Unified Gravity Through the Repulsion Graviton Space Model

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Abstract

The Repulsion Graviton Space Model (Re:GraviS Model) proposes a unified gravitational theory that explains galactic dynamics, cosmic accelerated expansion, and black hole interiors without relying on dark matter or dark energy as separate entities. This model asserts that **"Gravity arises as an entropic-driven repulsion of space countering the spherically symmetric repulsive force exerted by gravitons on space"**, leading to scale-dependent modifications of the gravitational potential that naturally reproduce observed phenomena. At galactic scales, graviton-induced spatial repulsion accurately explains the flattening of galactic rotation curves. The proposed gravitational potential demonstrates strong agreement with high-resolution observational data of galactic rotation curves, providing a physically consistent alternative explanation without invoking dark matter. At cosmological scales, the same repulsive mechanism drives cosmic accelerated expansion, with the derived energy density quantitatively matching observed dark energy values. This suggests that the repulsive force of gravitons acts as dark energy, contributing to the large-scale repulsive dynamics of the universe. In extreme gravitational fields, the equilibrium between graviton repulsion and spatial collapse prevents the formation of classical singularities within black holes. Instead, a quantum spatial region emerges, where gravitons act as interdimensional information carriers, resolving the information paradox and providing a unified explanation for quantum effects near singularities. The model predicts finite core radii, eliminating the problem of infinite density and curvature. These results demonstrate that the proposed framework consistently describes Einstein's field equations at small scales, cosmic expansion at large scales, and black hole environments under extreme conditions. By redefining gravity as an entropic-driven repulsion of space countering the repulsive force of gravitons, this work challenges the limitations of conventional gravitational theories and provides a groundbreaking pathway toward a quantum theory of gravity that unifies relativity and quantum mechanics.

Keywords: Gravity, Gravitons, Quantum Gravity, Dark Matter, Dark Energy, Galactic Rotation Curve, Cosmic Expansion, Black Holes, Black Hole Singularities, Repulsion

1 Introduction

The persistent mysteries of modern physics—dark energy, dark matter, and black hole singularities—pose significant challenges to our understanding of the universe. General relativity, as one of the most successful theories in physics, explains gravity as the curvature of spacetime caused by matter and energy. However, at quantum scales and on cosmological scales, its predictions diverge from observational realities, necessitating modifications or extensions to the theory. Repulsion Graviton Space Model (Re:GraviS Model) is proposed as a unified gravitational framework to address these discrepancies, offering explanations for galactic rotation curves, the accelerating expansion of the universe, and the finite nature of singularities.

1.1 Challenges in Galactic Rotation Curves

The observed flatness of galactic rotation curves is among the most compelling challenges to general relativity. While the Newtonian potential predicts declining orbital velocities at large distances, observations show that velocities remain nearly constant beyond the visible matter distribution [1, 2]. This discrepancy has traditionally been addressed by introducing dark matter, an unseen substance that interacts gravitationally but not electromagnetically. However, the Re:GraviS Model offers an alternative explanation by incorporating a modified potential driven by graviton-induced spatial repulsion. The model predicts the flattening of rotation curves without invoking dark matter, using a parameterized form of the potential that aligns with observational data [3].

1.2 The Accelerating Universe and Dark Energy

The discovery of the accelerating expansion of the universe [4, 5] led to the concept of dark energy, accounting for approximately 68% of the universe's total energy density [6]. In the standard Λ-CDM model, this phenomenon is explained by a cosmological constant (Λ), which acts as a uniform energy density permeating space. However, the nature of Λ remains enigmatic, raising questions about its origin and magnitude [7]. The Re:GraviS Model introduces a modified gravitational potential with a logarithmic term, derived from graviton effects, that naturally produces a spatially dependent repulsive force at large scales. This mechanism offers a physically grounded explanation for dark energy as the cumulative effect of graviton-induced repulsion, predicting an energy density consistent with observational estimates.

1.3 Black Hole Singularities and Finite Radii

General relativity predicts the formation of singularities—points of infinite density and curvature—within black holes [8]. While singularities signal the breakdown of classical physics, they also indicate the necessity for a quantum theory of gravity. In the Re:GraviS Model, the interaction between graviton-induced repulsion and spatial entropy-driven counterforces establishes an equilibrium that prevents the formation of zero-dimensional singularities. Instead, finite radii for black hole cores are predicted, providing a resolution to the singularity problem and potentially explaining quantum effects within black holes. This framework aligns with recent efforts to resolve the black hole information paradox through quantum gravity [9].

1.4 Repulsion Graviton Space Model Framework

In this study, it is assumed that the repulsive force exerted by gravitons on space acts spherically symmetrically, and the model is constructed based on this assumption.

The Repulsion Graviton Space Model (Re:GraviS Model) is grounded in a modified gravitational potential, given by:

$$\Phi(r) = -\frac{GM}{r} - \frac{GMR^2 g_{\mathrm{rp}}^{-1}}{2} \ln\left(\frac{R^2 + r^2}{2R^2}\right).$$

where the first term represents the classical Newtonian potential and the second term introduces a graviton-driven repulsion. Here, g_{rp} is the graviton repulsion constant that characterizes the efficiency of the repulsive force, with a unit of $[\mathrm{m}^3]$. Empirically, $g_{\mathrm{rp}} \approx 10^4$, determined from galaxy rotation curve fittings. This potential describes a seamless transition between local gravitational interactions and large-scale repulsive effects, addressing phenomena across multiple scales.

1. **Small Scales (Solar System):** The Newtonian term dominates, ensuring consistency with classical tests of general relativity, such as perihelion precession and light bending [10].

2. **Intermediate Scales (Galaxies):** The logarithmic term becomes significant, explaining the flattening of galactic rotation curves without requiring dark matter [3].

3. **Large Scales (Cosmology):** The logarithmic term drives spatial repulsion, naturally accounting for the accelerating expansion of the universe, with a predicted energy density matching that of dark energy [4, 6].

4. **Extremely Strong Gravitational Fields (Black Holes):** The balance between graviton repulsion and spatial entropy prevents infinite curvature, leading to finite singularities with calculable radii.

1.5 Objectives and Scope

This study aims to establish the Re:GraviS Model as a unified gravitational framework capable of explaining:

1. **Galactic Rotation Curves:** Reproducing observed data without invoking dark matter.

2. **Cosmic Acceleration:** Predicting dark energy density consistent with observational estimates.

3. **Black Hole Singularities:** Providing finite radii and resolving the singularity problem.

The subsequent sections present a detailed derivation of the modified potential, its theoretical implications, and comparisons with observational data.

2 Theoretical Framework and Derivation of the Re:GraviS Model

2.1 Gravitational Dynamics and Limitations of the Newtonian Potential

The classical Newtonian potential, given by:

$$\Phi_{\text{Newton}}(r) = -\frac{GM}{r},$$

has been instrumental in describing gravitational interactions on small scales, such as within the Solar System. However, on larger scales, such as galaxies and the universe, the limitations of this potential become evident. Specifically:

1. **Galactic Rotation Curves:** The Newtonian potential predicts a rapid decline in orbital velocities with increasing radius, inconsistent with the observed flat rotation curves of galaxies [1, 2].

2. **Cosmic Acceleration:** The potential lacks any mechanism to produce a repulsive force, necessary to explain the observed accelerating expansion of the universe [4, 5].

3. **Black Hole Singularities:** At extreme gravitational fields, the potential diverges, leading to infinite density and curvature, which is physically untenable [8].

These challenges highlight the need for a modified gravitational framework that extends the Newtonian potential while remaining consistent with general relativity and observational data.

2.2 Re:GraviS Model: Modified Gravitational Potential

To address these issues, the Re:GraviS Model introduces a modified potential:

$$\Phi(r) = -\frac{GM}{r} - \frac{GMR^2 \cdot 10^{-4}}{2} \ln\left(\frac{R^2 + r^2}{2R^2}\right).$$

2.2.1 Structure of the Modified Potential

The Re:GraviS potential consists of two terms:

1. **Newtonian Term** $-\frac{GM}{r}$**:**
This term governs gravitational interactions at small scales, ensuring consistency with classical tests of general relativity, such as perihelion precession, light bending, and the Shapiro delay [10].

2. **Logarithmic Term** $-\frac{GMR^2 \cdot 10^{-4}}{2} \ln\left(\frac{R^2+r^2}{2R^2}\right)$**:**
This term introduces a distance-dependent repulsive force, significant at larger scales. It reflects the influence of gravitons, conceptualized as inducing repulsion against space.

2.2.2 Key Properties of the Potential

(1) Small Scales ($r \ll R$): For small distances, the logarithmic term becomes negligible:

$$\Phi(r) \approx -\frac{GM}{r}.$$

Thus, the Re:GraviS potential reduces to the classical Newtonian form, ensuring that the model accurately describes Solar System dynamics.

(2) Large Scales ($r \gg R$): For large distances, the potential simplifies to:

$$\Phi(r) \sim -\frac{GMR^2 \cdot 10^{-4}}{2} \ln\left(\frac{r^2}{2R^2}\right).$$

This logarithmic term dominates, introducing a repulsive force that counteracts gravitational attraction. The repulsive nature aligns with the observed phenomena of flat galactic rotation curves and cosmic acceleration.

2.3 Derivation of the Gravitational Dynamics

The modified potential's implications for gravitational dynamics can be explored by calculating the forces it produces. The force corresponding to $\Phi(r)$ is given by:

$$F(r) = -\frac{d\Phi(r)}{dr}.$$

2.3.1 Newtonian Term

The derivative of the Newtonian term is:

$$F_{\text{Newton}}(r) = -\frac{d}{dr}\left(-\frac{GM}{r}\right) = \frac{GM}{r^2}.$$

2.3.2 Logarithmic Term

The derivative of the logarithmic term is:

$$F_{\log}(r) = -\frac{d}{dr}\left(-\frac{GMR^2 \cdot 10^{-4}}{2} \ln\left(\frac{R^2 + r^2}{2R^2}\right)\right).$$

$$F_{\log}(r) = \frac{GMR^2 \cdot 10^{-4}}{2} \cdot \frac{2r}{R^2 + r^2}.$$

Thus, the total force becomes:

$$F_{\text{total}}(r) = \frac{GM}{r^2} + \frac{GMR^2 \cdot 10^{-4} \cdot r}{R^2 + r^2}.$$

2.3.3 Potential Term

To reproduce the Re:GraviS potential, the scalar potential $V(\phi)$ is chosen as:

$$V(\phi) = -\frac{GM}{r} - \frac{GMR^2 \cdot 10^{-4}}{2} \ln\left(1 + \phi^2\right),$$

where $\phi(r)$ is related to the distance r by:

$$\phi(r) = \frac{r}{R}.$$

2.3.4 Modified Einstein Equations

The inclusion of the scalar field modifies the Einstein tensor $G_{\mu\nu}$ on the left-hand side of the field equations. Assuming a static, spherically symmetric spacetime with metric:

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2),$$

the Einstein equations become:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{df}{dr}\right) - \frac{2}{r^2}f(r) + 8\pi G\left(\rho_m + \rho_\phi\right) = 0,$$

where:

- ρ_m: Matter density.
- ρ_ϕ: Energy density of the scalar field, given by:

$$\rho_\phi = \frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + V(\phi).$$

2.3.5 Field Equation Solution and Connection to Re:GraviS Potential

By solving the coupled scalar field and Einstein equations, the effective gravitational potential $\Phi(r)$ emerges as:

$$\Phi(r) = -\frac{GM}{r} - \frac{GMR^2 \cdot 10^{-4}}{2}\ln\left(\frac{R^2 + r^2}{2R^2}\right),$$

recovering the Re:GraviS potential. The first term originates from the classical matter distribution, while the second term arises naturally from the scalar field dynamics.

2.4 Implications of the Field Equations

1. **Small Scales ($r \ll R$):** The scalar field's contribution becomes negligible, and the system reduces to classical general relativity, ensuring consistency with Solar System tests.
2. **Intermediate and Large Scales ($r \sim R$ to $r \gg R$):** The scalar field induces a repulsive force proportional to $\ln(R^2 + r^2)$, explaining the flattening of galactic rotation curves and the accelerating expansion of the universe.
3. **Extreme Gravitational Fields (Black Holes):** The interplay between attractive and repulsive forces prevents the formation of singularities, resulting in finite radii for black hole cores.

2.5 Summary

The Re:GraviS Model establishes a unified gravitational framework by extending the Einstein field equations to include a scalar field ϕ, representing graviton-induced spatial repulsion. Through these equations, a modified gravitational potential is derived:

$$\Phi(r) = -\frac{GM}{r} - \frac{GMR^2 \cdot 10^{-4}}{2}\ln\left(\frac{R^2 + r^2}{2R^2}\right),$$

which seamlessly integrates gravitational phenomena across multiple scales. The model demonstrates:

1. **Consistency with Classical Tests:** At small scales ($r \ll R$), the potential reduces to the Newtonian form, ensuring agreement with Solar System observations such as perihelion precession and light bending.
2. **Galactic Rotation Curves:** At intermediate scales ($r \sim R$), the logarithmic term dominates, naturally explaining the observed flattening of galactic rotation curves without invoking dark matter.
3. **Cosmic Acceleration and Dark Energy:** At large scales ($r \gg R$), the repulsive force induced by the logarithmic term accounts for the accelerating expansion of the universe, with a predicted energy density consistent with observational estimates of dark energy.
4. **Black Hole Singularities:** In extreme gravitational fields, the balance between graviton repulsion and spatial entropy prevents the formation of infinite singularities, resulting in finite radii for black hole cores.

The scalar field dynamics, governed by the potential $V(\phi)$ and incorporated into the Einstein field equations, provide a theoretical foundation for these phenomena. This unified framework not only reconciles gravitational observations across scales but also offers new insights into the nature of dark energy, galactic dynamics, and black hole interiors, addressing some of the most profound mysteries in modern physics.

3 Observational Applications of the Re:GraviS Model

3.1 Galactic Rotation Curves

3.1.1 Background

The observed flatness of galactic rotation curves has long defied explanations solely based on visible matter [1, 2]. While the dark matter hypothesis posits an unseen component to account for the discrepancy, the Re:GraviS Model offers an alternative explanation by incorporating a repulsive term in the gravitational potential.

The Re:GraviS potential predicts the rotational velocity as:

$$v(r)^2 = \frac{GM}{r} + \frac{GMR^2 \cdot 10^{-4} \cdot r^2}{R^2 + r^2}.$$

At large radii ($r \gg R$), the second term dominates, leading to a flattened velocity profile:

$$v(r) \sim \sqrt{\frac{GMR^2 \cdot 10^{-4} \cdot r^2}{R^2 + r^2}}.$$

3.1.2 Fitting to Observed Data

To validate this model, we applied it to a sample of galaxies from the SPARC dataset [2], which provides high-resolution rotation curves for 175 galaxies. Nonlinear least-squares fitting was employed to optimize the parameters $\alpha\,[\mathrm{m^2\,s^{-2}}] = GMR^2 \times 10^{-4}$ and $R\,[\mathrm{kpc}]$ [3].

Key Results:

1. **Parameter Ranges:**
    - $\alpha \sim 1000$ to 100000, showing a strong Pearson correlation coefficient of 0.956 with the observed rotational velocities [3].
    - $R \sim 0.5$ to $5\,\text{kpc}$, representing the scale of graviton effects [3].
2. **Adjusted R^2 Values:** The fitting results to the 175 galaxy rotation curve data from the SPARC dataset showed that the average adjusted R^2 was 0.802, while the median adjusted R^2 was 0.964 [3]. This indicates that the Re:GraviS potential accurately reproduces the observational data.
3. **Residual Analysis:** Residuals were randomly distributed with a mean near 0, confirming the absence of systematic errors and the robustness of the model [3].

3.1.3 Implications

The Re:GraviS Model not only reproduces galactic rotation curves without invoking dark matter but also provides a physical interpretation of α as a measure of graviton concentration and R as the scale of graviton effects. This interpretation bridges the gap between galactic dynamics and quantum gravity.

3.2 Graviton-Driven Spatial Repulsion

3.2.1 Definition of the Repulsive Force

The dynamics of galaxies and large-scale structures can be influenced by repulsive forces originating from graviton-space interactions. To model this effect, we define the graviton-driven spatial repulsion $F_{\text{rp}}(\alpha)$, which does not interact with matter but acts solely on space, consistent with established theoretical frameworks describing gravitons as massless particles that interact indirectly through spacetime curvature [11, 12, 3]. This repulsion depends on the graviton concentration parameter α and is expressed as:

$$F_{\text{rp}}(\alpha) = \alpha \cdot g_{\text{rp}},$$

where:

- α represents the graviton concentration, with units $[\text{m}^2\,\text{s}^{-2}]$,
- g_{rp} is the graviton repulsion constant, empirically determined to be $g_{\text{rp}} \approx 10^4$ from galactic rotation curve fittings, with units $[\text{m}^3]$.

The dimensionality of $F_{\text{rp}}(\alpha)$ is then calculated as:

$$[F_{\text{rp}}(\alpha)] = [\alpha] \cdot [g_{\text{rp}}] = [\text{m}^5\,\text{s}^{-2}].$$

This unique dimensionality reflects the spatial and dynamical scaling properties of graviton-driven repulsion. The term $[\text{m}^5]$ indicates the spatial extent of the repulsive influence, while $[\text{s}^{-2}]$ represents its dynamical contribution. These properties suggest that graviton-driven repulsion acts across various scales, influencing galactic dynamics and potentially contributing to phenomena such as the flattening of galactic rotation curves and cosmic accelerated expansion.

3.2.2 Connection to Higher-Dimensional Theories

The dimensional characteristics of $F_{\mathrm{rp}}(\alpha)$ resonate with higher-dimensional gravity theories, such as the Randall-Sundrum model [13] and the ADD model [14]. In these models, gravity propagates into higher dimensions, while other forces remain confined to a four-dimensional brane. This framework predicts modifications to gravitational interactions at different scales, which align with the spatial scaling behavior exhibited by $F_{\mathrm{rp}}(\alpha)$. These characteristics suggest that gravitons may interact with extra-dimensional space, serving as carriers of information and facilitating interactions beyond the four-dimensional spacetime. This interplay indicates that graviton-driven dynamics could contribute to the preservation of information across spacetime, offering intriguing implications for the resolution of longstanding gravitational puzzles.

3.2.3 Observational Relevance

The introduction of $F_{\mathrm{rp}}(\alpha)$ provides a framework for explaining the scaling behavior of galactic rotation curves without invoking additional dark matter components. The parameter α is optimized based on observational data, and its interaction with g_{rp} offers a new perspective on the spatial distribution of gravitational effects. Further exploration of this model's predictions may deepen our understanding of large-scale dynamics and the role of graviton interactions in cosmic phenomena.

3.3 Dark Energy and the Re:GraviS Framework

3.3.1 Introduction: Revisiting the Cosmological Constant

The discovery of the accelerating expansion of the universe [4, 5] has been a cornerstone of modern cosmology. This phenomenon is often attributed to the cosmological constant Λ, introduced in Einstein's field equations, which corresponds to a uniform energy density known as dark energy. The energy density of dark energy is conventionally expressed as:

$$\rho_\Lambda = \frac{\Lambda c^2}{8\pi G},$$

where Λ is the cosmological constant, c is the speed of light, and G is the gravitational constant. Observations, including those from the Planck satellite [6], suggest $\rho_\Lambda \sim 10^{-26}\,\mathrm{kg/m^3}$, aligning closely with the theoretical predictions of the Λ-CDM model.

However, the cosmological constant problem arises due to the discrepancy between this observed value of ρ_Λ and the much larger vacuum energy density predicted by quantum field theory. This discrepancy, sometimes described as a mismatch of up to 10^{120} orders of magnitude, highlights the profound theoretical challenges in reconciling vacuum energy predictions with observations [15, 7]. Within the Re:GraviS framework, this discrepancy can be addressed by considering how graviton-induced spatial repulsion transitions between small and large scales, providing a seamless explanation for the observed value of ρ_Λ.

3.3.2 Modified Field Equations in the Re:GraviS Model

The Re:GraviS model extends the Einstein field equations to include scalar field contributions, allowing the cosmological constant Λ to emerge naturally as a result of graviton-induced repulsion. The modified field equations are written as:

$$\nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} \nabla^\lambda \phi \nabla_\lambda \phi - g_{\mu\nu} V(\phi) + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu},$$

where:

- ϕ: Scalar field associated with gravitons.
- $V(\phi)$: Potential of the scalar field, which depends on the spatial scale.
- Λ: Cosmological constant, representing large-scale repulsion.
- $T_{\mu\nu}$: Energy-momentum tensor of matter.

The dynamics of the scalar field are further constrained by the field equation:

$$\Box\phi - \frac{\partial V(\phi)}{\partial \phi} = 0,$$

where $\Box\phi$ represents the propagation of the scalar field in curved spacetime, and $-\frac{\partial V(\phi)}{\partial \phi}$ accounts for the scalar potential's local effects.

The effective potential $\Phi(r)$, which includes both the standard Newtonian term and the scalar field contribution, is expressed as:

$$\Phi(r) = -\frac{GM}{r} - \frac{GMR^2 \cdot 10^{-4}}{2} \ln\left(\frac{R^2 + r^2}{2R^2}\right).$$

Here, the first term represents the Newtonian potential, while the second term arises naturally from the scalar field ϕ, introducing a logarithmic correction that becomes significant at large spatial scales $r \sim R$.

At small scales ($r \ll R$), the scalar field contributions are minimal, and the equations reduce to the standard Einstein field equations. At larger scales ($r \sim R$), the scalar field ϕ and its associated potential $V(\phi)$ play a dominant role, leading to an effective Λ that drives cosmic acceleration. This dual behavior bridges the gap between the quantum vacuum energy at small scales and the observed cosmological behavior at large scales.

3.3.3 Small Scales: Reproducing the Vacuum Energy

At small spatial scales, where $r \ll R$, the second term of the Re:GraviS potential becomes negligible:

$$\Phi_{\text{rep}}(r) \sim \ln\left(1 + \frac{r^2}{R^2}\right) \to 0.$$

The dynamics of the scalar field ϕ at these scales are governed by the field equation:

$$\Box\phi - \frac{\partial V(\phi)}{\partial \phi} = 0,$$

where:

- $\Box\phi = g^{\mu\nu} \nabla_\mu \nabla_\nu \phi$: The d'Alembert operator represents the propagation and curvature-driven dynamics of the scalar field ϕ.

- $-\frac{\partial V(\phi)}{\partial \phi}$: The scalar potential's gradient governs the field's local behavior.

In the small-scale regime ($r \ll R$), the scalar field behaves as a static and spherically symmetric field. This leads to the following approximations:

- The Laplacian of ϕ diminishes: $\nabla^2 \phi \to 0$, as spatial gradients are negligible in static configurations.
- The scalar potential $V(\phi)$ and its gradient $\frac{\partial V(\phi)}{\partial \phi}$ also become negligible as their dependence on r decreases.

As a result, the d'Alembert operator reduces to $\Box \phi \approx 0$, and the modified field equations simplify to the standard Einstein field equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}.$$

In this regime, the energy density of dark energy is effectively uniform, matching the observed ρ_Λ:

$$\rho_\Lambda = \frac{\Lambda c^2}{8\pi G}.$$

This uniformity reflects the local behavior of vacuum energy and its interaction with gravitons, ensuring consistency with quantum mechanical predictions.

3.3.4 Large Scales: Graviton-Induced Repulsion and Cosmic Acceleration

At larger scales, where $r \sim R$, the Re:GraviS potential's second term dominates, introducing a repulsive force that scales with the logarithmic term:

$$\Phi_{\text{rep}}(r) \sim \ln\left(\frac{r^2}{R^2}\right).$$

This repulsive force naturally explains the accelerating expansion of the universe, with the scalar field contributions becoming significant. The effective Λ in this regime emerges as a dynamical quantity related to the graviton-induced spatial effects, seamlessly connecting quantum vacuum energy at small scales with large-scale cosmic dynamics.

3.3.5 Implications for the Cosmological Constant Problem

The Re:GraviS framework offers a natural resolution to the cosmological constant problem by unifying small-scale and large-scale behavior:

1. **Small Scales ($r \ll R$):** The graviton-induced repulsion is negligible, and the Einstein field equations dominate. The uniform vacuum energy density ρ_Λ matches the observed value.
2. **Large Scales ($r \sim R$):** Graviton-induced effects drive cosmic expansion, with Λ emerging as a large-scale phenomenon. This dynamic interpretation avoids the severe discrepancy predicted by quantum field theory.
3. **Transition Across Scales:** The Re:GraviS potential provides a continuous transition between small and large scales, ensuring consistency across domains. This continuity eliminates the need for artificial tuning, offering a physically grounded explanation for ρ_Λ.

3.3.6 Conclusion: Resolving the Cosmological Constant Problem

The Re:GraviS framework provides a seamless explanation for the cosmological constant problem by integrating the effects of graviton-induced repulsion across scales. At small scales, such as those of unit volumes, it reproduces the observed vacuum energy density, while at large scales, spanning the vast expanse of the universe, it transitions to a dynamical interpretation of Λ that drives cosmic acceleration. This unified approach not only resolves the long-standing discrepancy between quantum field theory and cosmological observations but also strengthens the Re:GraviS model as a comprehensive theory of gravity.

4 Finite Radius of Black Hole Singularities: Analysis and Implications

4.1 Gravitational Force Balance

The Re:GraviS potential introduces a balance between the classical Newtonian attraction and a graviton-induced repulsive force. The total force $F(r)$ is derived as:

$$F(r) = -\frac{d\Phi(r)}{dr} = \frac{GM}{r^2} + \frac{GMR^2 \cdot 10^{-4} \cdot r}{R^2 + r^2}.$$

In extreme gravitational fields, such as those near a black hole core, the equilibrium condition for the radius r_s is defined by the equality of these two forces:

$$\frac{GM}{r_s^2} = \frac{GMR^2 \cdot 10^{-4} \cdot r_s}{R^2 + r_s^2}.$$

4.2 Equilibrium Radius Derivation

Rewriting the equilibrium condition without GM, we obtain:

$$\frac{1}{r_s^2} = \frac{R^2 \cdot 10^{-4} \cdot r_s}{R^2 + r_s^2}.$$

This equation describes the relationship between the Schwarzschild radius R and the finite radius r_s of the black hole core. Solving this numerically yields the finite radius.

4.3 Numerical Solution

Using the following physical parameters:

- Gravitational constant: $G = 6.674 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$,
- Solar mass: $M = 1.989 \times 10^{30}\,\mathrm{kg}$,
- Schwarzschild radius: $R = 2949.9\,\mathrm{m}$ (calculated as $R = \frac{2GM}{c^2}$ for a solar-mass black hole),

the cubic equation is solved, yielding:

$$r_s \approx 21.54\,\text{m}.$$

This result shows that the singularity is replaced by a finite core with a radius of approximately 21.54 m, governed by the equilibrium between attractive and repulsive forces in the Re:GraviS framework.

4.4 Physical Interpretation and Implications

The finite radius r_s represents the equilibrium point where the Newtonian gravitational attraction is counteracted by the graviton-induced repulsion. This finite radius resolves the classical singularity problem, eliminating infinite density and curvature at the core.

Key implications include:

1. **Singularity Resolution:** The Re:GraviS Model ensures that no zero-dimensional singularities form, instead predicting a dense but finite core.

2. **Quantum Gravity Relevance:** The finite radius provides a scale where quantum effects may dominate, serving as a potential testing ground for quantum gravity theories.

3. **Astrophysical Observables:** The finite radius could influence gravitational wave signals and tidal disruption events, providing observational signatures unique to the Re:GraviS framework.

Furthermore, the combination of this finite radius r_s with the higher-dimensional implications of graviton-induced spatial repulsion, as discussed in Section 3.2, suggests a novel framework for understanding the interiors and singularities of black holes. Specifically, the extension of gravitational interactions into extra dimensions offers new insights into the preservation of information within black holes and at their singularities. This approach directly addresses the black hole information paradox, which arises from the apparent loss of information during black hole evaporation a process governed by Hawking radiation [16]. By incorporating higher-dimensional effects, the Re:GraviS Model provides a potential pathway toward resolving this paradox.

4.5 Comparative Analysis with Classical Models

In contrast to general relativity, which predicts singularities with zero radius, the Re:GraviS Model introduces a natural cutoff due to the logarithmic term in the potential. This modification aligns with modern efforts to reconcile classical and quantum descriptions of gravity, offering a unified perspective on black hole interiors.

> *The prediction of a finite radius for black hole cores offers a groundbreaking resolution to the singularity problem, with potential implications spanning astrophysics, cosmology, and fundamental physics.*

5 Predictions and Applications of the Re:GraviS Model

5.1 Predictions for Galactic Dynamics

5.1.1 Stellar Halo Dynamics

In addition to explaining the flatness of galactic rotation curves, the Re:GraviS Model predicts the behavior of stars in galactic halos, where the density of visible matter is extremely low. The repulsive force induced by the logarithmic term of the potential becomes dominant at these scales:

$$F_{\log}(r) = \frac{GMR^2 \cdot 10^{-4} \cdot r}{R^2 + r^2}.$$

This force suggests that the motion of stars in galactic halos should exhibit systematic deviations from Newtonian predictions, particularly in low-density regions of dwarf galaxies and galaxy outskirts.

Observational Test: Future deep surveys, such as those conducted by the Vera C. Rubin Observatory (LSST), can provide high-precision stellar velocity measurements in galactic halos, offering a direct test of this prediction.

5.1.2 Interactions in Galaxy Clusters

In galaxy clusters, gravitational interactions occur on scales where the logarithmic term may influence the overall dynamics of the cluster. The Re:GraviS potential predicts weaker gravitational binding than traditional models, leading to a lower velocity dispersion of galaxies within clusters.

Observational Test:

- Comparison of cluster velocity dispersions with those predicted by Λ-CDM and Re:GraviS models can validate or constrain the influence of the logarithmic term.
- Data from large surveys like the Sloan Digital Sky Survey (SDSS) and the upcoming Euclid mission can be used for this analysis.

5.2 Predictions for Cosmic Expansion

5.2.1 Redshift-Dependent Hubble Parameter

The Re:GraviS Model offers a dynamic alternative to the cosmological constant by describing dark energy as a manifestation of graviton-induced repulsion. The logarithmic term in the potential implies a gradual evolution of the Hubble parameter with redshift, which can be expressed as:

$$H(z)^2 = H_0^2 \left[\Omega_m(1+z)^3 + \Omega_{\mathrm{DE}}(z)\right],$$

where $\Omega_{\mathrm{DE}}(z)$ evolves due to the distance dependence of the repulsive force.

Observational Test:

- Precision measurements of the Hubble parameter $H(z)$ at high redshifts ($z > 1$) using baryon acoustic oscillations (BAO) or quasar time delays could distinguish the Re:GraviS prediction from the constant Λ in Λ-CDM.

5.2.2 Evolution of Cosmic Structures

The Re:GraviS potential predicts slower growth rates for large-scale structures compared to Λ-CDM, as the repulsive force counteracts gravitational attraction. This can lead to observable differences in:

1. The clustering of galaxies.
2. The amplitude of matter power spectrum peaks.

Observational Test:

- Measurements of the matter power spectrum using cosmic microwave background (CMB) data from Planck and upcoming surveys like CMB-S4.

5.3 Black Hole Interiors and Gravitational Waves

5.3.1 Gravitational Wave Echoes

The finite radius of black hole cores predicted by the Re:GraviS Model suggests modifications to the gravitational wave signal near merger events. Specifically, gravitational wave echoes may arise from the interaction between the event horizon and the repulsive core.

Observational Test:

- Analysis of gravitational wave data from LIGO, Virgo, and KAGRA could reveal subtle deviations or echoes in the post-merger signals, providing indirect evidence for finite black hole radii.

5.3.2 Tidal Disruption Events

For black holes with finite radii, the dynamics of tidal disruption events (TDEs), where stars are torn apart near the black hole, may deviate from standard predictions. The gravitational potential near the core would alter the distribution of debris, affecting the observed light curve.

Observational Test:

- Monitoring TDEs with instruments like the James Webb Space Telescope (JWST) or the Extremely Large Telescope (ELT) could provide insights into the gravitational potential near black holes.

5.4 Implications for Fundamental Physics

5.4.1 Gravitational Constant at Different Scales

The inclusion of a scale-dependent term in the Re:GraviS potential suggests that the effective gravitational constant G_{eff} may vary with distance. At large scales, this variation could manifest as a correction to Newtonian gravity:

$$G_{\text{eff}}(r) = G + \Delta G(r),$$

where $\Delta G(r)$ accounts for the influence of the logarithmic term.

Experimental Test:

- Precision measurements of G in weak gravitational fields using satellite experiments or space-based missions like MICROSCOPE could reveal scale-dependent variations.

5.4.2 Connection to Quantum Gravity

The logarithmic term of the Re:GraviS model naturally incorporates graviton effects, bridging the gap between classical and quantum descriptions of gravity. This suggests that gravitons may play a dual role:

1. Mediating gravitational interactions.
2. Driving large-scale repulsion (dark energy).

Theoretical Exploration:

- Investigating the quantization of the scalar field ϕ in the Re:GraviS framework could yield insights into quantum gravity and the unification of forces.

5.5 Summary

The Re:GraviS Model predicts a wide range of phenomena, including:

1. **Galactic Dynamics:** Deviations in stellar motions and cluster velocity dispersions.
2. **Cosmic Evolution:** Redshift-dependent Hubble parameters and altered growth rates of structures.
3. **Black Hole Physics:** Gravitational wave echoes and modified tidal disruption events.
4. **Fundamental Physics:** Scale-dependent gravitational constants and connections to quantum gravity.

These predictions, many of which are testable with current or upcoming observational platforms, demonstrate the Re:GraviS Model's potential to provide a unified description of gravitational phenomena while advancing our understanding of the universe.

6 Discussion

6.1 Theoretical Implications of the Re:GraviS Model

6.1.1 Unified Framework Across Scales

The Re:GraviS Model represents a significant step toward unifying gravitational phenomena across multiple scales. By introducing a logarithmic term in the gravitational potential, the model successfully bridges small-scale Newtonian mechanics, galactic dynamics, and large-scale cosmic acceleration. This framework eliminates the need for ad hoc entities such as dark matter and the cosmological constant, replacing them with graviton-induced repulsion:

- **Small Scales** ($r \ll R$): The Newtonian term dominates, maintaining consistency with classical tests of general relativity.
- **Intermediate Scales** ($r \sim R$): The logarithmic term accounts for the flattening of galactic rotation curves.
- **Large Scales** ($r \gg R$): The repulsive term explains the accelerating expansion of the universe.

This multi-scale applicability highlights the Re:GraviS Model's potential to provide a unified explanation for gravitational phenomena.

6.1.2 Resolution of Singularities

In contrast to classical general relativity, which predicts the formation of singularities in black holes, the Re:GraviS Model introduces a finite radius for black hole cores. This is achieved through the equilibrium between the attractive Newtonian term and the repulsive logarithmic term. The finite radius not only resolves the singularity problem but also aligns with recent theoretical efforts to reconcile quantum mechanics and general relativity. These finite regions may exhibit quantum effects, potentially serving as a testing ground for quantum gravity theories.

6.1.3 Connection to Quantum Gravity

The graviton-induced repulsive term in the Re:GraviS Model provides a natural link between classical and quantum gravity:

1. **Graviton Field Representation:** The scalar field ϕ encapsulates graviton effects, introducing a dynamical component to the gravitational interaction.
2. **Dark Energy as a Quantum Effect:** The large-scale repulsion, interpreted as dark energy, may arise from the collective behavior of gravitons across cosmic distances.

This perspective suggests that the Re:GraviS Model could be extended to a full quantum gravity theory, incorporating quantized gravitational fields and their interactions with spacetime.

6.2 Limitations and Challenges

6.2.1 Dependence on Parameter Estimation

The Re:GraviS Model introduces two key parameters: α (graviton concentration) and R (graviton effect range). While these parameters are well-constrained by galactic rotation curve data, their physical interpretation requires further exploration. In particular:

- How does α scale with galaxy mass and structure?
- Is R truly universal, or does it vary with environmental factors?

6.2.2 Cosmological Implications

The model assumes a spherically symmetric potential, which may not fully capture the anisotropic nature of cosmic structures. Extending the Re:GraviS Model to include anisotropies or coupling with large-scale structure formation models is a necessary step for cosmological applications.

6.2.3 Quantum Gravity Integration

While the Re:GraviS Model provides a bridge to quantum gravity through the scalar field ϕ, it stops short of offering a fully quantized gravitational framework. Further development is needed to integrate the model with existing approaches to quantum gravity, such as string theory or loop quantum gravity.

6.3 Future Directions

6.3.1 Observational Tests

- **High-Precision Stellar Kinematics:** Targeting galactic halos and dwarf galaxies to test deviations in stellar motion.
- **Cosmic Surveys:** Using BAO and high-redshift supernovae to refine the model's predictions for the Hubble parameter.
- **Gravitational Wave Analysis:** Searching for echoes or deviations in post-merger signals from black holes.

6.3.2 Theoretical Extensions

- **Coupling with Large-Scale Structure Formation:** Investigating how the Re:GraviS potential affects galaxy formation and clustering.
- **Quantization of the Scalar Field:** Developing a quantum version of the Re:GraviS Model to explore graviton interactions at Planck scales.

6.4 Summary of Discussion

The Re:GraviS Model offers a unified and elegant framework for addressing gravitational phenomena across scales. Its ability to explain galactic rotation curves, cosmic acceleration, and black hole interiors with a single potential highlights its theoretical robustness and observational relevance. While challenges remain, the model provides a promising foundation for advancing our understanding of gravity and its role in shaping the universe.

7 Conclusion

The Re:GraviS Model represents a significant advancement in gravitational physics, offering a unified framework capable of addressing some of the most profound mysteries in modern science. By introducing a modified gravitational potential that incorporates graviton-induced repulsion, the model bridges classical and quantum descriptions of gravity while maintaining consistency with observations across vastly different scales.

7.1 Key Contributions

1. **Galactic Rotation Curves:** The model accurately reproduces the flatness of galactic rotation curves without requiring dark matter, providing a physical interpretation for the scale-dependent repulsion.

2. **Cosmic Acceleration:** The repulsive logarithmic term explains the accelerating expansion of the universe, predicting a dark energy density consistent with observational estimates.

3. **Black Hole Interiors:** The balance between gravitational attraction and graviton-induced repulsion resolves singularities, predicting finite radii for black hole cores and suggesting testable implications for gravitational wave signals and tidal disruption events.

4. **Unified Framework:** The Re:GraviS Model seamlessly connects phenomena across small (Solar System), intermediate (galaxies), and large (cosmological) scales, offering a unified explanation for gravitational dynamics.

7.2 Scientific Implications

The Re:GraviS Model challenges the conventional paradigm by proposing that dark matter and dark energy are not separate entities but rather manifestations of graviton effects. This perspective has profound implications:

- **For Cosmology:** The model provides an alternative to the Λ-CDM framework, suggesting that cosmic acceleration arises naturally from modified gravitational dynamics.

- **For Astrophysics:** The model predicts measurable deviations in galactic and cluster dynamics, enabling direct observational tests.

- **For Fundamental Physics:** By incorporating the repulsive force of gravitons on space into the gravitational framework, the model paves the way for a quantum gravity theory.

7.3 Future Prospects

While the Re:GraviS Model successfully addresses key phenomena, several challenges and opportunities for future research remain:

1. **Observational Validation:**
    - High-precision measurements of stellar motions in galactic halos.
    - Analysis of high-redshift Hubble parameters using upcoming cosmological surveys.
    - Detection of gravitational wave echoes from black hole mergers.
2. **Theoretical Refinement:**
    - Developing a quantized version of the scalar field ϕ.
    - Coupling the model with large-scale structure formation theories.
    - Extending the potential to include anisotropic effects for cosmological applications.
3. **Interdisciplinary Connections:** The Re:GraviS Model offers potential connections to string theory, loop quantum gravity, and other approaches in fundamental physics, providing a fertile ground for interdisciplinary exploration.

7.4 Outlook

The Re:GraviS Model is a promising step toward unifying gravitational physics, offering a framework that connects small-scale mechanics, galactic dynamics, and cosmological expansion under a single theoretical umbrella. By bridging the gap between classical and quantum gravity, it opens new pathways for understanding the universe and its underlying laws. As observational data and theoretical insights continue to evolve, the Re:GraviS Model provides a robust foundation for addressing the unresolved questions of modern physics.

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This study referenced the SPARC database to evaluate the theoretical model but did not include external datasets or figures from it.

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