state over a (classical / allowed) path. One then needs to combine all the different amplitude changes of the wavefunction, decomposed in the states associated to the paths, by summing / integrating over all the possible / allowed (classical) paths.

From this, knowing that (6) describes an infinitesimal operator, we recover Feynman's principle captured by Equation (7.7) in [75].

Note that (6), inspired by [83] names L and \mathcal{S} as respectively (a) Lagrangian and the action. We rather motivate it by the (classical) principle of least action: for the path(s) where $\delta \epsilon = 0$. On that path, the reasoning of section 2 can be repeated. It ensures that L and \mathcal{S} are correctly identified respectively as (a) Lagrangian and the action.

¹⁰ That is the typical partition function in path integrals by analogy to quantum statistical field theory.

[74,94] provides a very similar derivation based on Hamiltonian mechanics. As such it is restricted to cases where classical Hamiltonian mechanics works. The Hamiltonian represents the energy of the system¹¹.

We note that our proposed derivation of the principle of least action in section 2, automatically implies a path integral, without having to compromise too much in circularities about Schrödinger equation versus Dirac Equation (or Proca), or QFT, or knowing the action, or knowing or having existence of a Lagrangian (in our proof with our model it always exists), or of the Hamiltonian (Dirac's proof – we saw that Hamiltonians may not always exist).

Quantum superposition ensures that a quantum system can follow all the paths simultaneously.

As is well known, we recover the classical principle of least action when $\hbar \rightarrow 0$, with the dominating contributions that are not destructively interfering being limited to where the phase is extremized, i.e., path for which:

$$\delta S (\text{path}) = 0 \tag{7}$$

(7) is the true and rigorous derivation of the classical principle of least action. Of course, this coincides with no uncertainty principle, further confirming the tie to the approach of section 2.

Deriving more require some quantum physics formalism already in place and this is where there are some circularities. The objective here is not to rigorously resolve that, we may in the future. So we will assume that it is accepted that a system is characterized by a wave function, who evolution is the result of a unitary operator applied to the wave function and the wavefunction lives in a phase space that is an Hilbert space, where of course superpositions exist.

Non-physical paths, e.g., with spacelike portions would be forbidden with our proposal. This justifies the choices made in [1], versus other proposals in the literature that also support non-physical paths. Note that non-physical does not mean only classical. As is well known, instantons and quantum tunneling involves non classical paths that become physical with the uncertainty principle [252]. On the other hand supra luminous paths are not supported period. The approach of [1] is the correct one, and other analysis, or even practices of QFT are incorrect as discussed in [1]. They are only (usually) good approximations, which may support easier computations.

Recent works focused on mapping the most common paths of electrons in a superconductor and confirmed that they mostly follow, at the same time, the multiple paths of extremized action, as predicted [100-104].

We note that the relationship between uncertainty and non-commutativity [77,187,188] and the reasoning presented above recovers Quantum Physics of [77,83,184-186], core to DeWitt, Peierls and Schwinger's characterization of Quantum Physics / QFT [77]. The multipath analysis is therefore equivalent to Quantum Physics.

3.2 An Euclidian Detour

¹¹ This is assuming that the theory admits a Lagrangian and a Hamiltonian. We know that when the commutator of the involved variable is very large for a given theory, then it will not be possible to apply Dirac's approach, as no corresponding classical formulation exists, and no Lagrangian (or action) formulation may exist or be known [80]. The path integral is then taken as a principle, using the definitions of section 2, even if not well formulated, unless if no action can exist. With fundamental interactions and complete system, an action always exists, as we already explained. Multiple Lagrangians, dissipative, strongly coupled and phase transitions lead to further challenges as already discussed, but they are always due to limitations of the models, not incorrectness of the underlying foundations.

The approach of section 2, and analysis of section 3.1 that led to multiple paths being walked, can be looked at from a different perspective.

Let us consider a system at temperature T. Let us consider that the lowest energy state correspond to the path(s) of least action satisfying (7).

For any other path, it can be seen as a state with energy $\delta\epsilon$, above the lowest energy level. In general, this is a continuous spectrum.

In statistical physics, any such path has a probability defined by the maxwell-Boltzmann distribution [95], with a probability in:

$$\frac{1}{Z} e^{\frac{-\delta\epsilon}{kT}} \quad (8),$$

where Z is the partition function that normalizes the distribution.

If we model $\delta\epsilon$ as the energy received as Q, the heat from the outside to be in that state (path), and remember that for such a system, the entropy δS of following this path instead of the last action path is:

$$\delta S = \frac{Q}{T} \quad (9)$$

If we do a change of variable

$$\frac{1}{kT} \rightarrow \frac{it}{\hbar} \quad (10)$$

we recover the probability of each path as the effect of following the path expressed as (6). The change of variable is known as a Wick rotation [67-69]. [96] shows that such a process constructs a well-defined QFT, by ensuring that post Wick rotation the Wightman axioms are satisfied, which ensures proper and rigorous reconstruction of QFT.

We recover the path integral automatically.

The analysis captured in this section is not something that we have encountered so far in the literature. It matters because:

- It provides a new way to derive the path integral and principle of least action with essentially no quantum theory a priori assumptions beyond the uncertainty principle and linking wave functions or fields to probability distributions via the squared amplitudes, like Born did.
- It provides a unique insight into the relationship between entropy and action. Something that is conventionally known, but through different scenarios and reasonings. Here we see that the action is really reflective of the entropy associated to each path, even when considering a single particle.
- It also justifies the practice with path integrals to assume all paths equiprobable, and with a phase change impact¹².

The close link between the use of the certainty principle to justify the path integrals is also behind the fact that the path integral rigorously follows the uncertainty principle, as key consistency requirements as strongly harped on by

¹² That is different from extra paths as proposed with multi-folds, which are not resulting from the principle of least action algorithm, but instead extend the notion of path integrals with multi-fold mechanisms that have their proper kinematics and dynamics, and where paths can be added [1,9,10,218].

[93]. It is satisfied by the Feynman path integral, and by suitable fractional path integrals¹³ as discussed in [1,32] and reference therein. Fractional variational analysis is also well defined. See for example [139,140].

3.3 God's Equation

Despite our comments on the absence of Lagrangians, or the absence of Lagrangian or the presence of multiple Lagrangians, the action principle, or related path integrals apply to (most) fundamental physics (and many non-fundamental theories also), including Quantum physics, relativistic or not, QFT [143], Strings and LQG (Loop Quantum gravity. After all, the Standard Model (SM) is fully described by its, long, Lagrangian¹⁴. With Wick rotations, Statistical Physics and Statistical field theory follows a similar mathematical framework. When perturbatively expanded it leads to the Feynman diagrams and virtual particles, another concept directly related to the uncertainty principle.

Technically our model just continue to define it as the combination of $K - U$ integrated over paths in Hilbert space or configuration space. This is why, when learning about it while studying at University, the author decided to nickname Feynman's action based path integral as God's equation: it can explain and model everything; as long that we have a formulation for the action that is [264]. With this paper we have gone one step further, and generically proven the action principle.

Paths of extremized action will always be the main contributor to the path integrals, and so dominate the models of physics: the principle of least action is at the core of the path integral formalism. For example, papers on black holes quantum extremal surfaces, as discussed in [97-99] and references therein, obtain results with approximations that essentially amount to sticking to the path of least action (and compute it in Euclidian space, post Wick rotation).

Recent works focused on mapping the most common paths of electrons in a superconductor and confirmed that they mostly follow, at the same time, the multiple paths of extremized action, as predicted [100-104].

Because the reasoning that we present is quite generic, we know the path integral can be expanded to many other situations as generalized in [1], way beyond what is conventionally motivated as discussed for example in [105]. This is important. Rigorously, path integrals are functional integrals (See [1] for references therein). Yet the applicability of action path integrals, and the least action principle, beyond fundamental interactions and basic quantum physics, has not really been provided. It's rather a generalization of Feynman's work, hoping that it applies, and it always seems to apply so far. With our approach in sections 2 and 3, we feel that we have actually provided a way stronger justification for applying path integrals say to topology, group theory, spacetime (e.g., LGQ [130-132,133], or below), black holes (See references for quantum extremal surfaces and the information paradox in [97-99] and references therein), and other domains, i.e., to domains without a model otherwise justifying path integrals. At least we have convinced ourselves of the suitability of the principle and approach, which has proven itself again and again.

With this, we can justify [41,42], where it is shown using path integral and Wick rotation that a De Sitter isotropic, homogeneous universe with a small positive constant is the most probable. Our interpretation of the wick rotation may also help others feel more comfortable with what the Wick rotation and resulting entropy characterizes.

¹³ Otherwise, the small scale reconstruction of a multi-fold spacetime via random walk of massless particles could be in trouble [1,32].

¹⁴ Even if the widely marketed version of it is plagued with excessive, and incorrect, additional + h.c. terms.

In any case the result reinforced our analysis that a multi-fold universe must be of positive cosmological constant [1,11-15].

In any case, even if there were concerns with our derivation, it matches the widely accepted results, and remaining repeatable in other circumstances, broadens and reinforces the universality of the path integral and principle of least action.

Note added in December 13, 2024:

3.4 Path Integrals and Discontinuous Paths

In [a50], we show, in particular relying on [a56], that discontinuous paths of the path integrals imply a discrete and non-commutative spacetime.

Because of the above, and of the fact that paths in path integrals appear as random walks [a53-a55], any Quantum Physics theory implies a discrete and non-commutative spacetime.

[190,240] then imply that our universe, and any spacetime with a positive, i.e., possible also null, cosmological constant, including therefore our real universe [190] or multi-fold universes, cannot be supersymmetric, nor host to broken supersymmetry [a50], confirming the non-physicality of superstring, supergravity and M-theory [1,8-20,22,190,209,215,240].

3.5 A view on non-perturbative approaches and resurgence

3.5.1. Perturbative Feynman Diagram

Feynman diagram provide a systematic way to compute interaction/scattering amplitudes. They are a perturbative S matrix method [254].

Feynman diagrams are one of the main tool of QFT and experimental physics, surpassed only sometimes by non-perturbative techniques like lattice QCD when available, and perturbative methods are not applicable, too inaccurate, or too difficult. They are part of any QFT class and book. For a detailed derivation see for example [a57-a64,a70]. A simplified description is at [a68]. [a59] shows exactly the link to path integral [247-254,260].

Feynman introduced his diagram for QED; however, its applicability extends to the whole SM (perturbative), and beyond the SM [265,266]. For some good examples, see the discussions in references of [60], where perturbative and lattice QCD are involved.

By definition, Feynman diagrams conventionally mean a perturbative method.

In [63], we provide considerations on Feynman diagrams in conventional and multifold universe, and how they relate to classical vs. quantum contributions.

3.5.2 Divergent and Non-perturbative Feynman Diagrams

When considering the path integral, we are dealing with a non-perturbative formulation of QFT, or EFT when limited to a certain domain and energy scale. The path integrals sum over all possible, and physical [1,215], paths. It means the perturbative and the non-perturbative paths.

Non-perturbative paths are well characterized by perturbative Feynman diagrams, just don't ask if the series converges. This may require renormalization steps to make it so. However, in a perturbative approach to a quantum theory, we may also have to explicitly include topological solutions associated $n \rightarrow +\infty$, never encountered for finite n , where h is the order of the diagrams up to which the series is computed. These topological solutions are non-perturbative. Such non-trivial topological solutions, and their variations, must be included in the path integral computations [252]. They include what are known as instantons [254,267], with a physical interpretation, in the case of tunneling e.g., between semi classical vacua, provided in [252]. They are located at a specific spacetime point, hence the name. Their inclusions are key to computing a correct path integral, and shown to be sometimes key contributors to the renormalization processes, recovering underlying symmetries, or resolving anomalies, as discussed in [254] and references therein. However they do not solve all the problems, and they do not help with QCD, because the contributions of the instanton saddle points become large, divergent and misbehave.

Ensuring the convergence of the perturbative model, also lead to encounters of renormalons [254,255], which correspond to real axis pole (positive). [254] discuss low energy (IR) normalons encountered in QCD, that prevent good perturbative low energy computations in QCD.

However, we should also note that the path integral is usually seen to decrease in a rough Borel way till a certain order then resurge beyond.

But, In [258], it is shown, based on Dyson's paper [269], that the radius of convergence of the Feynman diagram perturbation theory is 0, making it an asymptotic series that does not converge, but that nevertheless can be made useful [268]. It does not seem to make too much physical sense considering how physically intuitive the series are, how good they are at some experimental precision confirmations, and key to our arguments about the fact that particles and virtual particles are equal citizens to fields in Quantum Physics [1,164,215], or even discussing them in [17].

The key is to see them as such asymptotic series with physical meanings

We model path integral as: (see instantons as in [252])

$$PI = PI_{\text{perturbative}} + PI_{\text{non-perturbative effects}} \quad (11)$$

PI denotes the action path integral of the system.

The effects associated to $PI_{\text{perturbative}}$ are to be complemented with non-perturbative paths (topological instantons, problematic renormalons, confinement, etc.). Note that the latter does not mean unphysical paths. We continue to argue that we need to follow the caveats and re-definitions of PI from [1,215], and forbid, for examples, paths with spacelike portions. Yes, it changes some conventional QFT results, which are now rather seen as approximations of what they should be.

(11) can be rewritten as:

$$PI = PI_{\text{converging series}} + PI_{\text{rest}} \quad (12)$$

Where $PI_{\text{converging series}}$ covers the (finite) set of Feynman diagrams that behave like a converging Borel series [254]. As explained above it will be up to a certain order, while the rest is accounted for by PI_{rest} , and can be seen as

representative of all non-perturbative and divergent effects (not handled by renormalization). Typically we could just decide to use, as first term, all the terms up to the largest order where they continue to decrease in value, or converge as a convergent Borel transformation. The rest, possibly with renormalization, is put in the second terms. It includes divergent terms of the asymptotic series, non-perturbative contributions, at least when pathologic like divergent/misbehaving instanton contributions in QCD.

Because of our derivation of the action path integral so far, Physics demands that physically correct actions (and Lagrangians / Hamiltonians), capture all the aspect of the theory, for EFT within the domain of validity, and then correctly “transitions” to another model. Non-perturbative effects appear as extra paths, like for example instanton for an infinite order, and never as a finite order. Therefore, it makes sense to expect that P_{rest} is indeed also carrying physical information, about everything that is non-perturbative. It should not be disregarded, and there must be some framework under which that information can be extracted, or the path integrals extremization can be modeled. Such a prediction is confirmed with the resurgence program [253,270-272], using E. Calle’s framework [270]. We may publish a paper in the future on this, linking it more closely to multi-fold considerations.

On the other hand, $PI_{\text{converging series}}$, can go to relatively high order before diverging as argued in [258], e.g., 100th order for QED (way lower when we have string coupling as in QCD). 100th order is way below many of the computation done these days, even for QED (up to 5th order), and therefore unaffected by this. QED remains the most precise model of the world out there. And with our explanation, à la (11) and (12), we know that we can just pursue computing $PI_{\text{converging series}}$, for situations that are perturbatively acceptable, and model, or approach non-perturbative aspects differently. Meanwhile the resurgence program may soon give us, ways to bridge the two.

The analysis shows that non-perturbative effects are due to collective effects, strong coupling, quantum effects like renormalons/divergences, tunneling/instantons, confinement and topological effects. It can guide modeling and computations, in terms of what to use where, and what can be neglected. Maybe it will provides recipes on how to process, modify and combine the lower order Feynman diagrams, or add quantum effects to account for them, a bit as in [63]. While resurgence is not yet giving us this, we suspect that it could be a possible outcome, if one push to the extreme that everything is based on particle, except maybe when it is about solitons and collective effects. But even then, we see this as a very promising direction for QFT.

It is important to again repeat that none of the PI terms (in (11) and (12)), include unphysical paths, like paths with space like segments, as discussed in [1,215].

3.6 Feynman Diagrams and Efficiency

That being said, computing higher order is a nightmare, pushing to the extreme current computing capabilities. There are reasons why lattice QCD is only slowly coming up as still suffering at just approximatively answering questions as in [64,69].

The reason, why so many Feynman diagrams must be computed, comes the symmetries of the theory. While the theory enable easy extraction of Lagrangians (and action as a result) or Hamiltonians, and sometimes guarantees their uniqueness (pattern-wise, e.g., for Yang Mills / QCD), which explains why Group theory is so central to particle physics, by defining these symmetries and their implications, Feynman diagrams are such a (beautiful and intuitively great) pain, because their expansions up to a given order hides these symmetries that can only be recovered when all orders are considered.

Approaches have been explored to reduce the amount of Feynman diagrams to consider, or the complexity of their computations, for example by adding helicity considerations, and other simplifications as hinted in [258].

Another envisaged approach to compute scattering amplitudes, relies on the recent notion of amplituhedron, a geometrical object, which has been discovered by chance, and shown to contain the amplitude of Feynman diagrams in ways easier to compute [273,274]. More precisely, the volume of the amplituhedron corresponds directly to scattering amplitudes, providing a more efficient way to compute these quantities without explicit reference to spacetime dynamics, or virtual particles. The amplituhedron is defined within the mathematical structure known as the positive Grassmannian, which consists of configurations where all Plücker coordinates are positive. However the approach is limited in terms of the QFTs to which it can apply. QFTs must be supersymmetric and the approach is the most developed and understood for the planar $N = 4$ supersymmetric Yang-Mills theory. It renders the tool useless as such QFT theory is unphysical [190,202,240], except as inspiration, motivation and guidance to look for another geometrical approach, and maybe in the context of the AdS/CFT correspondence conjecture, which again does not apply to our universe and concrete computations of Feynman diagrams/scattering amplitudes.

Interestingly, the inspiration may have paid off with the very recent introduction of surfaceology as another way to efficiently compute the amplitude for all-loop, all-genus, all-multiplicity Feynman diagrams by drawing the relevant diagram on a surface, then following predefined recipes [275-283].

It is a brand new approach, without much review papers, or tutorials. The only intuitive material that we know is [288]. The best tutorials are [275,283]. Surfaceology expresses scattering amplitudes for colored/Yang Mills theories, including colored fermions, as integrals over combinatorial objects constructed from surfaces decorated with kinematic data [283]. They amount to grouping differently the Feynman diagrams in ways that avoid having to list them all. The grouping is in fact not how it is derived. Instead, the derivation shows integration on all possible prescribed curves recovers all the associated Feynman diagrams and their amplitudes. This approach provides a more efficient alternative to the traditional Feynman diagram method going to complexity in $O(n^2)$ instead of in $O(4^n)$, for n involved particles.

We disagree with statements in say [283] or even [275] that the results would mean that spacetime is not a relevant physical concept. Instead we see this as combinatorial rearrangements in ways that does not involve spacetime considerations: surfaceology still carries the Feynman integral and diagram spacetime and unitary considerations, and therefore underlying dependencies. We think it is a better view than the click bait of [283] claiming independence from spacetime.

It is closely related to the double copy duality discussed in [17,63,170,176,284-287] and references therein, as all the theories that support double copy, share amplitude zeros/minima, i.e., forbidden scatterings. Because of the established link between gravity, and the multi-fold theory [17,63,170,176], we may follow-up with future papers, giving insights. We know that multi-fold gravity results from the entanglement of pairs of Yang Mills / SM particles, hence the double copy gravity between Yang Mills (gauge theories) and gravity. Then, double copy dual theories in turn share special properties through the surfaceology. In the context of [253], that result is trivial. Indeed, in multi-fold theory, the gravity being the interaction with a pair of entangled particle and antiparticle, with opposite momentum, a forbidden scattering with one of them is a forbidden scattering with both. Zeros are conserved and zeros are common to all the duals (gauge theories) to gravity, all sharing the same result. Another sign of the suitability of the multi-fold theory and its multi-fold mechanisms [1,8-10,22,191,209,215,229]?

4. Physical Action and Entropy

4.1 Analogies

The approaches that we followed in section 3, to derive the path integral formalism, illustrate another interesting point: the analogy and relationship between physical action and entropy.

These analogies are well known. See for example [106,107], and references therein. For example, consider for example the use of Wick rotations and Euclidian path integrals in the replica trick and island formula approach to black holes, and their quantum extremal surfaces mentioned earlier with [97-99,107] and references therein.

In fact, [107] computes equivalently the entropy of paths, versus their action, and also derives Schrödinger's equation in reversible, non-relativist cases¹⁵.

Accordingly, up to a sign, a pure convention, a change in the entropy is equivalent to a change in the physical action.

The derivation of [107], is based on knowing already the path integral formalism¹⁶, and recovers our interim results derived by our model. We however identified explicitly that the distribution of paths, thermal, based on Maxwell-Boltzmann, is the ensemble to consider behind the stochastic behavior of quantum physics, at least when it comes to the evolution of the wave function¹⁷.

4.2 Information Theory

Information theory also introduces the notion of information entropy as defined by channel. It is the core concept behind encoding, compression, transmission capacity, error corrections and encryption, and now even more important for quantum computing [109-111], relating entropy and Qubit / entanglement

In the references found in [112-114], entropy and action are linked to the learning process of AI systems; also a topic related to information theory. The relationship is analogous to what we have encountered here, when modeling for example neural network as optimization of physical systems and encountering in the processes the same equations as Thermodynamics, GR and Quantum physics [115-119].

For now, we refer to the papers referenced here for the details. We encountered the information entropy in [1,7,97-99,112-114], as also mentioned in the next section.

4.3 Entanglement entropy

In the quantum world, von Neumann introduced the Von Neumann entropy or entanglement entropy [137,138].

We encountered it in [120] and in [97-99], along with the references therein, in particular in terms of the relationships between Thermodynamic entropy, entanglement / von Neuman entropy, coarse grained entropy and fine grained entropy, the latter two refer to observational entropies or the lack of knowledge of a system by performing measurements of the system [189]. We refer to these papers for the details.

The references in the previous paragraph discuss the use of path integrals with Wick rotation (and the replica trick) to rely on entropy estimations along a path of least action (i.e., the dominant contributions to the path integrals) to

¹⁵ And it does that assuming a discrete spacetime, something predicted by the multi-fold theory [1,31,32,58].

¹⁶ And so again missing out on the insight of our derivation of section 4.

¹⁷ In a multi-fold universe, the W-type multi-fold hypothesis then adds to the stochastic interpretation of quantum physics and the wavefunction [108].

compute the evolution of black holes or universes spacetime, and potentially resolve, among others, the black hole information paradox. It show cases the link between all these notions of entropy, including back hole entropy [1,141,142], spacetime entropy [1,7,120,134], entanglement entropy, information entropy [7], and the Rényi entropy formula [144] (see [97-99]). [7] even includes a plausible justification for this relationship, where spacetime may even relate to the exchange of information.

However, a recent result is worth mentioning. It does not seem that there is a second law of entanglement analogous to the second law of thermodynamics: entanglement seems to be irreversible [121-122]. In the multi-fold theory, we predicted such irreversibility due to multi-fold deactivation during disentanglement [1,8-10,22,a19,a25,a39]. We had already encountered such analogies to Thermodynamics in [14].

4.4 Expanding on the Second Law of Thermodynamics, Including Irreversible Systems

Can our reasonings of section 2 and 3 be used to derive new or richer results for the entropy or the evolution of systems modeled by it?

Sections 2 and 3 imply that there is an optimization principle behind the second law of thermodynamics:

$$\Delta S \geq 0 \tag{13}$$

With equality for reversible systems/evolutions.

Indeed, the thermodynamic systems and the surroundings interacting with, if any, is driven by quantum physics (or classical). When reduced to its fundamental components, it is therefore always ultimately a system that can be modeled with action path integrals and (7).

To satisfy ourselves that this is probably true, consider a system characterized by (13).

Let us first remember that only reversible systems are on their own easily characterized by an action, as we have seen that dissipative systems, i.e., irreversible require modelling the external system too or other tricks.

If we have:

$$\delta S = 0 \tag{14}$$

In the phase space we have for the density function the Liouville theorem that says [75,123]:

$$\frac{d\rho}{dt} = 0 \tag{15}$$

The (13) applied to the $\frac{dS}{dt}$, with S as Gibbs Entropy formula [124]¹⁸, averaged over p and x, implies:

$$\frac{dS}{dt} = \frac{d(-k \int \rho \ln(\rho) dpdq)}{dt} = -k \int (\ln(\rho) + 1) \frac{d\rho}{dt} dq dp \Rightarrow \tag{16}$$

¹⁸ It can be seen as an information entropy for the system. It is beyond this paper to discuss the implications of this; but one day we may be tempted to go into more details. *Note added on December 12, 2024: See also [223], where we discuss the link between information and mass.*

(15), the Gibbs entropy, provides the thermodynamic entropy of a system characterized by a statistical distribution of statistical microstates¹⁹. Then, (15) allows the system to be modeled by a Hamiltonian and as a result, to be associated to an action and Lagrangian [125]. In other words, we show that (10) implies:

$$\delta \mathcal{S} = 0 \quad (17)$$

for some function \mathcal{S} , that is the action.

An alternate, more detailed and formal proof is provided in [125], where the entropy of a system is shown to be a Noether invariant, associated with invariance for an infinitesimal non-uniform time translation $\eta \hbar / kT$, at least for isolated systems or in adiabatic evolutions [126]. In this case, the translation is strangely reminiscent of (10). Our physical interpretation is as follows: *when we perform infinitesimal shifts in time, we encounter the uncertainty principle, even for classical systems where one may then also blame measuring inaccuracies and number imprecisions [301-306] (even if aspects of this are controversial and not endorsed yet by us), and we are subject to the multiple paths of the path integral. Reversible systems are systems that are not impacted by them, and so revert to the principle of least action (as for classical systems).*

If the system is not reversible, we know that direct use of the action of the system does not work, that is because dissipation or entropy change in the external system takes place so that:

$$\delta \mathcal{S}_{\text{total}} = \delta \mathcal{S} + \delta \mathcal{S}_{\text{ext}} = 0 \quad (18)$$

That total system is always modeled by an Action and principle of least action based on our results so far. It is also what we mean when speaking of a complete model. Tricks amount to approximating (18).

If we can assume that the external system is fully modellable as classical or quantum particles and fundamental interactions, then $\delta \mathcal{S}$ follows also a principle of least action.

In other words, while we may not know it, there is an underlying extremization principle for $K - U$.

That assertion, a theorem in fact, is in fact in agreement with several situations encountered with irreversible systems and Thermodynamics. Let us see how.

Consider α one of the parameters characterizing the system

$$\delta_{\alpha} \mathcal{S} = \delta_{\alpha} \int (K - U) dt = 0 \quad (19)$$

$$\delta_{\alpha} \frac{d\mathcal{S}}{dt} = \delta_{\alpha} L = 0 \quad (20)$$

With:

$$\mathcal{S} \leftrightarrow S \quad (21)$$

Where the double arrow means equivalent mapping up to some constant,

Then

$$\delta_{\alpha} \frac{dS}{dt} = 0 \quad (22)$$

(22) is equivalent to:

¹⁹ It is an average: fluctuations are not captured with this reasoning.

- The Onsager maximum principle for the difference between $\frac{dS}{dt}$ and the Onsager dissipation function that describes the evolution when the system is displaced from equilibrium [126-128]. It is known to be equivalent to the symmetry condition imposed by microscopic reversibility²⁰. Onsager symmetry principle is at the center and the starting point of all models of irreversibility.
- Prigogine's minimum entropy production principle, which states that stationary non-equilibrium states satisfy a minimum entropy production (while compatible with its constraints) [129].
- Of course reversible systems satisfy (22)

From a statistical physics point of view, it makes sense, without even having to model the system. *If the system is to is away from its entropy maximum, it will statistically first occupy the microstates that occupy the largest volume and therefore indeed maximize the rate of change of entropy.*

Our result asserts that (22) is a very generic theorem, and that it can extend to any system which can be completely described or complemented. There are more generic irreversible situations than just Onsager's and Prigogine cases. In all these cases, the production of entropy is extremized, and it correspond to a principle of least action²¹. Interestingly, it means that an irreversible system tends to evolve following the steepest gradient (climb, not descent) of its entropy, or said otherwise, to change as fast as possible its entropy. Again this theorem is general, generic, and adds a dynamic behavior to what is typically known at the second Law of Thermodynamics.

With respect to [125], fundamentally, what we have done when

$$\Delta S > 0 \tag{23}$$

is apply the reasoning to an updated system (complete, with external or with a trick) to be able to derive the same result (17), even with (23), then separate the action into external and system action, and get modified action \overline{S} that follows:

$$\delta \overline{S} = 0 \tag{24}$$

This should be valuable to any physicist, chemist or engineer dealing with irreversible systems or systems away from equilibrium. We are not sure how well the dynamic aspect is understood.

5. Multi-fold theory and Path Integrals

Even if we have been mentioning considerations, or lessons learned, from our work on the multi-fold theory, sections 2 to 4 apply to conventional Physics, i.e., without assumptions of being in a multi-fold universe. In this section, and in sections 7 and 8, we will add what the multi-fold theory allows us to relevantly conclude so far.

The multi-fold theory relies on the path integral formalisms, and a few principles enumerated in [1,8,9,10,22, 209,214,215,218,225,229] to open some paths available to the path integral in the presence of entanglement between systems. It is a modification of spacetime with multi-fold as extension linking the entangled particles. Multi-folds appear only when we respect the non-hierarchical principle for entangled systems: the entangled systems have been in contact, or they have been in contact with an entangled intermediary in the past.

²⁰ i.e., the analog to our completeness or external modeling statements.

²¹ In fact, it may be suitable for an additional theory associated to the second Law of Thermodynamics.

When the entangled systems are real, attractive effective potential appear between them. It is the result of these extra paths and the proposed multi-fold mappings mechanisms [25,26]. When considering virtual particles emitted by an energy source, these effective potentials amount to gravity, and can recover GR at larger scales [1,6,22].

The multi-fold theory is an example where we do not have the full formulation of the Lagrangian or Action, but it is built on the path integral formalism, adding the multi-fold mechanisms. It is unclear if a concrete closed form Lagrangian or Action formulation will ever be found for it.

To a large extent, the multi-fold theory is really taking path integral seriously, which include stating that paths are required to support entanglement effects, and path, aka interactions, are a form of communication (not necessarily exchanging messages, we don't want to violate other no-go theorems), that, per [5,6], create spacetime extensions: the multi-fold that results into gravity / gravity-like effects. Now the paths create spacetime to support them, a step further in the formalism of the path integral; acceptable and consistent if we elevate the formula to "God's equation". The theory is self-consistent: per section 3, fluctuations may include new topologies of spacetime or fluctuations outside spacetime. Per the principle of least action, these topologies prevail, when activated by entanglement of two systems, initially in local interaction.

The associated space time is, at Planck scales, discrete and mainly 2D growing to 4D, during which it is Lorentz invariant, constructed fractal or multi-fractal by random walks and non-commutative [1,4,16,28,31,32,72,240].

The W-type multi-fold hypothesis [108] then suggests that multi-folds may also support instant jump of the particle (system components) across the support of its wavefunction, and thereby justifying the Born rule and the meaning of the wave function. Doing so also across paths, it explains why all paths are actually simultaneously used by the system: a particle will randomly jump across its wavefunction support with the probability the square of the amplitude of the wave function at that location and that is what explains the jumps across path, while traveling all paths simultaneously.

6. Review of the Paper [70] that Allegedly Explains All of Physics

This present paper was inspired by reading the preprint [70]²². It proposes 9 principles that would explain/justify most of Today's Physics. We will not try to discuss all of them just the issues or clarifications that we believe should be made.

- No issues with $L(1)$ ²³, the principle of least action. We have discussed it in the previous sections. We believe that we have significantly expanded the analysis.
- $L(2)$ the principle of no supra luminosity is also well established, especially in a Lorentz symmetric universe. It is at the core of the multi-fold theory [1,8-10,22,209,215,229].
- $L(3)$, about maximal force, seems to be a particular focus, or pet project, of the author. It may not be the best criteria to select. Instead, the Thermodynamic derivation of GR [135] ($L'(3)$) is certainly more appealing as would be the Lagrangian formulation of GR [85-88] ($L''(3)$).

²² We are aware of the tendency of the author of [70] to push pet projects (e.g., maximal force), we know, we are guilty as charged with the Multi-fold theory. But it means that we do not necessarily agree with all the assertions or derivations. We like the overarching approach and want to repeat it for multi-fold theory, while also commenting at time on the paper from a conventional point of view, i.e., non multi-fold.

²³ $L(x)$ refers to row or line x among the nine lines discussed by [70] and presented in its table 1.

- L(4) directly results from the principle of uncertainty [91] with its dimension and the reasoning of sections 2 and 3, However, the minimum action quanta is $\hbar/2$, as also encountered differently in [106]²⁴. As we already mentioned, when $\hbar \rightarrow 0$, we recover classical physics. As this is so closely linked to the uncertainty principle, indeed, as already justified, Action quantization amounts to quantum physics. We believe that section 3 also goes a long way to show how quantum behaviors result.
- L(5) is confirmed by sections 3 and 4, but we have greatly expanded the implications, to explain the Wick rotation, and motivate the statistical behavior of multiple paths, as well as to irreversible Thermodynamics. The minimum value is obvious from the notion of bit or Qubit and information entropy.

It is worth noting that L(1), L(4), L(5), conventional result from the principle of least action, as should a rephrased L'(3) (L'(3) indirectly by recovering GR or from black hole entropy, as detailed for example in the reconstruction algorithms discussed in [1,6,215]). L(2) imposes the Lorentz symmetry on Lagrangians and actions. L(6) to L(9) are much less interesting as [70] solely indicates that these are open issues to complete the characterization of modern Physics. It gives a succinct list of what is missing: all is not that bad. And yes, L(6) and L(7) (+ L(2) and L(3)) provide the SM interactions + GR.

One could probably argue that the open ideas are disappointing, but at least [70] compiles what still needs to be characterized to get to a TOE.

[70] also provides some predictions based on the analysis above. Before discussing these, we want to present how we see the 9 rows/lines (L(i)) evolve in a multi-fold universe.

7. Multi-fold Contributions to a Least Action Driven TOE

[1,8-10,22,209,215,218,229] describes the core multi-fold theory, including L(2), and brings in path integrals as in L(4) and L(5), no supra luminosity as in L(2), and other principles like non-hierarchy of entanglement, and multi-fold tenancy models [1,9-10].

Interestingly, [1,6,215] recovers GR, and multi-folds seem encountered in GR at Planck scales [1,6], this covers L(2). The multi-fold spacetime is discrete, which implies also the quantization of the action L(5) in [106] and (*).

In section 1, and in Appendix A, we present some highlights of the multi-fold theory and the resulting SM_G , the standard model with gravity effects non-negligible at its scale, and how the approach seems to be able to contribute to solutions to multiple open issues with the SM and Λ CDM (the standard cosmological model [136]) [1,4-40,43-66]. Among those:

- The 7D multi-fold space time matter induction and scattering model allows us to encounter most of the SM particles, and their mass, and one may conjecture the ability to similarly extract all their other quantum numbers [23,31,54,56,72,73]. This addresses L(8) and part of L(9).
- The same mechanisms allow us to justify the SM (or SM_G) symmetry: $U(1) \times SU(2) \times SU(3) \times SL(2,C)$ [23,a32], matching the gravity and SM symmetries, and defining the 4 fundamental interactions, therefore recovering L(6), L(7), and L(8).

²⁴ (*) Note also that, as discussed in [106], a discrete spacetime also leads to such a result. As we mentioned, a multi-fold universe is discrete [1,8-10,22,209,215,229,230]. In fact, if we hadn't looked up the units of the uncertainty principle, we would have used the discrete spacetime argument to obtain that limit, with the view that $K - U$ is bounded by c and the minimum length as is the integration over the minimum time.

- A justification, or contribution for dark matter [1,35-38,40,43], or dark energy effects [1,27,34,43], including comments on web versions at [8].

With this, we can cover a lot of L(6) to L(9) except maybe for mixing angles and coupling constant. However, neutrino mixing could be handled by non-commutativity considerations as discussed in [1,31,72,73].

Note that we have shown possibilities to get these values. Systematic work to extract has not taken place. It is for future work²⁵.

8. About the [70] Paper's Predictions

Adding our multi-fold considerations, we have opinions and agree²⁶, or disagree, with some of the predictions provided in [70]:

- Prediction 1 is aligned with the Trans Planckian Censorship Conjecture in [135].
- Prediction 2 is to be more carefully interpreted. Clearly in a multi-fold universe, spacetime is discrete [1,4,16,28,31,32,72,230]. However, if we interpret [70] as saying that, if discrete, its discreteness is not observable, then we agree maybe with different scale considerations: its discreteness is not observable at the SM scales and above. In a multi-fold universe, we would prefer to state that multi-fold spacetime is discrete (and generated by random walks, (multi)fractal, Lorentz invariant and non-commutative [1,4,16,28,31,32,72,230]), but at larger scales where it is 4D, one does not notice its discreteness, albeit it has significant implications [65,a1,16,230]. [6,230] hints very strongly that our real / GR-based universe is discrete.
- Prediction 3 is directly aligned with our desert of new fundamental particle and no new Physics, other than the Ultimate Unification [1,48,59-62,64,65,72,99]. Although we note that the paper choices of L(7), L(8), and L(9) [70] implies such conclusions (circular arguments), while in our case, these conclusions are derived.
- Prediction 4 matches the model of multi-fold gravity, SM_G , UU and 2D spacetime that we encountered with the multi-fold theory: gravity, SM_G , UU and discrete 2D spacetime emerge from entanglement and multi-folds [1,6,8-10,22,24,190,230]. So yes quantum gravity is in our view fully defined, once we have the multi-fold mechanisms and principles, and GR remains valid till way small scales resulting in particular in the SM_G . We do not think that [70] was envisaging this interpretation though.
- Prediction 5 matches [6] that encounters multi-folds at Planck scales, and that GR and multi-folds continue to apply to the smallest scales, and spacetime continue to apply. However, in our case spacetime becomes 2D, at smallest scales and discrete. It's true that nothing in the reasoning of [70] mandates 4D. We may not read the prediction of [70] correctly. However, we also interpret the inclusion of k as representative of section 3 as well as capturing the essence that spacetime results, from random walk by massless Higgs bosons and that particles result from patterns or condensations of Higgs bosons [1,4,16,72-73,230].
- Prediction 6 is confusing us, in terms of the actual reasoning behind it. We would bluntly reject it based on our analysis of prediction 5 (Higgs boson behind particles and spacetime) and section 3.

²⁵ And depends on setting up a suitable team to do so.

²⁶ By this mean that we agree with the implications, not necessarily the way that they are phrased, or derived, in [70]. It rather means that in the multi-fold theory, we reached the same conclusions.

10. Conclusions

This paper started from [70] to provide the multi-fold variation of it, where we show that the multi-fold theory can explain quantum physics, relativity, Quantum gravity and potentially much of the SM / SM_G , as well as help with Λ -CDM. Across all related physics, the path integral and principle of least action reigns. In fact the multi-fold theory and its mechanisms are built on the path integral.

Inspired with how [70] attempts to summarize the whole of physics today with 9 principles, we decided to elevate the path integral / principle of least action to the same level and show how it can explain almost everything in a conventional and in a multi-fold universe. Furthermore, we know from [6] and other works compiled at [8], that our real universe may be multi-fold.

What can be explain brings us closer to the ambition of a Theory of Everything (TOE); hence the title. And the ability of path integrals to model most of this leads us to persist in naming the action path integral as God's equation²⁷.

We discussed some recent developments of the Feynman diagrams both in terms of resurgence and surfaceology. In particular we showed that the multi-fold theory trivially implies that all gauge theory double copy dual to gravity share the same forbidden scatterings, a result recently conventionally encountered. We see this as, again, a good sign for the multi-fold theory.

In that context, we managed to provide a new and more physical, by opposition to purely mathematical / analytic and classically mechanical, derivation of the principle of least action, which can then be used more broadly, when analytic mechanics does not model the system. We also used it to provide two new derivation of the path integrals. These are more physical and general than the derivation usually encountered that typically are circular with quantum mechanics and limited to it. Again, our derivation allow us to justify the reliance on path integral across Physics, where, in our view, the mechanical derivation may not really rigorously offer a foundation, unless if we invoked that it would be an example of non-provable statements foreshadowed by Gödel's incompleteness theorems [241,242,264]. With our derivation, which is no more the case and the incompleteness theorem should rather extend to ultimately proving the right TOE action. This may change if we can actually find it in the future.

Our work led us to also link physical Action and entropy. On one end to show that the stochastic interpretation of quantum physics and the wavefunction have a direct relationship and a thermodynamic analogy. On another end to derive a least action principle for the second law of thermodynamic beyond non-dissipative / reversible systems. Our result encountered the extremum principles proposed by Onsager and Prigogine for particular use cases involving irreversible systems, and a new theorem about the entropy evolution along the greatest entropy rate of increase for irreversible/out of equilibrium systems.

Appendix A. Overview of the Multi-fold Theory

In a multi-fold universe [1,8-10,22,209,229,234-236], gravity emerges from entanglement through the multi-fold mechanisms. As a result, gravity-like effects appear in between entangled particles [1,24,25], whether they are real or virtual. Long range, massless gravity results from entanglement of massless virtual particles [1,26]. Entanglement of massive virtual particles leads to massive gravity contributions at very small scales [1,27]. It is at the base of the

²⁷ Without implying or denying the existence of any God, what God would be or whose religion would relate. The statement is more aspirational, and purely symbolic.

E/G Conjecture [24], and the main characteristics of the multi-fold theory [22]. Multi-folds mechanisms also result in a spacetime that is discrete, with a random walk fractal structure and non-commutative geometry [6,32], in a range of spatial scales, which is Lorentz invariant and where spacetime nodes and particles can be modeled with microscopic black holes [1,4]. All these recover General Relativity (GR) at large scales, and semi-classical model remain valid till smaller scale than usually expected. Gravity can therefore be added to the Standard Model (SM) resulting into what we define as SM_G : the SM with gravity effects non-negligible at its scales. It can contribute to resolving several open issues with the Standard Model, and the standard cosmological model, without New Physics²⁸ other than gravity [1,4-40,43-66,71-73,92,97-99,108,112-114,120,135,136,145-183,190-230,232]. These considerations hint at an even stronger relationship between gravity and the Standard Model, as finally shown in [23]. It leads us to consider that the multi-fold theory gives good insight to conventional Physics, that our real universe may be well modeled as a multi-fold universe [1,4-40,43-66,71-73,92,97-99,108,112-114,120,135,136,145-183,190-230,232], something that we have done in this paper.

Among the multi-fold SM_G discoveries, the apparition of an-always in-flight, and hence non-interacting, right-handed neutrinos, coupled to the Higgs boson, generated by chirality flips by gravity of the massless Weyl fermions, induced by 7D space time matter induction and scattering models, and hidden behind the Higgs boson or field at the entry points and exit points of the multi-folds. Massless Higgs bosons can be modeled as minimal microscopic black holes mark concretized spacetime locations. They can condensate into Dirac Kerr-Newman soliton Qballs to produce massive and charged particles below the energy scales of the multi-fold electroweak symmetry breaking [1,4], and as random walk patterns above these scales [1,29,36,37], thereby providing a microscopic explanation for a the multi-fold kinematics and dynamics and associated unconstrained Kaluza Klein mechanisms [23,33,34,63,71,99,191,217], Higgs driven inflation [1,27], the electroweak symmetry breaking, the Higgs mechanism, the mass acquisition [139], and the chirality of fermions (and spacetime); all resulting from the multi-fold gravity electroweak symmetry breaking [1,4,17,23,29,32-34,63,66,98,99,72,202]. The multi-fold theory has concrete implications on New Physics like supersymmetry, superstrings, M-theory and Loop Quantum Gravity (LQG) [1,8-21,8,167,190,212,230].

Multi-folds are encountered in GR at Planck scales [5,6] and in Quantum Mechanics²⁹ (QM) if different suitable quantum reference frames (QRFs) are to be equivalent relatively to entangled, coherent or correlated systems [7]. This shows that GR and QM are different facets of something that they cannot well model: multi-folds [1,62,170,176].

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²⁸ Conventional physicists may argue it is New Physica. We consider that it isn't because no new particles or interactions are introduced. We just add gravity, ss we know it should be, and multi-fold mechanism and let conventional Physics unfold with the considerations. It does change conventional results or explanations, usually with same observables, and it does live in a discrete spacetime etc. as a result of conventional analysis of these consequences. We also do not cover stable field effects like Skyrmions [252], that we prefer to see as a collective effect for the theory. Beside the SM particles, there are other collective solutions/solitons in gauge theories, they have behaviors as particles but they are quasi particles composed of collective effects of a large set of particles. We see them as Qballs or patterns that can appear by multi-fold space time matter induction and scattering, under specific circumstances, nothing more. They are topological solitons and can appear in BEC, as expected with massless Higgs boson condensates [289,300].

²⁹ Standing in for Quantum Physics in general.

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