On the Flaws in a Resent Extension of Schwarzschild Geometry to Accelerated Masses

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Abstract

In this article, I attempted to demonstrate that generalizing the Schwarzschild solution to cases involving the motion of the gravitational source by decomposing the definition of the metric tensor, transforming the differential elements between reference systems and then reassembling it in the desired system, as presented in some recently published papers on arXiv and elsewhere, is an incorrect approach.

It appears that some scholars and writers, especially those whose interests and expertise focus solely on the pure mathematical aspects of the general theory of relativity, tend to confuse some fundamental concepts involved in the formulation of the basic mathematical relationships in this theory. This confusion may partly arise from the limitations in the mathematical symbol system used to express these relationships.

From special relativity, we learned that quantities such as time intervals and lengths differ from one inertial reference frame to another. We can determine how a particular quantity will appear in one reference frame if we know its value in another frame and understand the relationship between the two frames, specifically the velocity between them. For example, if a rod has a length L in the first frame , its length in another frame , moving with velocity relative to the first frame, is denoted by L'. The relationship between L and L' depends on the relative velocity between the two frames.

The prime mark in these notations indicates the reference frame to which the quantity belongs. However, what must be understood is that, in the context of general and special relativity, not every quantity marked with such a mark signifies a transformation of the quantity from one reference frame (unmarked) to another.

The differential elements of distance and time $d\mathrm{x}$, $d\mathrm{y}$, $d\mathrm{z}$ and dt used in defining the metric tensor in reference frame S are not related by any transformation equations to their counterparts dx' , dy' , dz' and dt' in the other reference frame S'. Treating such quantities as analogous to time intervals and lengths associated with specific events would lead to catastrophic results that contradict the principles of relativity, as will be demonstrated.

These quantities simply represent the differential elements of coordinates in the coordinate system. They are determined entirely by the knowledge of system and has no inherent relationship to the differential elements of their counterparts in another reference frame. The metric tensor is defined through the relationships among the differential elements within each reference frame. Thus, the metric tensor represents the reference frame itself, not a physical quantity observed from different frames.

This can be clarified with a simple example. Suppose we want to compute the metric tensor in empty flat space using a Cartesian coordinate system. The line element is:

$$\mathrm{ds}^2 = \sum g_{ij}\, \mathrm{dx}^{\mathrm{i}}\, \mathrm{dx}^{\mathrm{j}}$$

Then the metric is:

$$g_{ij} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

From the basic principles of special relativity, it is evident that if we were to calculate the metric tensor in this same space using another Cartesian reference frame S', moving with a constant velocity v along x -axis relative to S, the metric tensor's value should remain unchanged. This is because inertial frames are equivalent, and no frame should have a privileged value such as that shown above, while others have distorted values.

However, if we insist on decomposing the definition of the metric tensor and transforming its elements using Lorentz transformations between the reference frames, we arrive at a different and contradictory result.

By improper use of transformation relations:

$$\mathrm{dt}' = \mathrm{dt} \,/\, \gamma$$

And

$$dx' = \gamma\ dx$$

Substituting these transformation equations into the metric tensor definition leads to a result that the metric tensor in S' differs from its value in S:

$$g_{ij}' = \begin{pmatrix} -\frac{1}{\gamma} & 0 & 0 & 0 \\ 0 & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} !$$

This is incorrect, not only from the principles of relativity but also from general mathematical principles. Notably, if we start by defining the metric tensor in frame S' and then transform it back to frame S, we obtain the opposite result, highlighting the error in this approach.

Unfortunately, these observations seem to have been overlooked by the authors of a recent scientific paper* that aims to generalize the Schwarzschild solution to include cases of uniformly moving or accelerating gravitational sources. The entire paper is based on the flawed premise of taking the differential elements of the space used in the definition of the metric in the reference frame where the mass is stationary, transforming these elements to other moving frames, and then redefining the metric tensor in the new frames based on these transformations. The authors appear to have been preoccupied with the extensive mathematical computations stemming from this flawed premise without critically examining or testing its validity in simpler systems.

The troubling aspect of this paper is its reference to other many scientific works that rely on the same flawed method and have been published years earlier, indicating that this error has become widespread.

While innovative ideas are not exclusive to anyone, it is perplexing that the authors believe such a simplistic generalization of the Schwarzschild solution could elude experts in the field, such as Schwarzschild himself who devised the solution or Droste and Hilbert, who developed and refined the concept further.

The method of transforming the elements used in the definition of metric tensor from one reference frame to another to determine its value in a new frame only works in one specific scenario: changing the type of coordinate system, not its state of motion. For example, converting from a Cartesian coordinate system to a polar one or vice versa. Here, we can transform the differential elements of space from one type to another. This process enables determining the metric tensor's value in either coordinate system based on its value in the other. However, this is not a transformation in the relativistic sense; rather, it resembles a change in the language of expression, with no physical impact other than simplifying equations in a particular coordinate system. Therefore, (transforming) the metric tensor between these systems is acceptable, as the authors did in the initial step of converting the standard polar form of the Schwarzschild line element into its Cartesian form. This, however, is the only correct result in the entire paper.

References

* **arXiv:2405.01932[gr-qc**] The extension of Schwarzschild line element to include uniformly accelerated mass.Ranchhaigiri,Brahma.A.K.Sen.