Fibonacci Numbers and Gravitational Radiation

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Abstract

We report an arithmetic approximation that analyzes the current formalism with which the study of the phenomenon of gravitational waves attempts to unravel and measure the physical effect that gravitational waves have.To do this we have resorted to the use of the well known Fibonacci series of natural numbers and the logarithmic spiral associated with the sequence of such numbers.

Keywords *Gravitational waves, h strain,Fibonacci numbers,logarithmic spiral,Lyman-alpha line.*

Some deductions

Fibonacci series [1] , in which each number is the result of the sum of the two previous ones, is an infinite sequence of natural numbers. The general definition is

$$f_0 = 0\,,\ f_1 = 1,\ f_n = f_{n-1} + f_{n-2} \tag{1}$$

so that the list of first numbers would be :

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610\ldots \tag{2}$$

It is well known that the ratio between two Fibonacci numbers approaches the well-known golden ratio [2] , namely

$$\varphi = \frac{1+\sqrt{5}}{2} \qquad (3)$$

The larger the numbers in the series of Fibonacci we choose, the closer its ratio will be to φ.
What we are going to do is an inverse operation, namely, applying the inverse of φ

$$F_S = \frac{1}{N}\sum_{n_0}^{N}\frac{f_n}{f_{n+1}} \qquad (4)$$

The series approaches $\frac{\sqrt{5}-1}{2}$ when the numbers chosen are larger.

For example when $n_0 = 8$ and N= $n_8 = 233$

$F_S \simeq 0.618 \simeq \frac{\sqrt{5}-1}{2}$ as seen below:

$$\frac{1}{8}\sum[\frac{8}{13} + \frac{13}{21} + \frac{21}{34} + \frac{34}{55} + \frac{55}{89} + \frac{89}{144} + \frac{144}{233} + \frac{233}{377}] = 0.618 \quad (5)$$

so that series $F_S \simeq 0.618$ is equivalent to the inverse of the golden ratio $\frac{1}{\varphi} = \frac{\sqrt{5}-1}{2}$ (6)

Gravitational waves

Based on the postulates of his general theory of relativity, Einstein proposed that from the dynamic interaction of matter and space-time it would be possible to observe, despite their very small intensity, what is known today as gravitational waves.[3]
Einstein's deductions attempted to show that just as a charged particle moving with a certain acceleration emits electromagnetic radiation, an accelerated mass should emit *space-time* radiation.
But the gravitational interaction is much weaker than the electromagnetic interaction, therefore it is required that the source emitting gravitational waves must have an immense mass and move at a relativistic speed *close* to the speed of light.
By applying basic physical concepts, a formula can be outlined that accounts for how large the impact of a gravitational wave is ,when measuring the proportion or ratio between two lengths; this ratio is symbolized by **h** and explains the **strain** or *wave strain* that can be expected from the action of a gravitational wave.
Devices built to detect gravitational waves use laser interferometry technology.

The scenario (**figure 1**) involves a binary system, two massive stellar bodies (A) and (B), moving at a relativistic speed one towards the other describing a logarithmic spiral.
The result is that both bodies *merge* into one *central point*, which translates into a huge increase in the intensity of the gravitational waves emitted.

Wave strain

We could start doing a dimensional analysis and using physical constants along with the dynamics of the emitting source of gravitational waves.Applying Fibonacci series to a concise formula for *h* strain or *wave strain* reads

$$h \sim \frac{2G\, np_m}{c^4\, mr_p}\left[\frac{1}{N}\Sigma\left(\frac{f_{in}}{f_{in+1}}\right)\frac{1}{N}\Sigma\left(\frac{f_{jn}}{f_{jn+1}}\right)\right]k[L\,T^{-1}]^2 \sim \frac{4\pi\Delta L}{L} \quad (7)$$

Explanation of the terms of the previous equation :

$G = 6.674\, x\, 10^{-11} m^3 kg^{-1} s^{-2}$, *Newtonian constant*

$c = 299792458\, ms^{-1}$, *speed of light in vacuum*

p_m : *proton mass* $= 1.6726\, x\, 10^{-27} kg$

r_p: *proton radius* $= 8.4\, x\, 10^{-16} m$ [4]

f_{in}: *a number of Fibonacci series of spiral A*

f_{in+1} : *the number after the previous one*

f_{nj} : *a number of Fibonacci series of spiral B*

f_{nJ+1} : *the number after the previous one* (see figure 1)

$n = 10^x$, *number of protons that make up both stars*

$m : 10^y$ *orders of magnitude between* r_p *and the observer*

$[L\,T^{-1}]^2$: L^2 *and* T^{-2} *are the components of length squared and acceleration that multiplied by the mass represent the kinetic energy of a body in accelerated motion.*

$k : 10^z$ *orders of magnitude of* $[L\,T^{-1}]^2$

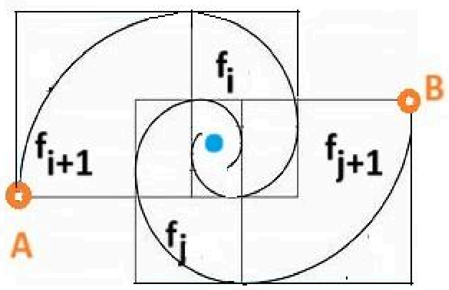

Figure 1.

Since

$$\left[\frac{1}{N}\sum\left(\frac{f_{in}}{f_{in+1}}\right)\frac{1}{N}\sum\left(\frac{f_{jn}}{f_{jn+1}}\right)\right]=\left[\frac{1}{\varphi}\right]^2 \qquad (8)$$

$$\left[\frac{1}{\varphi}\right]^2=\left[\frac{\sqrt{5}-1}{2}\right]^2=\left[\frac{3}{2}-\frac{\sqrt{5}}{2}\right] \qquad (9)$$

symbolizing with greek capital letter theta Θ

$$\Theta = \left[\frac{3}{2} - \frac{\sqrt{5}}{2}\right] \tag{10}$$

then

$$h \sim \frac{2G\, np_m}{c^4\, mr_p} \; [\Theta]\, k[L\, T^{-1}]^2 \sim \frac{4\pi\Delta L}{L} \tag{11}$$

the value of $\frac{4\pi\Delta L}{L}$ depends on the value of n, m, k . Needless to say, it will always be a very small number.
For this purpose, the device that measures the passage or impact of a wave is equipped with a laser interferometry system [5], so that the light wave inside will suffer a shift in its wavelength when a gravitational wave passes through the system and *modifies the space* in which the laser beam is emitted and reflected.

<u>A practical case</u> :

For instance we are looking for the hypothetical emission of gravitational waves from a stellar system with these parameters:
$n \sim 10^{58}$, about ten times the mass of the sun
$m \sim 10^{40}$ times the proton radius, so that $mr_p \sim 10^{24} m$, equivalent to 100 megaparsec (1megaparsec$\sim 10^{22} m$)

$k \sim 10^{17} m^2 s^{-2}$

Given this set of variables, it is expected that the wave strain will be

$h \sim 4\pi\, x\, 10^{-22}$ a very small dimensionless number.

An algebraic sketch

It's worth to note that when, for instance,

$n = 10^{65}$, the number of protons that make up the star, 10^8 times the mass of the sun

$k = 10^{18} \Rightarrow [LT^{-1}]^2 = 10^{11} m^2 s^{-2}$

$r = 10^{12} m$

$$\left[\left(\frac{G\,(10^{65}\, p_m)}{c^4\; 10^{12} m}\right) 10^{18} m^2 s^{-2}\right] - 1 \simeq \Theta \qquad (12)$$

$$\left[\left(\frac{G\,(10^{65}\, p_m)}{c^4\; 10^{12} m}\right) 10^{18} m^2 s^{-2}\right] = 1.382 \qquad (13)$$

once recalling equation (9) :

$$\Theta = \left[\frac{1}{\varphi}\right]^2 = \left[\frac{3}{2} - \frac{\sqrt{5}}{2}\right] = 0.382$$

therefore

$$[1.382] - 1 \simeq \Theta \tag{14}$$

taking into account alternative orders of magnitude, including *proton radius* :

$n = 10^{60}$; $k = 10^{11} \Rightarrow [LT^{-1}]^2 = 10^{11} m^2 s^{-2}$

$r = m\, r_p = 10^{36} r_p$ (about 100 megaparsec ~ $10^{24} m$)

the *wave strain* formula reads

$$h \sim \left[\left(\frac{2G\,(10^{60}\, p_m)}{c^4 \;\; m\, r_p}\right) 10^{11} m^2 s^{-2}\right] \Theta = 4\pi\, x\, 10^{-22} \tag{15}$$

since

$$\Theta = \left[\frac{3}{2} - \frac{\sqrt{5}}{2}\right] \text{ or } \left(\frac{f_n}{f_{n+1}}\right)^2 = 0.382$$

Lyman-alpha line

Lyman-alpha is a spectral line of hydrogen [6]. When electron of hydrogen atom transitions from an n=2 orbital to ground state n=1 emits a photon whose wavelength is

$1.21567\, x\, 10^{-7} m$

New scenario : two massive celestial bodies with a mass of about 10^8 times the mass of the Sun rotate around each other until they merge, emitting gravitational waves that pass through an *hydrogen cloud* located about $10^{12} m$ away. Once the appropriate $n\,,\ r\ and\ k$ values has been set :

$$n = 10^{65}$$
$$r = 10^{12} m$$
$$k = 10^{18} \Rightarrow 10^{18} m^2 s^{-2}$$

then

$$\left[\frac{G\,(10^{65}\,p_m)}{c^4\,10^{12} m} 10^{18} [m^2 s^{-2}]\right] - 1 = \frac{\pi\,(\lambda+\delta\lambda)}{L_0} \qquad (16)$$

$$\lambda = Ly\alpha = 1.21567\ x\ 10^{-7} m$$
$$\delta\lambda \simeq 0.00023\ x\ 10^{-7} m$$
$L_0 = 10^{-6} m$, a micron.

The equation (16) tells us the hypothetical wavelength shift of photons emitted in the ultraviolet range, specifically the Lyman-alpha line, which suffers a tiny displacement of $0.00023\ x\ 10^{-7} m$.

Conclusion

Two concepts, physical and geometric, have been used to attempt an arithmetic approximation to the subject of gravitational waves. The known series of Fibonacci and the golden ratio, can be combined to outline a geometric approximation to the physical system formed by two massive stellar bodies moving at very high speed.
Appropriate technology is capable of measuring the very tiny interaction of the gravitational waves emitted.
Based on the fundamentals of this topic, we have tried to make a simple arithmetic analysis.

References

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