

Maxwell's Demon and Psychoanalytic Theory

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Abstract.

The aim of this theoretical, interdisciplinary article is to extend the work initiated in the article, “The Einstein Model of a Solid as a Model of the Mental Apparatus from the Economic Perspective of Psychoanalytic Theory.”, by T. Aliferis (2023). In this article I referred to equilibrium fluctuations of the physical system Einstein Solid (ES). Here, I describe the progression of the ES contra the Law of Approach to Equilibrium and the Second Law of thermodynamics based on the concept of Maxwell’s Demon. I juxtapose with the deviation from equilibrium according to psychoanalysis. If this depiction is accomplished, the only case which may, or may not, be left open for further investigation involves the progression of the ES towards equilibrium starting from a non-equilibrium state. Upon success, we would have established a complete correspondence between the physical model and the mental apparatus psychoanalytically. This article is technical, and the psychoanalytic part is interwoven with the physical. One of the key findings is that the ego will be considered as correlated with the environment in accordance with the requirement dictated by the Reality Principle.

Keywords: Maxwell’s Demon, Einstein Solid, Psychoanalysis, Ego, Reality Principle.

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Introduction.

I intend to apply the concept of Maxwell’s Demon, classically, to a one-dimensional Einstein solid (ES), in the high-temperature (semiclassical) limit, isolated and “separated” into two, (L)eft or (R)ight compartments. The two compartments share q energy quanta, and the total number of Quantum Harmonic Oscillators (QHOs) is N . Both q and N remain constant in every configuration, so that the total energy and number of particles is conserved.

Allow me to note that L or R do not express a geometrical or spatial division of the system. They only have an algebraic meaning and attributes.

The main book of reference is “The Road to Maxwell’s Demon”, by Hemmo and Shenker, CUP (2012), henceforth referred as: Hemmo and Shenker (2012).

The temperature (T) of an ES in the high-temperature limit, is given by Eq. (1):

$$T = \frac{q}{N} \quad (1)$$

See Aliferis (2023), Eq. (5.1.12). The Configuration Space (abbreviation, C_S) or multiplicity of an ES is given by:

$$\Omega(q, N) = \frac{(q + N - 1)!}{q! (N - 1)!} \quad (2)$$

or in the high – temperature limit:

$$\Omega(q, N) \approx \left(\frac{eq}{N}\right)^N = (eT)^N, \text{ Eq. (5.3.2)}$$

$$C_{S(m,r)} \equiv \begin{cases} \Omega\left(\frac{q}{2} \pm m, \frac{N}{2} \pm r\right) \Omega\left(\frac{q}{2} \mp m, \frac{N}{2} \mp r\right) \\ \qquad \qquad \qquad \eta \\ \Omega\left(\frac{q}{2} \pm m, \frac{N}{2} \mp r\right) \Omega\left(\frac{q}{2} \mp m, \frac{N}{2} \pm r\right) \end{cases} \quad (3)$$

Where for every sign in (3), we have a different value for $C_{S(m,r)}$. A total of four different values for $C_{S(m,r)}$.

$C_{S(m,r)}$ expresses all possible cases of division of the configuration space into two separate ESs, L or R. Allow me to note here that the information that the system is in a specific microstate, belonging to the R or L compartment, uniquely determines both the variables (m, r) and the sign of $C_{S(m,r)}$.

$C_{S(m,r)} \cap C_{S(m',r')} = \emptyset$ for every $m \neq m'$ or $r \neq r'$. The macrostates are mutually exclusive.

Proof.

If they shared a microstate then it would be that $m = m'$ and $r = r'$. That is because the common microstate belongs to both $C_{S(m,r)}$ and $C_{S(m',r')}$ so it uniquely determines both (m, r) and their signs. We

observe though that there are two different states of $C_{-S(m,r)}$ for each (m,r) given their signs. It follows that the L compartment of a $C_{-S(m,r)}$ could have a common microstate with the R compartment of another $C_{-S(m,r)}$. However, the L and R compartments are distinct. Consequently, we have reached an impossibility. That means that we have proved the claim that $C_{-S(m,r)} \cap C_{-S(m',r')} = \emptyset$ for every $m \neq m'$ or $r \neq r'$.

The general equation that expresses the total number of microstates for each configuration determined by the number of energy quanta and QHOs (constraints m,r , respectively), between L, R, compartments, called $C_{-S(\text{generally})}$, is:

$$\begin{aligned}
C_{-S(\text{generally})} &= \Omega\left(\frac{q}{2} + m, \frac{N}{2} + r\right) \Omega\left(\frac{q}{2} - m, \frac{N}{2} - r\right) + \\
&\quad \Omega\left(\frac{q}{2} - m, \frac{N}{2} - r\right) \Omega\left(\frac{q}{2} + m, \frac{N}{2} + r\right) + \\
&\quad \Omega\left(\frac{q}{2} + m, \frac{N}{2} - r\right) \Omega\left(\frac{q}{2} - m, \frac{N}{2} + r\right) + \\
&\quad \Omega\left(\frac{q}{2} - m, \frac{N}{2} + r\right) \Omega\left(\frac{q}{2} + m, \frac{N}{2} - r\right) = \\
&= 2 \times \Omega\left(\frac{q}{2} + m, \frac{N}{2} + r\right) \Omega\left(\frac{q}{2} - m, \frac{N}{2} - r\right) + \\
&\quad + 2 \times \Omega\left(\frac{q}{2} + m, \frac{N}{2} - r\right) \Omega\left(\frac{q}{2} - m, \frac{N}{2} + r\right). \\
&0 \leq m \leq \frac{q}{2}, \quad 0 \leq r \leq \frac{N}{2}, \quad m, r \in \mathbb{N}
\end{aligned} \tag{4}$$

Proof:

a. The configuration space is comprised of four addends, each represented by a product of two factors, such that all possible combinations of the signs of m,r , are covered. The compartments are distinct, L or R, so the possible addends are four and not two.

b. In each product, e.g., $\Omega\left(\frac{q}{2} \pm m, \frac{N}{2} \pm r\right) \Omega\left(\frac{q}{2} \mp k, \frac{N}{2} \mp l\right)$, N, q , must be conserved, so that for m, r, k, l , it must be: $m = k, r = l$.

c. If we assume that we necessarily need additional variables m', r' independent of m, r , so that all possible microstates are covered, then, e.g.:

$$C_{-S}(\text{generally}) = 2 \times \Omega\left(\frac{q}{2} + m', \frac{N}{2} + r'\right) \Omega\left(\frac{q}{2} - m', \frac{N}{2} - r'\right) + 2 \times \Omega\left(\frac{q}{2} + m, \frac{N}{2} - r\right) \Omega\left(\frac{q}{2} - m, \frac{N}{2} + r\right) \quad (5)$$

Assuming that:

$$\left. \begin{aligned} -\frac{q}{2} \leq m', m \leq \frac{q}{2}, \\ -\frac{N}{2} \leq r', r \leq \frac{N}{2} \end{aligned} \right\} \quad (6)$$

And in order for all the possible cases to be covered, it must be that:

$0 \geq m + m' \geq -q$ και $0 \geq r' - r \geq -N \xrightarrow{q \geq (m+m') \geq 0 \wedge 0 \geq r-r' \geq -N} 0 \geq m' + m \geq 0$ και $0 \geq r - r' \geq 0 \Rightarrow m' = -m, r' = r$. So m', m and r', r , are dependent.

From now on we will be considering $m, r \geq 0$. That is allowed and necessary because $C_{-S}(\text{generally})$ is symmetric under the exchange of signs of m, r and we must also avoid a double counting of the same occurrences.

Statistical Mechanical Treatment of the Classical and Quantum Harmonic Oscillators.

Some of the treatment can be found in:

http://li.mit.edu/Archive/CourseWork/Ju_Li/MITCourses/8.333/1997/solutions/sol6.pdf

“In thermodynamics, recall that equilibrium is a state in which the thermodynamic magnitudes are constant over time, and the laws and magnitudes of standard classical thermodynamics are defined for such states only.” Hemmo and Shenker (2012), Section 7.4, p. 159. For the chemical potential μ we obtained in Aliferis (2023), Eq. (5.1.13), that $\mu = -kT \ln(T)$. Since the chemical potential depends solely on temperature it follows that the system is in equilibrium if and only if $T_L = T_R$.

I refer to the term: $\Omega\left(\frac{q}{2} + m, \frac{N}{2} + r\right) \Omega\left(\frac{q}{2} - m, \frac{N}{2} - r\right)$. We have that:

$$T_L \equiv \frac{q_L}{N_L} = \frac{\frac{q}{2} + m}{\frac{N}{2} + r} = T_R \equiv \frac{q_R}{N_R} = \frac{\frac{q}{2} - m}{\frac{N}{2} - r} \Leftrightarrow$$

$$\Leftrightarrow \left(\frac{q}{2} + m\right) = \frac{\left(\frac{q}{2} - m\right)\left(\frac{N}{2} + r\right)}{\frac{N}{2} - r} \Leftrightarrow (1 + \alpha)m = \frac{q}{2}(a - 1) \Leftrightarrow \quad (7)$$

$$\Leftrightarrow m = \frac{\frac{q}{2}(a - 1)}{a + 1} = \frac{q}{N} \times r$$

$$a = \left(\frac{N}{2} + r\right) / \left(\frac{N}{2} - r\right) \geq 1$$

Eq. (7) is valid for $m = (\Delta q) \wedge r = (\Delta N)$, see Eq. (18).

For the values of m, r given from Eq. (7) it follows that:

$$T_L = T_R = \frac{q}{N} \quad (8)$$

Remember that I referred to the term $\Omega\left(\frac{q}{2} + m, \frac{N}{2} + r\right)\Omega\left(\frac{q}{2} - m, \frac{N}{2} - r\right)$.

For the second term $\Omega\left(\frac{q}{2} + m, \frac{N}{2} - r\right)\Omega\left(\frac{q}{2} - m, \frac{N}{2} + r\right)$, (8) is also valid, since it follows from the first by changing the sign of variable r :

$$m = \frac{q}{N}(-r) \Rightarrow -\frac{q}{2} \leq m \leq 0$$

Since we exclude negative values for m, r we have for the second term that:

$$T_L = T_R = \frac{q}{N} \wedge m = r = 0 \quad (9)$$

Consequently, the maximum value of the configuration space of the system, $C_{S(max)}$, is given by Eq. (21), $m = r = 0$.

I refer to the term: $\Omega\left(\frac{q}{2} + m, \frac{N}{2} + r\right)\Omega\left(\frac{q}{2} - m, \frac{N}{2} - r\right)$.

From Eq. (7), we have that for a given r' there is a unique m' so that $T_L = T_R (= \frac{q}{N})$. Consequently, between two consecutive equilibrium points ($T_L = T_R$), we have that the value of the entropy, $S = S_L + S_R$, decreases (or increases) by an increasing (or decreasing, respectively) $|\Delta T| = |T_L - T_R|$. This means that before (or beyond) a maximum value of $|\Delta T|$ the entropy of the combined system, L, R, increases and finally reaches a maximum of $k \ln(\Omega_{max})$, see Eq. (16).

From Figure 1, (Wolfram Mathematica) and Eq. (7), it is obvious that there are two equal maximum values of $|\Delta T|$ between any two consecutive equilibrium points, i.e., to the corresponding values of m for

r in the intervals $[r, r + 1]$ or $[r - 1, r]$. Allow me to clarify that m, r , are considered strictly as integers.

According to Eq. (7), (12), (16), entropy reaches its maximum value for $T_L = T_R$. This implies that between two consecutive equilibrium points the entropy decreases, reaches one or more minimum values and then increases up to thermal equilibrium. We can infer that there are two equal minimum values for the entropy of the system between two consecutive equilibrium points and that each corresponds to each of the two equal maximum values of $|\Delta T|$. See Eq. (16).

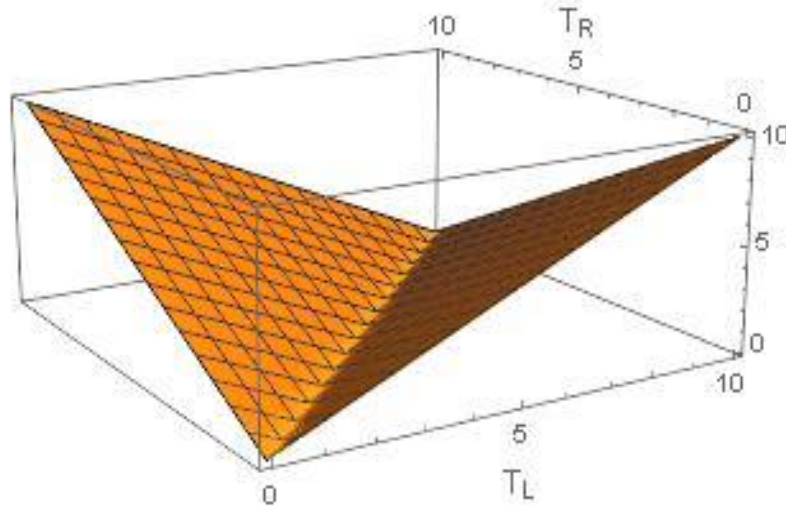


Figure 1 $|\Delta T| = |T_L - T_R|$

We have the right to consider that entropy can decrease since Hemmo and Shenker (2012), Chapter 13, and the fact that our model satisfies all the requirements that lead to this conclusion as I further examine in the Section “Deviation from Thermal Equilibrium”.

Gaussian Distribution of the Multiplicity Ω .

According to the First Law of Thermodynamics and particularly in the case of the ES, we obtain:

$$dU = TdS + \mu dN \quad (10)$$

For the chemical potential μ we have obtained Eq. (5.1.13), Aliferis (2023), $\mu = -kT \ln(T)$. This equation was derived from the Boltzmann entropy since it follows from the equation $\mu = -T \left(\frac{\partial S}{\partial N} \right)_U$. From the equipartition theorem (derived using statistical mechanics) we get:

$$U = NkT \Rightarrow dU = kTdN + kNdT \Rightarrow \frac{dU}{T} = kdN + kN \frac{dT}{T} \quad (11)$$

$$\frac{dQ}{kT} = \frac{dS}{k} = \frac{dU}{kT} - \frac{\mu}{kT}dN = \frac{dU}{kT} + (\ln T)dN \stackrel{(11)}{\Rightarrow} \quad (12)$$

$$\Rightarrow \frac{dS}{k} = dN + N \frac{dT}{T} + (\ln T)dN = dN + d[N \ln T] = d[N \ln(eT)] = d[\ln(eT)^N] = d[\ln(\Omega)]$$

Where Ω is given by Eq. (2) and not Eq. (14). This is because Eq. (12) refers separately to each of the weakly interacting L, R, compartments.

I have just proven that, in the case that I examine, the thermodynamic entropy is equivalent to the statistical mechanical entropy. This derivation was based on the First Law, the thermodynamic definition of entropy and principles and theorems of statistical mechanics.

For the total variation of the entropy of the system we have:

$$\begin{aligned} \frac{\Delta S_L + \Delta S_R}{k} &= \left(\frac{N}{2} + (\Delta N) \right) \ln \left(e \times \frac{\frac{q}{2} + (\Delta q)}{\frac{N}{2} + (\Delta N)} \right) \\ &+ \left(\frac{N}{2} - (\Delta N) \right) \ln \left(e \times \frac{\frac{q}{2} - (\Delta q)}{\frac{N}{2} - (\Delta N)} \right) \Rightarrow \\ &\Rightarrow \frac{\Delta S}{k} = -2 \left(\frac{(\Delta N)}{\sqrt{N}} - \frac{\sqrt{N}(\Delta q)}{q} \right)^2 \\ &= -2N \left(\frac{(\Delta N)}{N} - \frac{(\Delta q)}{q} \right)^2, \frac{2(\Delta N)}{N} \ll 1, \frac{2(\Delta q)}{q} \\ &\ll 1 \end{aligned} \quad (13)$$

Alternative derivation:

We have that:

$$\Omega = \left(\frac{eq_A}{N_A} \right)^{N_A} \times \left(\frac{eq_B}{N_B} \right)^{N_B} = \left(\frac{eq_A}{N_A} \right)^{N_A} \times \left(\frac{e(q-q_A)}{N-N_A} \right)^{(N-N_A)} \quad (14)$$

Setting:

$$N_A = \frac{N}{2} + y, q_A = \frac{q}{2} + x, \frac{2y}{N} \ll 1, \frac{2x}{q} \ll 1 \quad (15)$$

We get:

$$\begin{aligned} \Omega &= \left(\frac{e^{\left(\frac{q}{2}+x\right)}}{\frac{N}{2}+y} \right)^{\left(\frac{N}{2}+y\right)} \times \left(\frac{e^{\left(\frac{q}{2}-x\right)}}{\frac{N}{2}-y} \right)^{\left(\frac{N}{2}-y\right)} = \\ &= \left(\frac{e^2 \left(\frac{q^2}{4} - x^2 \right)}{\frac{N^2}{4} - y^2} \right)^{\left(\frac{N}{2}\right)} \times \left(\frac{q+x}{\frac{q}{2}-x} \right)^y \times \left(\frac{\frac{N}{2}-y}{\frac{N}{2}+y} \right)^y \approx \\ &\approx \left(\frac{e^2 \left(\frac{q^2}{4} - x^2 \right)}{\frac{N^2}{4} - y^2} \right)^{\left(\frac{N}{2}\right)} \times e^{\frac{4xy}{q}} \times e^{\frac{-4y^2}{N}} \Rightarrow \\ \Rightarrow \ln \Omega &\approx \left(\frac{N}{2}\right) \times \ln \left(\frac{e^2 \left(\frac{q^2}{4} \left(1 - \frac{4}{q^2} x^2 \right) \right)}{\left(\frac{N^2}{4} \left(1 - \frac{4}{N^2} y^2 \right) \right)} \right) + \frac{4xy}{q} - \frac{4y^2}{N} \approx \\ &\approx N \ln eT - 2 \left(\frac{y}{\sqrt{N}} - \frac{\sqrt{N}x}{q} \right)^2 \Rightarrow \\ \Rightarrow \Omega &\approx (eT)^N \times e^{-2 \left(\frac{y}{\sqrt{N}} - \frac{\sqrt{N}x}{q} \right)^2} = \\ &= \Omega_{max} \times e^{-2 \left(\frac{y}{\sqrt{N}} - \frac{\sqrt{N}x}{q} \right)^2} \xrightarrow{y=r, x=m} \\ C_{-S}(m,r) &= \Omega = \Omega_{max} \times e^{-2N \left(\frac{r}{N} - \frac{m}{q} \right)^2}, \frac{2r}{N} \ll 1, \frac{2m}{q} \ll 1 \end{aligned} \quad (16)$$

Consequently:

$$\begin{aligned} C_{-S}(m,r) = \Omega_{max} &\Leftrightarrow m = \frac{q}{N} \times r \stackrel{(18)}{\Leftrightarrow} (\Delta q) \\ &= \frac{q}{N} \times (\Delta N) \end{aligned} \quad (17)$$

A result that is equivalent to Eq. (7).

$C_{-S}(m,r)$ is defined in (3).

In the proof of (13), (16) I used the approximation:

$$\ln(1+x) \approx x, |x| \ll 1$$

Let me notice that from Eq. (18), in the thermodynamic limit $N \rightarrow \infty$ we conclude that:

When two large Einstein solids are in thermal equilibrium, any random fluctuations away from the most likely macrostate will be utterly unmeasurable.

In Eq. (14) the multiplicity Ω , of the system of the weakly interacting ESs, is defined according to statistical mechanics, Boltzmann entropy, Eq. (2). Multiplicity Ω , reaches its maximum value according to the equations (7), (17). In this case the system is, by definition, in $[M_{eq}]$. From what we can infer from Eq. (17), and when it is satisfied, then $T_L = T_R = \frac{q}{N}$ and vice versa. Consequently, I proved that equilibrium according to statistical mechanics ($dS = 0$) is equivalent to the thermodynamical perspective (system is considered as being always in equilibrium, i.e., uniform temperature and chemical potential throughout the system). The introduction of the constraints (m, r) in Boltzmann entropy, is performed in Eq. (16).

Furthermore and from Eq. (12) I concluded that entropy variation according to the thermodynamic viewpoint ($dS = \frac{dQ}{T}$) is equivalent to the expression for entropy variation in statistical mechanics ($dS = d[\ln(\Omega)]$), where Ω is given by Eq. (2). This result has been derived by proving the equipartition theorem in the grand canonical ensemble, see expression for the average energy (U, \bar{E}), Aliferis (2023), Eq. (5.4.9). Obviously, this result is also valid for an ES divided into two weakly interacting L, R compartments, exchanging particles (QHOs) and energy. See Eq. (13).

The standard deviation is given by:

$$\sigma_N \equiv \sqrt{(\Delta N)^2} \equiv (\Delta N) = \frac{\sqrt{N}}{2} \quad \wedge \quad \sigma_q \equiv \sqrt{(\Delta q)^2} \equiv (\Delta q) = \frac{q}{2\sqrt{N}} \quad (18)$$

Minimum - Maximum Configuration Space.

I examine the case where the configuration space (of the total system) takes its minimum value (see Eq. (4), setting $m = \frac{q}{2}$ και $r = \frac{N}{2} - 1$):

$$C_{S_{(min)}} \approx 2 \times \Omega(q, N) \quad (19)$$

The limit of the least Lebesgue measure is for $\frac{2m}{q} \rightarrow 1$ and $\frac{2r}{N} \rightarrow 1$. In this limit the term $2 \times \Omega\left(\frac{q}{2} + m, \frac{N}{2} + r\right) \Omega\left(\frac{q}{2} - m, \frac{N}{2} - r\right)$ dominates and particularly:

$$\Omega\left(\frac{q}{2} + m, \frac{N}{2} + r\right) \gg \Omega\left(\frac{q}{2} - m, \frac{N}{2} - r\right) \rightarrow 1. \quad (20)$$

Comment: The systems involved must be macroscopical, condition that I infringe at this point. I must also prove that we can neglect terms with small multiplicity.

The configuration space reaches its maximum value:

$$\begin{aligned} C_{S_{(max)}} &= 4 \times \Omega(q/2, N/2) \Omega(q/2, N/2) = \\ &= 4 \times \Omega^2(q/2, N/2) = 4 \times \Omega(q, N) \end{aligned} \quad (21)$$

Eq. (21) is derived from (4), setting $m = r = 0$. In the last step Eq. (22) has been used:

$$\Omega^2(q/2, N/2) = \left(e \times \frac{q}{\frac{N}{2}} \right)^{\frac{N}{2} \times 2} = \Omega(q, N) \quad (22)$$

The product of the multiplicities ($\Omega(q/2, N/2)$) of two identical compartments is equal to $\Omega(q, N)$. More generally this result is also valid when the compartments have the same temperature. Referring to equations (1), (2), we get:

$$\begin{aligned} \Omega &= \Omega_L \Omega_R = (e \times T)^{N_L} (e \times T)^{N_R} = (e \times T)^{N_L + N_R} = \\ &(e \times T)^N = \Omega(q, N) \end{aligned} \quad (23)$$

From (19) and (21) we notice that: $C_{S_{(max)}} \approx 2 \times C_{S_{(min)}}$.

Principle of Detailed Balance.

See Schroeder, Daniel V, (1999). "Introduction to thermal physics", pp. 57-58: "What we're assuming is that the microstates that *do* occur, over "long" but not unthinkably long time scales, constitute a representative sample. We assume that the transitions are "random", in the sense that they have no pattern that we could possibly care about". From "Stochastic Processes in Physics and Chemistry", Kampen, Elsevier (2007), p. 109: "Equation (4.1) merely states the obvious fact that in equilibrium the sum of all transitions per unit time into any state n must be balanced by the sum of all transitions from n into other states n' ".

Detailed balance is the stronger assertion that for each pair n, n' separately the transitions must balance”, Eq. (4.2):

$$W_{nn'}p_{n'}^e = W_{n'n}p_n^e \quad (24)$$

Where p_n^e is the equilibrium distribution $p_n^e = p_{n'}^e = \frac{1}{\Omega}$, and $W_{nn'}$ is the transition probability per unit time from n' to n .

If the number of energy quanta of a QHO increases or decreases by a small amount this variation can be reversed equiprobably. So, $W_{nn'} = W_{n'n}$ and Eq. (24) is valid.

Virial Theorem.

According to the Virial Theorem, quantum mechanically:

$$2\langle T \rangle = \sum_n \langle X_n \frac{dV}{dX_n} \rangle \quad (25)$$

Where $\langle T \rangle$ is the average kinetic energy, $\langle V \rangle$ is the average potential energy and X_n is the position operator.

In the case of a one-dimensional Einstein solid,

$$\begin{aligned} V &= \sum_n \frac{1}{2} kX_n^2 \xrightarrow{V_n=f(X_n)} \frac{dV}{dX_n} = \frac{dV_n}{dX_n} = kX_n \Rightarrow X_n \frac{dV}{dX_n} = kX_n^2 \quad (26) \\ &= 2V_n \Rightarrow \sum_n \langle X_n \frac{dV}{dX_n} \rangle = \sum_n \langle 2V_n \rangle \\ &= 2\langle V \rangle \stackrel{(25)}{\Rightarrow} 2\langle T \rangle = 2\langle V \rangle \Rightarrow \langle T \rangle = \langle V \rangle = \frac{NkT}{2} \end{aligned}$$

In the last step I used Eq. (5.4.8), (5.4.9), proved in Aliferis (2023) and the Eq. $\bar{E} = \langle T \rangle + \langle V \rangle$. Additionally, $V_n = \frac{1}{2} kX_n^2 = f(X_n)$, where f is a function of one variable. (This means that V_n is a function of one variable).

In classical statistical mechanics, Eq. (26) is proved in the Section “Statistical treatment of the Classical and Quantum Harmonic Oscillators”.

Loschmidt reversal.

Loschmidt reversal is compatible with the classical harmonic oscillator (CHO) and consequently with an isolated, thermodynamic system of

one-dimensional, non-interacting, identical, distinguishable, CHOs. That is because:

The force F acting on a CHO obeys Hooke's Law, Eq. (28):

$$F = \dot{p} = -\frac{\partial H}{\partial x} = -kx$$

And as such it is conservative.

The equations of motion are symmetric with respect to time and velocity reversal.

The degrees of freedom of the system are finite.

“Notice that, in a sense, Zermelo's objection to the H-theorem is weaker than Loschmidt's objection. Loschmidt's argument means that to every trajectory segment that evolves from a lower-entropy macrostate to a higher-entropy macrostate corresponds a segment that decreases entropy; whereas the Poincaré recurrence theorem means that every trajectory includes entropy-decreasing segments, without saying that the number of entropy-decreasing segments is equal to or larger than the entropy-increasing segments.” Hemmo and Shenker (2012), Section 7.10, p. 171.

Liouville's Theorem.

Liouville's theorem states that the phase space distribution function, λ , remains constant along the trajectories of the system in phase space.

I will apply Liouville's theorem for N identical CHOs in the classical phase space. For the Hamiltonian of the system, we have:

$$H = \sum \left(\frac{1}{2} kx_i^2 + \frac{1}{2} \frac{p_i^2}{m} \right) = E \quad (27)$$

$$\begin{aligned} \dot{x}_i &= \frac{\partial H}{\partial p_i} = \frac{p_i}{m} \Leftrightarrow p_i = m\dot{x}_i \\ \dot{p} &= -\frac{\partial H}{\partial x_i} = -kx_i, \text{Hooke's Law.} \end{aligned} \quad (28)$$

The phase space current vector J can be written as:

$$J = \left(\frac{\partial H}{\partial p_1}, -\frac{\partial H}{\partial x_1}, \dots, \frac{\partial H}{\partial p_N}, -\frac{\partial H}{\partial x_N} \right) \quad (29)$$

The evolution of the distribution function λ in phase space is governed by the continuity equation:

$$\frac{\partial \lambda}{\partial t} + \nabla(\lambda \cdot \mathbf{J}) = 0 \quad (30)$$

In a Hamiltonian system, the divergence of the phase space current \mathbf{J} is zero:

$$\nabla \mathbf{J} = \sum_i \frac{\partial \dot{x}_i}{\partial x_i} + \frac{\partial \dot{p}_i}{\partial p_i} = 0 \quad (31)$$

Additionally:

$$\begin{aligned} & (\nabla \lambda) \mathbf{J} = \\ = 0 & \Leftrightarrow \left(\frac{\partial \lambda}{\partial x_1}, \frac{\partial \lambda}{\partial p_1}, \dots, \frac{\partial \lambda}{\partial x_N}, \frac{\partial \lambda}{\partial p_N} \right) \left(\frac{\partial H}{\partial p_1}, \frac{\partial H}{\partial x_1}, \dots, \frac{\partial H}{\partial p_N}, \frac{\partial H}{\partial x_N} \right) = \\ & = 0 \Leftrightarrow \frac{p_i}{m} \frac{\partial \lambda}{\partial x_i} = k x_i \frac{\partial \lambda}{\partial p_i} \end{aligned} \quad (32)$$

To solve the partial differential equation (PDE), (32) I start by recognizing that this is a first-order PDE involving the function λ of the variables x_i and p_i . One approach to solving this is by using the method of characteristics.

The method of characteristics involves solving the following ordinary differential equation (ODE):

$$\begin{aligned} \frac{1}{m} \frac{dp_i}{(-x_i)} = k \frac{dx_i}{p_i} & \Rightarrow k \sum_i \int x_i dx_i = -\frac{1}{m} \sum_i \int p_i dp_i \Leftrightarrow \\ & \Leftrightarrow \sum_i \left(\frac{1}{2} k x_i^2 + \frac{1}{2m} p_i^2 \right) = C = \text{constant} \end{aligned} \quad (33)$$

Eq. (33) is valid, for $C \equiv H$.

So, we get that: $\lambda = g \left(\sum \left(\frac{1}{2} k x_i^2 + \frac{1}{2m} p_i^2 \right) \right) = g(H)$, for any g , arbitrary, differentiable function. We can infer that Eq. (32) is validated and additionally that $\frac{\partial \lambda}{\partial t} = 0$. Consequently, from Eq. (30):

$$\frac{d\lambda}{dt} = 0 \Leftrightarrow \lambda(t) = \text{const.} \quad (34)$$

We have that:

$$\begin{aligned}\lambda &= g\left(\sum\left(\frac{1}{2}kx_i^2 + \frac{1}{2}\frac{p_i^2}{m}\right)\right) = g(H) \equiv \\ &\equiv \frac{\alpha}{h^N} \iint_{E \leq H} dx_1 dp_1 \dots dx_N dp_N \approx \alpha \Omega(q, N),\end{aligned}\quad (35)$$

$$H = qhf \text{ (ES, high – temperature limit)}$$

Function g is differentiable so it can be defined as an integral. I defined λ to express the volume of a hypersphere $\Omega(q, N)$ multiplied by an arbitrary positive constant α . I chose $1 \geq \alpha > 0$. From Eq. (30) every $\alpha\lambda$ is an acceptable distribution function. For the derivation of Eq. (35) see the Section “Statistical treatment of the Classical and Quantum Harmonic Oscillators”.

The Liouville equation reflects that λ is conserved along the trajectories of the system in phase space and is constant over time.

Probability Rule, measure ρ .

Based on Hemmo and Shenker (2012), p. 132, footnotes:

A measure ρ (referred as “ μ ” in Hemmo and Shenker (2012)) on a set X is a map $\rho: A \rightarrow [0, \infty]$ on a σ – algebra A of X such that:

- (a) $\rho(\emptyset) = 0$;
- (b) If $\{A_i | i \in \mathbb{N}\}$ is a countable family of mutually disjoint sets in A , meaning that $A_i \cap A_j = \emptyset$ for $i \neq j$, then:

$$\rho\left(\bigcup_i A_i\right) = \sum_i \rho(A_i) \quad (36)$$

(Lebesgue measure $\equiv \|\ \|\$). I will make the assumption, which is necessary from the clinical data, that P' :

$$P'([M_1], t | [M_0], t) \equiv \frac{\rho(\|[M_1]\|, t)}{\rho(\|[M_0]\|, t)} = \frac{\|[M_1]\|}{\|[M_0]\|}, \quad (37)$$

$$[M_1], [M_0]: B(t) \cap [M_j] \neq \emptyset, \quad j = \{0, 1\}$$

expresses the probability that the dynamical blob will collapse to $[M_1]$ at time t , instead of collapsing to $[M_0]$, at the same point in time.

According to the above we end up to one of the possible definitions of the Transition Probability P and measure ρ :

$$P([M_1], t_3 | [M_0], t_1) \equiv \rho([M_1], t_3) \equiv \frac{\|[M_1]\|}{\sum_i \|[M_i]\|}, \quad (38)$$

$$[M_i]: B(t_3) \cap [M_i] \neq \emptyset, \forall i$$

Eq. (38) expresses also the observed Relative Frequency of succession of macrostates, see the Section “Relative Frequency, Clinical Data”. $B(t_3)$ is a function of $[M_0], t_1$.

ρ , as defined in Eq. (38), satisfies all the conditions of the definition of measure. Additionally, it is normalized and $0 \leq \rho \leq 1$, so that it expresses Probability Measure, (Hemmo and Shenker (2012), p. 132, footnotes).

“This idea that larger macrostates are invariably associated with a higher probability would entail the prediction that systems evolve directly to the macrostate with the largest measure. This prediction would be of course false for the following reason. By and large, the observed phenomenon in thermodynamic systems is that systems evolve to equilibrium via macrostates that have gradually increasing sizes. Therefore, this equilibrium ought to receive high probability. But the above prediction gives it only small probability. Moreover, equating the probability of a macrostate with its size is also in clear contradiction to the laws of dynamics that govern the evolution of mechanical systems by dictating their trajectories [...] By contrast, the Probability Rule takes into account the dynamics in a way that meshes very well with the fact that systems evolve to equilibrium gradually, that is via macrostates with increasing entropies.” Hemmo and Shenker (2012), Section 6.4, p.136.

By defining ρ in Eq. (38) we observe that the macrostate with the maximum probability of occurrence is the one with the maximum Lebesgue measure, from the set of the macrostates that intersect the dynamical blob at a specific point in time. This definition/conclusion does not necessarily contrast what was mentioned in the paragraph above, since the dynamical blob evolves to equilibrium gradually. Only in the case that the dynamical blob intersects $[M_{eq}]$, is the system probable to be found in $[M_{eq}]$. In this case $[M_{eq}]$ has the maximum probability of occurrence compared to the rest macrostates that overlap with the dynamical blob. See Eq. (38). Furthermore, my definition of ρ is compatible with the conditions (I) – (III), for the Law of Approach to Equilibrium as defined in Hemmo and Shenker (2012), Section 7.5, p. 162. See also Section “v-measure”.

As we notice in Section “Liouville’s Theorem”, the dynamical blob is defined classically. Additionally, the dynamical blob has a constant size $\|B(t)\| = \alpha\Omega(q, N) \approx \alpha(eT)^N = \alpha\Omega_{max} = \|[M_0]\|$, Eq. (2), (35). “Now, one of the properties of the Lebesgue measure that makes it so convenient is that it is conserved under the dynamics of classical mechanics.”, Hemmo and Shenker (2012), Section 6.9, p. 151.

Hemmo and Shenker (2012), p. 131: “Formally, the rule for calculating transition probabilities in statistical mechanics is the following:

(Probability Rule) $P([M_1], t_2 | [M_0], t_1) = \rho(B(t_2) \cap [M_1])$

Hemmo and Shenker (2012), Section 6.6, p. 139: “This means that the probability that a system that starts in macrostate $[M_0]$ at time t_1 will end in macrostate $[M_1]$ at time t_2 is given by the relative size ρ of the dynamical blob $B(t_2)$ which overlaps with the macrostate $[M_1]$.”.

I do not follow this approach in this study. That is because the Probability Rule, must express the Relative Frequency of occurrence of future macrostates which is given by the clinical data. And the clinical data dictate that ρ refers to one time instance, t_1 , (more accurately and to a time t_2 , humanly possible next to t_1), it is proportional to the value of the Lebesgue measures of each macrostate that overlaps with the dynamical blob at the same time, by the sum of the Lebesgue measures of the rest macrostates that overlap with the blob. And it is not depending on the measure of the overlap of the dynamical blob with any macrostate.

“Road, Section 6.6, p. 139”. “Our above account of probability in statistical mechanics implies that the probability assigned to future macrostates depends on the initial macrostate that one assigns to the system, and on the calculation of the dynamical blob, given that initial macrostate.” From a statistical mechanical point of view: “The knowledge that the microstate of G is within $B^*(t_2)$ is a combination of three items: (i) the observation that at t_1 the macrostate of G was $[M_0]$, (ii) the observation that at t_2 the macrostate of G was $[M_1]$, and (iii) the calculation that, owing to the equation of motion of G, the dynamical blob that starts out in $[M_0]$ at t_1 evolves into $B(t_2)$ at t_2 .”, Hemmo and Shenker (2012), Section 6.6, p. 140.

ρ according to its definition Eq. (38), is a function of the dynamical blob, of $[M_1]$, but also of the initial microstate $[M_0]$: The Lebesgue measure of the dynamical blob is equal to/determined by $\|[M_0]\|$, $\|[M_1]\|$, at the respective points in time t_1, t_2 , when a measurement occurred. This definition of ρ agrees with statistical mechanics and Hemmo and Shenker (2012), except for the definition of the Probability Rule

depending on the size of the overlap of the dynamical blob and each macrostate, at an instance.

“Consider again the two observers O and Q (from Section 6.6) who differ only in their observed history (see Figure 6.4). Observer O has prepared the system in the macrostate $[M_0]$ at t_1 . Observer Q has no knowledge about this preparation. At t_2 both O and Q find the system in the macrostate $[M_1]$. As we saw, at t_2 O calculates the probabilities for the macrostates at t_3 on the basis of the blob $B^*(t_3)$, while Q calculates the probabilities for the macrostates at t_3 on the basis of the blob $\hat{B}(t_3)$. Observer O assigns probabilities to any macrostate $[M_i]$ at t_3 given $[M_0]$ at t_1 by $P([M_i], t_3 | [M_0], t_1) \approx a_i$, while Q assigns the probabilities to any macrostate $[M_i]$ at t_3 given $[M_1]$ at t_2 by $P([M_i], t_3 | [M_1], t_2) \approx b_i$.” Hemmo and Shenker (2012), Section 6.8, p. 149. According to statistical mechanics, a_i , b_i , are not necessarily equal because the calculation of the evolution of the dynamical blob, (both of its Lebesgue Measure and its evolution in time), might differ according to the data/information: $[M_0], t_1$ or $[M_1], t_2$. See Eq. (38).

“We can express the fact that the thermodynamic regularities are independent of observed history as the idea that $a_i = b_i$. This immediately implies that $\rho(a):\rho(b) = \rho(\gamma):\rho(\delta)$ (see Figure 6.8).” Additionally, “A very special case, which may or may not hold in thermodynamic evolutions, is that in which the probability of a macrostate is equal to its measure: $\rho(\alpha):\rho(\beta) = \rho(\gamma):\rho(\delta) = \rho([M_1]):\rho([M_2])$ ”. Hemmo and Shenker (2012), Section 6.8, p. 149. This is almost the case that I examine. Specifically, the probability of a macrostate is proportional to its Lebesgue measure: $\rho([M_i], t) \propto \|[M_i]\|$, $[M_i]: B(t) \cap [M_i] \neq \emptyset, \forall i$, (see Eq. (37), (38)). This means that my definition of ρ is not necessarily contrary to thermodynamics, ($a_i = b_i$), “It is a fact that the thermodynamic regularities are robust in the sense that they are independent of observed history.” Hemmo and Shenker (2012), Section. 6.8, p. 149.

Commentating.

The sum, $\sum_i \|[M_i]\|$, expresses the total size in phase space of the macrostates that intersect the dynamical blob. The Lebesgue measure $\|[M_1]\|$, divided by $\sum_i \|[M_i]\|$, expresses approximately the probability of the collapse of the dynamical blob into $[M_1]$. It is still necessary though to consider as valid the principle that: “Equal “a priori probability” is assigned to equal volumes of phase space”. Amnon Katz, “Principles of Statistical Mechanics”, p. 31.

In every dynamical blob corresponds a unique set of macrostates $[M_i]$. The reverse statement is not valid though since for instance many (equal in Lebesgue measure) “distorted” versions of the dynamical blob can intersect the same macrostates, $[M_i]$.

We can conclude from the above that Eq. (38) is valid taking into consideration the available clinical data.

Relative Frequency, Clinical Data.

I define that each arrangement of the total amount of libido (q energy quanta) over the ideas (N , QHOs) residing in the ego, represents a mental state. I.e., that each microstate is a mental state.

Macrostates are observable while microstates are not.

I argue that as for the time evolution of the mental apparatus it is possible for the Relative Frequency of the observed macrostates to be measured. For the physical definition and role of the Relative Frequency see Hemmo and Shenker (2012), Section 6.5.

I assume that the practitioner demonstrates to the patient the largest in Lebesgue measure macrostate from those that the patient mentions or implies. I.e., by definition the most “liberating”, “broader”, “existing” choice. “Existing” here refers to the macrostates which overlap with the dynamical blob. This implies that the probability of finding the current state of the ego within $[M_i]$, $[M_i]$: $B(t) \cap [M_i] \neq \emptyset, \forall i$, depends on its Lebesgue measure and $B(t)$.

We conclude from the above that Eq. (38) is valid.

v-measure.

In the Section “Gaussian Distribution of the Multiplicity Ω ”, I proved that equilibrium according to statistical mechanics ($dS = 0$) is equivalent to the thermodynamic perspective, system always in equilibrium, (uniform temperature and chemical potential) and that with the identification, $C_{\mathcal{S}(m,r)} \equiv \Omega(q, N)$, see Eq. (23) and $m, r, (m', r')$ are related by Eq. (7). From (12):

$$\begin{aligned} \frac{\Delta S_{\{stat\}}}{k} &= \ln \left(C_{\mathcal{S}(m',r')} / C_{\mathcal{S}(m,r)} \right) = \ln(\Omega(q, N) / \Omega(q, N)) = 0 \quad (39) \\ &= \frac{\Delta S_{\{thermo\}}}{k} \end{aligned}$$

The outcome is that $C_{\mathcal{S}(m,r)}$, is appropriate as v-measure. See Hemmo and Shenker (2012) Section 7.2, pp. 157, 158. A more rigorous demonstration follows right below.

In statistical mechanics entropy can be defined for any macrostate and not specifically in equilibrium. By application of (1), (2) we get:

$$\frac{S}{k} = (S_R + S_L)/k = N \ln(eT_R) + N_L \ln\left(\frac{T_L}{T_R}\right) \quad (40)$$

Let us consider that we have an evolution from a macrostate A in equilibrium to a macrostate B, also in equilibrium. According to thermodynamics it must be, $T_A = T_{L,A} = T_{R,A} = T_{L,B} = T_{R,B} = T_B$. In statistical mechanics when the previous Eq. is satisfied, we have additionally that $T_A = \frac{q}{N} = T_B$, (see Eq. (8)). The variation of the statistical entropy is $\Delta S_{A \rightarrow B} = S_B - S_A$, by substituting to Eq. (16) and due to (17) is zero. Consequently, starting from the thermodynamic perspective (uniform temperature/chemical potential) we infer the statistical mechanical notion of entropy.

In the Section “Gaussian Distribution of the Multiplicity Ω ” Eq. (14), (15), (16), (17), I established an algebraic relationship for $C_{-S(m,r)}$. If the statistical entropy at equilibrium ($dS = 0 \Rightarrow S = \max.$), i.e., Eq. (7), (17) is valid, we also have that $T_A = \frac{q}{N} = T_B$, Eq. (8), we end up in the validation of the thermodynamic version (as I outlined there).

In conclusion, I proved that the statistical mechanical notion of equilibrium is equivalent to its thermodynamic counterpart.

Alternative Approach.

In this alternative approach (which is wrong according to Hemmo & Shenker (2012), Section 9.2, p. 194: “...in classical mechanics there are no microscopic measurements: a classical account of measurements has to be carried out in terms of macrostates.”).

The number of energy quanta (eigenvalue n) of the average-energy-QHO(s) are given by the equation $kT = \langle n \rangle hf$ see Aliferis (2023), Eq. (5.5.8), (5.5.9). Consequently, the number of energy quanta determines whether a QHO belongs to L or R among comparison with $\langle n \rangle$. This implies that we gradually broaden the temperature difference between T_L, T_R , as it is the case in the original version of Maxwell’s Demon for an ideal gas.

The Observer.

The observer (henceforth abbreviated as O) is an entity that according to the theory of psychoanalysis resides within the ego, see “The Ego and the Mechanisms of Defence”, by Anna Freud, Karnac Books

(1993), pp. 6-10. To the best of my knowledge, *O* lacks a precise definition in the theory of psychoanalysis.

O in physics is equipped with memory and the ability to conduct measurements over the ES (ego or more generally the Cs.). *O* processes and manipulates information. The physical interaction between *O* and the ES is negligible.

According to Hemmo and Shenker (2012), Section 5.8, *O* is physically correlated both with the environment and the isolated system, the part of the ego (or ES) in our case, that contains ideas and libido. At least in the latter case correlation occurs after a measurement takes place. This means that *O* links the environment with the mental apparatus. Thus, the ego in its abstract version (*O*, ES) cannot be considered as being isolated from the environment.

Physicalism.

Physicalism dictates that the mental apparatus is physical. I.e., since the mental apparatus is a conceivable entity then as such it is in space and time. This does not necessarily mean that the material counterpart of the mental apparatus is a coherent physical system or has any physically useful sense as it was intended. Just that it is located in the universe as any other thought entity.

The Ego as an Open System.

An open system according to physics does exchange energy and matter with its surroundings.

My objective was to depict the abstract model of the mental apparatus as defined in the theory of psychoanalysis, to that of a valid abstract model in theoretical physics, i.e., to make psychoanalysis (and consequently psychology) scientific in an abstract level.

However, in an abstract model of the mental apparatus (may it be according to theoretical physics or psychoanalysis) the concepts and the attributes of libido and ideas may not have any physical/spatial/bodily equivalents, may it be energy and “particles” (energy carriers) as Freud requested in the “Project”. For instance, it seems that it could be difficult to depict ideas (abstractly defined as QHOs in my approach) to any somatic counterpart.

Still, there could be place for a depiction of the abstract to the material even without a detailed correspondence between their respective constituents. It would be necessary though for the bodily representation of the mental apparatus to be somehow located and confined in

physical space and time and to specify its boundaries, the precise way of inner functioning, and furthermore interacting with its environment. Such a proof is beyond the scope of the present research, and I am unaware of its existence by the time of this writing.

Another way of viewing the same issue is the possibility that the abstract can be interconnected with the physical directly and moreover that there can be an interaction taking place between them. That, without the need for a somatic equivalent of the mental apparatus being present in the interaction. I am unaware of such a proof by the time of this writing.

The above analysis does not contradict what was mentioned in the Sections “The Observer” and “Physicalism”.

Deviation from Thermal Equilibrium.

I examine the deviation from equilibrium by a gradually increasing/decreasing temperature difference $|\Delta T| = |T_L - T_R|$ between the compartments L, R. This can be considered as a deviation contra the Second Law when occurring by a fixed r , see Eq. (16).

It would be very helpful to refer to the Section “Statistical Mechanical Treatment of the Classical and Quantum Harmonic Oscillators” for an introductory physical study of what follows.

Given an arrangement of QHOs between the two distinct compartments, L, R, there can be, in an algebraically valid way, heat transfer against the Second Law (as defined by Clausius).

The second approach that has to do with deviation from equilibrium is Maxwell’s Demon as described by Hemmo and Shenker (2012), Chapter 13 and in the present research in the Section “Maxwell’s Demon”. In this (second) approach deviation from equilibrium does not have to do directly with temperature difference (or heat transfer) between the two compartments L, R. Instead, it is based on a v -measure decrease in each cycle of operation.

According to Hemmo and Shenker (2012), Section 2.4, we are allowed to consider both approaches as equivalent as for the invalidity of the Second Law. This means that the later objection to the Second Law implies that we have the right to consider as valid and the former objection, i.e., that $|\Delta T|$ can increase given a specific arrangement of the oscillators between L, R, i.e., by keeping r constant. That is because the two definitions of the Second Law are equivalent. “Since the entropy formulation assumes those of Kelvin and Clausius, and the formulations of the Second Law by Kelvin and Clausius, in turn, can be

derived from the entropy formulation of the entropy law, all three formulations are equivalent.”, Hemmo and Shenker (2012), Section 2.4.

As temperature is linked to the quota of affect, see Aliferis (2023), it would seem reasonable to correlate heat transfer contra the Second Law between L and R, with mood imbalance.

Measurement in Maxwell’s Demon.

I describe measurement as applied in the special case of Maxwell’s Demon.

In our case, there is no measuring device D necessary, however for reasons of compatibility with Hemmo and Shenker (2012) we do not disregard it.

In what follows the term gas (G) can and should be replaced by the term ES without any conflict.

(A) Pre-measurement.

Initially, before the measurement, O is in some microstate O_S and G is in the macrostate $[L + R]$ relative to O_S . This pre-measurement situation is illustrated in Figure 9.2, where the horizontal axis represents the states of G , the vertical axis represents the states of D , and, as usual, O is perpendicular to the page. The states of these three elements of this set up are correlated as follows. (i) The set $[S]$ is correlated with the microstate O_S of O , so that O experiences D as being in $[S]$. (ii) The dynamical set up is such that when D is in $[S]$, G is somewhere in the box rather than anywhere outside the box: Therefore, the system is always at L or R . The Lebesgue measure of the phase space before the first measurement is $\alpha\Omega(q, N)$ (iii) The microstates of D in $[S]$ are not correlated with the position of G in the box, so that O cannot read off from the pre-measurement state $[S]$ of D whether G is in the left-hand side or the right-hand side of the box. At this stage we must point out that information is always concerning the macrostate of the system and ignorance (with respect to the microstate within that macrostate). Conditions (ii) and (iii) together mean that the pre-measurement macrostate of $D + G$ (relative to O) is $[S, L + R]$, as illustrated in Figure 9.2.

(B) Split

In Figure 9.3, the Lebesgue measure of the union of $[0, L]$ and $[1, R]$ is equal to the Lebesgue measure of $[S, L + R]$. Notice that at the split stage, the relevant macrostates of G are $[L]$ and $[R]$ (and no longer the single macrostate $[L + R]$) because O can distinguish between these sets of microstates of G , by looking at D . Once again, the correlations between O , D , and G are objective and physical since they are brought

about by the interactions between these systems and expressed by the accessible region which is affected by external constraints and other objective limitations.

(C) Outcome

By the end of the measurement, the actual microstate of D+G is either in the macrostate [0, L] or in the macrostate [1, R]; and the macrostate in which the actual microstate of D + G happens to be is the macroscopic outcome of the measurement. Let us now add O's state to this picture. The macrostates [S], [0], and [1] of D, are sets of microstates of D that are correlated with three microstates of O, let us call them o_s , o_0 , and o_1 , and so the actual microstate of D + G determines whether O's microstate will be o_0 or o_1 . Since, by our assumption (discussed in Chapter 5), O's microstate gives rise to O's experience, by the end of the evolution O has either the experience that D is in [0] or the experience that D is in [1]; and from this, together with an acquaintance with the correlations between D and G (acquaintance which induces O to use D as a measuring device of the position of G), O infers whether G's actual position is in [L] or [R]. Thus, the final state is this: either O is in o_0 and has the experience that D + G is in [0, L], or O is in o_1 and has the experience that D + G is in [1, R]. Notice (and this will turn out to be important) that since O's final microstate is correlated with the entire macrostate [0, L] (or [1, R], as the case may be), O assigns the entire macrostate [0, L] to D + G, so that O describes the final state as in Figure 9.6. (O knows that D + G are in [0, L] and no probabilities are involved.)

These are the three conditions that an interaction must satisfy to be a measurement.

Maxwell's Demon.

In Eq. (3), (4), (19) and (21), we set the configuration space as algebraically divided by two after each consecutive measurement. If the number of consecutive measurements is n then the configuration space is arithmetically divided by 2^n .

The Lebesgue measure of the dynamical blob must be equal to the Lebesgue measure of the configuration space after n successive measurements, i.e., $\|B(t)\| \equiv \frac{\alpha\Omega(q,N)}{2^n}$. Nevertheless, Liouville's theorem is still valid, see Hemmo and Shenker (2012), Section 13.3, pp. 276, 277.

Both of q, N , are conserved. The probability of every microstate, $P_{(microstate)}$, after each consecutive measurement is:

$$0 \leq P_{(\text{microstate})} = \frac{2^n}{\alpha\Omega(q, N)} \leq 1 \Rightarrow \quad (41)$$

$$\Rightarrow 0 \leq n \leq \frac{\ln(\alpha\Omega(q, N))}{\ln 2}$$

This means that the probability of occurrence of each microstate (mental state) increases exponentially by each consecutive measurement. On the opposite, the number of microstates decreases exponentially. Psychologically, this means that the number of “options” and “choices” has “shrunk”:

“Idealization is the other defense important to note in depressive patients. Because their self-esteem has been damaged by the effects of their experiences (either by feeling chronically empty or feeling secretly bad), the admiration with which they view others is correspondingly increased. Self-perpetuating cycles of holding others in excessively high regard, then feeling diminished in comparison, then seeking idealized objects to compensate for the diminution, feeling inferior to those objects, and so on, are typical for depressive people.”. N. McWilliams, “Psychoanalytic Diagnosis”, The Guilford Press (2011), p. 240.

Additionally, “This is substantially what Freud had hypothesized in his time (1921) as a periodic reaction of rebellion of the ego against the burden of the ego ideal. If we consider this explanation from a common-sense perspective, it may appear quite reasonable to view mania, with its sense of liberation from constraints and limitations, as a kind of violent insurrection against an oppressor, which breaks the paralyzing stasis of depressive inhibitions, although it is nevertheless destined to fail miserably at a certain point. For these authors, the ego’s reaction of opposition does indeed provide relief, but is actually to be considered a defense against the difficult, although necessary, restoration of the bond with the depressive object (whether external or internalized), which allows for overcoming the depression by overcoming the ambivalence.”. See, Giancarlo Ventimiglia (2018): A psychoanalytic interpretation of bipolar disorder, International Forum of Psychoanalysis, DOI: 10.1080/0803706X.2018.1518595, “Constraints” and “limitations” increase and ambivalence prevails.

From the analysis above I imply that we may assume that Maxwell’s Demon may also apply to the cycles observed in bipolar disorder. See also the Section “Deviation from Thermal Equilibrium” for a reference to what I describe as “mood imbalance”.

Hemmo and Shenker (2012), Section 12.8 p. 268: “It is clear that blending does not erase the memory of O . Blending only prevents

inferring the actual macrostate in the past $[M_1]$ or $[M_2]$ from the present macrostate regardless of whether or not O remembers the past macrostate.”. I assume that the patient does not necessarily forget the past macrostate after each cycle of operation of Maxwell’s Demon.

The rest of the treatment of Maxwell’s Demon for an ES and O can be found approximately intact for the case that I examine, see Hemmo and Shenker (2012), Chapter 13.

Psychoanalytic Notes.

The number of ideas (N) and the number of energy quanta (q), of the Cs. is conserved. There is no interaction with the $Ucs.$ considered.

The size of the configuration space expresses the number, multiplicity (Ω), of the arrangements of q energy quanta over the N ideas (QHOs) of the ego (ES). I.e. the number of all possible mental states. Each mental state is defined as a microstate (arrangement). Each microstate is represented by a point in the classical phase space. It (microstate/mental state) belongs to a macrostate part of the ES/ego. See Figure 5.6, Section 5.4, Hemmo and Shenker (2012).

I assume that the innate human awareness and cognition of the self, of the environment and the arrow of time, is according to the Law of Approach to Equilibrium and the Second Law. This includes that the thermodynamic notion of entropy corresponds to exploitability of energy despite the fact that, “...the thermodynamic notion of entropy will no longer correspond to exploitability of energy. In other words, in a Demonic world, in which Maxwellian Demons exist, the very definition of entropy will have to change: the size of a macrostate will no longer be inversely correlated with the degree of exploitability of energy.”. Hemmo and Shenker (2012), Section 7.12, p. 180. That of course does not rule out the possibility of occurrence of Maxwell’s Demon in the mental apparatus. On the contrary, I raise the question whether Maxwell’s Demon is equivalent to pathological splitting of the ego. See Section “Propositions for further interdisciplinary study”.

The configuration space, comparatively (Relatively), “ RC_S ” to its initial size ($\|B(t)\| \equiv \alpha\Omega(q, N)$), is decreasing exponentially in every cycle of Maxwell’s Demon.

$$RC_S = \frac{2^{(-n)}\alpha\Omega(q, N)}{\alpha\Omega(q, N)} = 2^{(-n)} \quad (42)$$

“The fact that in Lanford’s theorem, and in all attempts to underwrite the thermodynamic regularities by mechanics, the direction of time has to be inserted by hand, is no accident, and cannot be avoided, owing to the time symmetry of mechanics.”. Hemmo and Shenker (2012), Section 7.12, p. 178.

I consider the following two cases:

1. “Tami will fail in her predictions time and again. If she adheres to the theorem despite these failures, she will conclude that her actual trajectory is atypical: despite the high likelihood of entropy increase, she happens to experience the unlikely cases, repeatedly. But such a consistent failure will happen only if Tami adopts the measure ρ of probability and the measure ν of entropy that fit our experience.” Hemmo and Shenker (2012), Section 7.12. In conclusion, deviation from equilibrium is unwanted by the patient.
2. “...If she replaces them with measures that reflect the relative frequencies that she observes and the degree to which she can exploit energy, her theory will be good and useful – albeit anti-thermodynamic relative to our measures.” Hemmo and Shenker (2012), Section 7.12. The patient is in agreement with the deviation from equilibrium.

Propositions for further interdisciplinary study.

The notion of the “observer” in psychoanalysis. Reality Principle and the correlation of the observer with the environment.

The ego as encapsulating the observer and the isolated system of ideas and libido. Ego, “identification” and their connection to the concepts of microstates, macrostates, dynamical blob and measurement.

Maxwell’s Demon may be related to pathological splitting of the ego.

The equivalence of the three formulations of the Second Law in thermodynamics, and the counterexample of Maxwell’s Demon, may imply a theoretical unification of the definitions of the spectrum of psychoses.

The arrow of time and regression.

“Omnipotent control” and information, exploitability of energy.

The psychological mechanism of “undoing” and the concept of erasure, as defined in Hemmo and Shenker (2012), Chapter 12.