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Kepler's constant in celestial mechanics, in electromagnetism and in cosmology.

Abstract. *It is well known that the Kepler constant* ***R^3/T^2*** *is a constant of celestial mechanics. It is shown here that the ratio of the cube of the distance to the square of time is a constant for many physical objects and is of a universal nature. The cosmological equation* $\boldsymbol{GM_UT_U^2=R_U^3}$ *is obtained, which includes the constant* $\boldsymbol{R_U}$***^3/*** $\boldsymbol{T_U}$***^2*** *as a ratio of the parameters of the Universe. The cosmological equation combines 4 parameters of the Universe: mass* $\boldsymbol{M_U}$*, radius* $\boldsymbol{R_U}$*, time* $\boldsymbol{T_U}$ *and the Newtonian constant of gravitation* ***G****. In the cosmological equation, the constant* $\boldsymbol{R_U}$***^3/*** $\boldsymbol{T_U}$***^2*** *is a constant of the Universe. The electrodynamic equation* $\boldsymbol{e^2t_0^2=m_er_e^34\pi\varepsilon_0}$ *is obtained, which includes the constant* $\boldsymbol{r_e}$***^3/*** $\boldsymbol{t_0}$***^2*** *as a ratio of the parameters of the electron. The equation combines 4 parameters of the electron: mass* $\boldsymbol{m_e}$*, classical radius of the electron* $\boldsymbol{r_e}$*, characteristic time* $\boldsymbol{t_0}$*, electric charge* ***e****. In the equation containing the electron parameters, the constant* $\boldsymbol{r_e}$***^3/*** $\boldsymbol{t_0}$***^2*** *is a constant of electromagnetism. These equations show that the limits of applicability of Kepler's ratio* ***R^3/T^2*** *go far beyond planetary mechanics. Ratio* ***R^3/T^2*** *is a constant not only of celestial mechanics, but also a constant of the Universe and even a constant of the electron.*
Keywords: *large numbers, cosmological equations, Kepler's Third Law, Kepler's constant, electron, parameters of the observed Universe, Newtonian constant of gravitation G.*

1. Introduction

In astronomy, Kepler's third law is used as an empirical relationship for describing an idealized orbit of a planet. The ratio R^3/T^2, called the Kepler constant, directly follows from Kepler's third law [1, 2]. This value is a constant for a specific central body. For example, the ratio R^3/T^2 is the same for all planets in the solar system. It is believed that the ratio R^3/T^2 is applicable to celestial mechanics for planets, asteroids, and comets. It is shown here that the ratio of such parameters of an object as the cube of its size to the square of its characteristic time is a constant for many physical objects from an electron to the Universe and is of a universal nature. To demonstrate this, we will use the coincidence of large numbers.

2. The law of scaling of large numbers.

The revealed patterns of formation of large numbers, which follow from the ratios of dimensional parameters of the Universe, made it possible to derive the law of scaling of large numbers [3].

The law of scaling of large numbers has the form (Fig. 1):

$$D_i = (D_{20})^i = (\sqrt{\alpha D_0})^i$$

$$i = 0, \pm1, \pm2, \pm3, \pm4, \pm5, \pm6, \pm7, \pm8, \pm9$$

Fig. 1. The scaling law of large numbers. D_0 is a large Weyl number (D_0 = 4.16561...x 10^{42}), α - fine structure constant.

The scaling law provides a new method for calculating the values of large numbers. Its advantage is that large numbers are obtained from dimensionless constants. The scaling law generates large numbers up to a scale of 10^{180} with high accuracy. Large numbers obtained from the scaling law are close in accuracy to the Newtonian constant of gravitation G. This makes it possible to obtain the parameters of the Universe by a mathematical method with high accuracy, close to the accuracy of the Newtonian constant of gravitation G. The values of large numbers and formulas for their calculation are shown in Fig. 2.

$$(\sqrt{\alpha D_0})^0 = 1$$
$$D_{20} = (\sqrt{\alpha D_0})^1 = 1.74349...\bullet 10^{20}$$
$$D_{40} = (\sqrt{\alpha D_0})^2 = 3.03979...\bullet 10^{40}$$
$$D_{60} = (\sqrt{\alpha D_0})^3 = 5.29987...\bullet 10^{60}$$
$$D_{80} = (\sqrt{\alpha D_0})^4 = 9.24033...\bullet 10^{80}$$
$$D_{100} = (\sqrt{\alpha D_0})^5 = 16.1105...\bullet 10^{100}$$
$$D_{120} = (\sqrt{\alpha D_0})^6 = 28.088...\bullet 10^{120}$$
$$D_{140} = (\sqrt{\alpha D_0})^7 = 48.972...\bullet 10^{140}$$
$$D_{160} = (\sqrt{\alpha D_0})^8 = 85.383...\bullet 10^{160}$$
$$D_{180} = (\sqrt{\alpha D_0})^9 = 148.86...\bullet 10^{180}$$

Fig. 2. Large numbers and formulas for their calculation from the law of scaling of large numbers.

The table in Fig. 3 shows the relationships of dimensional quantities that lead to large numbers. Many coincidences of large numbers make it possible to derive new cosmological equations.

Ratios of dimensional constants	**Scale**
$\frac{Gm_e^2}{r_e\alpha^2\hbar H} = \frac{G\hbar}{r_e^3Hc^2} = \frac{Gm_e}{r_e^2\alpha Hc} = \frac{Gm_e^3c}{\alpha^3\hbar^2H} = \frac{c^2}{M_UR_UG\Lambda} = \frac{c^3}{M_UGH} = \frac{c^2R_U}{M_UG} = \frac{l_{Pl}^4}{\Lambda r_e^6} = \frac{c^3T_U}{M_UG} = \frac{\Lambda c^2}{H^2} = \frac{c^3T_U^3}{R_U^3} = \frac{H^2}{M_UR_UG\Lambda^2} = .\frac{cr_e^3A_0}{G\hbar} = \frac{c^4}{M_UR_UH^2G} = \frac{M_UR_UHA_0G}{c^5} = 1$	10^0
$D_{20} = \frac{r_e}{l_{Pl}} = \frac{t_0}{t_{Pl}} = \frac{\alpha\, m_{Pl}}{m_e} = \frac{cl_{Pl}}{r_e^2H} = \frac{l_{Pl}R_U}{r_e^2} = \frac{c^2l_{Pl}}{r_e^2A_0} = \sqrt{\alpha D_0}$	10^{20}
$D_{40} = \frac{T_U}{t_0} = \frac{R_U}{r_e} = \frac{m_ec^2}{\alpha\hbar H} = \frac{1}{t_0H} = \frac{r_e^2}{l_{Pl}^2} = \frac{t_0^2}{t_{Pl}^2} = \frac{\alpha^2m_{Pl}^2}{m_e^2} = \frac{m_ec^3 4\pi\varepsilon_0}{He^2} = \frac{c^2}{r_eA_0} = (\sqrt{\alpha D_0})^2$	10^{40}
$D_{60} = \frac{T_U}{t_{Pl}} = \frac{R_U}{l_{Pl}} = \frac{M_U}{m_{Pl}} = \frac{c}{l_{Pl}H} = \frac{r_e^3}{l_{Pl}^3} = \frac{t_0^3}{t_{Pl}^3} = \frac{c^3}{Gm_{Pl}H} = \frac{c^2}{l_{Pl}A_0} = (\sqrt{\alpha D_0})^3$	10^{60}
$D_{80} = \frac{R_U^2}{r_e^2} = \frac{HM_U^2\alpha G}{c^3m_e} = \frac{c^2}{r_e^2H^2} = \frac{cr_e}{Hl_{Pl}^2} = \frac{1}{r_e^2\Lambda} = (\sqrt{\alpha D_0})^4$	10^{80}
$D_{100} = \frac{m_ec^3}{l_{Pl}\alpha\hbar H^2} = \frac{r_e\alpha M_U}{l_{Pl}m_e} = \frac{HM_U^2\alpha Gr_e}{c^3m_el_{Pl}} = \frac{R_U^2}{r_el_{Pl}} = \frac{1}{r_el_{Pl}\Lambda} = (\sqrt{\alpha D_0})^5$	10^{100}
$D_{120} = \frac{T_U^2}{t_{Pl}^2} = \frac{R_U^2}{l_{Pl}^2} = \frac{M_U^2}{m_{Pl}^2} = \frac{c^2}{l_{Pl}^2H^2} = \frac{R_U^3}{r_e^3} = \frac{M_Uc^2}{\hbar H} = \frac{GM_U^2}{\hbar c} = \frac{c^5}{G\hbar H^2} = \frac{c^3}{G\hbar\Lambda} = \frac{1}{l_{Pl}^2\Lambda} = (\sqrt{\alpha D_0})^6$	10^{120}
$D_{140} = \frac{r_e^2m_ec^3}{l_{Pl}^3\alpha\hbar H^2} = \frac{r_e^3\alpha M_U}{l_{Pl}^3m_e} = \frac{R_U^3}{l_{Pl}r_e^2} = \frac{1}{t_{Pl}t_0^2H^3} = \frac{c}{l_{Pl}r_e^2H\Lambda} = \frac{c^2}{l_{Pl}r_e^2A_0\Lambda} = (\sqrt{\alpha D_0})^7$	10^{140}
$D_{160} = \frac{M_UR_Uc^2\alpha^2}{Gm_e^2} = \frac{M_U^2R_UG\alpha}{c^2r_e^2m_e} = \frac{1}{r_e^4\Lambda^2} = (\sqrt{\alpha D_0})^8$	10^{160}
$D_{180} = \frac{r_e^4m_ec^3}{l_{Pl}^5\alpha\hbar H^2} = \frac{r_e^5\alpha M_U}{l_{Pl}^5m_e} = \frac{R_U^3}{l_{Pl}^3} = \frac{c^3}{l_{Pl}^3H^3} = \frac{c}{l_{Pl}^3H\Lambda} = \frac{c^2}{l_{Pl}^3A_0\Lambda} = \frac{GM_UT_U^2l_{Pl}}{\Lambda r_e^6} = (\sqrt{\alpha D_0})^9$	10^{180}

Fig. 3. Set of coincidences of large numbers. M_U is the mass of the observable Universe, α is the fine structure constant, ħ is Planck's constant, G is the Newtonian gravitational constant, Λ is the cosmological constant, R_U is the radius of the observable Universe, T_U is the time of the Universe, H

is the Hubble constant, A_0 is the cosmological acceleration, r_e is the classical radius of the electron; c - speed of light in vacuum; $t_0=r_e/c$, m_e - electron mass, D_0 - large Weyl number, t_{pl} - Planck time, l_{pl} - Planck length, m_{pl} - Planck mass.

3. Kepler’s constants for the Universe.

From the coincidence of large numbers on the scale of 10^{180}, an equation was obtained that connects 4 parameters of the Universe: mass, radius, time, Newtonian constant of gravitation G:

$$\mathbf{G M_U T_U^2 = R_U^3} \quad (1)$$

where: M_U is the mass of the observable Universe, G is the Newtonian gravitational constant, R_U is the radius of the observable Universe, T_U is the time of the Universe.

From this equation follows the formula (Fig. 4) for the Newtonian constant of gravitation G:

$$\boxed{G = \frac{R_U^3}{M_U T_U^2} \qquad (2)}$$

Fig. 4. Formula connecting 4 parameters of the Universe. M_U - mass of the observable Universe; R_U - radius of the observable Universe; T_U - time of Universe.

Formula (2) contains the parameters of the Universe and is very close in appearance to the well-known equation that follows from a combination of Kepler's law and Newton's Law of Gravitation for the motion of planets in a circular orbit:

$$G = \frac{4\pi^2 R^3}{MT^2} \qquad (3)$$

where: G is the Newtonian gravitational constant, M is the mass of the central body, R is the semi-major axis of the elliptical orbit, T is the period of revolution of the body.

The difference between equation (2) and equation (3) is that instead of frequency in radians per second, frequency in Hz is used in equation (2). Other differences are that instead of the mass of the planet, the mass of the Universe is included in the formula, and instead of the parameters of the orbit of the planet, the parameters of the Universe are included. Formula (2) includes the ratio R_U^3/T_U^2, which contains the parameters of the Universe. Ratio R_U^3/T_U^2 refers to an object that does not have orbital motion. We will call the ratio R_U^3/T_U^2 Kepler's constant for the Universe. This is not the only example. Kepler's law is used in [5] in relation to the gravitational interaction in atoms. The formula for the hydrogen atom also contains the Kepler ratio [3]. These are not all the examples demonstrating the universality of the ratio of the cube of the distance to the square of time. This constant has found itself in the equation of electromagnetism.

4. Kepler's constants for the electron.

The Kepler ratio for the electron r_e^3/t_0^2 is included in the formula for the Newtonian constant of gravitation G (Fig. 5) [4]:

$$G = \frac{r_e^3}{t_0^2 m_e D_0} \qquad (4)$$

Fig. 5. Newtonian constant of gravitation G. r_e - classical electron radius; $t_0=r_e/c$; c - speed of light in vacuum; m_e - electron mass; D_0 is a large Weyl number (D_0 = 4.16561...x 10^{42}).

Formula (4) (Fig. 5) includes the Kepler ratio, presented as r_e^3/t_0^2. We will call the ratio r_e^3/t_0^2 Kepler's constant for the electron. Planets and electrons are very different objects. But for the proper parameters of such different objects, the ratio of the cube of the distance to the square of time is valid, which is a constant similar in form to the Kepler constant.

From the coincidence of large numbers on the scales of 10^{160}, 10^{120}, 10^{40} follows an equation relating the mass of an electron to its electric charge:

$$e^2 = m_e \bullet \frac{r_e^3}{t_0^2} \bullet 4\pi\varepsilon_0 \qquad (5)$$

where: e is the electric charge of an electron, m_e is the electron mass, r_e is the classical radius of the electron, $t_0=r_e/c$, c is the speed of light in vacuum.

Formula (5) includes the Kepler ratio for an electron, represented as r_e^3/t_0^2. As applied to planets, the Kepler law and the Kepler constant include the orbital parameters. As applied to the Universe, the R_U^3/T_U^2 constant includes the Universe parameters. As applied to an electron, the r_e^3/t_0^2 constant includes the electron parameters.

5. Standard gravitational parameter of the Universe.

Formula (2) includes 4 basic parameters of the Universe M_U, R_U, T_U, G. Other parameters of the Universe (Λ, A_0, H) are calculated using the parameters R_U and T_U:

$$\mathbf{\Lambda T_U^2 = 1/c^2; \quad A_0 T_U = c\ ; \quad HR_U = c; \quad \Lambda H^2 R_U^2 T_U^2 = 1} \qquad (6)$$

where: Λ is the cosmological constant, R_U is the radius of the observable Universe, T_U is the time of the Universe, H is the Hubble constant, A_0 is the cosmological acceleration, c is the speed of light in vacuum.

Equation (2) can be written as (Fig. 6):

$$\mu_U = GM_U = \frac{R_U^3}{T_U^2} = 7.69868...\text{x } 10^{42}\ \text{m}^3\text{s}^{-2} \qquad (7)$$

Fig. 6. Standard gravitational parameter of the Universe.

The product GM_U is called the Standard gravitational parameter of the Universe. The Standard gravitational parameter of the Universe is equal to the Kepler constant for the Universe. Here we use the concept of "gravitational parameter of the Universe", which was first introduced by Haug E. G. [6].

Coincidences of large numbers allow us to obtain equivalent formulas for the Standard gravitational parameter of the Universe. Fig. 7 shows 12 equivalent formulas.

$$\boldsymbol{\mu_U} = \left\{ \begin{matrix} GM_U, & \dfrac{R_U^3}{T_U^2}, & \dfrac{A_0}{\Lambda}, & \dfrac{R_U^3 A_0^2}{c^2}, \\ c^2 R_U^3 \Lambda, & R_U^3 H^2, & \dfrac{R_U H^2}{\Lambda}, & c^2 R_U, \\ \dfrac{r_e^2 \alpha D_0}{t_0^2}, & c^3 H, & \dfrac{Hc}{\Lambda} & \dfrac{A_0 c^2}{H^2}. \end{matrix} \right\} = \mathbf{7.69868...x\ 10^{42}\ m^3s^{-2}}$$

Fig. 7. Equivalent formulas for calculating the Standard gravitational parameter of the Universe. M_U is the mass of the observable Universe, α is the fine structure constant, G is the Newtonian gravitational constant, Λ is the cosmological constant, R_U is the radius of the observable Universe, T_U is the time of the Universe, H is the Hubble constant, A_0 is the cosmological acceleration, re is the classical radius of the electron; c - speed of light in vacuum; $t_0=r_e/c$, D_0 - large Weyl number.

All formulas give the same value of the Standard gravitational parameter of the Universe, equal to the Kepler constant for the Universe.

$$\boldsymbol{\mu_U} = GM_U = \frac{R_U^3}{T_U^2} = \frac{A_0}{\Lambda} = \frac{R_U^3 A_0^2}{c^2} = c^2 R_U^3 \Lambda = R_U^3 H^2 = \frac{R_U H^2}{\Lambda} = c^2 R_U =$$

$$= \frac{r_e^2 \alpha D_0}{t_0^2} = c^3 H = \frac{Hc}{\Lambda} = \frac{A_0 c^2}{H^2} = 7{,}69868...\text{x } 10^{42}\ \text{m}^3\text{s}^{-2} \qquad (8)$$

There is a relationship between the Kepler constant for the Universe and the Kepler constant for the electron. The standard gravitational parameter of the Universe can be represented by the Kepler ratio for the electron and a large scale number of 10^{40}.

$$\mu_U = GM_U = \left(\frac{r_e^3}{t_0^2}\right) \bullet (\alpha D_0) = 7.69868... \bullet 10^{42} m^3 s^{-2} \qquad (9)$$

Formula (9) shows that the Kepler constant for the Universe (R_U^3/T_U^2) is a scaled Kepler constant for the electron (r_e^3/t_0^2). The scaling factor is the large number D_{40} = 3.03979… x 10^{40} (Fig. 2).

6. Standard gravitational parameter of the электрона.

The product Gm_e will be called the Standard gravitational parameter of the electron.

$$\mu_e = Gm_e = \frac{r_e^3}{t_0^2 D_0} = 0.607987... \bullet 10^{-40} m^3 s^{-2} \qquad (10)$$

Fig. 8 shows 9 equivalent formulas for the Standard gravitational parameter of the electron.

$$\mu_e = \left\{ \begin{matrix} Gm_e, & \dfrac{r_e^3}{t_0^2 D_0}, & \dfrac{c^2 r}{D_0}, \\ \dfrac{m_e c^5 t_{Pl}^2}{\hbar}, & \dfrac{c^4 m_e}{F_0 D_0}, & \dfrac{\hbar c m_e}{m_{Pl}^2}, \\ \dfrac{\alpha\hbar c}{m_e D_0}, & \dfrac{c^3 r_e^2 m_e}{\alpha\hbar D_0}, & \dfrac{c^3 l_{Pl}^2 m_e}{\hbar}. \end{matrix} \right\} = \mathbf{0.607987... x\ 10^{-40}\ m^3 s^{-2}}$$

Fig. 8. Equivalent formulas for calculating the Standard gravitational parameter of the electron. G is the Newtonian gravitational constant, F_0 is the force constant, α is the fine structure constant, ħ is Planck's constant, r_e is the classical radius of the electron; c is the speed of light in vacuum; $t_0= r_e/c$, me is the electron mass, D_0 is the large Weyl number, t_{pl} is the Planck time, l_{pl} is the Planck length, m_{pl} is the Planck mass.

The Standard gravitational parameter of the Universe (R_U^3/T_U^2) is a scaled Standard gravitational parameter of the electron (r_e^3/t_0^2Do). The scaling factor is the large number αD_0^2 .
The obtained Kepler constant for the Universe R_U^3/T_U^2 and the Kepler constant for the electron r_e^3/t_0^2 define the range of possible Kepler constants for other objects in the Universe. The range occupies the area from elementary particles to the Universe. In the interval between R_U^3/T_U^2 and r_e^3/t_0^2 there should be Kepler constants for elementary particles, atoms, planets, galaxies [7].

7. Conclusion

The cosmological equation of the Universe, the equation of electromagnetism for the electron, Kepler's law for planets, the equation for the Newtonian constant of gravitation G are equations of different nature. All of these equations include the ratio of the cube of the distance to the square of time as a constant. This shows the universal nature of the ratio. The universality of Kepler's constant places it in the rank of the most important constant of physics and cosmology. Such involvement of Kepler's constant in phenomena of different nature requires deep study.

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