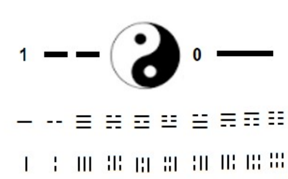
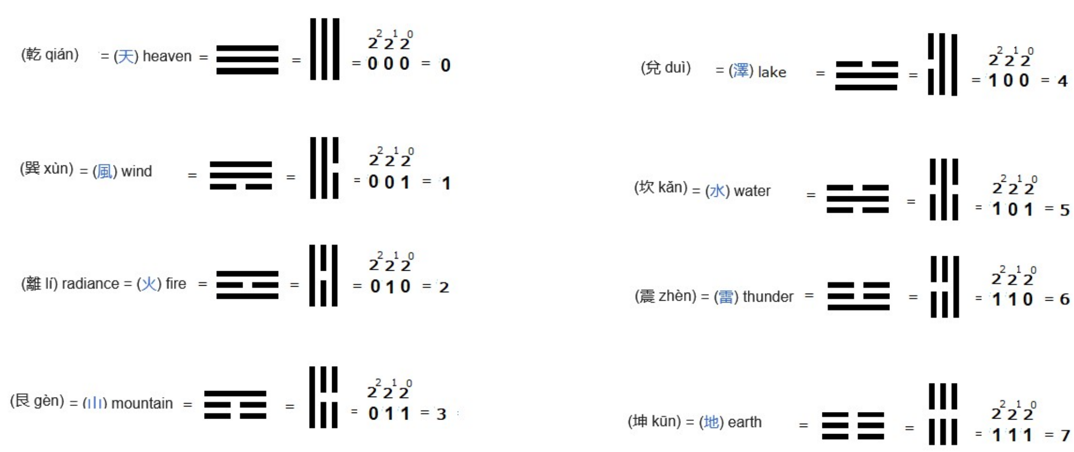
I Ching hexagram groups and subgroups

Groupings and subgroups exist between the hexagrams of the I Ching

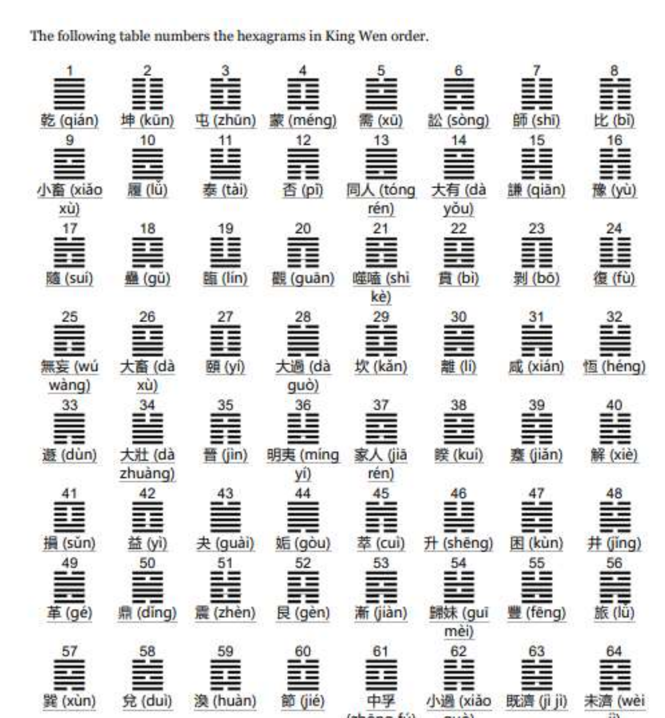
The I Ching (Yijing) (Book of Changes) is an ancient Chinese divination text that is manual in the Western Zhou period (1000–750 BC). Over the course of the Warring States and early imperial periods (500–200 BC).
Thus, the I Ching Zhou yi originated around 5000 years ago.
The Zhou yi was traditionally ascribed to King Wen of Zhou and the Duke of Zhou, and also associated with the legendary Fuxi.

The I Ching is formulated using hexagrams, in-turn made-up of trigrams made from the yin and yang.
yin and yang are represented by broken and solid lines: the yin broken (- -) and yang solid (—) forming a binary system.
trigrams are the eight triplets of yin/yang elements stacked vertically; forming a modulo seven reduced residue system.





hexagrams are trigrams, again stacked vertically.
Thus, the base hexagrams are the 64 combinations of stacked trigram pairs.
The following diagram displays the King Wen ordering of hexagrams:



Adjacent hexagrams in the King Wen ordering are related in an interesting way:
Let: $\mathbf{X}$ represent a trigram, and $\overline{\mathbf{X}}$ be the same trigram with the yin's transformed to yang's , and vice-versa.
Further, let: $\overleftrightarrow{\mathbf{X}}$ be that trigram where the top and bottom lines are exchanged.
So:

$0 = \overline{7} = \overleftrightarrow{0}$	$4 = \overline{3} = \overleftrightarrow{1}$
$1 = \overline{6} = \overleftrightarrow{4}$	$5 = \overline{2} = \overleftrightarrow{5}$
$2 = \overline{5} = \overleftrightarrow{2}$	$6 = \overline{1} = \overleftrightarrow{3}$
$3 = \overline{4} = \overleftrightarrow{6}$	$7 = \overline{0} = \overleftrightarrow{7}$

and: $\overline{\overleftrightarrow{\mathbf{X}}} = \overleftrightarrow{\overline{\mathbf{X}}}$

with the accompanying variations.

Using this, the hexagrams may be arranged into eight groups:

$\mathbf{XX}$	(0,0)	(5,5)
$\overline{\mathbf{X}}\overline{\mathbf{X}}$	(7,7)	(2,2)

$\mathbf{XY}$	(5,6)	(7,3)	(7,4)	(2,6)	(0,6)	(0,3)	(5,3)
$\overleftrightarrow{\mathbf{Y}}\mathbf{X}$	(3,5)	(6,7)	(1,7)	(3,2)	(3,0)	(6,0)	(6,5)

$\mathbf{XY}$	(7,5)	(0,2)	(2,7)	(5,0)
$\mathbf{YX}$	(5,7)	(2,0)	(7,2)	(0,5)

$\mathbf{XY}$	(1,0)	(3,7)	(1,2)	(4,0)	(4,5)	(4,2)	(6,2)	(1,5)	(4,7)
$\mathbf{Y}\overleftrightarrow{\mathbf{X}}$	(0,4)	(7,6)	(2,4)	(0,1)	(5,1)	(2,1)	(2,3)	(5,4)	(7,1)

$\mathbf{X}\overline{\mathbf{X}}$	(7,0)	(5,2)
$\overline{\mathbf{X}}\mathbf{X}$	(0,7)	(2,5)

$\mathbf{XY}$	(4,6)	(4,3)	(3,4)	(1,3)
$\overleftrightarrow{\mathbf{Y}}\overleftrightarrow{\mathbf{X}}$	(3,1)	(6,1)	(1,6)	(6,4)

$\mathbf{XX}$	(6,6)	(1,1)
$\overleftrightarrow{\mathbf{X}}\overleftrightarrow{\mathbf{X}}$	(3,3)	(4,4)

$\mathbf{X}\overleftrightarrow{\mathbf{X}}$	(1,4)	(3,6)
$\overline{\overleftrightarrow{\mathbf{X}}}\overline{\mathbf{X}}$	(6,3)	(4,1)

Note: this grouping can be reduced, as:

$$\mathbf{Y} = \overline{\mathbf{X}} \Rightarrow \overline{\mathbf{X}}\mathbf{X} = \mathbf{YX}$$
$$\mathbf{Y} = \mathbf{X} \Rightarrow \overleftrightarrow{\mathbf{X}}\overleftrightarrow{\mathbf{X}} = \overleftrightarrow{\mathbf{Y}}\overleftrightarrow{\mathbf{X}}$$
$$\mathbf{Y} = \overleftrightarrow{\mathbf{X}} \Rightarrow \overline{\overleftrightarrow{\mathbf{X}}}\overline{\mathbf{X}} = \overline{\mathbf{Y}}\overline{\mathbf{X}}$$

reducing the total grouping to four (with subgroups)

stated more concisely, the binary base-pairs number corresponds to the hexagram number in King Wen ordering :

$(\mathbf{XX}, \overline{\mathbf{X}}\overline{\mathbf{X}})$	01	29

$(\mathbf{XY}, \overleftrightarrow{\mathbf{Y}}\mathbf{X})$	03	15	19	21	25	33	39

$(\mathbf{XY}, \mathbf{YX})$	07	13	35	05

$(\mathbf{XY}, \mathbf{Y}\overleftrightarrow{\mathbf{X}})$	09	23	37	43	47	49	55	59	45

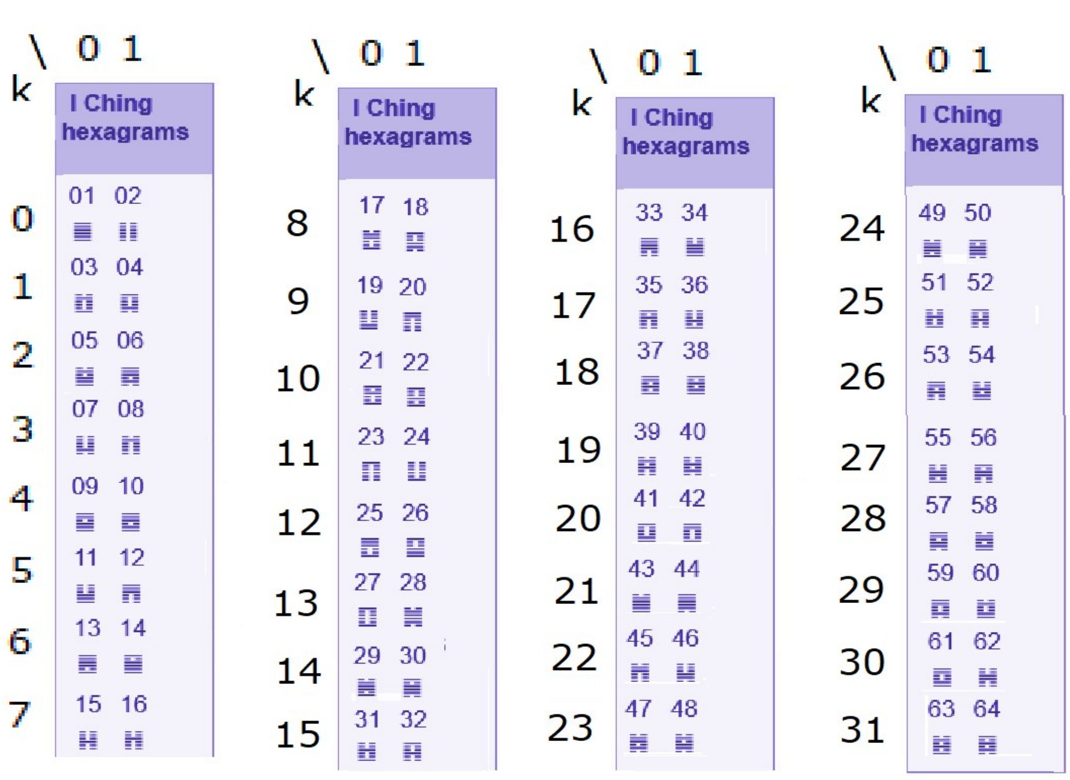
$(\mathbf{X}\overline{\mathbf{X}}, \overline{\mathbf{X}}\mathbf{X})$	11	63

$(\mathbf{XY}, \overleftrightarrow{\mathbf{Y}}\overleftrightarrow{\mathbf{X}})$	17	31	41	53

$(\mathbf{XX}, \overleftrightarrow{\mathbf{X}}\overleftrightarrow{\mathbf{X}})$	51	57

$(\mathbf{X}\overleftrightarrow{\mathbf{X}}, \overline{\overleftrightarrow{\mathbf{X}}}\overline{\mathbf{X}})$	61	27

using the table/matrix:



(n,k) ⇔ (2k+n+1)

notice all the base-pair numbers are odd: (**n** $= 0$) $\Leftrightarrow$ (**n** $= 1$) corresponds to even numbers

n is the table/matrix column number. (**n** $\in \{0, 1\}$)

k is the table/matrix row number.

2k+**n**+**1** is the number of the hexagram in King Wen ordering.

for example:

$(0,9) \Leftrightarrow 2 \cdot 9 + 0 + 1 = 19 \Leftrightarrow (19,20)$

54 is even corresponds to an **n**=1 $\Rightarrow 54 = $ 2**k**+**n**+1 $\Rightarrow$ **k** $= (54 - 2)/2 = 26 \Rightarrow 54 \Leftrightarrow (1,26)$

(thus, an above hexagram group is readily determined from a King Wen order hexagram number)

So, relationships between I Ching hexagram groups (and subgroups - as noted above) may be investigated.

Further, one may wonder on the mathematical insight of the initial developer of the yinyang-trigram-hexagram system.