Forecasting Financial Growth using Differential Equations

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1 Abstract

Although previous studies have established the predictive power of Lotka-Volterra models in financial markets, they assume static demand that does not account for exogenous factors or variable growth rates. The U.S. Domestic Airline industry, however, is an example of a dynamic market that is heavily reliant on customer demand and travel availability, which is why this study introduces non-autonomous models to accurately forecast future growth. Furthermore, only a few firms control most of the market share in this industry, and our study predicts Avelo Airline's viability in direct competition with these larger, well-established airlines. Our approach not only studies the behavior of small companies in oligopoly markets, but also provides these firms with a framework to react to the growth rate of market-setters and determine the necessary growth to achieve long term viability in the industry. The model examines several methodologies for financial forecasting into one comprehensive model that enhances the predictive capabilities of Lotka-Volterra models, providing valuable insight for participating firms.

2 Introduction

In 2023, the domestic United States airline industry generated approximately 211.152 billion US dollars in revenue. However, only four United States airlines accounted for around 64.4% of that revenue generation ("Data elements - financial", n.d.). Those airlines were United Airlines, Delta Airlines, American Airlines, and Southwest Airlines. This massive concentration of revenue into only a few companies creates a state of limited competition that is commonly referred to as an oligopoly. These market conditions make it difficult for new competitors to arise in the market, as consumers are more likely to remain loyal to established brands. Other airline industry-specific challenges include issues with regulatory compliance, the exorbitant costs associated with the airline business, and market saturation. All of these obstacles mean that new airlines face a long and difficult road towards long-term viability.

As a result of this, the launch of the new commercial airline Avelo Airlines on April 28, 2021 was in many ways a novelty, as the airline represented the first launch of a new mainstream U.S. airline in almost 15 years. In 2024, the airline announced a notable milestone when it turned its first profit since it began operations in 2021. Also, they have announced the addition of five new airplanes over the course of 2024 (rachelmplumb, 2024a). Due to this positive financial news from Avelo, we have decided to model this promising young company within a typically oligopolistic market.

Through the use of Lotka-Volterra models, this paper will model the U.S. domestic airline market and forecasted growth of Avelo Airlines, as well as the optimal financial decisions for the small firm through numerous simulations of realistic market possibilities. This represents a major addition to previous literature, as the majority of prior research on this topic merely focused on the larger companies within oligopoly markets, while we attempt to model the smaller companies. To do this, we first outline the equations needed to simulate the competition between the four largest airline companies with inclusion of outside goods. Then, we predict future financial quarters for these companies. After that, we outline the equation describing the market share of Avelo Airlines and forecast the firm's future growth.

3 The Proposed Model

The proposed Lotka-Volterra model is a refinement of previous literature and accounts for exogenous changes in the market and random consumer buying behavior. The traditional Lotka-Volterra model is used to display predator-prey actions in nature, and has been modified countless times in order to model multiple different types of competition. However, these models are often flawed when it comes to depicting the complex changes and variability associated with volatile financial markets such as the airline industry in recent years. In order to circumvent this, some previous models (Lakka et al., 2013) are able to exclude market changes from the study when considering industries with relatively persistent consumer behavior. The airline industry however, experienced significant changes due to COVID-19 and company mergers. For example, the total industry's 2019 Q3 earnings were 45.58 billion US dollars and the total industry's 2020 Q3 earnings were 19.79 billion US dollars, demonstrating a 56.58% decrease (Marasco et al., 2016). Additionally, there have been several major U.S. Airline mergers in the past few decades, which further decrease the accuracy of a model with constant growth rates. In order to capture these changes in demand and the time-dependent growth rates of various companies, the proposed Lotka-Volterra models are non-autonomous. Specifically, they utilize utility functions $f_i(t)$ which represent the derivative of the company growth functions $g_i(t)$. These utility functions encapsulate various factors like ticket prices, consumer preferences, and brand recognition (Marasco et al., 2016). Therefore, we consider the following system of non-autonomous Lotka-Volterra equations

$$\frac{dx_i(t)}{dt} = x_i(t)\left[g_i(t) - \sum_{j=1}^{N} g_j(t)x_j(t)\right], i = 1, \cdots, N \tag{1}$$

Where t represents financial quarters since 2000 Q1. $x_i(t)$ represents the market share of i-th airline at time t, ranging between 0 and 1, where $x_i(t) = 0$ corresponds to a lack of presence in the market and $x_i(t) = 1$ indicates total control of the market. $g_i(t)$ represents the growth rate of airlines, and N represents the number of competing companies. The term $\sum_{j=1}^{N} g_j(t)x_j(t)$ encapsulates the competitiveness from all airlines, respective to their current market share. This equation identifies the interaction between an airline's intrinsic growth potential and the competitive pressure of the industry. By applying this model to major U.S. airlines, we can understand how market shares grow over time, and provide accurate financial forecasts. By applying this model to the major U.S. Domestic Airlines, we find their equations represented as

$$\frac{dx_1(t)}{dt} = x_1(t)[g_1(t) - (x_1(t)g_1(t) + g_2(t)x_2(t) + g_3(t)x_3(t) + g_4(t)x_4(t))] \tag{2}$$

$$\frac{dx_2(t)}{dt} = x_2(t)[g_2(t) - (g_1(t)x_1(t) + g_2(t)x_2(t) + g_3(t)x_3(t) + g_4(t)x_4(t))] \tag{3}$$

$$\frac{dx_3(t)}{dt} = x_3(t)[g_3(t) - (g_1(t)x_1(t) + g_2(t)x_2(t) + g_3(t)x_3(t) + g_4(t)x_4(t))] \tag{4}$$

$$\frac{dx_4(t)}{dt} = x_4(t)[g_4(t) - (g_1(t)x_1(t) + g_2(t)x_2(t) + g_3(t)x_3(t) + g_4(t)x_4(t))] \tag{5}$$

where $x_1(t)$ is the market share of United Airlines, $x_2(t)$ is the market share of Delta Airlines, $x_3(t)$ is the market share of American Airlines, and $x_4(t)$ is the market share of Southwest Airlines. Note that the initial conditions $x_1(0)$, $x_2(0)$, $x_3(0)$, and $x_4(0)$ are 0.137, 0.129, 0.125, and 0.053 respectively. The differential equations (2), (3), (4), and (5) can be solved to give us the multinomial logit model (Marasco et al., 2016), which is a well known economic framework for modeling choice probability among alternatives. The model determines the utility as the attractiveness of a firm, or the likelihood of the business being chosen by a consumer compared to the rest of the industry. This approach is useful in the airline industry, where the consumer choices are captured by various factors represented in the utility functions. By solving the differential equations, we find

$$x_1(t) = \frac{\exp(f_1(t))}{1 + \exp(f_1(t)) + \exp(f_2(t)) + \exp(f_3(t)) + \exp(f_4(t))} \tag{6}$$

$$x_2(t) = \frac{\exp(f_2(t))}{1 + \exp(f_1(t)) + \exp(f_2(t)) + \exp(f_3(t)) + \exp(f_4(t))} \tag{7}$$

$$x_3(t) = \frac{\exp(f_3(t))}{1 + \exp(f_1(t)) + \exp(f_2(t)) + \exp(f_3(t)) + \exp(f_4(t))} \tag{8}$$

$$x_4(t) = \frac{\exp(f_4(t))}{1 + \exp(f_1(t)) + \exp(f_2(t)) + \exp(f_3(t)) + \exp(f_4(t))} \tag{9}$$

Where $f_i(t)$ represents the utility functions of airlines and $\frac{df_i(t)}{dt} = g_i(t)$. Note that in equations (6), (7), (8), and (9), companies will be affected by all other companies' utility functions, as there is a limited amount of demand that companies are competing for. Utility functions encapsulate various factors such as the size of company, customer satisfaction, service quality, reputation, and affordability. (Marasco et al., 2016). Because the U.S. domestic airline industry is not solely made up of these four companies, we must also consider the function for the outside good which represents all the other airlines, $x_0(t)$:

$$x_0(t) = 1 - \sum_{i=1}^{4} x_i(t) \tag{10}$$

Using the same models (Marasco et al., 2016), we also determine the utility points of each function, normalizing the utility of the outside good to be 0:

$$f_i(t) = \ln x_i(t) - \ln x_0(t), i = 1, \cdots, 4 \tag{11}$$

Considering the discrete utility points, we can extract the utility function of best fit using the *FindFit* function from Mathematica®. This process involves identifying the equation that most closely models the utility $f_i(t)$ of a company at any given time. By fitting the observed data, we extract coefficients that give accurate representation of the airline utility over time. The fitted utility functions were found to be

$$f_1(t) = 0.00019085t^3 + 0.0116302t^2 - 0.140290t + 0.986380, t \leq 29 \tag{12}$$

$$f_2(t) = 0.00014411t^3 - 0.00626932t^2 + 0.0567349t - 0.76337499, t \leq 29 \tag{13}$$

$$f_3(t) = 0.00020718t^3 - 0.01044641t^2 + 0.13567537t - 1.18995762, t \leq 29 \tag{14}$$

$$f_4(t) = 0.00016133t^3 - 0.00712246t^2 + 0.06181285t - 1.11943963, t \leq 29 \tag{15}$$

Note that the study analyzed polynomials of various orders and sinusoidal functions, and found that third degree polynomials had the greatest predictive power. This can be further proved by the low error values of the resulting graphs. One of the major limitations of previous literature is that it fails to consider long-term growth when forecasting firms' future growth, because equations (12), (13), (14), and (15) only have a limited predictive window (Marasco et al., 2016). To circumvent this, we utilize a change point at $t = 29$ and switch to a long-term logarithmic model to more accurately forecast long term company market shares:

$$f_1(t) = f_1(29) + \frac{1}{\alpha}\int_{29-\alpha}^{29} f_1'(x)dx * \ln(1 + (t - 29)), t > 29 \tag{16}$$

$$f_2(t) = f_2(29) + \frac{1}{\alpha}\int_{29-\alpha}^{29} f_2'(x)dx * \ln(1 + (t - 29)), t > 29 \tag{17}$$

$$f_3(t) = f_3(29) + \frac{1}{\alpha}\int_{29-\alpha}^{29} f_3'(x)dx * \ln(1 + (t - 29)), t > 29 \tag{18}$$

$$f_4(t) = f_4(29) + \frac{1}{\alpha}\int_{29-\alpha}^{29} f_4'(x)dx * \ln(1 + (t - 29)), t > 29 \tag{19}$$

where α denotes the number of quarters that have passed since the most recent significant external change in the industry. Using these logarithmic functions, we are able to model long-term growth in ways that previous studies have been unable to. In order to model the future market share for these airlines, we assume that over time they will have relatively stable logarithmic growth. For the U.S. Domestic Airline industry, we utilize $\alpha = 5$ to capture the state of the market post COVID-19. This α value is reinforced by the low MSE and MAPE, as well as the high FS of the equation. MSE standards for Mean Squared Error, MAPE stands for Mean Absolute Percentage Error, and FS stands for forecasting strength. The high confidence present throughout the numerous error metrics indicates the reliability of equations (12-15) and (16-19), potentially demonstrating predictive uses of our proposed model.

4 Results

By dividing the operating revenue of each company by the operating revenue of the entire U.S. domestic airline industry, we obtain market shares per quarter ("Financial AAL — American Airlines", n.d.)("Financials", n.d.)("Investor relations", n.d.)("Earnings Releases - United Airlines Holdings, Inc." n.d.)("Data elements - financial", n.d.). The market share $x_i(t)$ for each airline i at time t is given by:

$$x_i(t) = \frac{\text{Operating Revenue of Airline } i}{\text{Total Operating Revenue of U.S. Domestic Airline Industry}}$$

By modeling the market shares using our derived utility functions, we visualize the dynamics of the major airliens over time. This process includes plugging in utility functions (12), (13), (14), and (15) into the market share formulas (6), (7), (8), and (9) to plot the given functions on a graph and visualize their competitive position in the industry.

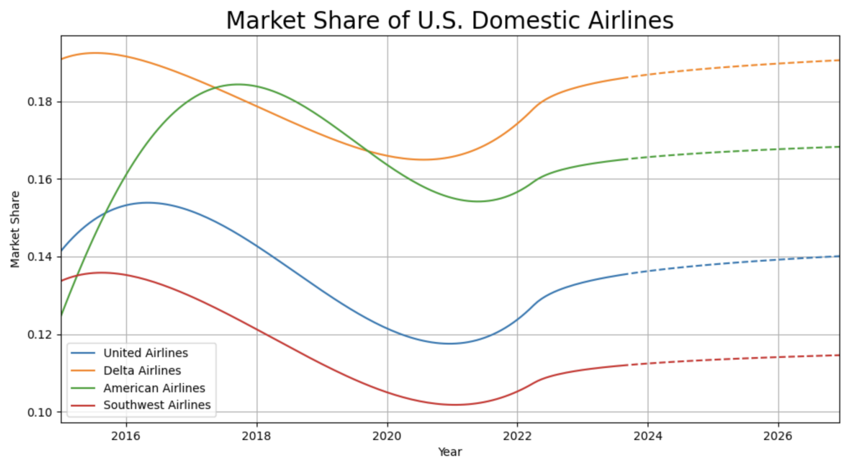

Figure 1: Market Shares of U.S. Domestic Airlines

Competitor	United	Delta	American	Southwest
MSE	0.093	0.095	0.108	0.093
MAPE%	1.91%	1.67%	1.50%	1.36%
FS	0.885	0.879	0.867	0.892

Table 1: Model Forecast Errors for Major Airlines

Figure 1 represents the volatility present in the airlines industry. It represents a major dip that was present throughout the pandemic for the major airlines. One likely reason that the other airlines (outside good) gained greater market share during the pandemic is due to their low operating costs and ability to scale back operations quickly in sub-optimal market conditions. Larger companies however, have ongoing costs such as employee retainment which forced them to sustain their profit losses during the pandemic. The graph also projects a realistic post-COVID growth rate, as we expect the major airlines will experience a leveling-off of their growth now that the volatility associated with COVID-19 has dissipated. There will likely be minor fluctuations present throughout the data we are attempting to predict, however the logarithmic utility functions normalizes these and provides the best representation for extended forecasting windows. Next, we will model what could happen to airline companies under different economic scenarios.

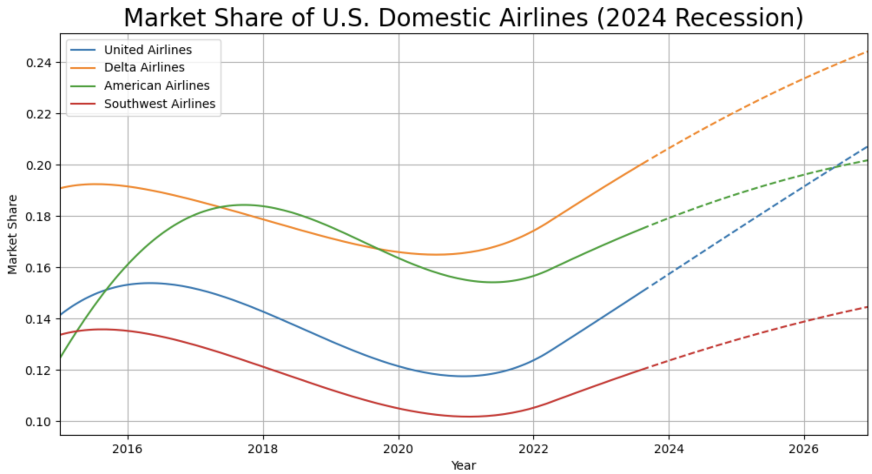

Figure 2: Market Shares of U.S. Domestic Airlines

Figure 2 shows that in the presence of a general economic recession, the market share of the major U.S. Domestic Airlines could increase, and the market share of other airlines (outside good) could decrease. This may be due to these larger airlines having larger reserve funds and the ability to sustain themselves in recessions, while smaller airlines lose market holding due to their need to generate profit every financial quarter in order to stay afloat.

One important consideration is that during COVID-19, the larger airlines lost market share because of the severe scale of the pandemic and its unexpected nature. A recession that occurs over a longer period of time however, results in larger airlines' consolidation of the smaller airlines market share.

5 Avelo Models

Using the previous models, we find the utility function and market share model of Avelo Airlines, which is $f_5(t)$ and $x_5(t)$ respectively. An important consideration is that market conditions such as standard of travel and ticket prices are typically set by larger firms, and we assume Avelo Airlines has no impact on the utility of other airlines. Therefore, we find that

$$x_5(t) = \frac{\exp(f_5(t))}{1 + \exp(f_1(t+30)) + \exp(f_2(t+30)) + \exp(f_3(t+30)) + \exp(f_4(t+30))} \tag{20}$$

where the denominator of the function represents the cumulative utility of market setters. Market setters are defined as major companies such as United, Delta, American, and Southwest, which set the industry standards for pricing, service, and marketing. These airlines shape the competitive industry and their utility largely impacts the market conditions. Therefore, the denominator is comprised of only these four major airlines.

Previous literature primarily focused on applying utility functions to larger companies, leveraging their extensive historical data to derive accurate models. However, this approach could not be applied to Avelo Airlines due to their limited data, as they launched in Q2 2022 and data is only available until Q4 2023, spanning just seven financial quarters. The limited timeframe made it challenging to derive a robust utility function using traditional methods. To address this, we employed a new methodology, which accounts for the shorter data span while still providing a close representation for Avelo Airlines' market performance. This study utilizes a new methodology in which we leverage the market share formula of Avelo Airlines in order to isolate for $f_5(t)$, and find that

$$f_5(t) = \ln\left[x_5(t)\left(1 + \exp\left(f_1(t)\right) + \exp\left(f_2(t)\right) + \exp\left(f_3(t)\right) + \exp\left(f_4(t)\right)\right)\right] \tag{21}$$

By solving for discrete points of $f_5(t)$ we can model a best fit quadratic function for Avelo Airlines' forecasted utility.

$$f_5(t) = 0.0332t^2 - 0.276t - 5.058 \tag{22}$$

This equation has the following MSE, MAPE, and FS values, which indicate that this equation is also reliable in matching the utility points. Graphing (22) gives us:

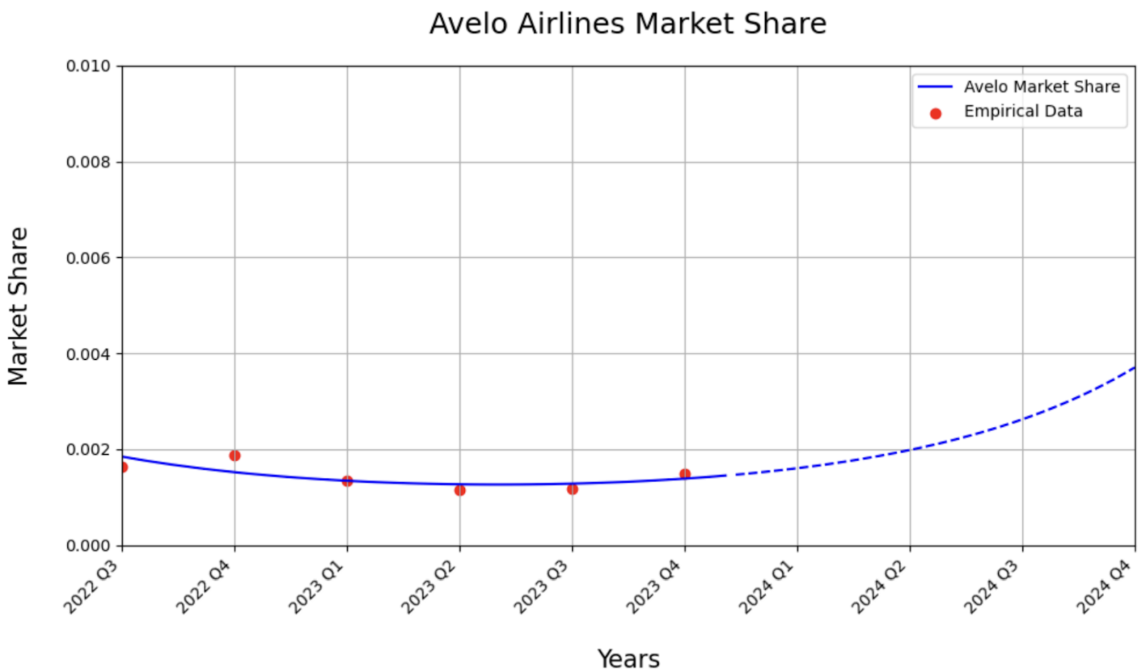

Figure 3: Forecasting Avelo Airline Growth

Competitor	Avelo Airlines
MSE	0.0161
MAPE%	1.885
FS	0.756

Table 2: Forecast Errors for Avelo Airlines

This graph demonstrates a possible short-term scenario through our model, in which Avelo Airlines continues their current pattern of growth throughout 2024. The doubling of market share in the model may seem to be an aggressive projection; however, it aligns with previously available data, as Avelo Airlines experienced 74 percent growth in revenue in 2024 Q1 (rachelmplumb, 2024a). Therefore, it is possible that Avelo could experience significant growth throughout the rest of 2024, possibly signified with major events that indicate their publicity such as flying NC State to the 2024 Men's and Woman's NCAA Basketball Final Fours, as well as turning their first profit in Quarter 1 2024 (rachelmplumb, 2024b).

Figure (4) shows a longer-term projection of what might happen if Avelo Airlines continues its current pattern of growth:

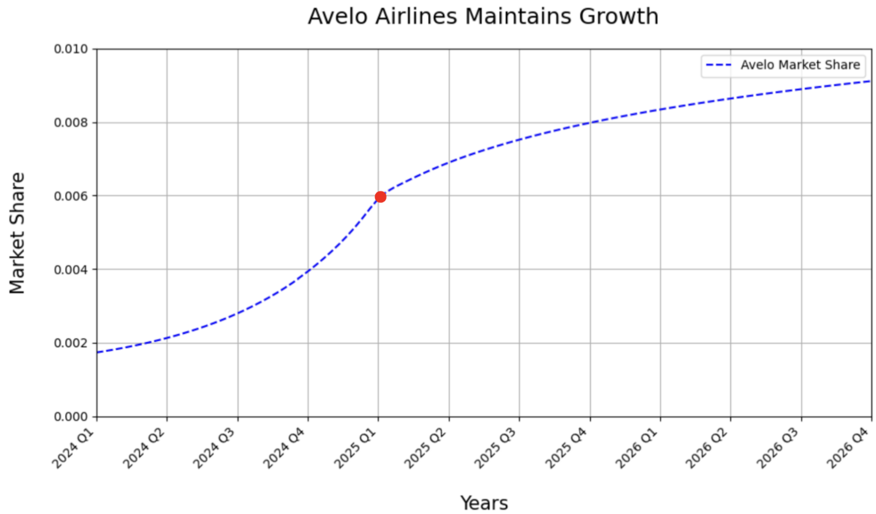

Figure 4: Avelo Airlines market share with maintained significant growth

Notice how at 2025 Q1, the model utilizes a switch point to a logarithmic function based off the slope of the previous graph in order to forecast growth as time goes to infinity. This is because the polynomial utility functions have only a few quarters of predictive capability. Many previous studies on this topic have restricted their modeling to only focusing on predicting the next year of growth. However, we use the logarithmic functions to model how larger companies tend to experience decreasing, relatively stable growth rates over time. As a whole, this graph represents our most realistic scenario for Avelo, as they experience rapid growth throughout 2024, but gradually level off as they experience increased competition and resource constraints.

Another common scenario over time within economic markets is an economic recession. Figure 5 is an example of what could happen to Avelo during a widespread economic recession.

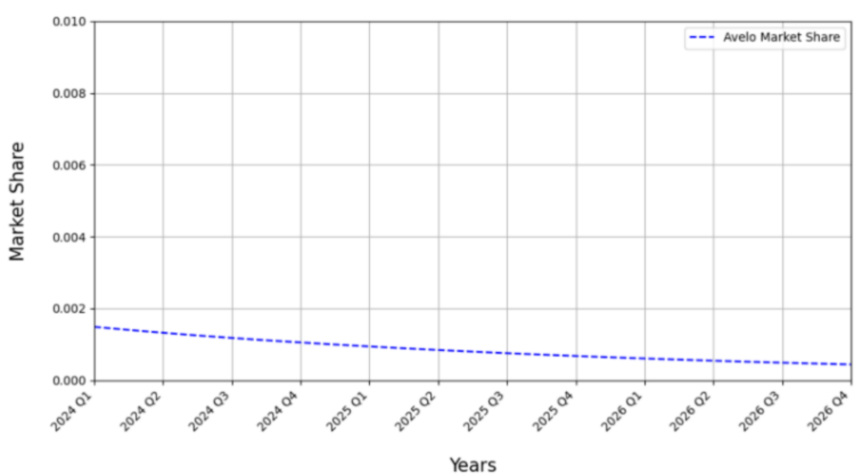

Figure 5: Avelo Airlines market share in decline

This figure represents a slow, but gradual decline of Avelo's market share. This is possible because longer economic recessions typically have a more pronounced impact on smaller companies in comparison to larger ones. This is due to smaller companies not having the requisite savings to weather a substantial decline in the use of their service. A major practical application of this section of the model for smaller businesses is modeling worst-case scenarios. Modeling potential negative scenarios is crucial for companies, as they must mitigate the risk if such events take place. Another possibility is that Avelo merges with another airline, which would give Avelo increased financial stability, significant network expansion, and enhanced market competitiveness through economies of scale. This causes a sudden spike in profits and drastically increase their market share, as represented in the figure below.

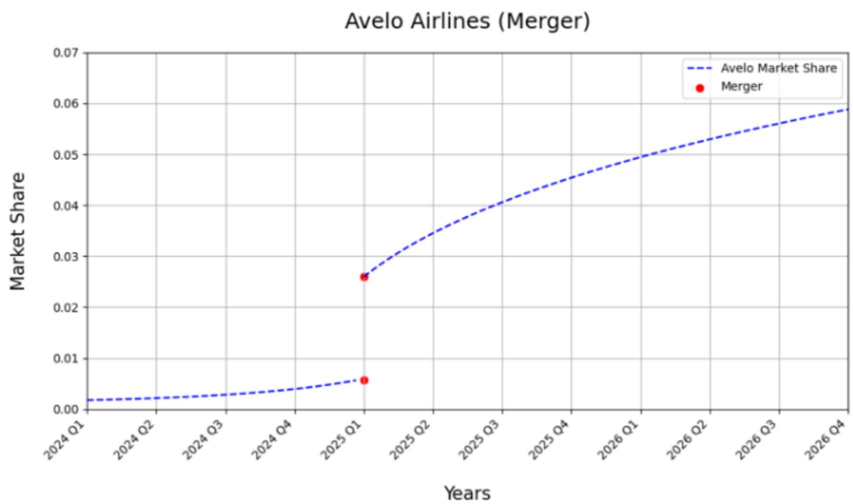

Figure 6: Avelo Airlines market share with merger

Note that in Figure 6 there is a more rapid increase in the rate of the market share, rather than just a one-time spike. This is due to the aforementioned benefits of the merger, and shows that our model is able to represent dynamic changes within the airlines market.

6 Conclusion

The objective of this paper was to create a mathematical model that could properly represent economic growth in an oligopoly market. The data (Figure 1) suggests that the market shares of all the major airlines could increase steadily, which indicates that smaller companies could face even more difficulties in the future. This data, coupled with planned mergers such as JetBlue with Spirit and Alaska Airlines with Hawaiian Airlines, suggests that the airline industry might continue to consolidate in the coming years. The ongoing trend of mergers might point to a future where fewer, larger airlines dominate the market. This makes our data relevant for companies such as Avelo Airlines. Currently for Avelo Airlines, the trends suggest an increase in market share over the next few years to be a realistic possibility indicating the company's possible viability for the foreseeable future (See Figure 4).

Some improvements we could have made were to use data points of smaller increments to find more accurate ways to get utility functions. Also, we could have directly incorporated Avelo into the Lotka-Volterra equations for the major airlines. Further research could involve other industries than the ones in this research paper, with a few other applicable industries including the credit card industry, service provider industry, and the mobile phone industry. Another idea for future research is to wait until Avelo's financial data for 2024 has been released, and then recalculate our utility functions accordingly. Due to Avelo being founded so recently, there is more potential for error in projections for them than for the four major airlines. More data would help to alleviate this problem, and allow for Avelo's future growth to have a more accurate model. Finally, we could also explore the model with the addition of another smaller airline company, and how competition with that company would affect Avelo. A specific example for a small airline we would use is Breeze Airways. Breeze is very similar to Avelo, as both airlines were founded in 2021, and have recently begun to experience positive growth. As a result, it would be interesting to model the growth of the two companies against each other, in order to suggest which airline has more potential for long-term financial viability.

In conclusion, our model provides a reasonable method of forecasting how different events within oligopoly markets would impact smaller companies. However, due to the changing nature of economic markets, our model will need continuous fine-tuning in order to maintain its predictive power. Ideally, small companies will be able to use our model to represent the best choices to survive in oligopoly markets. This would be a positive development for the future of the United States economy, as oligopolies could be harmful to innovation and fair competition.

7 Data

Year	Quarter	United Market share	Delta Market Share	AA Market share	SW Market Share
2000	1	0.137	0.129	0.125	0.053
2000	2	0.138	0.127	0.124	0.057
2000	3	0.126	0.124	0.129	0.058

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Year	Quarter	United Market share	Delta Market Share	AA Market share	SW Market Share
2000	4	0.128	0.122	0.125	0.059
2001	1	0.125	0.123	0.128	0.060
2001	2	0.129	0.121	0.125	0.065
2001	3	0.126	0.118	0.116	0.064
2001	4	0.111	0.120	0.119	0.069
2002	1	0.113	0.125	0.145	0.066
2002	2	0.118	0.122	0.143	0.072
2002	3	0.117	0.121	0.140	0.070
2002	4	0.110	0.123	0.136	0.071
2003	1	0.097	0.135	0.140	0.065
2003	2	0.096	0.132	0.139	0.069
2003	3	0.112	0.127	0.134	0.067
2003	4	0.103	0.130	0.131	0.066
2004	1	0.101	0.123	0.127	0.062
2004	2	0.104	0.122	0.124	0.066
2004	3	0.110	0.119	0.116	0.065
2004	4	0.100	0.116	0.115	0.065
2005	1	0.099	0.115	0.118	0.065
2005	2	0.101	0.117	0.120	0.068
2005	3	0.106	0.112	0.117	0.068
2005	4	0.101	0.109	0.116	0.069
2006	1	0.106	0.105	0.121	0.071
2006	2	0.109	0.112	0.119	0.077
2006	3	0.110	0.110	0.113	0.075
2006	4	0.101	0.107	0.113	0.077
2007	1	0.099	0.109	0.116	0.076
2007	2	0.105	0.112	0.114	0.081
2007	3	0.110	0.112	0.111	0.080
2007	4	0.100	0.107	0.111	0.079
2008	1	0.094	0.109	0.110	0.081
2008	2	0.100	0.107	0.110	0.084

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Year	Quarter	United Market share	Delta Market Share	AA Market share	SW Market Share
2008	3	0.105	0.104	0.109	0.085
2008	4	0.098	0.108	0.107	0.091
2009	1	0.093	0.114	0.110	0.090
2009	2	0.098	0.113	0.110	0.096
2009	3	0.104	0.108	0.108	0.094
2009	4	0.097	0.106	0.108	0.097
2010	1	0.100	0.170	0.112	0.097
2010	2	0.107	0.177	0.111	0.103
2010	3	0.107	0.177	0.107	0.103
2010	4	0.100	0.167	0.109	0.104
2011	1	0.096	0.168	0.104	0.102
2011	2	0.099	0.172	0.103	0.104
2011	3	0.103	0.173	0.103	0.102
2011	4	0.094	0.169	0.104	0.103
2012	1	0.152	0.172	0.107	0.102
2012	2	0.161	0.178	0.106	0.128
2012	3	0.162	0.179	0.104	0.125
2012	4	0.152	0.173	0.102	0.124
2013	1	0.154	0.173	0.107	0.123
2013	2	0.162	0.179	0.104	0.129
2013	3	0.161	0.182	0.104	0.125
2013	4	0.158	0.176	0.102	0.125
2014	1	0.149	0.180	0.111	0.122
2014	2	0.155	0.185	0.109	0.130
2014	3	0.157	0.186	0.106	0.125
2014	4	0.150	0.182	0.101	0.126
2015	1	0.143	0.188	0.107	0.127
2015	2	0.152	0.193	0.104	0.133
2015	3	0.154	0.190	0.177	0.137
2015	4	0.148	0.185	0.174	0.135
2016	1	0.143	0.193	0.163	0.136

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Year	Quarter	United Market share	Delta Market Share	AA Market share	SW Market Share
2016	2	0.160	0.207	0.194	0.148
2016	3	0.151	0.182	0.190	0.129
2016	4	0.146	0.181	0.184	0.130
2017	1	0.139	0.181	0.188	0.129
2017	2	0.148	0.185	0.186	0.134
2017	3	0.120	0.155	0.149	0.104
2017	4	0.143	0.184	0.182	0.125
2018	1	0.139	0.182	0.182	0.122
2018	2	0.151	0.189	0.178	0.126
2018	3	0.154	0.188	0.174	0.124
2018	4	0.150	0.178	0.175	0.126
2019	1	0.142	0.186	0.180	0.121
2019	2	0.155	0.197	0.180	0.126
2019	3	0.155	0.183	0.174	0.077
2019	4	0.151	0.187	0.177	0.122
2020	1	0.141	0.178	0.168	0.114
2020	2	0.067	0.087	0.102	0.073
2020	3	0.084	0.131	0.137	0.090
2020	4	0.094	0.138	0.133	0.083
2021	1	0.092	0.150	0.135	0.087
2021	2	0.107	0.171	0.170	0.111
2021	3	0.132	0.180	0.173	0.110
2021	4	0.127	0.178	0.168	0.112
2022	1	0.125	0.189	0.162	0.111
2022	2	0.142	0.197	0.177	0.118
2022	3	0.146	0.193	0.176	0.113
2022	4	0.147	0.193	0.178	0.112
2023	1	0.165	0.221	0.199	0.127
2023	2	0.153	0.198	0.177	0.123
2023	3	0.158	0.199	0.173	0.119
2023	4	0.157	0.194	0.173	0.124

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