Cooperative Neighboring Numbers

(Pascal's triangle – another view)

Marko V. Jankovic

Institute of Electrical Engineering “Nikola Tesla”, Belgrade, Serbia
Swiss Rockets, Belgrade, Serbia

Abstract In this paper, a modification and a generalization of the idea that was used for the creation of Pascal's triangle, is proposed. The proposed method is based on cooperative neighboring numbers that reside on the edges, diagonals and vertices of regular polygons. Cooperative strategy represents creation of the new number using addition.

1 Introduction

Here, the idea of cooperative neighboring numbers that reside on the edges (sides), diagonals and vertices of regular polygons (*n*-gons) is going to be explored. The numbers are considered neighbors only if they belong to the same edge. The proposed idea represents a modification and a generalization of the principle that is used for generation of Pascal's triangle [1], or principle that is used in Conway's game of life [2] (in that case cooperative and competitive principles were used – here we are skipping the competitive part). It will be shown that the sum of all numbers that are present in the i-th iteration is equal to the i-th power of n (n^i), if the numbers reside on the edges, diagonals and vertices of the regular n-gon. In the case $n = 2$, i-th iteration corresponds to $(i+1)$-th raw of Pascal's triangle. The basic idea relies on the following property of the regular n-gon (polygon) – if we denote the number of connectors in the n-gon with C (connectors represent diagonals and sides (edges) of the n-gon) and number of vertices with V ($V=n$), it is trivial to prove that the following equality holds:

$$2\mathrm{C} + V = n^2 .$$

In the initial itereation all vertices contain the number 1, and connectors are empty. The proposed method can be generalized in many directions, but that is beyond the scope of this paper.

2 Cooperating neighboring numbers that reside on a line segment

In this section the simplest case is going to be analyzed – the numbers that reside on a line segment. Numbers are going to cooperate in the sense that neighboring numbers are going to produce number (“offspring”), between them, that is equal to their sum. Once they produce the “offsprings”, all numbers from the previous iteration are going to be deleted, except the numbers on the vertices. It can be seen that this represents a small modification of the principle used for generation of the Pascal's triangle. The following figure depicts a few initial iterations of the method.

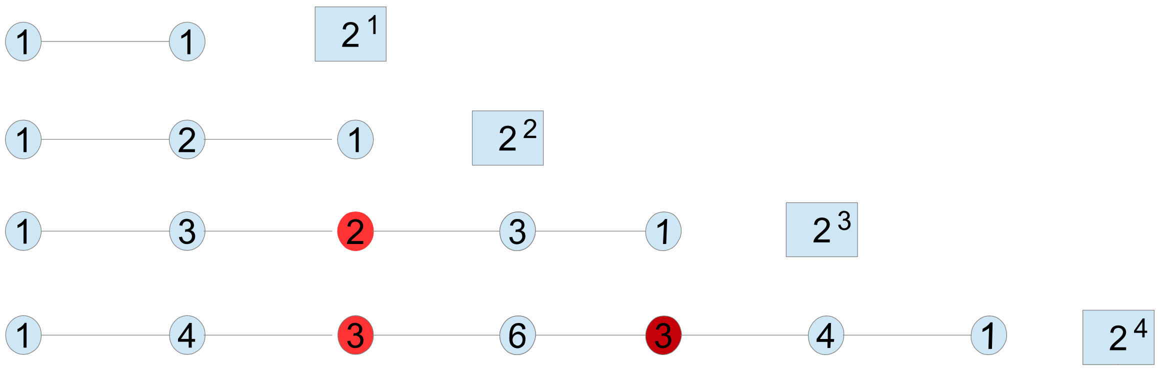

Fig. 1. The first 4 iterations of the proposed method applied on the numbers that reside on a line segment

Proof that sum of the numbers on a line segment in i-th iteration is equal to 2^i, follows directly from the proof that is used in the case of Pascal's triangle.

3 Cooperating neighboring numbers that reside on the regular triangle and square

Here we are going to present the proposed method for the numbers that reside on the connectors and vertices of a regular triangle and square. The numbers are considered neighbors only if they belong to the same edge.

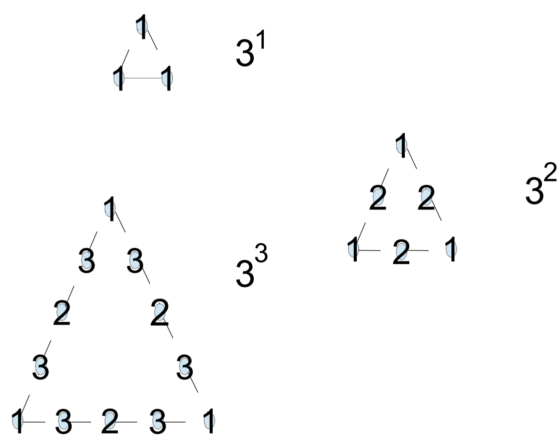

Fig. 2. The first 3 iterations of the proposed method applied on the numbers that reside on a regular triangle

In the caase of a regular triangle (Fig. 2), we do the same as in the case of a line segment, except that we do not remove numbers from the previous iteration, but rather keep them. It is not difficult to prove that the sum of all numbers in i-th iteration is equal to 3^i, since it is simple to understand that the sum of all numbers in the second iteration is equal to 3^2 ($2C + V = 3^2$, where C is the number of connectors $C = 3(3-1)/2$ and V is the number of vertices V=3), and then induction can be used.

In the case $n = 4$, the numbers reside on the connectors and vertices of a regular square. That also can be presented in 3D space – in that case numbers reside on the edges and vertices of a regular tetrahedron, which is easier for graphical presentation since there is no intersecting line segments (unfortunately this approach would be difficult to follow in the case of regular n-gons, where $n > 4$, since we would need to deal with 4D or higher dimensional graphics). Figure 3 depicts several iterations in this case. At the end, when number of the numbers on a single edge becomes big, the easiest way to represent process is to analyze what is happening on a single connector, since all connectors contain the same numbers (that is done after iteration 2).

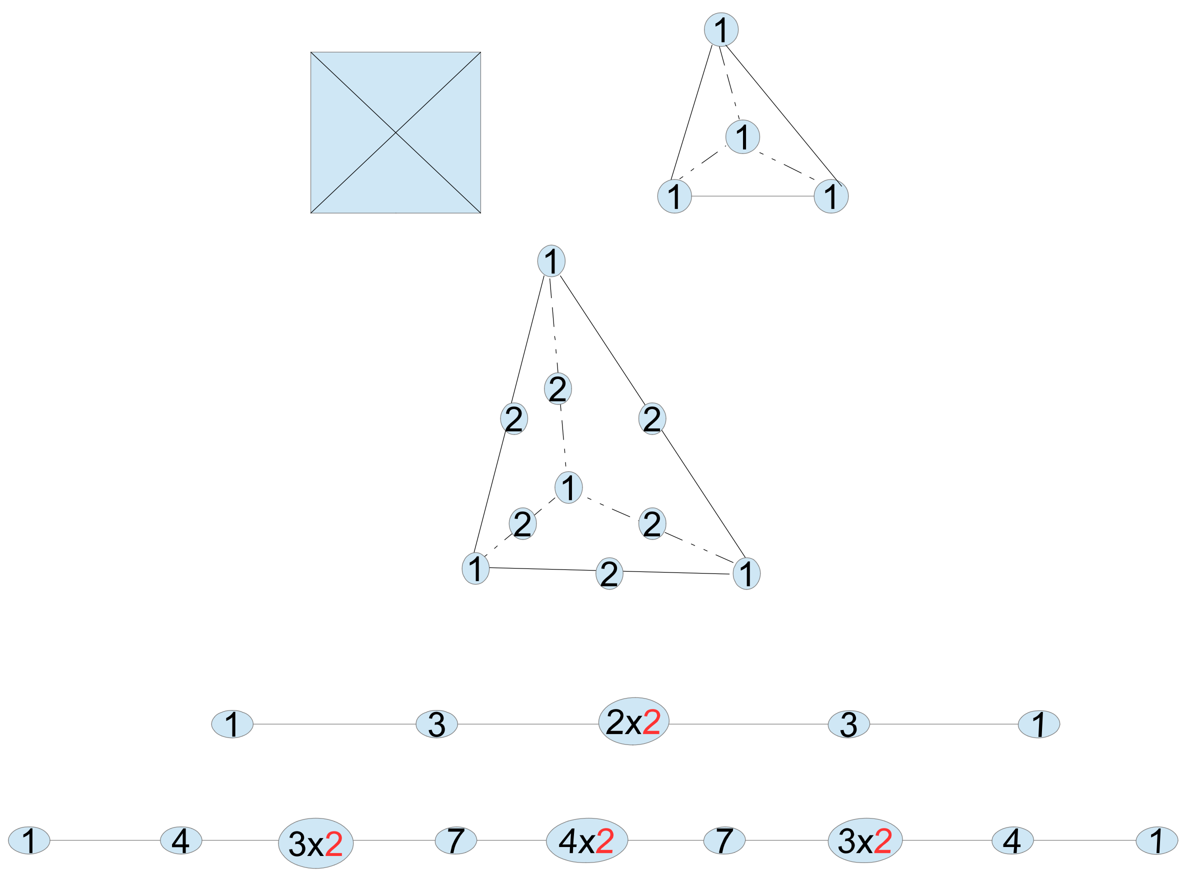

Fig. 3. The first 4 iterations of the proposed method applied on the numbers that reside on a regular tetrahedron (or connectors and vertices of regular square)

In this case, numbers from previous iteration create an “offspring” that is between them, and then they are doubled, except the numbers that are on the vertices – they stay the same. Again it is simple to prove that the sum off all numbers in i-th iteration is equal to 4^i.

4 Cooperating neighboring numbers that reside on a regular *n*-gon

In the general case of regular n-gon ($n > 2$), the proposed method can be illustrated by the following figure – only single connector is depicted in order to simplify representation (first 3 iterations are presented)

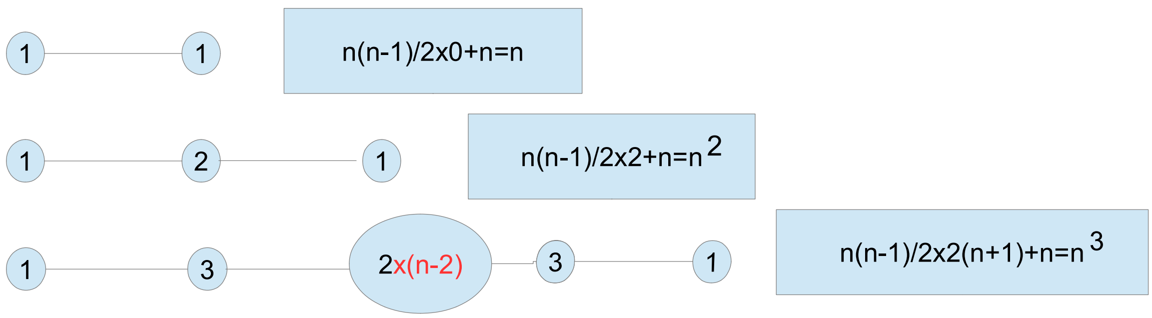

Fig. 4. The first 3 iterations of the proposed method applied on the numbers that reside on a regular n-gon

In this case, numbers from previous iteration produce “offspring” between them and they are multiplied by (n-2) after that, with the exception of numbers on vertices. Proving that sum of all numbers in the i-th iteration will be n^i is simple. A hint for the proof is given on the Figure 4.

5 Conclusion

In this paper, a simple modification and a generalization of the rule that is used to generate Pascal's triangle is presented. Generally speaking, it was shown that cooperation (based on addition) of the neighboring numbers that reside on the sides, diagonals and vertices of a regular polygon could be used for calculation of powers of natural numbers. Further generalization of the proposed method is not analyzed in this paper, and can go in the direction of different geometrical structures (e.g. chemical structures), different cooperation methods, different definition of neighboring numbers, introduction of non-neighboring cooperation (entanglement), introduction of asymmetry, addition of competition (like in the case of Conway's game of life), and so on.

One example would be to try to collapse the whole n-gon to a line segment and check what can be calculated. In that case we have "integration" of the polygon and the "desired" result is contained in the derivative of consecutive iterations (a partial "integration" case is also interesting, but is not going to be analyzed here). The cooperation rule that should be applied is the one that was used for the n-gon. The following figure depicts the first three iterations in the case $n > 2$.

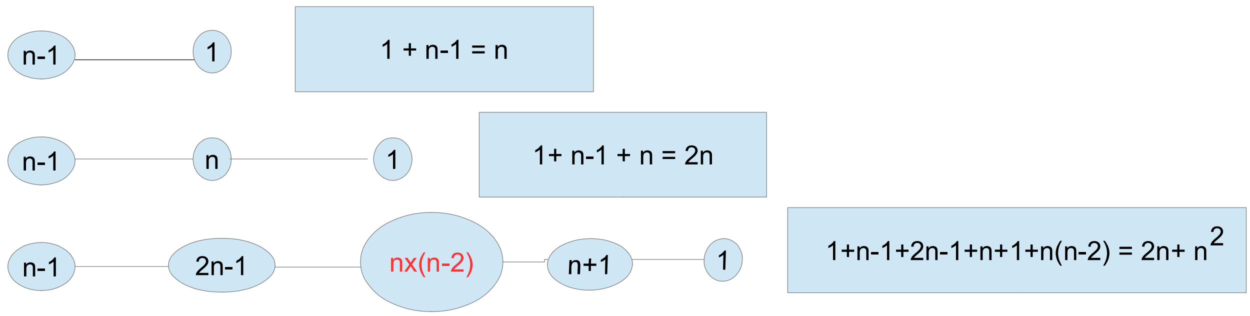

Fig. 5. The first 3 iterations of the proposed method applied on the numbers that reside on a line segment obtained by the collapse of a regular n-gon.

It is not difficult to show that the difference of the sum of the numbers in the i+1-th and the sum of the numbers in the i-th iteration is i-th power of n (n^i). It is interesting to be noticed is that the "space integration" can be reversed by "temporal derivation".

References:

1. Smith, K.J. (1973): Pascal's triangle. *The Two-Year College Mathematics Journal*, **4**(1), pp. 1-13.

2. Conway, J. (1970): The game of life. *Scientific American*, **223**(4), pp. 4-4.