A novel offline and online parameter identification technique of nonlinear fractional order systems using approximated fractional order derivative and the intelligent optimization methods

Chol-Sik Ryang*, Kyong-Min Yun, Tae-Song Kwan, Il Song Rim, Hyon Ho Han

*Faculty of Mathematics, **Kim Il Sung** University, Pyongyang , DPR Korea*

* *CS.Ryang@star-co.net.kp*

Abstract

This paper makes an accurate fractional model of the existing non-linear systems using fractional order theory and various intelligent optimization methods and proposes a novel method to identify time-varying parameters of the fractional non-linear system offline and online. More accurate mathematical model of the proposed system was made by applying approximated fractional derivative into the state space model of the classical non-linear system. The initial parameter values of the proposed non-linear fractional system were identified offline by using hybrid particle swarm optimization-genetic algorithm method that is a combination of particle swarm optimization(PSO) and genetic algorithm(GA) that are typical intelligent optimization methods. The time-varying parameters of the non-linear fractional order systems were identified online in real-time by using the output error technique and the recursive least square method. In order to verify the efficiency of the proposed identification technique, we made a simulation experiment for offline and online identification of the time-varying parameters in the existing nonlinear fractional Lorentz system and nonlinear fractional lithium-ion battery system. Simulation results show that the proposed novel identification method can be effectively used for offline and online parameter identification of many complicated non-linear fractional order systems in practice.

Keywords :

parameter identification; fractional order system; Particle Swarm Optimization; Genetic Algorithm; Recursive Least Square

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1. Introduction

Fractional calculus could be used in modeling many physical phenomena and engineering systems of the real-world in a more reasonable and accurate way compared with the classical integer calculus[1].

In recent years, fractional calculus has been widely researched by a lot of researchers and applied in various fields, especially in control engineering[2], electrochemistry[3], dielectric physics[4] , viscoelasticity[5] , nonlinear acoustics[6], diffusion theory [7], photoelectricity field ionization[8], human engineering [9,10], epidemicity contagious disease [11], fuzzy logic [12], artificial neural network [13], spline approximation[14], and so on. It has also been reported that many researches for fractional modeling and system identification of nonlinear systems are continuously being made [15-21]. Fractional calculus is a kind of classical integer calculus, which could be used in modeling the classical nonlinear systems to be more accurate fractional nonlinear systems [22].There are several definitions of fractional calculus and [23] describes 3 typical definitions; Grunwald – Letnikov, Riemann- Liouville and Caputo fractional calculus. Otherwise fractional operators have usually been approximated by high order rational models. General approximation of fractional operator in a limited frequency band is the recursive distribution of zeros and poles proposed by Oustaloup in [24].

Trigeassou et al [25] have proposed to use the integrator outside a certain frequency band instead of a gain and described the principle and method of approximation of fractional integration in [26].

The parameters of fractional nonlinear system could be identified offline and online by using several optimization methods such as intelligence optimization and least square method. Particle Swarm Optimization is the method proposed by Eberhart and Kennedy in 1995, which is the intelligence optimization method that imitates food-searching action of a group of insect such as ants and bees , a flock of birds and a shoal of fish[27].In [28], inertia weight has been suggested in order to consider the effects of the previous speed on the present speed of the particle. Because Particle Swarm Optimization has been known to be very sensitive to the size of particle population, inertia weight coefficient, moving step number, maximum velocity and maximum number of replacement, it could be used to identify the parameters of the system. However, if the number of particles is given to be large, it's possible to find optimal solution while it's time consuming and takes a large amount of calculations.

Genetic algorithm is a kind of intelligent optimization method, proposed by Holland in 1968, that imitates the natural genetic phenomena. With the rapid development of computer technology since 1990's, genetic algorithm is widely used in several aspects such as reservoir operation optimization, numerical model parameter optimization, path searching problem , battery management system and so on [29,30]. Meanwhile, the classical least square method is one of parametric identification method of the system and widely used in many fields because of its simplicity in principle and good performance in identification [31,32].

Also the recursive least square(RLS) is the approach which parameters are updated on every addition of new data, which is widely used in online real time parametric identification of the system[33].

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* Corresponding author

Email addresses: CS.Ryang@star-co.net.kp (Chol-Sik Ryang)

Recursive least square (RLS) identification is performed to add new observation data to the previous observation values and recalculate the normal least square estimation. This paper sets an accurate fractional model of the existing nonlinear systems using fractional theory and several intelligent optimization approaches and proposes a new approach for online real-time identification of the time varying parameters in fractional nonlinear system. Section 2 introduces the basic knowledge on the definition of fractional nonlinear system, the fractional derivative and fractional integration, and the approximation of fractional operators. Section 3 describes the offline and online parametric identification method of nonlinear fractional system using approximated fractional order derivative and the intelligence optimization method. Section 4 shows simulation results and verification for offline and online parametric identification in nonlinear fractional Lorentz system and nonlinear fractional lithium-ion battery system. Section 5 gives conclusions.

2. Knowledge Background

2.1. fractional nonlinear systems

Fractional calculus is the generalization of the classical integer order calculus.

This paper considers the general fractional nonlinear system as follows [22] :

$$\begin{aligned} &{}_0D_t^{\alpha_i} x_i(t) = f_i(x_1(t),\, x_2(t), \cdots,\, x_n(t),\, t) \\ &x_i(0) = c_i\,,\;\; i = 1, 2, \cdots, n \\ &y(t) = C[\, x_1 \quad x_2 \quad \cdots \quad x_n\,]^T \end{aligned} \tag{1}$$

where , x_i is for input, y for output, c_i for initial condition, α_i for fractional order and $C = [0 \;\; \cdots \;\; 0 \;\; 1]$.

$$f(X) = 0 \tag{2}$$

And $E^* = (x_1^*,\, x_2^*,\, \cdots,\, x_n^*)$ is supposed to be the equilibrium point of nonlinear system (1).

2.2. fractional derivative and fraction integration

Among several definitions on fractional derivative, [23] has proposed the Grunwald – Letnikov, Riemann- Liouville and Caputo fractional calculus definitions for absolute continuous functions, that are three most commonly used ones. The Riemann- Liouville derivative definition of the order α can be described as follows:

$${}_c^R D_t^{\alpha}\, f(t) = \frac{d^{\alpha}}{dt^{\alpha}} \left[\frac{1}{\Gamma(n-\alpha)} \int_c^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}}\, d\tau \right] \tag{3}$$

where, $n-1 \le \alpha < n,\;\; n \in N$ and $\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x}\, dx$ is the gamma function.

Caputo derivative is defined as follows;

$$ {}_c^C D_t^\alpha \; f(t) = \frac{1}{\Gamma(n-\alpha)} \int_c^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} \, d\tau \tag{4} $$

Grunwald-Letnikov derivative definition can be described as follows:

$$ {}_c^{GL} D_t^\alpha \; f(t)|_{t=kh} = \lim_{h \to 0} \frac{1}{h^\alpha} \sum_{j=0}^{[\frac{t-c}{h}]} \omega_j^{(\alpha)} f(kh - jh) \tag{5} $$

where, h is the sample time and $[\cdot]$ is the flooring function and the coefficient $\omega_j^{(\alpha)}$ is given below.

$$ \omega_j^{(\alpha)} = \frac{(-1)^j \Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)}, \qquad j = 0,1,\cdots \tag{6} $$

While Riemann-Liouville) integration definition on function $f(t)$ can be described as follows:

$$ I_\alpha(f(t)) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau) \, d\tau \tag{7} $$

where, α is the real positive.

2.3. approximation of fractional operators

Fractional operators are usually approximated by high order rational models. As the result, a fractional model and its rational approximation have the same dynamics in a limited frequency band.

General approximation of s^α in frequency band $[\omega_b , \omega_h]$ is the recursive distribution of zeros and poles proposed by Oustaloup. Trigeassou et al.[25] have suggested to use an integrator instead of a gain outside the frequency range $[\omega_b , \omega_h]$.

$$ I_\alpha(S) = \frac{1}{S^\alpha}, $$

$$ I_\alpha^*(S) = \frac{C_0}{S} \left(\frac{1 + S/\omega_b}{1 + S/\omega_h} \right)^{1-\alpha} \approx \frac{C_0}{S} \prod_{k=1}^{N} \frac{1 + S/\omega_k'}{1 + S/\omega_k} \tag{8} $$

The bloc diagram of fractional integration approximated by equation (8) can be shown in figure 1 [26].

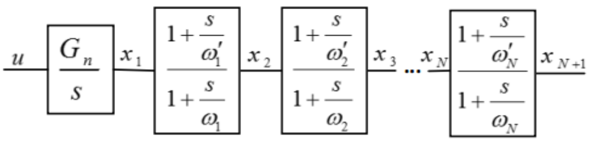

Figure 1. Block diagram of $I_\alpha^*(S)$

Operator $I_\alpha^*(S)$ is characterized by 6 parameters , where ω_1' and ω_N define the frequency range.

N is the number of cells(it is directly related to the quality of the needed approximation),and pulsations ω_i and ω_i' are related as follows;

$$\omega_i = \lambda\omega_i', \quad \lambda > 1$$
$$\omega_{i+1}' = \eta\omega_i, \quad \eta > 1 \tag{9}$$

Fractional order of operator is :

$$\alpha = 1 - \frac{\log(\lambda)}{\log(\lambda\eta)} \tag{10}$$

A larger number N is one of better approximations of the integrator $I_\alpha(S)$. Since operator $I_\alpha^*(S)$ is found by the product of cells, the state variables are defined as the output of each cell through figure 1 [26]. The state space model of this system is given as follows:

$$M\,\dot{x} = Ax + Bu$$

Equivalently it can be described as follows:

$$\dot{x} = A^* x + B^* u$$
$$A^* = M^{-1}A, \qquad B^* = M^{-1}B \tag{11}$$

where

$$M = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ -\lambda & 1 & 0 & \cdots & 0 & 0 \\ 0 & -\lambda & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & -\lambda & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ \omega_1 & -\omega_1 & 0 & \cdots & 0 & 0 \\ 0 & \omega_2 & -\omega_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -\omega_{N-1} & 0 \\ 0 & 0 & 0 & \cdots & \omega_N & -\omega_N \end{bmatrix},$$

$$B = \begin{bmatrix} G_N \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \qquad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N+1} \end{bmatrix}.$$

Hence, the block diagram of state space model on the operator of fractional integration is represented in figure 2.

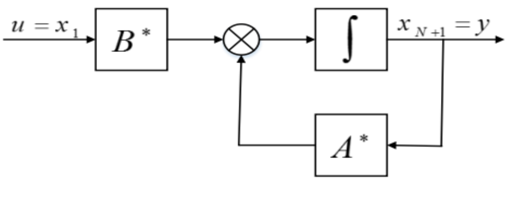

Figure 2. Block diagram of state space model on the operator of fractional integration

3. A novel offline and online parameter identification technique of non-linear fractional -order

systems

3.1. The offline parameter identification of non-linear fractional-order systems

3.1.1. output error method

Output error method is one of the visual parameter estimation methods which gives the same input to the estimation object and model, and minimizes the output error by means of least square.

In output error method the parameter is represented as fractional equation unlike in prediction error method, so that it is nonlinear for the parameter to find it difficult in its analytical solution.

As a result, nonlinear optimization iteration approach like gradient-based method should be available. On the other hand, because of the local minimum value in the output error method, applying the gradient-based method can converge to the local minimum value which is not the global minimum value. Fig. 3 shows the schematic diagram of the output error method.

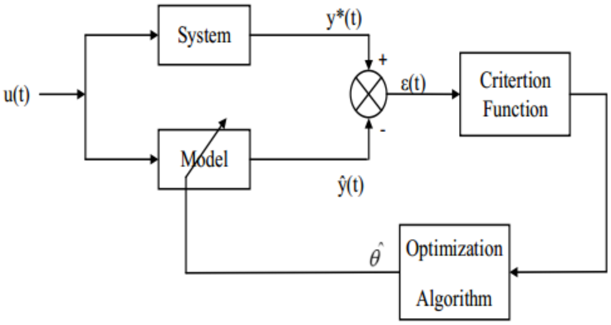

Figure 3 .Principle diagram of output error method

Parameter estimation of nonlinear system in the time domain is done with M data pairs $\{u_k, y_k^*\}$.

Where u_k is the input value and y_k^* is the output value of real object.

The objective function uses the following mean square error.

$$J = \frac{1}{M}\sum_{k=1}^{M}(y_k^* - \hat{y}_k(u, \hat{\theta}))^2 \tag{12}$$

Here, $\hat{y}_k(u, \hat{\theta})$ is the output value of model system based on input signal u and parameter estimation $\hat{\theta}$ .Parameter estimation $\hat{\theta}$ giving minimum to the objective function can be found by using PSO-GA optimization method.In general, in case that the system of the object is given as the linear system or nonlinear integer system, parameter estimation is simple and easy to solve.

3.1.2. Offline parameter identification by PSO method

In D-order solution search space each point represents a solution called 'particle'. At the initial stage the PSO method generates a random particle swarm and finds the optimal solution through several iterations.If the i -th particle position is represented as $X_i = [X_{i1}, X_{i2}, \cdots, X_{id}]$, flight velocity as $V_i = [V_{i1}, V_{i2}, \cdots, V_{id}]$, i -th particle optimum position as $P_i = [P_{i1}, P_{i2}, \cdots, P_{id}]$ and the optimum position among all particles as $P_g = [P_{g1}, P_{g2}, \cdots, P_{gd}]$, the velocity and position of a particle can be updated as follows;

$$V_i(t+1) = w \cdot V_i(t) + c_1 \cdot rand() \cdot (P_i - X_i(t)) + c_2 \cdot rand() \cdot (P_g - X_i(t)) \quad (13)$$

$$X_i(t+1) = X_i(t) + V_i(t) \quad (14)$$

Here, $i = 1, \cdots, N$

w is the constant inertia weigh coefficient

$c_1 > 0$ and $c_2 > 0$ are the moving step coefficients

$rand()$ is uniform random number in interval [0,1] .

In equation, w is coefficient to show the inertia on the previous flight of particle. c_1 and c_2 are coefficients to control the maximum step in the optimum position flight direction of each particle itself and population.

The flow chart of offline parameter identification by PSO method is shown below.

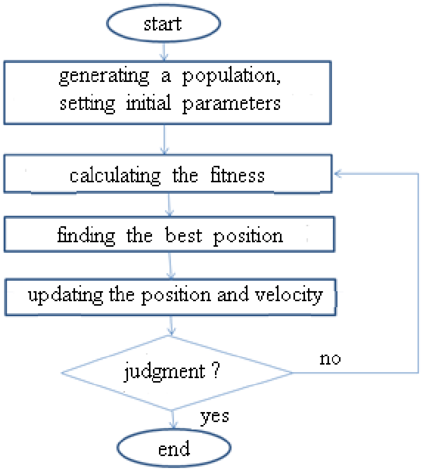

Figure 4. Flow chart of offline parameter identification by PSO method

3.1.3. Offline parameter identification by GA

Parameters of nonlinear fractional system can be effectively identified by using GA. First, we randomly create a group of parameter search and make an initial group by coding each individual in terms of decimal code. Second, we calculate the fitness value of each individual based on the analytic model of nonlinear system. Third, we get a next generation of population by using genetic operation (selection, cross-over and mutation) for each individual. After judgment of the terminal condition, if the evolution generation number reaches the maximum generation number of the initial step or if the optimal solution is obtained in a group of individuals, searching must be done or the above process be iterated. .Flow chart of offline parameter identification by GA is represented as follows;

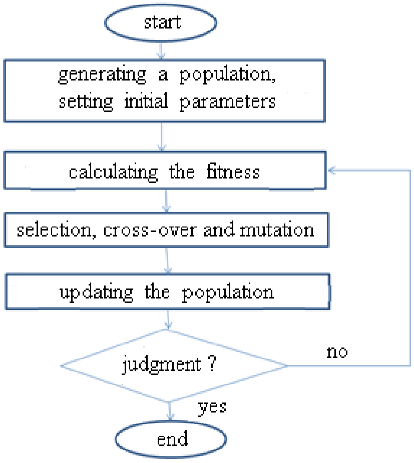

Figure 5. Flow chart of offline parameter identification by GA

3.1.4. Offline parameter identification by hybrid PSO-GA

Combination of GA and PSO can improve the convergence and global optimizing performance. The method that PSO is combined with GA is called the hybrid PSO-GA. Evolution operators of GA can be used to prevent premature convergence of parameter identification algorithm. Applying cross-over and mutation operations into PSO improves the global optimum and the population diversity. The hybrid PSO-GA can be used in offline parameter identification of nonlinear fractional system. First, we randomly generate the initial position and velocity of the population and set the maximum number of repetitions. Second, we calculate the fitness value of each individual of the population by using the fitness function. Third, we update the position and velocity of the particles by using equations (13) and (14). Fourth, by applying random selection, cross-over and mutation operations into the updated population, we get a new population and search for the optimal solution by using the fitness function.

Fifth, if we get a reasonable value after judging the terminal condition, we end the searching; otherwise, we iterate the above process. Figure 6 shows the flow chart of offline parameter identification using hybrid PSO-GA.

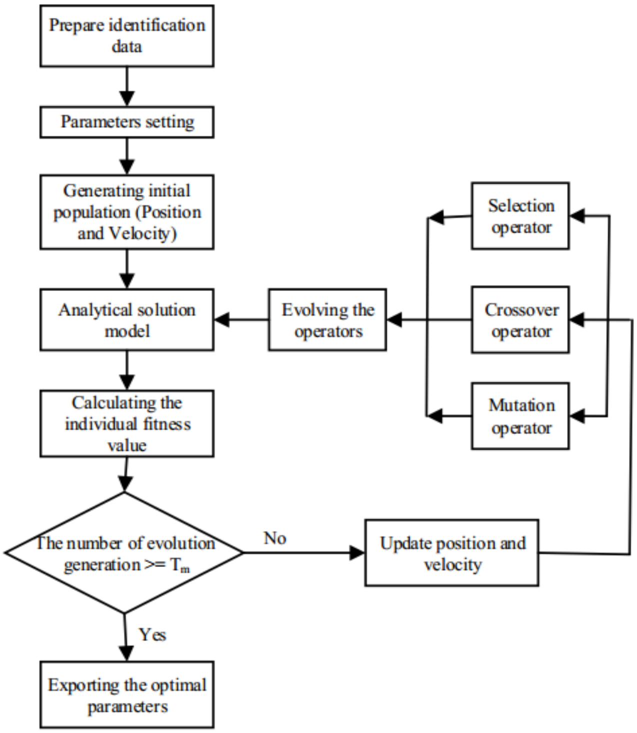

Figure 6. Flow chart of offline parameter identification based on hybrid PSO-GA

3.2. Online parameter identification of nonlinear fractional systems

3.2.1. Online parameter identification based on RLS

Using RLS, the parameters of nonlinear fractional system can be identified online in real time. At the time of $t-1$, the normal least square estimation of parameter is expressed as follows;

$$\hat{\theta}(t-1) = \left[\sum_{k=1}^{t-1} \varphi(k)\varphi^T(k)\right]^{-1} \sum_{k=1}^{t-1} \varphi(k) y(k) \tag{15}$$

Giving the following symbol,

$$R(t-1) = \sum_{k=1}^{t-1} \varphi(k)\varphi^T(k) \tag{16}$$

The following equations will be given;

$$R(t-1)\hat{\theta}(t-1) = \sum_{k=1}^{t-1} \varphi(k) y(k) \tag{17}$$

$$R(t) = \sum_{k=1}^{t} \varphi(k)\varphi^T(k) = R(t-1) + \varphi(t)\varphi^T(t) \tag{18}$$

$$R(t-1) = R(t) - \varphi(t)\varphi^T(t) \tag{19}$$

Using equations (17), (18) and (19), the following relation can be given;

$$\hat{\theta}(t) = \hat{\theta}(t-1) + R^{-1}(t)\varphi(t)[y(t) - \varphi^T(t)\hat{\theta}(t-1)] \tag{20}$$

Meanwhile, in case of $P_t = R^{-1}(t)$, the following recursive least square equation can be obtained.

$$\left.\begin{aligned} &\hat{\theta}(t) = \hat{\theta}(t-1) + P_t\varphi(t)[y(t) - \varphi^T(t)\hat{\theta}(t-1)] \\ &P_t = P_{t-1} - [1+\varphi^T(t)P_{t-1}\varphi(t)]^{-1} P_{t-1}\varphi(t)\varphi^T(t)P_{t-1} \\ &\hat{\theta}(0) = 0, \qquad P_0 = \gamma \cdot I, \qquad \gamma > 0 \end{aligned}\right\} \tag{21}$$

Therefore, using (21), the parameters of nonlinear fraction system are identified online in real time.

3.2.2. Online parameter identification based on hybrid PSO-GA and RLS

This paper presents a new online parameter identification method combining hybrid PSO-GA with RLS in order to improve the convergence and accuracy for online parameter identification of nonlinear fractional system. Figure 7 shows the flow chart of online parameter identification based on hybrid PSO-GA and RLS.

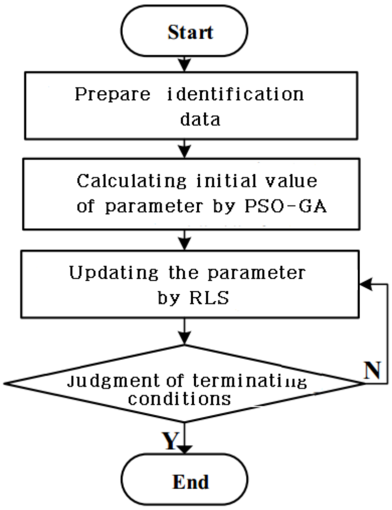

Figure 7. Flow chart of online parameter identification based on hybrid PSO-GA and RLS

4. Simulation results

4.1. Result of offline parameter identification of nonlinear fractional Lorentz system

The state space model of nonlinear fractional Lorentz system is represented as follows;

$$\begin{aligned} &{}_0^{GL}D_t^q\, x(t) = \sigma[y(t) - x(t)] \\ &{}_0^{GL}D_t^q\, y(t) = x(t)[\rho - z(t)] - y(t) \\ &{}_0^{GL}D_t^q\, z(t) = x(t)y(t) - \beta z(t) \end{aligned} \tag{22}$$

This fractional state space model is a hydrodynamical equation used to consider the flow of heat in meteorology. Here, x, y and z represent the spatial coordinates and σ, ρ, β and q are parameters to be identified.In simulation the true values of identification parameters are set to be $\sigma = 12$, $\rho = 5.67$, $\beta = 30$ and $q = 0.98$ and initial condition of coordinate set to be $(x_0, y_0, z_0) = (0.1, 0,1, 0.1)$ .Grunwald-Letnikov derivative is used and the step size is given as $h = 0.001$. In addition, the fractional derivative is approximated by $N = 2 \sim 7$ for different cells in frequency of $\omega_b = 10^{-3}\ rad/s$ and $\omega_h = 10^{1}\ rad/s$. The followings are set to be searching range of parameters.

$$0 \le \sigma \le 23,\ 0 \le \rho \le 8,\ 0 \le \beta \le 55,\ 0 \le q \le 1.2,$$
$$0 \le \omega_b \le 1,\ 0 \le \omega_h \le 100$$

In MATLAB 2021a programming language, the hybrid PSO-GA is implemented and the simulation experiment is made. In simulation $M = 50$ is given for the size of the initial population; $\omega = 0.85$, $c_1 = 3.64$ and $c_2 = 2.76$ for the initial values of the inertia weight coefficient and the moving step coefficients, respectively; 0.8 for cross-over probability; 0.7 for mutation probability; 500 for maximum genetic number. Tables 1, 2 and 3 show the simulation results using GA, PSO and hybrid PSO-GA, respectively.

Table 1. Simulation result based on GA

Cell number	σ	Std of σ	ρ	Std of ρ	β	Std of β	q	Std of q	J
True value	12	-	5.67	-	30	-	0.98	-	-
2	10.54	3.56	5.31	0.97	29.53	1.04	0.97	0.08	2.26
3	20.37	3.07	5.57	0.31	29.48	0.26	0.97	0.02	0.55
4	14.93	6.43	5.62	0.66	29.57	0.27	0.98	0.03	0.78
5	13.54	4.02	5.68	0.43	29.98	0.20	0.98	0.05	0.43
6	13.98	4.38	6.11	0.58	31.07	0.97	0.99	0.21	1.39
7	12.75	4.21	6.47	0.51	30.03	0.84	1.00	0.09	1.02

Table 2. Simulation result based on PSO

Cell number	σ	Std of σ	ρ	Std of ρ	β	Std of β	q	Std of q	J
True value	12	-	5.67	-	30	-	0.98	-	-
2	10.93	4.47	5.31	0.56	27.90	0.36	0.95	0.25	1.07
3	14.37	3.91	5.53	0.17	29.73	0.05	0.97	0.02	0.09
4	12.27	1.27	5.66	0.05	29.85	0.09	0.98	0.03	0.07
5	12.01	0.93	5.67	0.09	29.99	0.06	0.98	0.07	0.07
6	13.35	2.57	5.74	0.25	29.12	0.22	0.99	0.08	0.08
7	14.88	2.34	5.82	0.33	29.53	0.26	0.99	0.12	0.15

Table 3. Simulation result based on hybrid PSO-GA

Cell number	σ	Std of σ	ρ	Std of ρ	β	Std of β	q	Std of q	J
True value	12	-	5.67	-	30	-	0.98	-	-
2	11.97	1.53	5.60	0.32	29.14	1.25	0.99	0.12	0.05
3	13.32	2.16	5.62	0.11	31.05	1.73	1.00	0.08	0.04
4	12.05	1.27	5.69	0.18	30.05	0.21	0.98	0.26	0.04
5	12.01	1.12	5.67	0.13	30.01	0.15	0.98	0.03	0.01
6	12.04	1.35	5.67	0.22	30.08	0.38	0.99	0.07	0.02
7	12.03	1.74	5.68	0.17	30.16	0.20	0.97	0.14	0.03

In simulation experiment each method was repeated 30 times, respectively, with increase in the number of cells. As a result, in simulation experiment using hybrid PSO-GA, when the number of cells is 5, the least mean square error was given as $J = 0.01$. From the above simulation experiment results, it can be seen that offline parameter identification method by hybrid PSO-GA is more effective in accuracy and convergence and better in performance than the other methods.

4.2. Online parameter identification result of nonlinear fractional lithium-ion battery system

This paper proposes the data tested in the system of NCR18650-1,2,3 type lithium-ion battery. The above 3 types of batteries are characterized by 3.7V for rated voltage, 2.8V for cutoff voltage, and 2422 mAh , 2661 mAh, and 2855 mAh for their capacities, respectively. Figure 8 shows the electrochemical impedance spectroscopy (EIS) curves of lithium-ion battery under the different state of charges(SOC). In the electrochemical impedance spectroscopy (EIS) test the amplitude of test signal is set to be 5mV, frequency range is 0.005 Hz~5000 Hz, and the SOCs of lithium-ion battery are 40%, 60% and 80%, respectively. Figure 9 shows the hybrid pulse power characteristic (HPPC) test curve of lithium-ion battery. Discharge current of battery in the hybrid pulse power characteristic (HPPC) test is set to be 3A, the rest time is set to be 1h and the sample time of voltage and current is set to be 0.1s. Figure 10 shows the fractional order equivalent circuit model of lithium-ion battery.

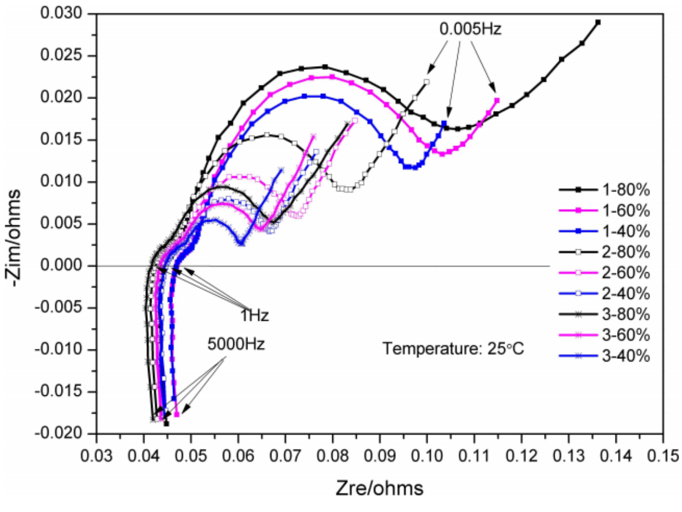

Figure 8. Electrochemical impedance spectroscopy (EIS) curves of lithium-ion battery under different state of charge(SOC)

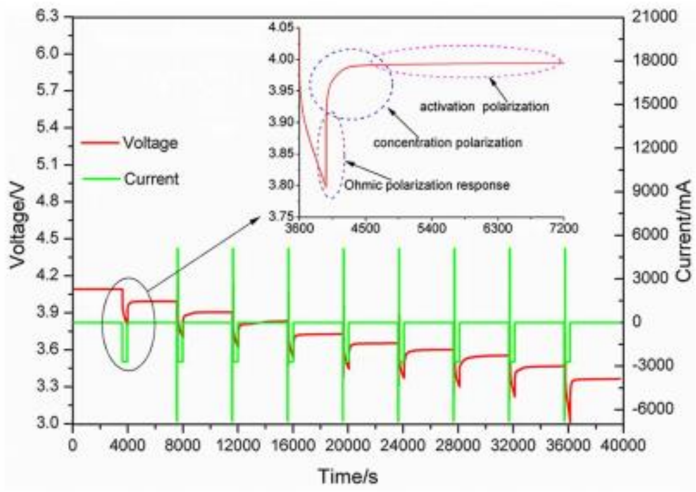

Figure 9. Hybrid pulse power characteristic (HPPC) test curve of lithium-ion battery

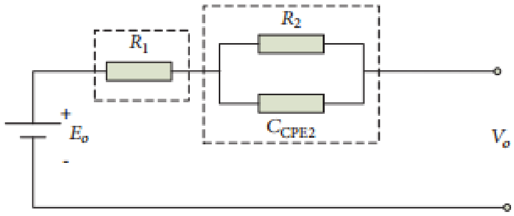

Figure 10. Fractional order equivalent circuit model of lithium-ion battery

In figure 10, R_1 is set for the ohmic resistor, R_2 for the concentration polarization resistor, CPE_2 for the constant phase element, E_0 for the open circuit voltage of the battery and V_0 for terminal voltage of the battery. Total impedance transfer function of the fractional equivalent circuit model of the lithium-ion battery is given as

$$Z(S) = R_1 + \frac{R_2}{1 + R_2 \cdot CPE_2 \cdot S^{\alpha}} \tag{23}$$

where, $\alpha \in R$, $0 < \alpha < 1$.

The state space model of nonlinear fractional lithium-ion battery system is represented as

$$ {}_{0}^{GL}D_t^{\alpha} V_2 = -\frac{V_2}{R_2 \cdot CPE_2} + \frac{I}{CPE_2} \tag{24}$$

$$V_0 = E_0 + IR_1 + V_2 \tag{25}$$

where, R_1, R_2, CPE_2 and α are parameters to be identified.

In simulation experiment the searching range of identification parameters is given as

$0 < R_1 < 0.2$, $0 < R_2 < 0.1$, $0 < CPE_2 < 4000$, $0 < \alpha < 1$.

In a similar way to the above, the simulation experiment of hybrid PSO-GA was also made in MATLAB 2021a programming language.In this simulation, the size of initial population was given to be $M = 200$, the initial values of inertia weight coefficient and moving step coefficient to be $\omega = 1.24$, $c_1 = 5.35$, $c_2 = 3.82$, the cross-over probability to be 0.75 and the maximum genetic number to be 1000. Table 4 shows the simulation result of the identification parameters of lithium-ion battery system under different SOCs. Figure 11 shows the online parameter identification result of nonlinear fractional lithium-ion battery system.

Table 4. Parameter identification result of lithium-ion battery system under different SOCs

SOC	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
R_1, Ω	0.0380	0.0387	0.0378	0.0383	0.0380	0.0378	0.0389	0.0402	0.0401
R_2, Ω	0.0083	0.0080	0.0101	0.0085	0.0102	0.0104	0.0113	0.0081	0.0082
CPE_2, KF	0.8021	0.8235	0.8783	0.8920	0.9012	0.9320	0.8149	0.9170	0.9348
α	0.7123	0.7015	0.6998	0.7310	0.7154	0.7062	0.6950	0.7012	0.7108

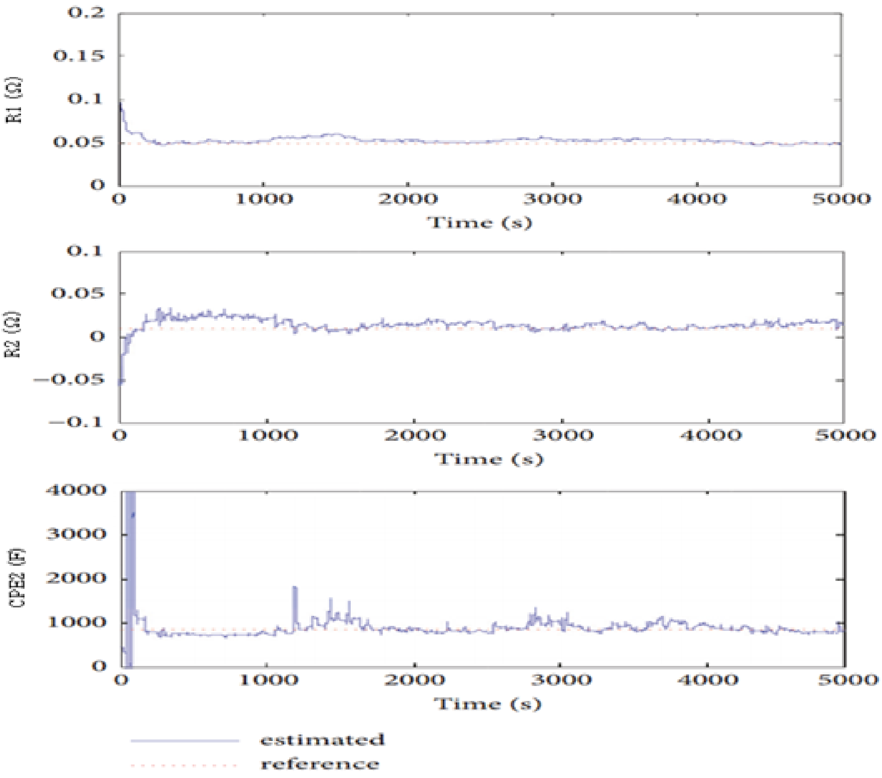

Figure 11. Online parameter identification result of nonlinear fractional lithium-ion battery system

As a result of the above simulation experiment, the method proposed in this paper could be found to be more effective in accuracy and convergence and better in performance compared with PSO and GA, respectively.

Conclusion

This paper establishes the fractional model of non-linear systems using approximated fractional-order derivative and intelligent optimization methods and proposes a novel method to identify time-varying parameters of the fractional non-linear system offline and online. More accurate fractional state space model is established by applying approximated fractional order derivative into the state space model of the previous non-linear system. In addition, the above modeled initial parameter values of non-linear fractional system is identified offline by using the hybrid particle swarm optimization-genetic algorithm method. Also the time-varying parameters of the non-linear fractional order system are identified online in real time by using the output error technique and the recursive least square method. Finally, a new identification technique presented in this paper is verified through the simulation experiment for offline and online identification of the time-varying parameters in the existing nonlinear fractional Lorentz system and nonlinear fractional lithium-ion battery system. The simulation results show that the proposed identification method could be effectively used for the offline and online parameter identification of many complicated non-linear fractional order systems in practice.

CRediT authorship contribution statement

Chol-Sik Ryang : Methodology, Writing-original draft. Hyok-Chol U: Writing–review -editing. Yu-Song Choe : Software.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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