A Modified Born-Infeld Model of Electrons as Foundation of a Classical Model of Point-Like Electrons

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Abstract

While the original Born-Infeld model describes electrons as solutions of classical field equations, there are also several classical particle models that describe electrons as point-like particles. Along the lines of research on these models, the present work proposes a new model of point-like electrons, which represents the peak of a rotating field solution of a modified Born-Infeld field theory by a relativistic, point-like particle. This new model is compared with a recently published neo-classical model of point-like electrons in order to clarify similarities and differences.

1 Introduction

The internal structure of electrons has been an open question since electrons were experimentally discovered more than 125 years ago. Since quantum physics does not answer this question, many classical models of the electron's internal structure have been proposed over the years. An example of classical field-theoretical models of electrons is the model by Born and Infeld [BIF34], as well as models derived from it, for example, a modified Born-Infeld model of electrons as rotating field solutions [Kra23]. Another group of classical models describe electrons as spinning, point-like particles; for example, the neo-classical electron model by Beck [Bec23] and other models referenced in Beck's work.

One interesting difference between the mentioned models is that the modified Born-Infeld model rotates with the Compton frequency as suggested by de Broglie's internal clock hypothesis [dB25], while the point-like electrons in Beck's model spin with twice the Compton frequency as suggested by the so-called Zitterbewegung of specific solutions of the Dirac equation. Which of these two frequencies is closer to reality has to be decided by physical experiments; thus, we are not concerned with this question here.

The main objective of this work is the presentation of a new model of point-like electrons in Section 3. This model represents the rotating peak of a field-theoretical, modified Born-Infeld electron model by a point-like particle, which spins at the Compton frequency. The new model is compared with the classical part of Beck's neo-classical electron model, which spins twice as fast. This comparison clarifies the implications of employing either frequency as discussed in Section 4 and might lead to future work as outlined in Section 5. First, however, Section 2 discusses relevant previous work.

2 Previous Work

2.1 Neo-Classical Model of Point-Like Electrons

Beck's neo-classical electron model [Bec23] is one of several models (see references in Beck's work) of point-like electrons spinning with the frequency of the so-called Zitterbewegung, i.e., twice the Compton frequency. In Beck's model, "the space-time path of an electron of mass m is the sum of the motion of an auxiliary point, the *spin center*, that describes the *global* motion corresponding to the bodily transport of the electron, plus an inherent *local* spin motion about the spin center." [Bec23] This spin motion is motivated by the so-called Zitterbewegung and, therefore, shares the same frequency: "For a free electron, viewed from its *rest frame* fixed at the spin center so that the global velocity of the electron is zero, the spin motion is circular at the speed of light c with angular frequency $\omega_0 = 2\,m\,c^2/\hbar$ and radius $r_0 = c/\omega_0 = \hbar/(2\,m\,c)$, half of the reduced Compton wavelength, giving an angular momentum about the spin center of $m\,c\,r_0 = \hbar/2$." [Bec23]

Since Section 3 compares the classical equations of motion of Beck's model with the equations of motion of the model proposed in this work, a summary of the various forms of Beck's classical equations of motion is provided next.

In order to describe the equations of motion, Beck employs proper time τ, i.e., "the time measured by a clock fixed in this rest frame [fixed at the spin center]" [Bec23]. "Using 4-vectors relative to an observer inertial reference frame X_o with its origin at O, the *total* motion $x(\tau)$ [$= (c\,t, \mathbf{x})$] of the electron of mass m and charge $q = -e$ (so $e > 0$ is the unit electronic charge) is modeled as the sum of: (i) a *local* spin motion $z(\tau)$ [$= (c\,t_z, \mathbf{z})$] where the electron moves in perpetual motion about its spin center C, and (ii) a *global* motion $y(\tau)$ [$= (c\,t_y, \mathbf{y})$] of the electron corresponding to the motion of this spin center" [Bec23]. With $F(x)$ denoting the electromagnetic field tensor at the electron's space-time coordinates x, overhead dots denoting derivatives with respect to proper time τ, employing standard Ricci calculus, and noting that $z^\mu = x^\mu - y^\mu$, Beck provides the following equations of motion [Bec23, Eq. (2.1)]:

$$
\begin{aligned}
\ddot{x}^\mu &= -\omega_0^2\,(x^\mu - y^\mu)\,, && (1)\\
\ddot{y}^\mu &= \frac{q}{m} F^{\mu\nu}(x)\,\dot{x}_\nu. && (2)
\end{aligned}
$$

Additionally, two constraints have to be satisfied [Bec23, Eq. (2.4)]:

$$
\begin{aligned}
C1: &\quad \dot{x}_\mu \dot{x}^\mu = 0, && (3)\\
C2: &\quad \ddot{x}_\mu \ddot{x}^\mu = -c^2\omega_0^2. && (4)
\end{aligned}
$$

Constraint $C1$ requires electrons to move at the speed of light c, while constraint $C2$ requires free electrons to orbit a spin center C. In the rest frame of the spin center C, the angular velocity has to be ω_0, and the trajectory has to be a circular path with radius $r_0 = c/\omega_0$.

With $u = \dot{x}$, the electromagnetic 4-vector potential $A = (V/c, \mathbf{A})$ defined such that $F^{\mu\nu} = \frac{\partial A^\nu}{\partial x_\mu} - \frac{\partial A^\mu}{\partial x_\nu}$, and the Lagrangian function L [Bec23, Eq. (2.18)] defined by

$$
L \stackrel{\text{def}}{=} \frac{m}{2} u^\mu u_\mu + q A^\mu(x) u_\mu - \frac{m}{2\,\omega_0^2}\dot{u}^\mu \dot{u}_\mu, \tag{5}
$$

Beck derives an equivalent fourth-order equation for the total motion $x(\tau)$ [Bec23, Eq. (2.20)]

$$
\ddddot{x}^\mu + \omega_0^2 \ddot{x}^\mu - \frac{q\,\omega_0^2}{m} F^{\mu\nu}(x)\,\dot{x}_\nu = 0. \tag{6}
$$

Introducing a state (x, u, y, π) with the momentum of the global motion $\pi = m\,\dot{y}$, Beck rewrites the equations of motion to 16 equations [Bec23, Eq. (3.18)]

$$
\begin{aligned}
\dot{x}^\mu &= u^\mu, && (7)\\
\dot{u}^\mu &= -\omega_0^2\,(x^\mu - y^\mu)\,, && (8)\\
\dot{y}^\mu &= \frac{1}{m}\pi^\mu, && (9)\\
\dot{\pi}^\mu &= q\,F^{\mu\nu}(x)\,u_\nu. && (10)
\end{aligned}
$$

Using a spin 4-tensor S [Bec23, Eq. (3.1)] with

$$
S^{\mu\nu} = -m\,(z^\mu u^\nu - z^\nu u^\mu)\,, \tag{11}
$$

these equations are further rewritten to equations of motion for the state (x, u, S, π) [Bec23, Eq. (3.19)]:

$$
\begin{aligned}
\dot{x}^\mu &= u^\mu, && (12)\\
\dot{u}^\mu &= \frac{4\,c^2}{\hbar^2} S^{\mu\nu}\pi_\nu, && (13)\\
\dot{S}^{\mu\nu} &= \pi^\mu u^\nu - \pi^\nu u^\mu, && (14)\\
\dot{\pi}^\mu &= q\,F^{\mu\nu}(x)\,u_\nu. && (15)
\end{aligned}
$$

The 4-tensor S is also used to express the spin-field interaction term Φ [Bec23, Eq. (3.12) and Eq. (3.13)]:

$$\Phi = \frac{1}{2}\left(z_\mu q\, F^{\mu\nu} u_\nu - z_\nu q\, F^{\mu\nu} u_\mu\right) = -\frac{q}{2\,m} F^{\mu\nu} S_{\mu\nu} = -\frac{q}{m}\left(\mathbf{B}\cdot\mathbf{s} + \frac{1}{c}\mathbf{E}\cdot\mathbf{d}\right) = U_m + U_e. \tag{16}$$

The magnetic interaction term U_m may also be expressed as $U_m = -\boldsymbol{\mu}\cdot\mathbf{B}$ with the magnetic moment $\boldsymbol{\mu} = \frac{q}{m}\mathbf{s}$.

According to Beck [Bec23, Eq. (3.16)], an electron's energy E in a non-relativistic approximation depends on $\Phi/2$:

$$E \approx m\,c^2 + \frac{1}{2} m\,\mathbf{V}^2 + \frac{1}{2}\Phi \tag{17}$$

with the velocity $\mathbf{V}$ of the spin center in the inertial reference frame X_o. Thus, only about half of U_m contributes to E, which means that the magnetic moment of the model is half of its value in Dirac's theory of electrons.

A similar discrepancy of factor 2 appears in the model's frequency-energy and wavenumber-momentum relations [Bec23, Eq. (2.13)] compared to the well-known relations by de Broglie [dB25].

2.2 Modified Born-Infeld Model of Electrons

The Lagrangian density $\mathscr{L}$ of the modified Born-Infeld field theory discussed here is defined as

$$\mathscr{L} \stackrel{\text{def}}{=} \frac{b^2}{\mu_0}\left(1 - \sqrt{1 - \frac{1}{b^2}(\partial^\mu A^\nu)(\partial_\mu A_\nu)}\right) \tag{18}$$

with the Born-Infeld parameter b specifying the maximum magnetic field strength, the vacuum permeability μ_0, and the electromagnetic four-potential $A = (V/c, \mathbf{A})$ [Kra24b].

The corresponding Euler-Lagrange equations were solved numerically in previous work [Kra23] resulting in a rotating field solution with a peak moving at the speed of light c on a circular orbit with radius $r_1 \stackrel{\text{def}}{=} \hbar/(m\,c)$, i.e., the reduced Compton wavelength. Thus, the angular frequency of this circular motion is $\omega_1 \stackrel{\text{def}}{=} m\,c^2/\hbar = c/r_1$, i.e., the angular Compton frequency, as required by de Broglie's internal clock hypothesis [dB25]. (Here, the symbols r_1 and ω_1 are employed to be consistent with Beck's work [Bec23].) While most features of the solution (electric charge, magnetic moment, radius of orbit, angular frequency) were imposed on it, the total field energy of the solution was matched to the rest mass energy of electrons by adjusting the Born-Infeld parameter b.

At large distances, the solution appears to show the same Lorentz-type interaction with electromagnetic fields as relativistic electrons [Kra24a].

In more recent work [Kra24b], the total field energy as well as the total momentum of the field solution were computed by numerically evaluating elements of the field's canonical stress-energy tensor. While the total field energy of a free electron is conserved, the total momentum of a free electron is a rotating vector pointing in the direction of the velocity of the peak of the rotating field solution with an absolute value close to $m\,c/2$. The intrinsic angular momentum of the solution was evaluated similarly and appears to match the spin of electrons $\hbar/2$.

3 Classical Model of Point-Like Electrons Based on the Modified Born-Infeld Model of Electrons

This section presents a new model of point-like electrons, which is based on the motion of the peak of a rotating field solution of the modified Born-Infeld model of electrons (see Section 2.2). The proposed model is closely related to Beck's neo-classical model (see Section 2.1), if the rotation center of the field solution is identified with the spin center of Beck's model.

In both models, the motion of a point-like electron is the sum of the motion of a spin center, which moves like a relativistic electron of mass m and charge $q = -e$, and a spin motion about the spin center. In the rest frame fixed at this spin center, the point-like electron moves at the speed of light c in a circle about the spin center. In the proposed model, this spin motion is the same as the motion of the peak of the mentioned rotating field solution, i.e., the radius of the circular orbit is the reduced Compton wavelength $r_1 \stackrel{\text{def}}{=} \hbar/(m\,c)$, and the angular frequency is the angular Compton frequency $\omega_1 \stackrel{\text{def}}{=} m\,c^2/\hbar = c/r_1$. (In Beck's model, by contrast, the radius is $r_0 \stackrel{\text{def}}{=} \hbar/(2\,m\,c)$ and the angular frequency is $\omega_0 \stackrel{\text{def}}{=} 2\,m\,c^2/\hbar = c/r_0$.)

In order to match the total momentum of the rotating field solution of about $m\,c/2$ (in the rest frame fixed at the spin center), the mass "participating" in the spin motion of the proposed model is set to $m/2$. This is motivated by the fact that not all field energy of the rotating field solution is concentrated in one point. Instead, the field energy and, therefore, the mass of the electron is spread out in a way that results in a total momentum of the rotating field solution that may be modeled by a point-like mass $m/2$ moving at the speed of light c. Since the radius of its orbit is $r_1 \stackrel{\text{def}}{=} \hbar/(m\,c)$ and the angular frequency is $\omega_1 \stackrel{\text{def}}{=} m\,c^2/\hbar = c/r_1$, a point-like mass of $m/2$ is consistent with an angular momentum of $r_1 c\,m/2 = \hbar/2$.

On the other hand, the whole charge q is assumed to be concentrated at the point-like electron. This is motivated by the relatively small (compared to r_1) volume around the peak of the field solution where nonlinear field strengths are reached. For a classical, point-like particle of charge q and mass m, the gyromagnetic ratio $\gamma(q,m)$ is expected to be $\gamma(q,m) = q/(2\,m)$. If, however, only half the mass $m/2$ contributes to the angular momentum, the value of the gyromagnetic ratio $\gamma(q,m/2)$ is doubled, leading to a spin g-factor of 2, which is consistent with Dirac's theory of electrons.

In order to state the equations of motion, Beck's notation is employed (see Section 2.1), i.e., τ is the proper time measured by a clock fixed in the rest frame of the spin center, overhead dots denote derivatives with respect to this proper time τ, and standard Ricci calculus is employed for 4-vectors and the electromagnetic field 4-tensor $F^{\mu\nu}$ as well as the spin 4-tensor $\tilde{S}^{\mu\nu}$. The total motion of a spinning electron of mass m and charge q is denoted by the 4-vector $x(\tau) = (c\,t, \mathbf{x})$, which is the sum of a local spin motion $z(\tau) = (c\,t_z, \mathbf{z})$ about the spin center C (with only mass $m/2$ "participating" in this local spin motion), and a global motion of this spin center $y(\tau) = (c\,t_y, \mathbf{y})$ corresponding to the standard motion of a relativistic electron of mass m and charge q. Thus, $x^\mu(\tau) = z^\mu(\tau) + y^\mu(\tau)$. The equations of motions are then:

$$\begin{aligned} \ddot{x}^\mu &= -\omega_1^2\,(x^\mu - y^\mu)\,, && (19)\\ \ddot{y}^\mu &= \frac{q}{m} F^{\mu\nu}(x)\,\dot{x}_\nu, && (20) \end{aligned}$$

which are the same equations as in Beck's model [Bec23, Eq. (2.1)] except for ω_1 replacing ω_0.

Additionally, two constraints have to be satisfied:

$$\begin{aligned} \dot{x}_\mu \dot{x}^\mu &= 0, && (21)\\ \ddot{x}_\mu \ddot{x}^\mu &= -c^2\omega_1^2. && (22) \end{aligned}$$

Apart from the replacement of ω_0 by ω_1, these are the same constraints as constraints $C1$ and $C2$ in Beck's model [Bec23, Eq. (2.4)].

An equivalent fourth-order equation of motion for $x(\tau)$ is:

$$\ddot{\ddot{x}}^\mu + \omega_1^2 \ddot{x}^\mu - \frac{q\,\omega_1^2}{m} F^{\mu\nu}(x)\,\dot{x}_\nu = 0, \tag{23}$$

which is again the same as in Beck's model [Bec23, Eq. (2.20)] but with ω_1 instead of ω_0. Using $u = \dot{x}$ and the electromagnetic 4-vector potential $A = (V/c, \mathbf{A})$ defined such that $F^{\mu\nu} = \frac{\partial A^\nu}{\partial x_\mu} - \frac{\partial A^\mu}{\partial x_\nu}$, this fourth-order equation may be derived from the Lagrangian function $\tilde{L}$ defined as

$$\tilde{L} \stackrel{\text{def}}{=} \frac{m}{2} u^\mu u_\mu + q A^\mu(x) u_\mu - \frac{m}{2\,\omega_1^2} \dot{u}^\mu \dot{u}_\mu, \tag{24}$$

which corresponds to the Lagrangian function L in Beck's model [Bec23, Eq. (2.18)] but using ω_1 instead of ω_0.

Introducing a state (x, u, y, π) with the momentum of the global motion $\pi = m\,\dot{y}$, the equations of motion may be rewritten in 16 equations:

$$\begin{aligned} \dot{x}^\mu &= u^\mu, && (25)\\ \dot{u}^\mu &= -\omega_1^2\,(x^\mu - y^\mu)\,, && (26)\\ \dot{y}^\mu &= \frac{1}{m}\pi^\mu, && (27)\\ \dot{\pi}^\mu &= q\,F^{\mu\nu}(x)\,u_\nu && (28) \end{aligned}$$

with the usual replacement of ω_0 by ω_1 relative to Beck's model [Bec23, Eq. (3.18)].

When introducing the spin 4-tensor $\tilde{S}$, it is important to remember that the mass "participating" in the local spin motion is only $m/2$ in the proposed model (instead of m in Beck's model). Thus, $\tilde{S}$ is given by

$$\tilde{S}^{\mu\nu} = -\frac{m}{2}\left(z^\mu u^\nu - z^\nu u^\mu\right), \tag{29}$$

which corresponds to Beck's 4-tensor S [Bec23, Eq. (3.1)] except for a factor 1/2.

The equations of motion for the state $(x, u, \tilde{S}, \pi)$ are then:

$$\dot{x}^\mu = u^\mu, \tag{30}$$

$$\dot{u}^\mu = \frac{2\,c^2}{\hbar^2}\tilde{S}^{\mu\nu}\pi_\nu, \tag{31}$$

$$\dot{\tilde{S}}^{\mu\nu} = \frac{1}{2}\left(\pi^\mu u^\nu - \pi^\nu u^\mu\right), \tag{32}$$

$$\dot{\pi}^\mu = q\,F^{\mu\nu}(x)\,u_\nu. \tag{33}$$

These equations correspond to similar equations in Beck's model [Bec23, Eq. (3.19)] but with $\tilde{S}$ instead of S and additional factors 1/2 where the locally spinning mass is involved (only indirectly as part of $\pi = m\,\dot{y}$ and not in the last equation where the whole mass of the electron is relevant).

When expressing the spin-field interaction term Φ [Bec23, Eq. (3.12)] using the 4-tensor $\tilde{S}$ (instead of S), the result is:

$$\Phi = \frac{1}{2}\left(z_\mu q\,F^{\mu\nu}u_\nu - z_\nu q\,F^{\mu\nu}u_\mu\right) = -\frac{q}{m}F^{\mu\nu}\tilde{S}_{\mu\nu} = -\frac{2\,q}{m}\left(\mathbf{B}\cdot\mathbf{s} + \frac{1}{c}\mathbf{E}\cdot\mathbf{d}\right) = \tilde{U}_m + \tilde{U}_e. \tag{34}$$

Thus, $\tilde{U}_m$ and $\tilde{U}_e$ are twice the values of U_m and U_e of Beck's model [Bec23, Eq. (3.13)] and, therefore, resolve a discrepancy of Beck's model with Dirac's theory of electrons.

Similarly, the discrepancy of Beck's model with de Broglie's frequency-energy and wavenumber-momentum relations [Bec23, Eq. (2.13)] is avoided. This is to be expected since the proposed model is based on a modified Born-Infeld model, which itself is based on de Broglie's internal clock hypothesis (see Section 2.2).

4 Discussion

The model presented in Section 3 appears to be at least as self-consistent as Beck's model [Bec23]. Moreover, it appears to be more consistent with the gyromagnetic ratio and the spin-field interaction of Dirac's theory of electrons as well as de Broglie's frequency-energy and wavenumber-momentum relations than Beck's model. Furthermore, the new model agrees with de Broglie's internal clock hypothesis.

The trade-off is that the frequency of the so-called Zitterbewegung does not appear in the proposed model. Whether this is an actual disadvantage is an open question since there is no general consensus yet on the physical reality of this phenomenon—nor on the reality of de Broglie's internal clocks. These questions have to be decided by future physical experiments.

Another open question is whether the new model may be related to the Dirac equation in a similar way as Beck's neo-classical model [Bec23]. However, Müller claimed that the Dirac equation can be derived from de Broglie's internal clock hypothesis in combination with a superposition principle [Mül14], which might be a more general way to relate the proposed model to the Dirac equation.

Moreover, Section 3 compares the proposed model only to Beck's model in order to keep the presentation short. Thus, Beck's model serves as a recently published representative of multiple models of point-like electrons that include a spin motion at the frequency of the so-called Zitterbewegung. Interested readers are referred to Beck's work [Bec23] for discussions of these models.

5 Conclusion and Future Work

This work presents a new, classical model of point-like electrons based on the motion of the peak of a rotating field solution of a modified Born-Infeld model of electrons [Kra23, Kra24b]. This foundation does not only motivate model parameters such as the radius $r_1 \stackrel{\text{def}}{=} \hbar/(m\,c)$ of the spin motion, its angular frequency $\omega_1 \stackrel{\text{def}}{=} m\,c^2/\hbar = c/r_1$, and the mass $m/2$ "participating" in the spin motion, but also provides a model for the physical process that keeps the electron in a circular spin motion, and thereby distinguishes itself from many other models of spinning, point-like electrons.

Another distinguishing feature is the angular frequency ω_1 since many other models of spinning, point-like electrons feature the angular frequency $\omega_0 = 2\,\omega_1$ of the so-called Zitterbewegung. Once there is more experimental evidence for a spin motion at ω_0, it might be interesting to compute and analyze a rotating field solution of the modified Born-Infeld field theory at this frequency, and construct a model of point-like electrons based on this solution analogously to the model described in Section 3.

Another topic for future work is the relation of the proposed model to quantum mechanics. To this end, a superposition principle for paths of point-like electrons as described by Müller [Mül14] might be necessary. Here, the concept of an underlying field solution could be useful again. To sketch a rough idea of how this process might work, consider a single electron passing through a double slit. Only parts of the full rotating field solution pass through the two slits, and only one of these two partial waves can include the particle-like peak of the full solution that is described by the proposed model. The other peak-less partial wave, however, is likely to interfere with the partial wave that includes the peak, which is able to absorb the peak-less partial wave due to the nonlinearity of the peak. If the two partial waves are not in-phase, this absorption is likely to result in a change of velocity of the spin center that the peak is orbiting about, which means that the spin center is likely to leave the path that it was on before absorbing the peak-less partial wave. This process might, therefore, reduce the probability of detecting particles on such paths and thus appear as destructive interference of paths that are not in-phase. A full development of this idea is certainly far beyond the scope of this paper and is therefore left to future work.

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A Revisions

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