# Computer Modeling of Plane Wave Propagation Through Dielectric Sections and Plasmas

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**Abstract**

A technique for modeling plane wave propagation through inhomogeneous media and plasma layers with different electron densities is presented. The technique is based on transmission line theory. Therefore, a general technique is introduced for solving complex transmission line systems. A data structure and algorithm for representing, and simultaneously solving for all nodes within the transmission line network is presented. The method is based on representing the network as a recursive tree structure and solving for the voltage, current, and impedance at each node using recursive programming techniques. First, all frequency dependent parameters within the tree structure are updated, then in a post-order traversing of the tree, the impedance at each node are computed followed by a pre-order traversing of the tree to compute node voltages and currents. For plane wave propagation, the reflection coefficient, the electric field and magnetic field are computed. The method is applied to normal incidence but can easily be extended to oblique incidence. A tapered transmission line model was used to verify the algorithm. In addition, an example was provided verifying the ability to compute the frequency response and impulse response of a system with a plasma. Finally, the application of the technique to model the heat tile and the plasma that develops on either aero-assist or spacecraft reentry is presented. This paper is based on work done at the Center for Communication and Signal Processing (CCSP) at North Carolina State University by the author in 1987. In addition, the paper presents work which was supported by NASA Langley Research Center under Contract NASI-1x925. The author was the Principal Investigator.

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# Chapter 1

## Introduction

In this paper we present a program for computer modeling and simulation of plane wave propagation in an homogeneous media. The technique is based on drawing an analogy between plane wave propagation and transmission line theory. Next an algorithm for solving complex transmission line networks is presented. This of course leads to the algorithm for solving for plane wave propagation. The network is represented in the computer by a recursive binary tree data structure. Using recursive programming techniques, the node voltage current, and impedance at each node within the tree structure is computed. For plane wave propagation, the reflection coefficient, the electric field and magnetic field are computed. In this manner, the frequency response of the network, from the source node to the receiving node is computed. The impulse response or the pulse response of the network is then calculated from the frequency response using Fast Fourier Transforms.

Computer programs for modeling transmission line networks have been written using ABCD parameters [5]. In this paper a technique in which the frequency response is simultaneously obtained at all nodes within the network is presented. This paper is based work done at the Center for Communication and Signal Processing (CCSP) at North Carolina State University by the author in 1987. In addition, the paper presents work performed by [7] which was supported by NASA Langley Research Center under Contract NASI-1x925. The author was the Principal Investigator. The work was also published in [8]. A major objective of this paper is to bring all the work into single comprehensive report. In addition, a lot of the original work was written in Word (1987) and has since become unreadable. So, all the work has been converted to Latex including equations and many of the original figures have been resurrected. For the Latex file visit http://aj7bf.com .

# Chapter 2

## Plane Wave / Transmission Line Analogy

Consider the plane wave propagation problem illustrated in Figure 1. For this geometry,

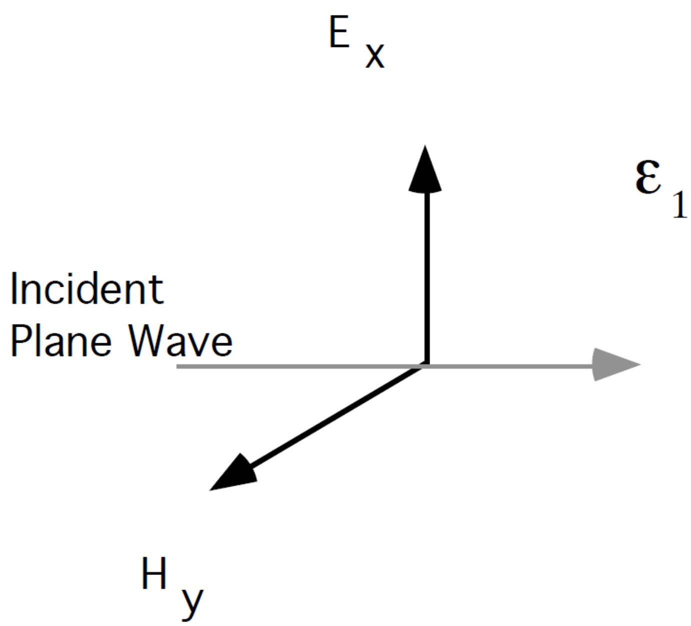


Figure 1: Plane Wave Propagation

$$E_y = E_z = 0 \quad H_x = H_z = 0 \quad \frac{\partial E_x}{\partial x} = \frac{\partial E_x}{\partial y} = 0 \quad \frac{\partial H_y}{\partial x} = \frac{\partial H_y}{\partial y} = 0 \tag{1}$$

Maxwell's equations reduce to

$$\frac{dE_x}{dz} = -j\omega\mu H_y \qquad \frac{dH_y}{dz} = -j\omega\mu E_x \tag{2}$$

These equations are similar to the differential equations for lossless transmission lines,

$$\frac{dV}{dz} = -j\omega LI \qquad \frac{dI}{dz} = -j\omega CV \tag{3}$$

Hence, the theory used to describe propagation in transmission lines applies equally well to plane-wave propagation. Next, consider the discontinuity at the boundary between two different medium with different dielectric constants. See Figure 2. This condition is similar to the discontinuity when two different transmission lines with different characteristic impedances are connected to each other as shown in Figure 3.

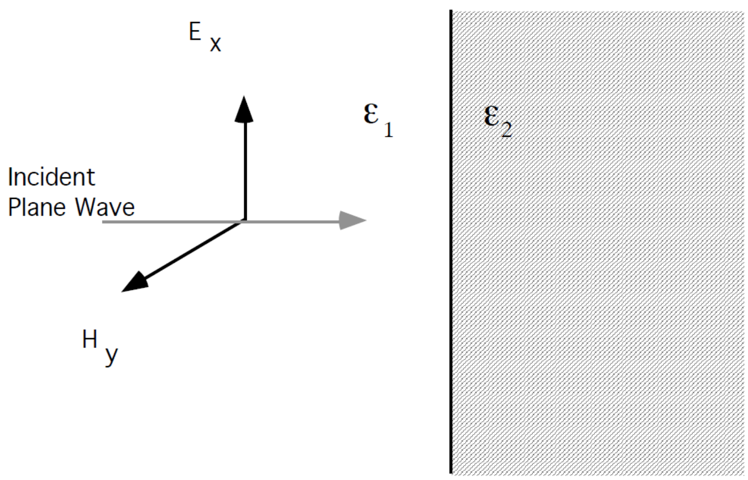


Figure 2: Plane wave discontinuity

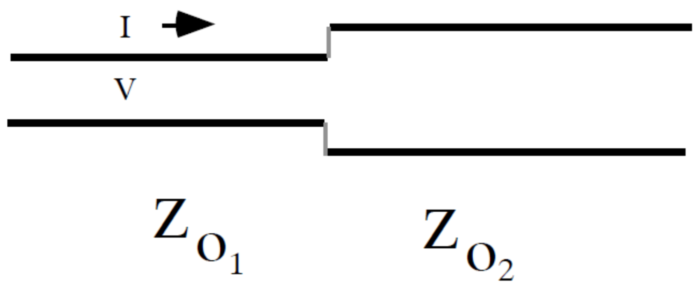


Figure 3: Transmission line discontinuity

The analogy between transmission lines and plane wave propagation ( normal incidence) is summarized in the Table 1 . Note that the mismatch at the discontinuity is represented by a reflection coefficient for each case.

| *Transmission Line Quantity* | *Symbol or Equation* | *Plane-Wave Quantity* | *Symbol or Equation* |
|---|---|---|---|
| Voltage | $V$ | Electric field Intensity | $E_x$ |
| Current | $I$ | Magnetic field intensity | $H_y$ |
| Inductance per unit length | $L$ | Permeability | $\mu$ |
| Capacitance per unit length | $C$ | Permittivity | $\varepsilon$ |
| Characteristic impedance | $Z_0 = \sqrt{\frac{L}{C}}$ | Intrinsic impedance | $\eta = \sqrt{\frac{\mu}{\varepsilon}}$ |
| Phase-shift constant | $\beta = \omega\sqrt{LC}$ | Phase-shift constant | $\beta = \omega\sqrt{\mu\varepsilon}$ |
| Velocity of propagation | $v = \frac{1}{\sqrt{LC}}$ | Velocity of propagation | $v = \frac{1}{\sqrt{\mu\varepsilon}}$ |
| Reflection Coefficient | $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$ | Reflection coefficient at boundry between $e_1$and $e_2$) | $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ |
| Incident wave power | $P^+ = \frac{\lvert V^+\rvert^2}{2Z_0}$ | Incident wave power density | $P^+ = \frac{\lvert E_x^+\rvert^2}{2\eta}$ |

Table 1: Plane Wave/ Transmission Line Analogy [3]

A summary of plasma parameters of interest is shown in Table 2.

| | |
|---|---|
| Permittivity, n electrons/cm$^3$ | $\varepsilon = \varepsilon_0 \varepsilon_r = \varepsilon_0 \left(1 - \frac{81\,n}{f^2_{kHz}}\right)$ |
| Intrinsic Impedance | $\eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{\eta_0}{\sqrt{1 - \frac{81\,n}{f^2_{kHz}}}}$ |
| Propagation Constant | $\gamma = \alpha + j\,\beta = j\,\omega\,\sqrt{\mu_0 \varepsilon_0}\sqrt{1 - \frac{81\,n}{f^2_{kHz}}}$ |
| Group Velocity | $v_g = \frac{d\omega}{d\beta} = c\sqrt{1 - \frac{81\,n}{f^2_{kHz}}}$ |
| Critical Frequency, e electron charge, m electron mass. | $\omega_p^2 = \frac{e^2 n}{m \varepsilon_0}$ |
| Introduction of loss due to collision frequency,$\nu$. | $\varepsilon = \varepsilon_0 \left[1 - \frac{\omega_p^2}{\omega^2 \left(1 - j\frac{\nu}{\omega}\right)}\right]$ |
| Attenuation Constant | $\alpha = \frac{\eta_0 n e^2 \nu}{2m\left(\omega^2 + \nu^2\right)}$ |

Table 2: Summary of Plasma Parameters of Interest

Based on the above discussion we can solve the problem of plane wave propagation through the system shown in Figure 4, by solving the equivalent transmission line problem also shown. Therefore, we will focus on solving general transmission line networks and return to plane wave propagation in a later chapter.

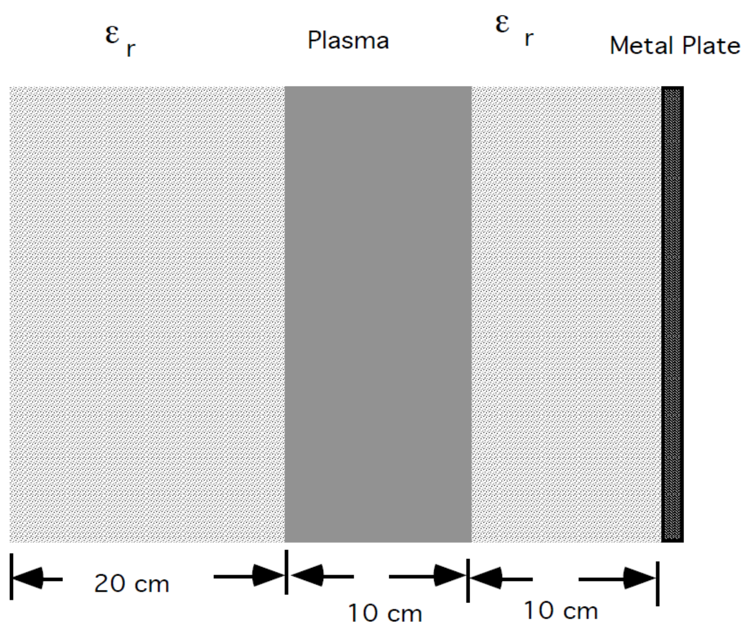


Figure 4: Modeling propagation through multiple layers as transmission line sections

# Chapter 3

## Transmission Line Networks

Consider the basic problem of simulating pulse transmission through a loaded transmission line. Assume that the pulse of interests is band limited with a cutoff frequency of $f_c$ . We can obtain the pulse response by first computing the frequency response of the network at equal intervals, then we perform a complex multiplication of the frequency response of the pulse and the transmission line network as calculated, and finally the inverse FFT of the result yields the time domain pulse response. Actually, the impulse response can also be obtained by computing the inverse FFT of the frequency response. Therefore, as a first step in calculating the frequency response of the network, we analyze the network response to a single sinusoid of frequency $f_0$. Consider the loaded transmission line connected to the generator $E_g$ through a source impedance $Z_s$ as shown in Figure 5 [2].

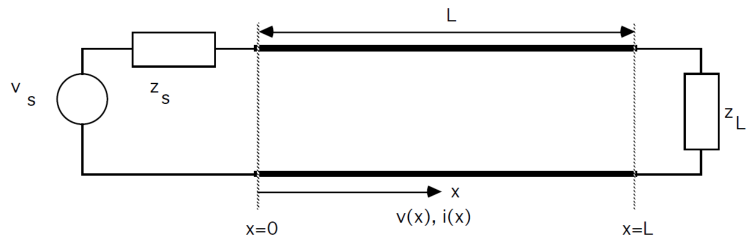


Figure 5: Generator connected to loaded transmission line

The voltage and current at any point on the transmission line can be obtained from the following expressions:

$$v(x) = \frac{v_s Z_0}{Z_0 + Z_s} e^{-\gamma x} \frac{1 + \Gamma_L e^{-2\gamma(L-x)}}{1 - \Gamma_s \Gamma_L e^{-2\gamma L}} \tag{4}$$

$$i(x) = \frac{v_s}{Z_0 + Z_s} e^{-\gamma x} \frac{1 - \Gamma_L e^{-2\gamma(L-x)}}{1 - \Gamma_s \Gamma_L e^{-2\gamma L}} \tag{5}$$

In the above expressions

$$\gamma = \sqrt{(r + j\omega l)(g + j\omega c)} \tag{6}$$

is the propagation constant and

$$Z_0 = \sqrt{\frac{r + j\omega l}{g + j\omega c}} \tag{7}$$

is the characteristic impedance of the transmission line. The expressions for the source and load reflection coefficients are,

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{8}$$

$$\Gamma_S = \frac{Z_s - Z_0}{Z_s + Z_0} \tag{9}$$

The expression for $v(x)$ includes the superposition of all waves reflecting from the source and load mismatches. This can be seen by a Taylor series expansion of (4)

$$v(x) = \frac{v_s Z_0}{Z_s + Z_0}[e^{-\gamma x} + \Gamma_L e^{-\gamma(L-x)} + \Gamma_L \Gamma_s e^{-\gamma(2L+x)} + \Gamma_L^2 \Gamma_s e^{-\gamma(3L-x)} + \Gamma_L^2 \Gamma_s^2 e^{-\gamma(3L+x)} + ...] \tag{10}$$

To obtain the shape of the pulse at the load we evaluate $v(L)$ at frequencies from $f = 0$ to $f = f_c$ in discrete steps where $f_c$ is the cutoff frequency of the band limited pulse. The number of points must be a power of 2 such that the inverse FFT may be used to obtain the sampled pulse response at the load.

Consider now the case where the boundary voltage and current are known on a section of transmission line. See Figure 6 . Evaluate $v(0)$ in (4) and then compute.

$$\frac{v(x)}{v(0)} = e^{-\gamma x} \frac{1 + \Gamma_L e^{-2\gamma(L-x)}}{1 + \Gamma_L e^{-2\gamma L}} \tag{11}$$

Also

$$\frac{i(x)}{i(0)} = e^{-\gamma x} \frac{1 - \Gamma_L e^{-2\gamma(L-x)}}{1 - \Gamma_L e^{-2\gamma L}} \tag{12}$$

Thus, using (11) and (12) the voltage and current can be evaluated at any point on the transmission line given the boundary voltage and current.

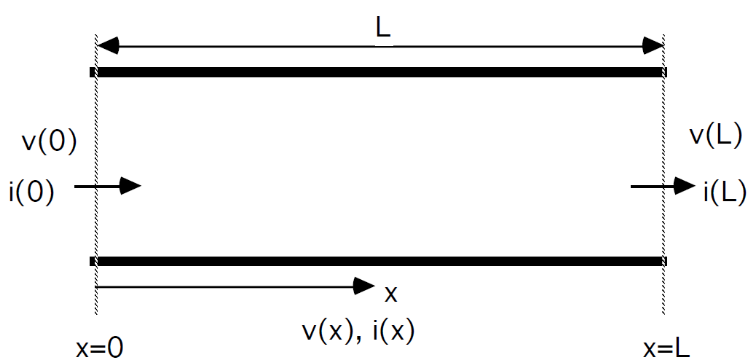


Figure 6: Section of transmission line with boundary voltages and currents

With the above preliminaries, we will examine the simple network in Figure 7 and present a methodology for its solution. In Figure 7, the nodes have been labeled n1 through n5. To solve this network, that is to obtain the voltage and current at each node and at any location within the network, consider equation (4). This equation suggests that if the impedance at node n1 was known then the voltage and current at node n1 can be calculated from the generator and source impedance. Thus, the first step is to obtain the impedance at n1. This impedance is seen to consist of the parallel combination of the impedance looking into n5 and n2 from n1.

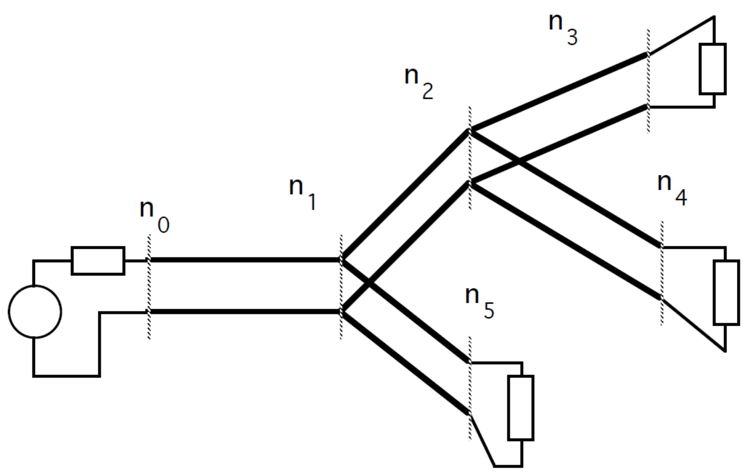


Figure 7: Example transmission line network

These impedances can be obtained by noting that (Figure 8),

$$Z_{in}(x) = \frac{1 + \Gamma_L e^{-2\gamma(L-x)}}{1 - \Gamma_L e^{-2\gamma(L-x)}} Z_0 \tag{13}$$

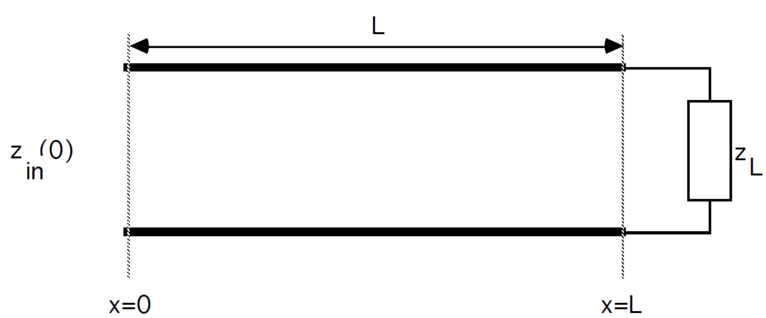


Figure 8: Input impedance of a loaded transmission line

Thus, the first step is to calculate the impedances looking into n3 and n4 from n2. The parallel combination forms the impedance at n2. The impedance at n1 is thus calculated by the parallel combination of the impedances looking into n2 and n5.

Therefore, the following methodology is suggested for solving the network. In the first pass, starting from the three loaded end nodes, the impedances are calculated and the parallel combination of these impedances at the parent node forms the parent node impedance. Working backward in this manner, the impedance at the root node (n1 in the example) is calculated. Using (4) the voltage and current at the root node n1 is calculated. Using (11) and (12) and the boundary voltages and currents, calculated at the parent node, the voltage and current at each node in the network can be calculated. Note that the current at each node is split into two currents flowing into each node.

In the case of propagation through layers of different medium, there are no branches. This situation is highlighted in Figure 9. In this case the impedance at node n5 is computed first. Next, the impedance at node n4 is computed and so forth until the total impedance looking into the network is obtained. In the next phase, the voltage and currents are computed starting from the source, node n1 and moving towards the load.

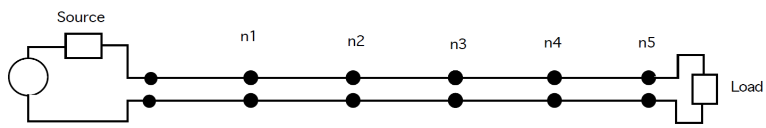


Figure 9: Network of cascaded sections

# Chapter 4

## Recursive Programming and Data Structures

To introduce the algorithm for solving a complex transmission line network, we first consider the case where the network is limited to the binary tree structure shown in Figure 10 . In the figure, the generator is connected to the root of the tree through a source impedance $Z_s$. The tree consists of nodes which are either parents or leaves. A leaf is a node which is terminated on a load. For example, n3, n4, n6, n7, n9, n11, and n12. Parent nodes have two branches. A left branch and a right branch. Nodes n1,n2, n5, n8, and n10 are parent nodes.

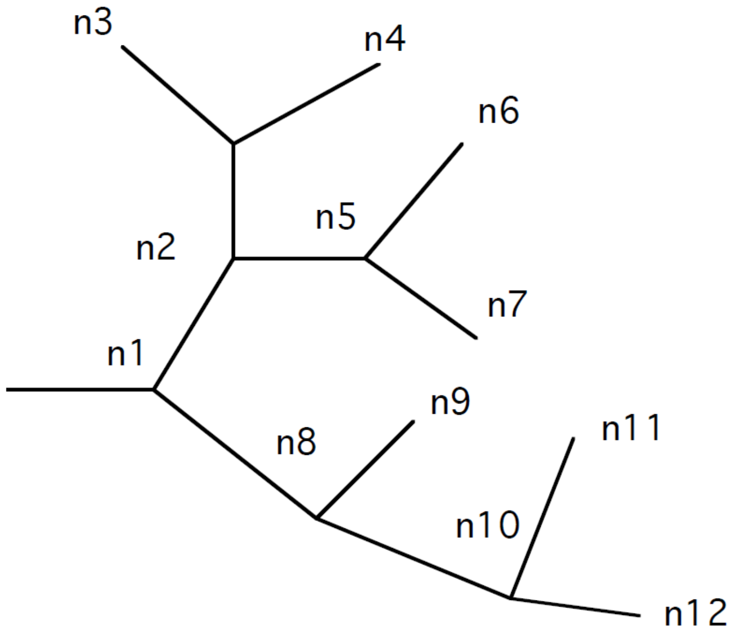


Figure 10: Transmission line network as a tree structure

In general, each branch represents a transmission line with different characteristics and lengths. Each section of transmission line is associated with the node on which it terminates. Thus, the section of transmission line from the generator to the root node n1 is described in the data structure pointed to by n1. This concept is described below. Each node has an associated data structure which occupies memory locations. A pointer can be defined which points to the data structure in memory. As nodes are added to the tree, memory is dynamically allocated for the data structure and a pointer is defined.

A detailed description of the algorithm to solve the binary tree representation of transmission line networks is available in [1] including using a TCL script interpreter. The original work is documented in [2].

# Chapter 5

## Accuracy of Modeling Continuous Taper with Cascaded Sections.

The purpose of this modeling and simulation is to determine whether the results produced by the Transmission Line Networking Algorithm and Program($TransNetCalc$) for a transmission line with a tapered characteristic impedance converges to the true solution as the number of sections is increased. In addition, it is desired to determine the accuracy to be expected in a given simulation as a function of the number of sections used in $TransNetCalc$. Consider the transmission line in Figure 11 with an exponentially tapered characteristic impedance.

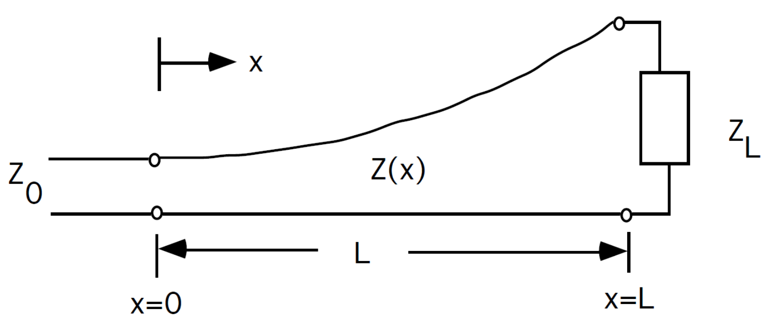


Figure 11: Exponentially tapered characteristic impedance transmission line

In our simulation we used a source impedance of 100Ω and a load impedance of 500Ω. Also, the total length $L = 10m$. We used sections of two wire cable with geometry shown in Figure 12.

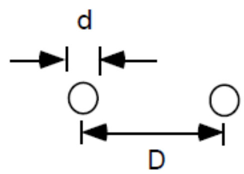


Figure 12: Two conductor parallel transmission line

The characteristic impedance is,

$$Z_0 = 120ln(\frac{2D}{d}) \tag{14}$$

For the continuous exponential taper

$$ln(Z) = \frac{x}{L} = ln(Z_L) \tag{15}$$

(We note that using the symbol "L" for Load, Inductance, and Length, will not be an issue hoping the context will provide identity). The exact differential equation relating the reflection coefficient to location along the line is (Riccati equation, see [4] )

$$\frac{d\Gamma}{dx} = j2\beta\Gamma - \frac{1}{2}(1-\Gamma^2)\frac{d[ln(Z_L)]}{dx} \tag{16}$$

The exact solution to this differential equation for the input reflection coefficient for the case of an exponential taper is

$$\Gamma_i = \frac{A\sin(\frac{BL}{2})}{B\cos(\frac{BL}{2}) + j2\beta\sin(\frac{BL}{2})} \tag{17}$$

$$A = \frac{ln(Z_L)}{L}, B = \sqrt{4\beta^2 - A^2} \tag{18}$$

In order to test the program $TransNetCalc$, we cascaded sections of two wire transmission lines. The program $TransNetCalc$ requires values of inductance L and capacitance C.

$$L = \frac{Z_0}{v_c} Z_L^{\frac{x}{L}} \tag{19}$$

$$C = \frac{1}{v_c Z_0} Z_L^{\frac{-x}{L}} \tag{20}$$

where $v_c$ is the velocity of light in free space. In Figure 13 a comparison of the results from $TransNetCalc$ and the exact solution to the Riccati equation is presented. The best comparison is in the Figure 14 which is a "zoomed in" view of Figure 13 . A simulation was performed with 500 sections and the result could not be distinguished from the exact solution. The conclusion is that *TransNetCalc* produces the exact solution asymptotically as the number of sections is increased. The reason for this asymptotic exactness is that *TransNetCalc* includes the effects of all reflections, not a simple approximation where second and higher order reflections are neglected.

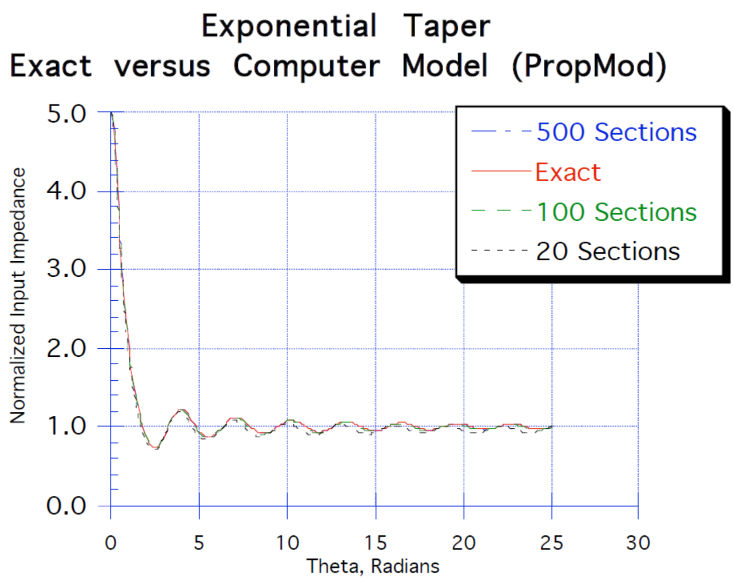


Figure 13: Comparison of *TransNetCalc* Computer Model and Exact Result

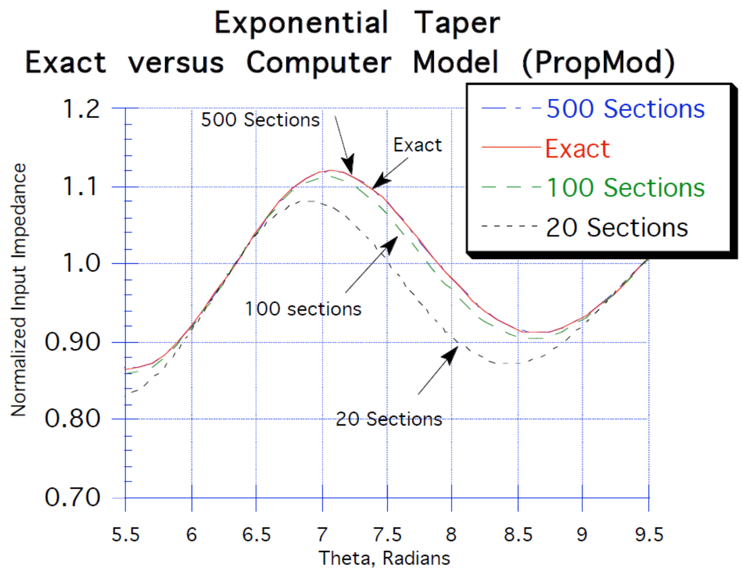


Figure 14: Zoomed In Comparison of *TransNetCalc* Computer Model and Exact Result

# Chapter 6

## Computer Simulation Results for Plane Wave Propagation Configurations with Plasmas

### 6.1 Plane Wave Propagation Through Plasma

In this section, the propagation of a radar signal through a medium with plasma will be analyzed. Figure 15 shows a plasma sandwiched between two media with dielectric constants $\epsilon_r = 1.0$. A metal plate is placed against the second slab. The electron density of the plasma is $3.16 * 10^{14} e/cm^3$ corresponding to a critical frequency of 160 GHz.

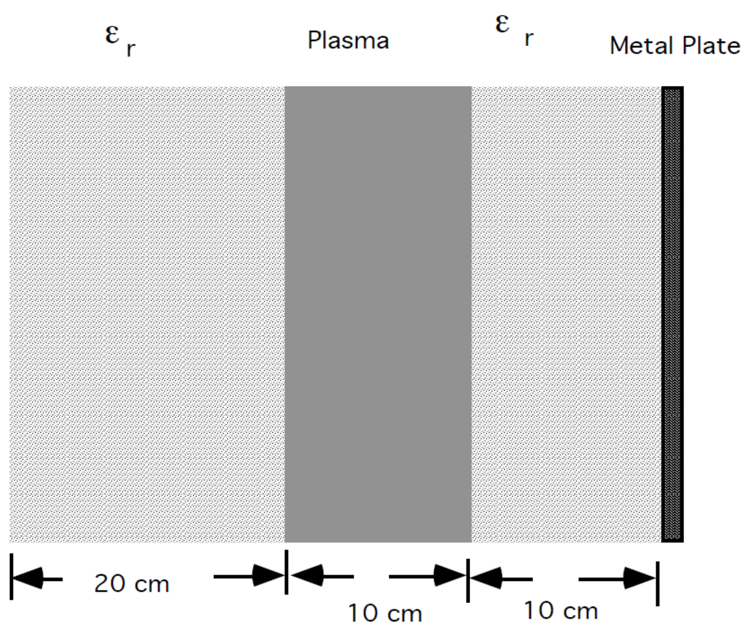


Figure 15: Plasma Propagation Model

The following material is from Gary Ybarra's Ph.D. Dissertation [7]. The principal investigator was Professor Sasan Ardalan and the NASA Tehnical Officer was Mr. Robert T. Neece (NASA Grant NAG-1-1219).

> Two cases are now presented. In the first case, the impulse response over a bandwidth of 20 GHz at a starting frequency of 40 GHz is considered. For the second case, the same bandwidth is used but the beginning frequency is increased to 100 GHz. In both cases, 512 frequency samples were obtained, which provides 1024 points in the resulting impulse response computed from a 512 point inverse FFT. The impulse response corresponding to the first case is shown in Figure 16. The time domain data has been converted to distance using the free-space velocity of propagation. In addition, consideration of the two-way travel time has been provided in the distance scale. Since the frequency span is 40 to 60 GHz, a strong reflection is observed from the plasma whose critical frequency is 60 GHz. The range to the reflection is 20 cm, precisely as expected from the channel geometry (Figure 15)[7].

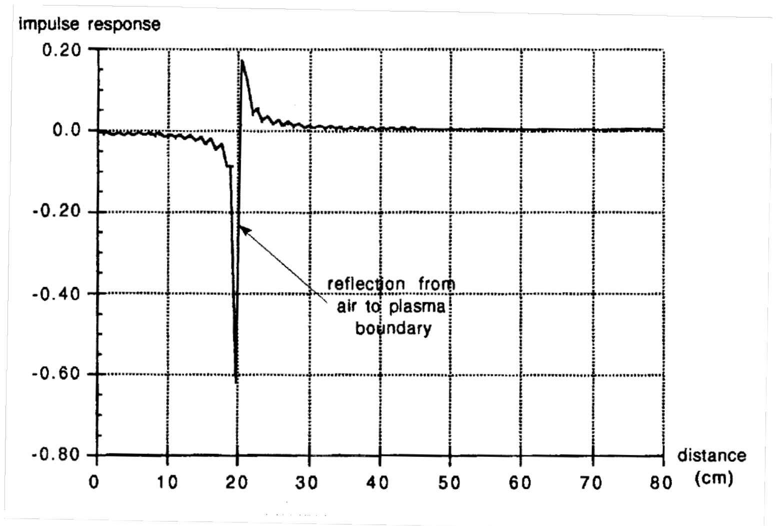


Figure 16: Impulse response of electron plasma sandwitched between air slabs and terminated in a metal plate. The starting freqiency is 40 GHz[7].

In the second case, the frequency span is from 100 GHz to 120 GHz. The resulting impulse response is shown in Figure 17. Most of the energy passes through the plasma because its critical frequency is 60 GHz. However, there are minor reflections due to the dielectric discontinuities at the air/plasma interfaces. At 100 GHz, the dielectric constant of the plasma is $\epsilon_r = 0.64$. The resulting reflections are shown in Figure 17. The distance to the main reflection is shown to be 40 cm which corresponds to the distance to the metal plate as indicated in Figure 15. The velocity of propagation within the plasma has been taken into account[7].

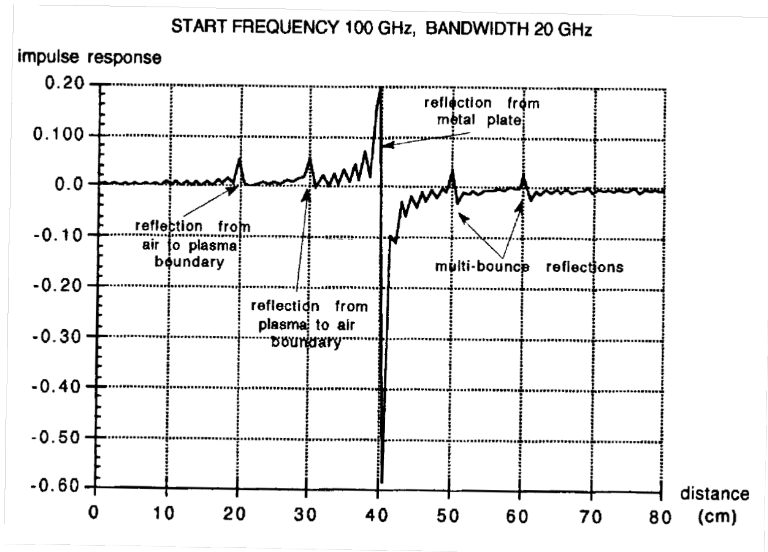


Figure 17: Impulse response of electron plasma sandwitched between air slabs and terminated in a metal plate. The starting freqiency is 100 GHz[7].

The impulse response corresponding to the first case has a reflection due to the plasma and the frequency span below the critical frequency. In the second case, the impulse response has no reflection due to the plasma but is reflected by the metal plate as the frequency span is higher than the critical frequency.

## 6.2 Radar Signal Plane Wave Propagation Through Heat Tile During Re-entry or Aeroassist

The following is based on [8] which illustrates the application of the Plane Wave Propagation approach in this paper to measure parameters in the plasma that develops in re-entry or aeroassist. See Figure 18. To quote the paper:

> The shuttle will deploy the AFE vehicle, which will then be accelerated to atmospheric entry velocity. During the 600 s aeropass, the high temperature in the proximity of the nonablating heat tiles will generate a dynamic plasma. The friction created will slow the craft, providing an aerobrake. Following the aeropass, the AFE vehicle will attain a low earth orbit to be retrieved by the shuttle. One purpose of this experiment is to ascertain the feasibility of using the atmosphere as an aerobrake, which requires accurate measurement of the plasma density profile. The plasma profile data will be used to confirm or improve the flow field predictions made by computational fluid dynamics.

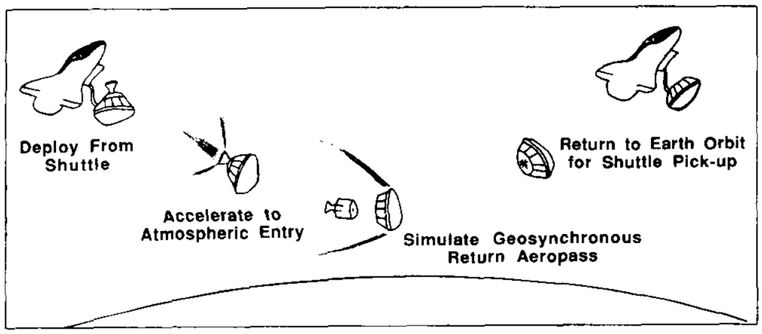


Figure 18: NASA Aeroassist Flight Experiment

Figure 19 shows the Radar measurement propagation path.

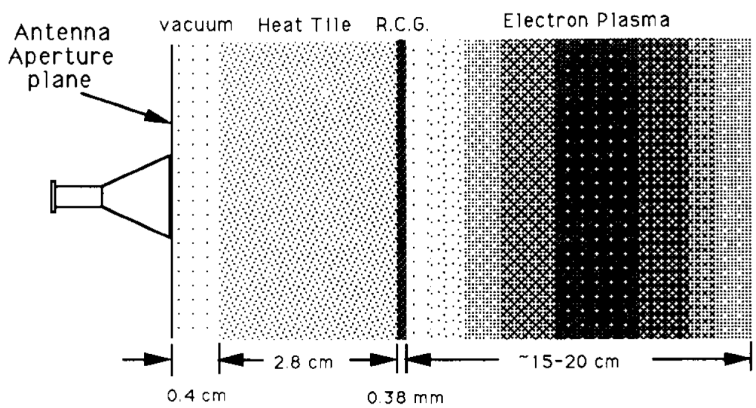


Figure 19: Microwave reflectometer ionization sensor (MRIS) propagation path.

There exist several predictions of the plasma density profiles that are to be measured. One such prediction, based on Computational Fluid Dynamics (CFD), is considered to be one of the better predictions presently available for the plasma that develops. An example of a CFD predicted profile is shown in Figure 20,

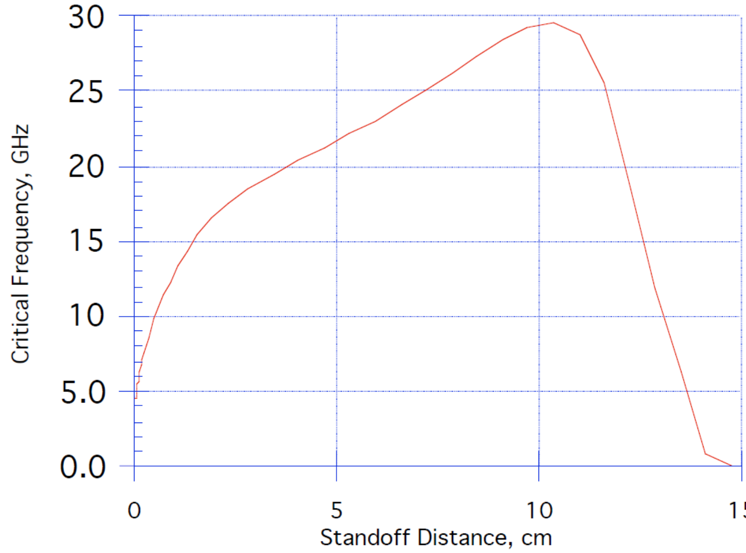


Figure 20: Profile for 4 cm Standof Distance at 20 GHz Critical Frequency

The next set of results provides great insight into the behavior of the plasma. Consider Figure 21 which shows the impulse responses obtained using the cascaded slab model for the Tile/RCG CFD profile system using 64 frequency steps at 64 MHz intervals. Each impulse response corresponds to a different starting frequency as indicated. The frequencies are stepped in 64 MHz increments up to 64 steps. Therefore, the bandwidth of each measurement sequence is 4.096 GHz (64 x 64 MHz). See [7] and [8].

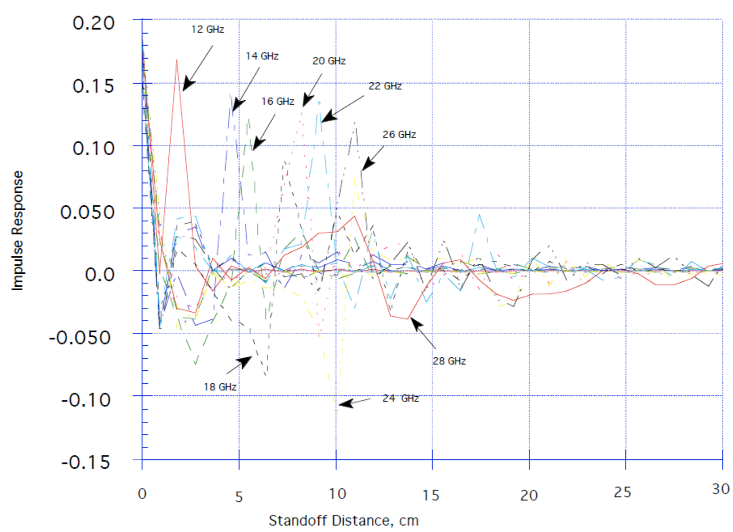


Figure 21: Impulse Responses as the Starting Frequency is Changed ( BW = 64x64 MHz) Profile 20.4

A major application of the Plane Wave Propagation Modeling in this paper is in probing the plasma that develops in spacecarft re-entry.

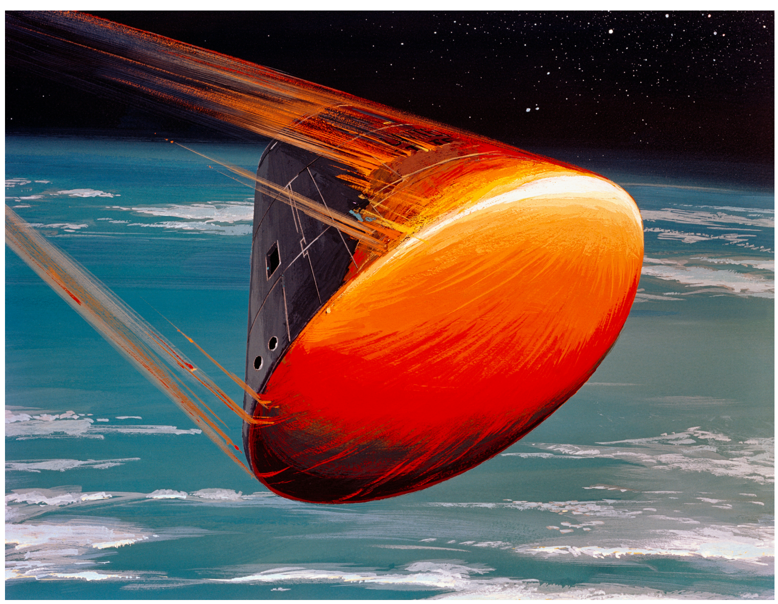

Figure 22: Apollo command module flying with the blunt end of the heat shield at a non-zero angle of attack By North American Rockwell [6]

# Chapter 7

## Conclusions

This paper presents a method to model and analyze plane wave propagation including medium with plasmas. A tapered transmission line model was used to verify the algorithm used to model the system. In addition, an example was provided verifying the ability to compute the frequency response and impulse response of a system with a plasma. Finally, the application of the technique to model the heat tile and the plasma that develops on either aeroassist or spacecraft re-entry was presented.

# Chapter A

## Appendix

### A.I Calculating Distance in the System

Since the program stores the data structure of the network, it is possible to build an array which corresponds to the distance away from the source at which various reflections occur. Thus, by taking into account the group velocity in each medium and noting that we know the length of each layer, we can calculate the time spent in each layer. Since $dt$ is invariant, we can calculate the distance corresponding to each sample through the following procedure:

```
/*
* Code segment for calculating the distance corresponding to each reflected
* sample in a layer
*/
if (!plasma) velocity =SPEED_OF_LIGHT/sqrt(epsr);
else
velocity =SPEED_OF_LIGHT*sqrt(epsr);
/*
* Calculate time spent in layer
*/
time = current_P->length/velocity;
/*
* Calculate the number of samples that are reflected from this layer
* Factor of 2 corresponds to round trip time
*/
j = (int) (2.0 *time/dt + 0.5);
dist=0.0;
i=0;
for (jj = 1; jj <= j; jj++) {
printf("%d %d length = %f vel = %e time = %e\n",plasmaFlag,j,
current_P->length,velocity,time );
dist += current_P->length/(float)j * 100.0; /* dist in cm */
distance_A[i] = dist;
fprintf(prof_F,"%d\t%e\t%f \n",i,dist,epsr);
i++;
if ( i >= npts ) break;
}
```

## A.II C Code for Distance Calculation

```
/*
* This routine is a special routine that creates
* an array of distance corresponding to lengths of
* different segments. The calculation adjusts the length for
* the speed of light in the medium.
* This routine ignores the right nodes and goes all the way to the
* load. Also a file is created for the dielectric constant as a
* function of distance
* at a fixed frequency.
*/
void BuildDistance(root_P, distance_A,dt,freq,npts)
struct node *root_P; /* pointer to node */
float distance_A[];
float dt;
float freq;
int npts;
{
float n,epsr,eps,nu;
float dist;
float time,accTime,totalTime;
float velocity;
int plasmaFlag=0;
int i,j,jj;
FILE *prof_F;
```

```
struct node *current_P; /* pointer to node */
prof_F = fopen("profile.dat","w");
if ( root_P == (struct node *) NULL ) RET;
totalTime = 0;
current_P = root_P;
dist=0;
i=0;
fprintf(stderr,"dt freq npts %e %e %d\n",dt,freq,npts);
/*
* go through layers
*/
do {
/*
* check if plasma
*/
if(strncmp(current_P->typeName,"plasma",6) ==0 ) {
plasmaFlag = 1;
nu=current_P->r;
n=current_P->l;
epsr = 1.0-81e6*n/(freq*freq);
if(epsr > 0) {
eps = EPS0* epsr;
} else {
eps = -EPS0* epsr;
epsr=1.0;
}
}
else {
eps=current_P->c;
epsr = eps/EPS0;
}
printf("%s epsr = %e \n",current_P->name,epsr);
if (!plasmaFlag) velocity =SPEED_OF_LIGHT/sqrt(epsr);
else
velocity =SPEED_OF_LIGHT*sqrt(epsr);
time = current_P->length/velocity;
j = (int) (2.0 *time/dt + 0.5);
for (jj = 1; jj <= j; jj++) {
printf("%d %d length =%f vel =%e time =%e\n",
plasmaFlag,j,current_P->length,velocity,time);
dist += current_P->length/(float)j * 100.0;
distance_A[i] = dist;
fprintf(prof_F,"%d\t%e\t%f \n",i,dist,epsr);
i++;
if ( i >= npts ) break;
}
} while ( (current_P = current_P->left_P) && (i < npts) );
if(i< npts) {
for(j=i; j<npts; j++) {
dist += dt * SPEED_OF_LIGHT * 100.0;
distance_A[j] = dist;
fprintf(prof_F,"%d\t%e\t%f \n",j,dist,epsr);
}
}
fclose(prof_F);
}
```

## A.III C Code for *TransNetCalc*

For the C Code for the Transmission Line Network Calculation (*TransNetCalc*) Program Visit:

https://www.ccdsp.org
also available at GitHub:
https://github.com/silicondsp/TransNetCalc

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# Chapter B

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