A Refutation of M. Detic

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Abstract. In this paper we refute M. Detic [1].

1 Introduction

M. Detic [1] poses a primality test, in which

$$2^{n-1} - 1 \equiv 2^n - 2 \pmod{n}$$

for prime values $n = 5$ and $n = 7$ but not the composite value $n = 9$. We demonstrate the invalidity of this primality test.

2 The Test

We have

$$\begin{aligned} 2^{n-1} - 1 &\equiv 2^n - 2 \pmod{n} \\ \iff 2^{n-1} + 1 &\equiv 2^n \pmod{n} \\ \iff 2^{n-1} + 1 &\equiv 2 \cdot 2^{n-1} \pmod{n} \\ \iff 1 &\equiv 2^{n-1} \pmod{n}. \end{aligned}$$

This is always true for primes p due to the well-known Fermat's Little Theorem [2].

3 Refutation

However, the converse due not hold for all composite numbers. In particular, it fails for the Carmichael numbers [3]. For example, $n = 561$ is one such number. We have

$$2^{560} - 1 \equiv 2^{561} - 2 \pmod{561} \iff 2^{560} \equiv 1 \pmod{561}.$$

One can check that this indeed holds.

4 Conclusion

We have thus refuted Detic's proposed primality test. By equivalence to Fermat's Little Theorem, this test is a *necessary* condition that all primes must satisfy but is not a *sufficient* condition.

5 References

[1] M. Detic, "Analysis of the Congruence Expression for testing if n is prime," viXra:2412.0003v1
[2] "Fermat's Little Theorem," Wikipedia
[3] N. J. A. Sloane, "Carmichael Numbers," OEIS:A002997