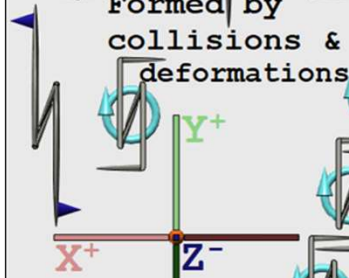
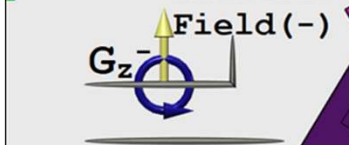


Shape&Space with existence contradiction, can spontaneously be created (Gödel's incompleteness theorem)

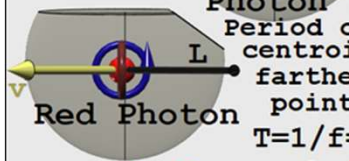


Entanglement is caused by non deformable particles



$|\omega_{Red}| < |\omega_{Blue}|$
 $I_{Red} > I_{Blue}$
 $|L_{Red}| = |L_{Blue}| = \hbar$

Polarization of photon
 $= L \times v$
 $L = I\omega$



$E_x = \omega_p \hbar = \hbar \omega$; $\omega_p = 2\pi dt/d\phi$

The theory Of Nothing(How everything works&How it was created

Clusters grow until Electric & $I_x = I_y + \delta$ ($\delta \rightarrow 0$), change Magnetic orientation & rotate about $\sim Z$, (Intermediate & Major axis theorem) More collisions in Z direction cause:

For frame with Z align to L:
 $\omega_z = \omega_z(t) = \omega_z(0)$
 If $\omega_x, \omega_y = q(t)$:
 $\partial^2 q / \partial t^2 = -\Omega^2 q$; $\Omega = \omega_z (I_x - I_z) / I_x$

3D Wave Equation: $\partial^2 \Psi / \partial t^2 = c^2 \Delta \Psi$; $\{\Delta = \partial^2 / \partial x^2 + \partial^2 / \partial y^2 + \partial^2 / \partial z^2\}$ is satisfied by $\Psi = q(t) f(r)$; if $f = -\Delta f c^2 / \Omega^2$; $\{\partial \Psi / \partial t^2 = f \partial^2 q / \partial t^2 = -\Delta f c^2 / \Omega^2 (-\Omega^2 q) = c^2 \Delta f = c^2 \Delta \Psi\}$ & Schrodinger equation: $\nabla^2 \Psi = -h^2 / (2m) \Delta \Psi + U \Psi = i \hbar \partial \Psi / \partial t$; satisfied by Ψ (If Ψ_1, Ψ_2 satisfy 3DWE (same c) $\Psi_1 \Psi_2$ also; & any wave can be built by $\Psi = C e^{i(K \cdot r - \omega t)}$; same $|v| = \omega / |K|$; waves; Deformation $\rightarrow E = mc^2 = pc$, & $E = hf = h|v|/\lambda$; $\lambda = h/|p|$; $E = KE + PE = \frac{1}{2} m |v|^2 + U = |p|^2 / (2m) + U$; $\Delta \Psi = -\Psi |K|^2 = -\Psi (2\pi/\lambda)^2 = -\Psi (|p|/\hbar)^2 = -\Psi m(E-U)/\hbar^2$; $\omega = 2\pi f = 2\pi E/h = E/\hbar$; $\partial \Psi / \partial t = -i\omega C e^{i(K \cdot r - \omega t)} = -i\omega \Psi = -i E \Psi / \hbar$

$L_w = \text{angular momentum (Quantum mechanics mistakenly consider } L_q \text{ as } L_w)$
 Deformation cause $v=c$ (stop deform at constant force)

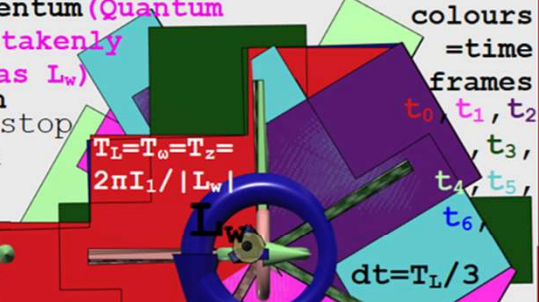


Magnetic Field Direction $= L_q$
 G_z^+
 G_y^-
 $v=c$
 Electric Field(+)

$E_x = \int F dx = \int F r d\phi$ [$\phi=0 \rightarrow 2\pi$]
 G_x^- Field $= \int F r \omega_p dt$ [$dt = T_L/3$]
 Direction $= 0 \rightarrow T_L = \omega_p \int dt$
 $= -L_q = \omega_p L = \omega_p \hbar = hf$; object radius; $D = \text{Distance}$; $k = \text{constant}$;

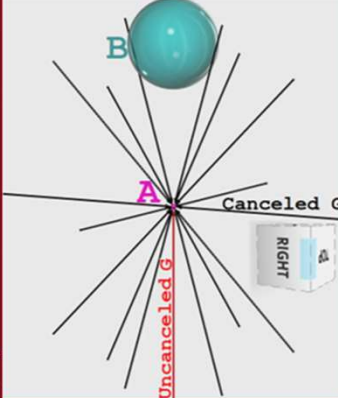
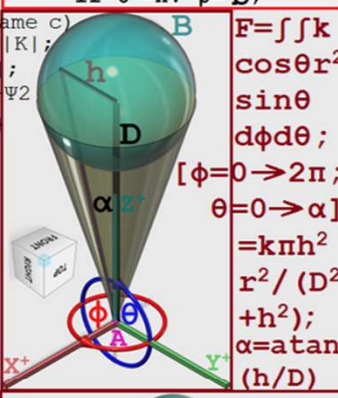
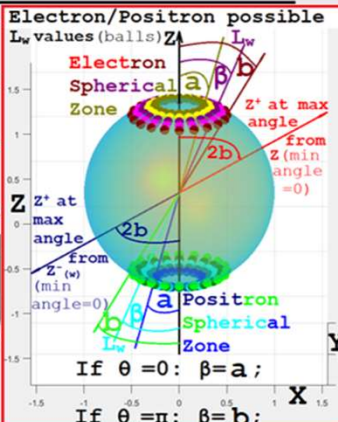
location \rightarrow phase
 $\frac{1}{2} I_z = I_x = I_y$
 $|\omega_x| < |\omega_y| < |\omega_z|$

colours = time frames
 $t_0, t_1, t_2, t_3, t_4, t_5, t_6$
 $dt = T_L/3$



For e^+ :
 $L_w = -L_w$
 $L_q = -L_q$

Gravity is caused by tiny particles' collision forces that don't cancel out, due to blockage by other object.
 Gravitation force $= kmh^2 r^2 / (D^2 + h^2)$; $h, r =$



Theory Of Nothing (How everything works & How it was created from nothing (By Guy Abitbol):

1-Nothing, shape&space with existence contradiction, is always being created

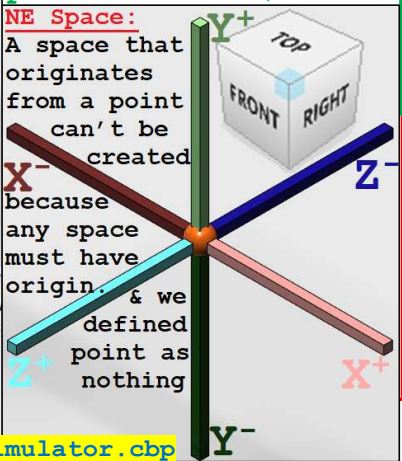
(with any relative velocity); {This step is the only non rigorously proved step in this theory; due to Gödel's incompleteness theorem; Alternatively: something, any shape in any dimension, is always being created, because nothing is not

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is never created, this theory is also never created)

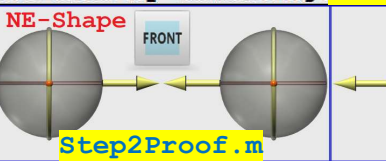
3-If 2G's collide, they bend at the collision point & rotate;



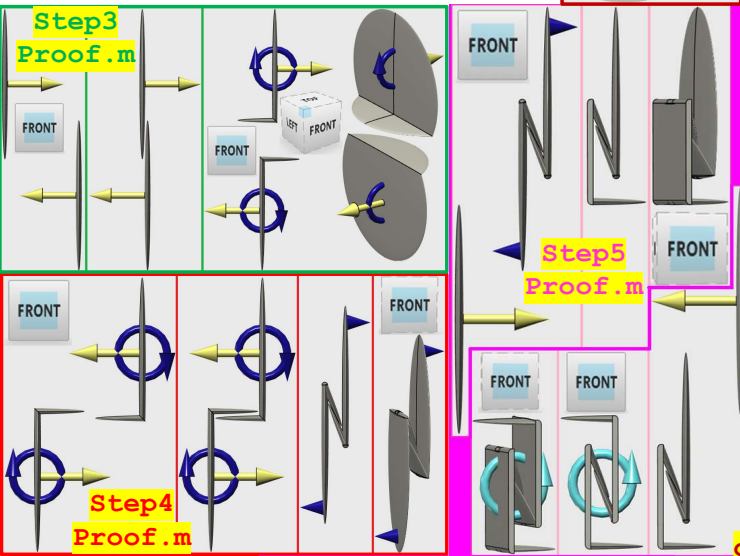
NE Space:
A space that originates from a point can't be created

NE (Non-Exsisting) Shape
0D (0 Dimension=Nothing) encloses (from begin to end) 1Do (o=open=has begin & end), which encloses 2Do which encloses 3Do...; So this shape'll always be enclosed by nothing. but any open shape can be enclosed by something.

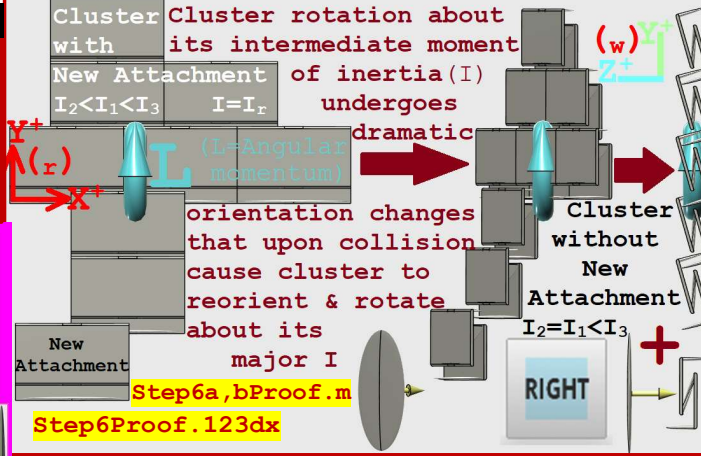
GASimulator.cbp



2-When 2 high velocity NE-Shapes collide, they deformed into 2 Squashed NE-Shapes (G) & move away from each other at lower speed
(by energy loss due to deformation & momentum conservation; For proof run Matlab file & then C file)



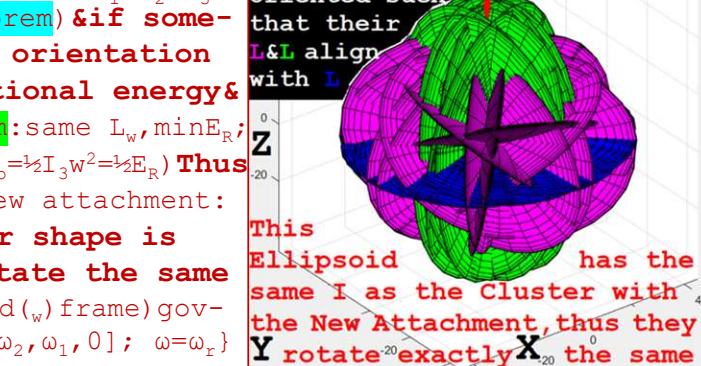
4-If 2 bent Gs collide, they can be attached to each other & rotate together
5-Further collisions, bendings & attachments create cluster:



6-As cluster become larger & larger, any new attachment increases its size by only small amount. But if it causes the cluster to rotate about intermediate I, the cluster change its orientation dramatically during the rotation

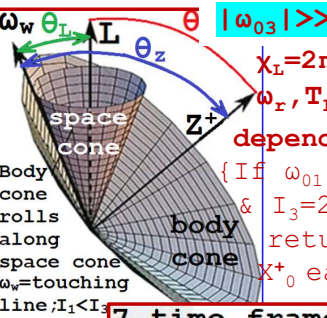
(even if $I_1 < I_2 < I_3$ & if something collide with it when its orientation is near orientation of major I (short axis; I_3) rotation, it'll loss rotational energy & start to rotate about its major I (Major axis theorem: same L_w , $\min E_R$; e.g. $L = [I_1 \omega_{01}; 0; 0]$, $E_R = \frac{1}{2} I_1 \omega_{01}^2 \rightarrow L_b = [I_3 w; 0; 0] = L$, $I_3 = 2 I_1$, $w = \frac{1}{2} \omega_{01}$; $E_{Rb} = \frac{1}{2} I_3 w^2 = \frac{1}{2} E_R$) Thus over time the cluster get $I_1 = I_2 < I_3$; (Probability of New attachment: breaking > incorporating) & $|\omega_{03}| > |\omega_{01}/2|$; ($\omega_{01} = \omega_{r1}(0)$) Cluster shape is used to emphasize asymmetry; bodies with same I rotate the same (for any body ω & R (Rotation matrix from rotated (r) to world (w) frame) govern only by solving: $I w' = I \omega \times \omega$; & $R' = R [0, -\omega_3, \omega_2; \omega_3, 0, -\omega_1; -\omega_2, \omega_1, 0]$; $\omega = \omega_r$)

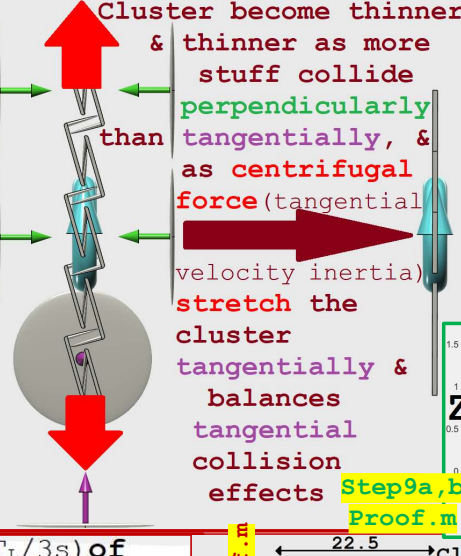
12 time frames (in 6.5s) of ellipsoid [45;32;2.6]; Mass=2531; Rotating ~about: Short/Intermediate/Long Axis Major/Intermediate/Minor I Angular velocity at time 0 at rotating frame $\omega_r(0)$: [0.05;0.05;1] No Orientation [0.07;1.4;0.07] [3;0.15;0.15] Oriented such that their L & L align with I

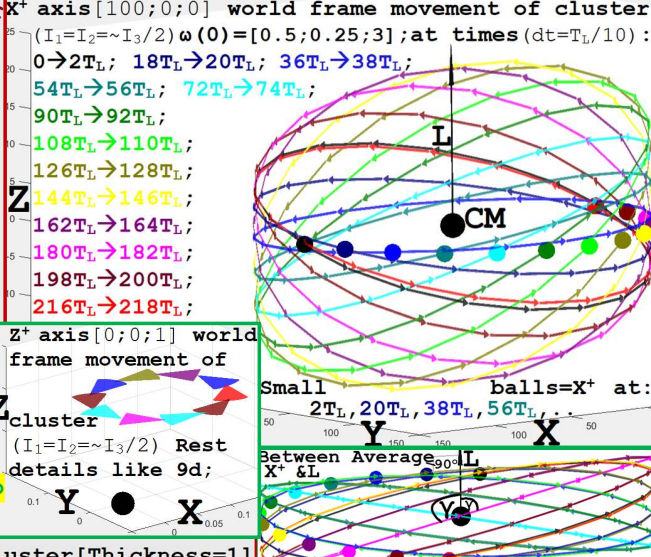


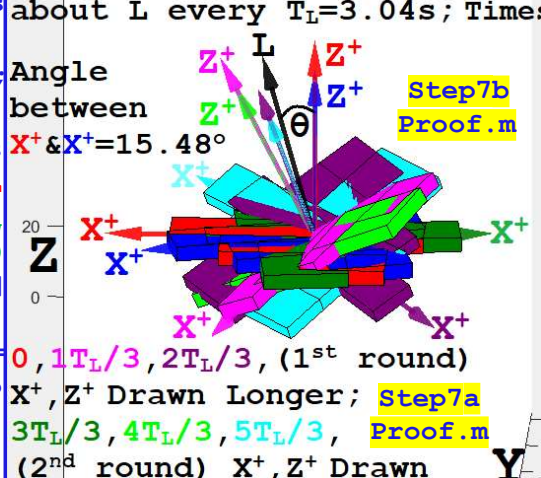
This Ellipsoid has the same I as the Cluster with the New Attachment, thus they rotate exactly the same

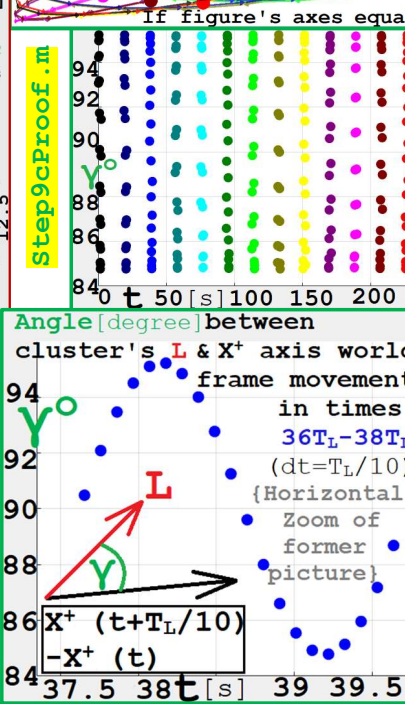
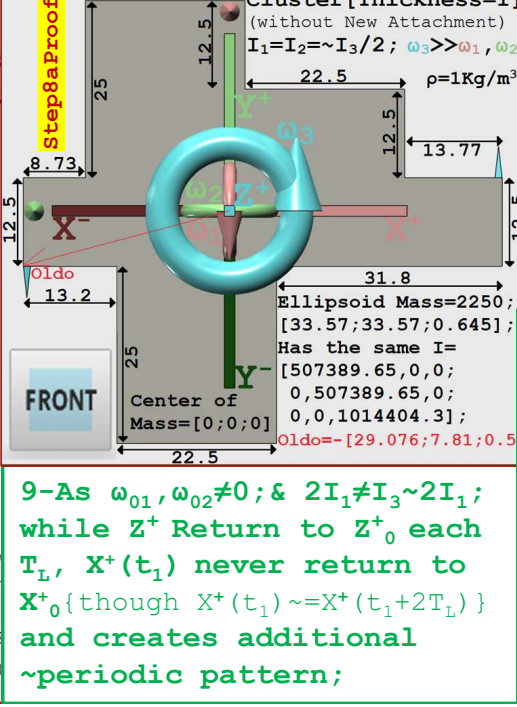
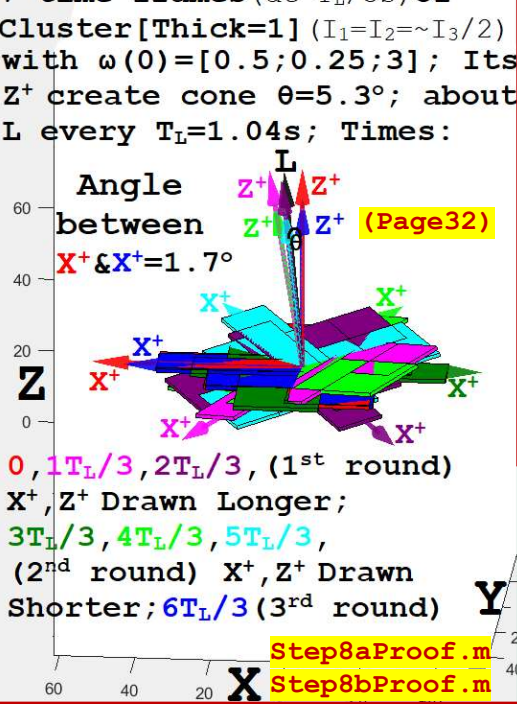
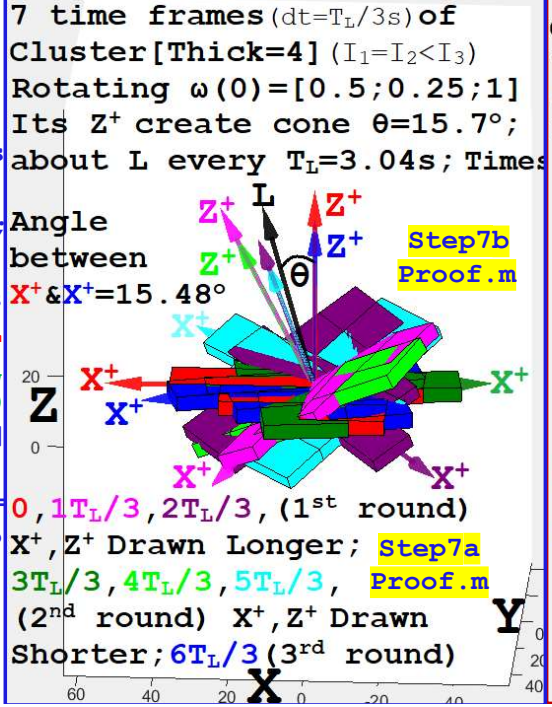
7-Formed cluster (without New Attachment) rotates exactly like axis symmetric body (in both $I_1=I_2$); $\omega_3'=0$; & $\omega_1''=-C^2\omega_1$; $\omega_2''=-C^2\omega_2$; are Simple Harmonic Oscillator; $C=\omega_{03}(I_1-I_3)/I_1$; $\omega_r=[\omega_{01}\cos(Ct)+\omega_{02}\sin(Ct)]$; $\omega_\theta=[\omega_{01}\sin(Ct)-\omega_{02}\cos(Ct)]$; ω_{03} ; if it has no initial orientation, to find its orientation in time t : rotate it $\Psi=Ct$; rad about its Z^+ axis $Z_r^+=Z^+$ & then rotate it $\phi=t|L_w|/I_1$; about $L_w=[I_1\omega_{01}; I_2\omega_{02}; I_3\omega_{03}]$. Thus Rotation matrix from (r) to (w) is $R=R_1R_2$; $R_2=[\cos\Psi, -\sin\Psi, 0; \sin\Psi, \cos\Psi, 0; 0, 0, 1]$; $L_u=L_w/|L_w|$; $L_uX=[0, -L_{u3}, L_{u2}; L_{u3}, 0, -L_{u1}; -L_{u2}, L_{u1}, 0]$; $(L_uXv=L_u \times v)$ Id $= [1, 0, 0; 0, 1, 0; 0, 0, 1]$; $R_1=Id+L_uX\sin\phi+(1-\cos\phi)L_uX^2$; $\omega_w=R\omega_r$; $Z^+=R[0; 0; 1]$; $X^+=R[1; 0; 0]$; ω_w & Z^+ Return to ω_0 & Z^+ each $T_L=2\pi I_1/|L_w|$; s But X^+ Never Return to X^+ ; ω_r Return to ω_0 each $2\pi/C$; s $\theta=\arccos(I_3\omega_{03}/|L_w|)$; $\theta_L=\arccos((L_w \cdot \omega_0)/(|L_w||\omega_0|))$; $\theta_z=\arccos(\omega_{03}/|\omega_0|)$; χ =Angle Between X^+ & X^+ = $\arccos(\cos\Psi(\cos\phi(1-L_{u1}^2)+L_{u1}^2)-\sin\Psi(\sin\phi L_{u3}+(\cos\phi-1)L_{u1}L_{u2}))$

χ_L =Angle between $X^+(t)$ & $X^+(t+2T_L)=2CT_L=4\pi\omega_{03}(I_1-I_3)/|L|$;
8-After reorientation, cluster thickness become thinner & thinner; getting $I_1=I_2 \sim I_3/2$; & If also $|\omega_{03}| \gg |\omega_{01/2}|$
 $\chi_L=2\pi=0$; R, ω_r, T_L don't depend on I_1 {If $\omega_{01}, \omega_{02}=0$; & $I_3=2I_1$; X^+ return to X^+ each $2T_L$ }


Cluster become thinner & thinner as more stuff collide perpendicularly, & as centrifugal force (tangential velocity inertia) stretch the cluster tangentially & balances tangential collision effects


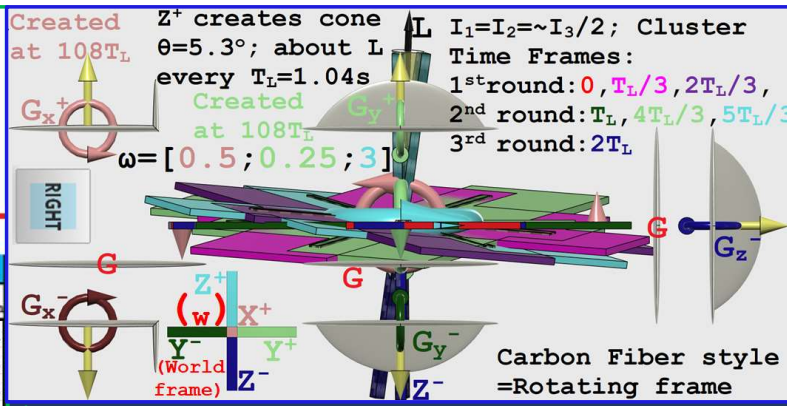


7 time frames ($dt=T_L/3s$) of Cluster [Thick=4] ($I_1=I_2 < I_3$) Rotating $\omega(0)=[0.5; 0.25; 1]$ Its Z^+ create cone $\theta=15.7^\circ$; about L every $T_L=3.04s$; Times: $0, 1T_L/3, 2T_L/3$, (1st round) X^+, Z^+ Drawn Longer; $3T_L/3, 4T_L/3, 5T_L/3$, (2nd round) X^+, Z^+ Drawn Shorter; $6T_L/3$ (3rd round) Shorter; $6T_L/3$ (3rd round) Shorter;




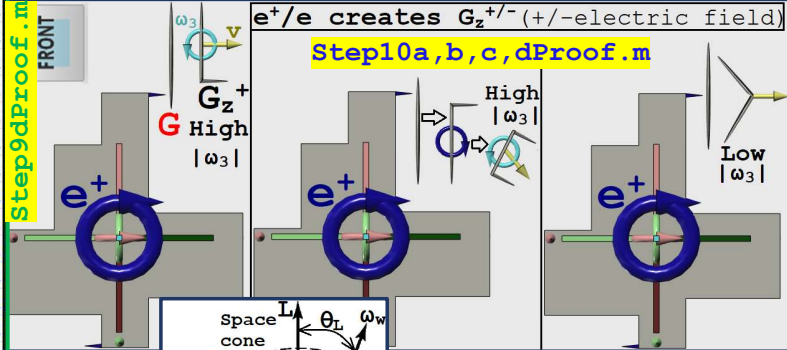
Velocity(V) of Cluster's($I_1=I_2=\sim I_3/2$)
 Axis: [100;0;0] or [0;0;1], world
 frame position at time t($S_{w1}(t)$ or
 $S_{w3}(t)$); at 20 time points($dt=T_L/10$);
 $V_{w1}=S_{w1}(t+dt)-S_{w1}(t)$; or
 $V_{w3}=S_{w3}(t+dt)-S_{w3}(t)$; $V_m=V_{max}$;

$V_{w1}[100;0;0]$				$V_{w3}[0;0;1]$			
$\omega(0)=[0.5;0.25;3]$				$\omega(0)=[0.5;0.25;3]$			
V(1)	V(2)	V(3)	V _m	V(1)	V(2)	V(3)	V _m
-4.9	30.9	-2.6	2	0.040	-0.041	-0.002	-2
-14	28	-2.3	2	0.056	-0.009	-0.004	1
-22	22.2	-1.8	-1	0.051	0.026	-0.005	1
-28	14.1	-1.1	-1	0.026	0.051	-0.004	2
-31	4.55	-0.4	-1	-0.009	0.057	-0.002	2
-31	-5.4	0.44	-1	-0.040	0.041	0.002	2
-28	-15	1.17	-1	-0.056	0.009	0.004	-1
-22	-22	1.79	-2	-0.051	-0.026	0.005	-1
-14	-28	2.25	-2	-0.026	-0.051	0.004	-2
-4.4	-31	2.49	-2	0.009	-0.057	0.002	-2
5.36	-31	2.49	-2	0.040	-0.041	-0.002	-2
14.8	-28	2.23	-2	0.056	-0.009	-0.004	1
22.8	-22	1.75	1	0.051	0.026	-0.005	1
28.6	-14	1.09	1	0.026	0.051	-0.004	2
31.4	-4.1	0.34	1	-0.009	0.057	-0.002	2
31	5.84	-0.4	1	-0.040	0.041	0.002	2
27.5	15.1	-1.1	1	-0.056	0.009	0.004	-1
21.4	22.7	-1.7	2	-0.051	-0.026	0.005	-1
13.3	28.1	-2.2	2	-0.026	-0.051	0.004	-2
3.96	30.9	-2.4	2	0.009	-0.057	0.002	-2



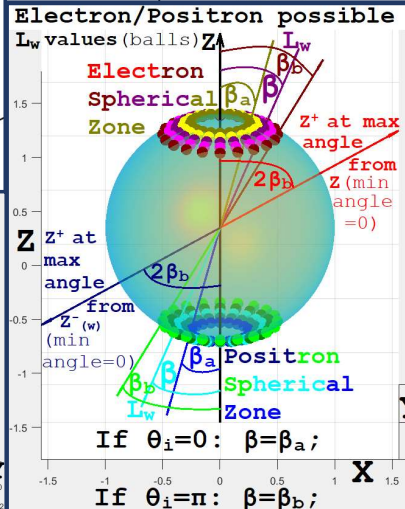
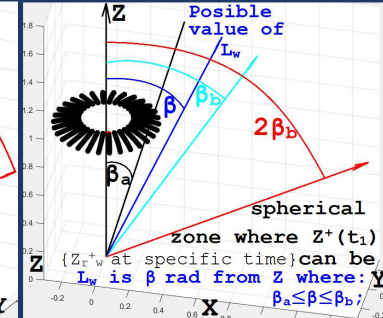
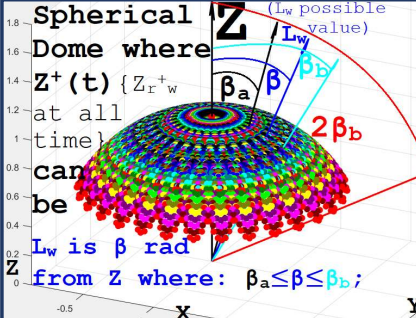
10-When the cluster collide with Gs, it bent them at the collision point, rotate them about z& xy axis& throw them with high speed creating $G_z^{+/-}$ & $G_{xy}^{+/-}$ which we call **E&B** {Electric & Magnetic field; charge = ω_3 ; (create $G_z^{+/-}$)}

11-Over time all the clusters in the universe throw $G_z^{+/-}$ & $G_{xy}^{+/-}$ on each other, which cause them to:attach/detach sub-particles, change orientation & L_w . The equilibrium of these collisions create types of clusters differing in the number of their sub-particles; The more the sub-particles, the more the inner movement& resistance, the cluster has (mass); But all clusters are asymmetric & have $I_1=I_2=\sim I_3, |\omega_3| \gg |\omega_{01/2}|$; (standard model massive particles)



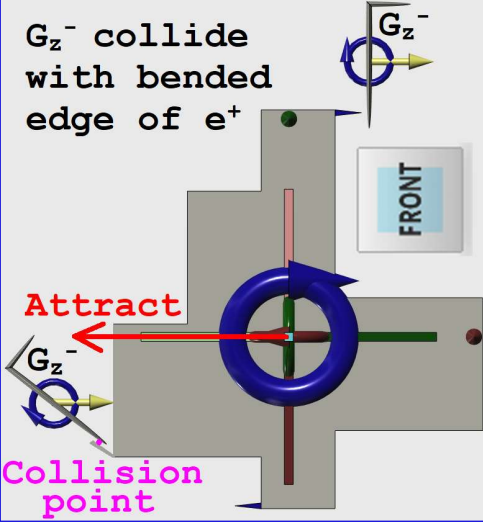
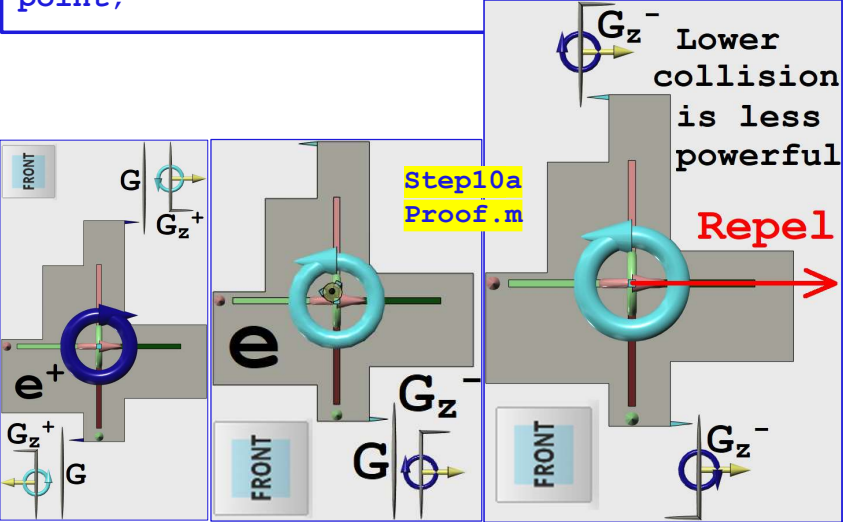
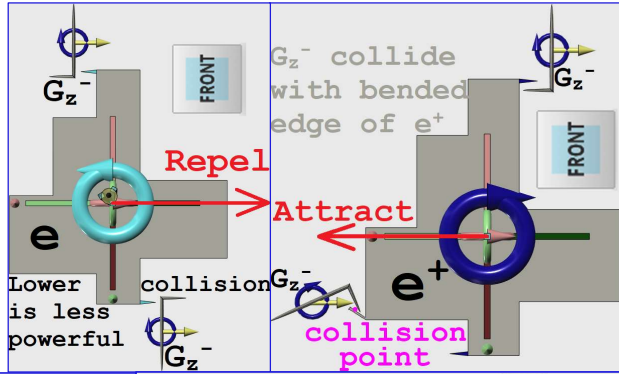
12-Because the clusters throw many G's on each other, G_{xyz} increase/reduce $|L_w|$ of small/large $|L_w|$ clusters. Such that at equilibrium all cluster $|L_w|$ =constant; & as $T_L=2\pi I_1/|L_w|$; For same type clusters {same I_1 ; e.g. electrons(e) & positrons (e^+)} Z^+ ($=Z_{r+w}$ =component of body Z^+ axis taken along w) always align with world frame Z axis ($Z^+_{(w)}$) at the same time ($t=0$) & their X^+ (X_{r+w}) has been aligned with $X^+_{(w)}$ at same specific time point (in inferred past). However, different e's have different L_w direction; Thus, all $Z^+(t)/Z^+(t_1)$ { Z_{r+w} at all/specific time} reside in spherical dome/zone; & the e/ e^+ L_w is also restricted to spherical zones (one for e & one for e^+); There's transformation between any spatial direction (L_q) to spherical zone direction (L_w). For e: $L_w(\alpha, \theta_i) = |L_w| L_d(\alpha, \theta_i)$; $L_d(\alpha, \theta_i) = [\sin\beta\cos\alpha; \sin\beta\sin\alpha; \cos\beta]$; $\beta = \beta_a + \theta_i$ ($\beta_b - \beta_a$)/ π ; $0 \leq \alpha \leq 2\pi$; $0 \leq \theta_i \leq \pi$; β_a, β_b =constants {unknown in range: $0^\circ < \beta_a \leq \beta \leq \beta_b < 30^\circ$; L_d =unit vector in spherical zone β_a, β_b given to any direction α, θ_i } Quantum mechanics mistakenly predicts that $L_w(\alpha, \theta_i)$ direction is $L_q(\alpha, \theta_i) = [\sin\theta_i\cos\alpha; \sin\theta_i\sin\alpha; \cos\theta_i]$; For e: Magnetic field $B_u = \mu = -L_q$; { $L_w = I_r\omega_0$; $I_r = [I_1, 0, 0; 0, I_1, 0; 0, 0, I_3]$; $I_r^{-1} = [1/I_1, 0, 0; 0, 1/I_1, 0; 0, 0, 1/I_3]$; $\omega_0 = I_r^{-1}L_w = |L_w| [\sin\beta\cos\alpha; \sin\beta\sin\alpha; \cos\beta]/I_1$ } For e^+ : $L_w(\alpha, \theta_i) = -|L_w| L_d(\alpha, \theta_i)$; $L_q(\alpha, \theta_i) = -[\sin\theta_i\cos\alpha; \sin\theta_i\sin\alpha; \cos\theta_i]$; For e^+ : $B_u = \mu = L_q$; $\theta = \arccos(I_3\omega_3/|L_w|) = \beta$; {For e/ e^+ : $\omega_3 = +/-$ }

Step12a,b,c,d,eProof.m
 See table page 5 for:
 $\omega_0, \omega_w, L_w, L_q, B_u$ values



13-While e^-/e^+ contains many bended edges (as e^-/e^+ contains many G_s), $G_z^+/-$ contains only 1 bended edge (as it contains 1 G); & because collision response is dictated by the collision point, we can see that when G_z^- (created by e^-) collide with e^- it repel it from its source (e^-); & when G_z^- (created by e^-) collide with e^+ it attracts it to its source (e^-); by symmetry e^+ attract/repel e^-/e^+ ; Explain Property#1: Like Charges Repel & Opposite Charges Attract; & $F_e = q_1 q_2 r^u / (\epsilon_0 4\pi |r|^2)$; $E_1 = F_e / q_2$; $\{F_e = \text{Electric force on charge } q_1; E_1 = E \text{ on } q_1; r = \text{vector from } q_2 \text{ to } q_1; r^u = \text{unit } r; q_2 \& q_1 = \text{non overlapping spheres stationary with respect to each other; } \epsilon_0 = \text{electric constant}\}; |F_e| \propto 1/|r|^2; (\propto = \text{proportional})$ as $G_z^+/-$ are geometrically diluted in 3D space (Sphere r Surface Area $= 4\pi |r|^2$); The bigger the q the more $G_z^+/-$ that don't cancel each other; {Bug of classic theory: if $|r| \rightarrow 0$ $F \rightarrow \infty$; But we know that any interaction between 2 particles: 1) is finite (even if $|r| \rightarrow 0$); 2) significantly changes when $|r|$ becomes smaller than their size (except gravitational interaction)}

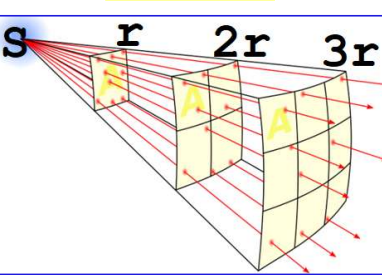
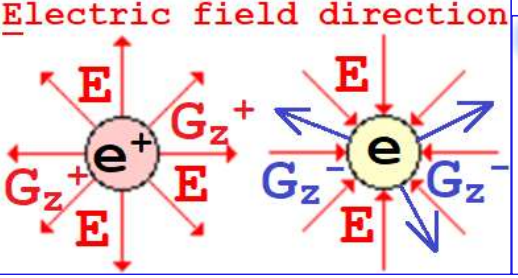
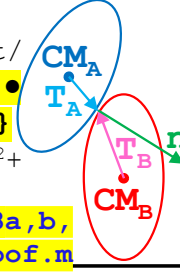
E direction at point is along/opposite to $G_z^+/-$ flying direction at that point;



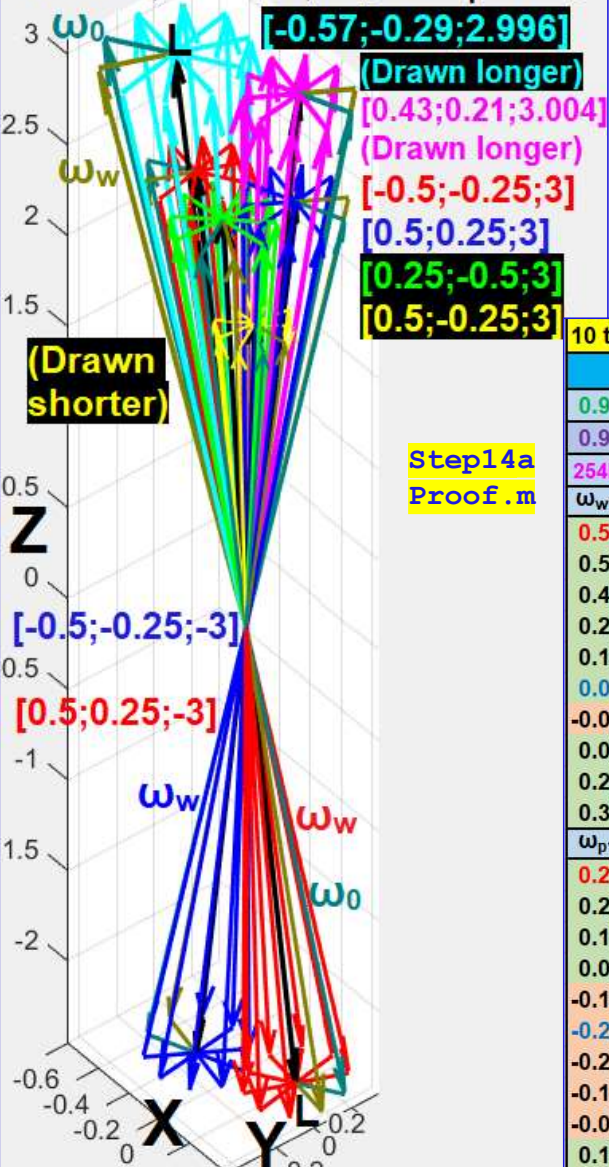
Collision Response: $n = \text{normal}; I = I_{CM} = \text{Inertia tensor at CM}; I \& \omega$ in rotating frame, rest in world frame; $R = \text{transform from rotating to world frame};$

No deformation ($T_A, T_B, I_A, I_B, n = \text{constant}$), **No breaking** ($M_A, M_B = \text{constant}$) $F = F_{A \rightarrow B} = -F_{B \rightarrow A} = |F|n$; $J = J_{A \rightarrow B} = \int F dt = -J_{B \rightarrow A} = n \int |F| dt = n j = \int p'_B dt = p_B(t_2) - p_B(t_1) = M_B v_B(t_2) - M_B v_B(t_1) = M_B v_{Bc} - M_B v_B$; $v_{Bc} = v_B + jn/M_B$; $v_{Ac} = v_A - jn/M_A$; $\int \tau_B dt = \int T_B \times F_{A \rightarrow B} dt = T_B \times \int F_{A \rightarrow B} dt = T_B \times J = \int L'_B dt = L_B(t_2) - L_B(t_1)$; $R_B^{-1}(T_B \times J) = R_B^{-1}(L_B(t_2) - L_B(t_1)) = I_B \omega_{Bc} - I_B \omega_B$; $Q_A = R_A^{-1}(T_A \times n)$; $Q_B = R_B^{-1}(T_B \times n)$; $\omega_{Bc} = \omega_B + jI_B^{-1}Q_B$; $\omega_{Ac} = \omega_A - jI_A^{-1}Q_A$; $v_n = v \cdot n = \text{amount of } v \text{ in } n \text{ direction};$ Coefficient of restitution $= \check{e} = (v_{Bcn} - v_{Acn}) / (v_{An} - v_{Bn}) = ((v_{Bc} + R_B \omega_{Bc} \times T_B) \cdot n - (v_{Ac} + R_A \omega_{Ac} \times T_A) \cdot n) / ((v_A + R_A \omega_A \times T_A) \cdot n - (v_B + R_B \omega_B \times T_B) \cdot n)$; $\{\check{e} \text{ determined experimentally; } \check{e} = 1; \text{elastic; rubber; no energy loss}\}$

$J_{n,A \rightarrow B} = \int F_{n,A \rightarrow B} dt = \int p'_{n,B} dt = M_B v_{Bcn} - M_B v_{Bn} = -J_{n,B \rightarrow A} = -(M_A v_{Acn} - M_A v_{An})$; as all on n : $M_B |v_{Bcn} - v_{Bn}| = M_B (|v_{Bcn}| - |v_{Bn}|) = -M_A (|v_{Acn}| - |v_{An}|)$; $W_{n,A \rightarrow B} = \sum \{ \int F_{in,A \rightarrow B}(r_{Bi}(t)) \cdot r_{Bin}'(t) dt \} = \sum \{ \int M_{Bi} a_{Bin} \cdot v_{Bin} dt \} = \sum \{ \frac{1}{2} M_{Bi} \int (|v_{Bin}|^2)' dt \} = \frac{1}{2} M_B |v_{Bcn}|^2 - \frac{1}{2} M_B |v_{Bn}|^2 = -W_{n,B \rightarrow A} = -(\frac{1}{2} M_A |v_{Acn}|^2 - \frac{1}{2} M_A |v_{An}|^2)$; $M_B (|v_{Bcn}| - |v_{Bn}|) (|v_{Bcn}| + |v_{Bn}|) = -M_A (|v_{Acn}| - |v_{An}|) (|v_{Acn}| + |v_{An}|)$; divide green: $(|v_{Bcn}| + |v_{Bn}|) = (|v_{Acn}| + |v_{An}|)$; all on n : $v_{Bcn} - v_{Acn} = v_{An} - v_{Bn}$; $\check{e} = 0$; (plastic; clay; all energy converted to heat/deformation) **solve for j :** $j = (\check{e} + 1) \{ (v_A - v_B) \cdot n + \omega_A \cdot Q_A - \omega_B \cdot Q_B \} / \{ M_A^{-1} + M_B^{-1} + I_A^{-1} Q_A \cdot Q_A + I_B^{-1} Q_B \cdot Q_B \}$; $\{ \text{If ellipsoid: } I^{-1} = [5/(s_2^2 + s_3^2), 0, 0; 0, 5/(s_1^2 + s_3^2), 0; 0, 0, 5/(s_2^2 + s_1^2)] / M \}$ **If collide:** $j = \text{positive};$ So n dictates v_{Ac} ; **Step13a, b, c, d Proof.m**



10 Timeframe: 0, $T_L/10, \dots$; of cluster ($l_1=l_2 \sim l_3/2$) ω_w & ω_p if ω_0 :

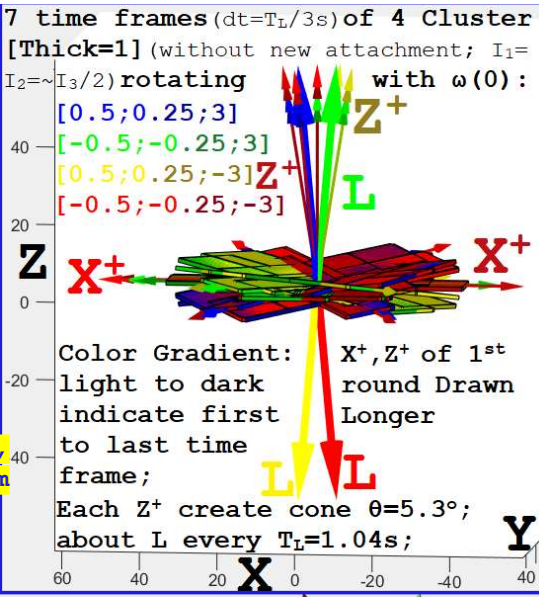
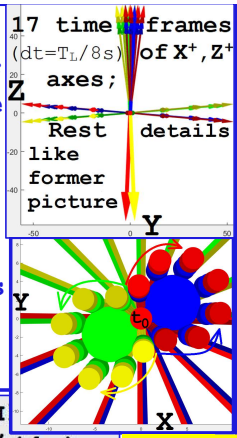


10 timeframe from 0 to T_L ; $\omega_p = \omega_w$ components perpendicular to L ; E_u ; L_q ; ω_0 ; $\beta_a = 0.04647 = 2.7^\circ$; $\beta_b = 0.1394 = 8^\circ$; $|L_w| = 3056402.465$; $l_{r1} = l_{r2} = 507389.65$; $l_{r3} = 1014404.3$;

e^*_A			e^*_B			e_C			e_D			e_E			e_F			e_G			e_H		
0.90	0.45	0.000	-0.90	-0.45	0.000	0.80	0.40	0.448	-0.80	-0.40	-0.448	-0.45	0.90	0.000	-0.90	-0.45	0.000	0.90	0.45	0.000	-0.90	-0.45	0.000
0.90	0.45	0.000	-0.90	-0.45	0.000	-0.80	-0.40	-0.448	0.80	0.40	0.448	0.45	-0.90	0.000	0.90	0.45	0.000	-0.90	-0.45	0.000	0.90	-0.45	0.000
254k	127k	-3M	-254k	-127k	-3M	-291k	-145k	3.04M	216k	108k	3.05M	127k	-254k	3M	254k	127k	3M	-254k	-127k	3M	254k	-127k	3M
ω_{w1}	ω_{w2}	ω_{w3}	ω_{w1}	ω_{w2}	ω_{w3}	ω_{w1}	ω_{w2}	ω_{w3}	ω_{w1}	ω_{w2}	ω_{w3}	ω_{w1}	ω_{w2}	ω_{w3}	ω_{w1}	ω_{w2}	ω_{w3}	ω_{w1}	ω_{w2}	ω_{w3}	ω_{w1}	ω_{w2}	ω_{w3}
0.50	0.25	-3.000	-0.50	-0.25	-3.000	-0.57	-0.29	2.996	0.43	0.21	3.004	0.25	-0.50	3.000	0.50	0.25	3.000	-0.50	-0.25	3.000	0.50	-0.25	3.000
0.53	0.08	-3.005	-0.53	-0.08	-3.005	-0.44	-0.43	3.002	0.32	0.32	3.007	0.37	-0.38	3.005	0.38	0.37	3.005	-0.38	-0.37	3.005	0.53	-0.08	3.005
0.45	-0.07	-3.018	-0.45	0.07	-3.018	-0.24	-0.46	3.019	0.18	0.34	3.016	0.40	-0.21	3.018	0.21	0.40	3.018	-0.21	-0.40	3.018	0.45	0.07	3.018
0.29	-0.15	-3.034	-0.29	0.15	-3.034	-0.07	-0.37	3.040	0.05	0.28	3.028	0.32	-0.06	3.034	0.06	0.32	3.034	-0.06	-0.32	3.034	0.29	0.15	3.034
0.12	-0.12	-3.047	-0.12	0.12	-3.047	0.02	-0.20	3.057	-0.02	0.15	3.038	0.17	0.02	3.047	-0.02	0.17	3.047	0.02	-0.17	3.047	0.12	0.12	3.047
0.00	0.00	-3.052	0.00	0.00	-3.052	-0.01	0.00	3.064	0.00	0.00	3.041	0.00	0.00	3.052	0.00	0.00	3.052	0.00	0.00	3.052	0.00	0.00	3.052
-0.02	0.17	-3.047	0.02	-0.17	-3.047	-0.14	0.14	3.057	0.11	-0.10	3.038	-0.12	-0.12	3.047	0.12	-0.12	3.047	-0.12	0.12	3.047	-0.02	-0.17	3.047
0.06	0.32	-3.034	-0.06	-0.32	-3.034	-0.34	0.17	3.040	0.25	-0.13	3.028	-0.15	-0.29	3.034	0.29	-0.15	3.034	-0.29	0.15	3.034	0.06	-0.32	3.034
0.21	0.40	-3.018	-0.21	-0.40	-3.018	-0.51	0.08	3.019	0.38	-0.06	3.016	-0.07	-0.45	3.018	0.45	-0.07	3.018	-0.45	0.07	3.018	0.21	-0.40	3.018
0.38	0.37	-3.005	-0.38	-0.37	-3.005	-0.60	-0.09	3.002	0.45	0.07	3.007	0.08	-0.53	3.005	0.53	0.08	3.005	-0.53	-0.08	3.005	0.38	-0.37	3.005
ω_{p1}	ω_{p2}	ω_{p3}	ω_{p1}	ω_{p2}	ω_{p3}	ω_{p1}	ω_{p2}	ω_{p3}	ω_{p1}	ω_{p2}	ω_{p3}	ω_{p1}	ω_{p2}	ω_{p3}	ω_{p1}	ω_{p2}	ω_{p3}	ω_{p1}	ω_{p2}	ω_{p3}	ω_{p1}	ω_{p2}	ω_{p3}
0.25	0.12	0.026	-0.25	-0.12	0.026	-0.28	-0.14	-0.034	0.21	0.11	-0.019	0.12	-0.25	-0.026	0.25	0.12	-0.026	-0.25	-0.12	-0.026	0.25	-0.12	-0.026
0.27	-0.05	0.021	-0.27	0.05	0.021	-0.15	-0.28	-0.027	0.11	0.21	-0.015	0.25	-0.13	-0.021	0.13	0.25	-0.021	-0.13	-0.25	-0.021	0.27	0.05	-0.021
0.19	-0.20	0.008	-0.19	0.20	0.008	0.05	-0.31	-0.010	-0.04	0.23	-0.006	0.27	0.04	-0.008	-0.04	0.27	-0.008	0.04	-0.27	-0.008	0.19	0.20	-0.008
0.04	-0.27	-0.008	-0.04	0.27	-0.008	0.22	-0.23	0.010	-0.17	0.17	0.006	0.20	0.19	0.008	-0.19	0.20	0.008	0.19	-0.20	0.008	0.04	0.27	0.008
-0.13	-0.25	-0.021	0.13	0.25	-0.021	0.31	-0.05	0.027	-0.23	0.04	0.015	0.05	0.27	0.021	-0.27	0.05	0.021	0.27	-0.05	0.021	-0.13	0.25	0.021
-0.25	-0.12	-0.026	0.25	0.12	-0.026	0.28	0.14	0.034	-0.21	-0.11	0.019	-0.12	0.25	0.026	-0.25	-0.12	0.026	0.25	0.12	0.026	-0.25	0.12	0.026
-0.27	0.05	-0.021	0.27	-0.05	-0.021	0.15	0.28	0.027	-0.11	-0.21	0.015	-0.25	0.13	0.021	-0.13	-0.25	0.021	0.13	0.25	0.021	-0.27	-0.05	0.021
-0.19	0.20	-0.008	0.19	-0.20	-0.008	-0.05	0.31	0.010	0.04	-0.23	0.006	-0.27	-0.04	0.008	0.04	-0.27	0.008	-0.04	0.27	0.008	-0.19	-0.20	0.008
-0.04	0.27	0.008	0.04	-0.27	0.008	-0.22	0.23	-0.010	0.17	-0.17	-0.006	-0.20	-0.19	-0.008	0.19	-0.20	-0.008	-0.19	0.20	-0.008	-0.04	-0.27	-0.008
0.13	0.25	0.021	-0.13	-0.25	0.021	-0.31	0.05	-0.027	0.23	-0.04	-0.015	-0.05	-0.27	-0.021	0.27	-0.05	-0.021	-0.27	0.05	-0.021	0.13	-0.25	-0.021

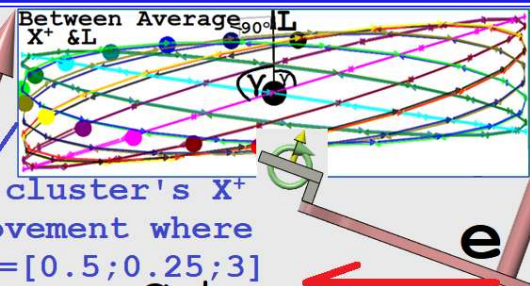
$14-A_s|\omega_3| \gg |\omega_{1/2}|$ different e/e^+ have roughly the same $+/-\omega_{w3}$, but very different $\omega_{w1,2}$; so while e 's ω_{w3} creates only G_z^- ; e 's $\omega_{w1,2}$ creates different $G_{xy}^+/-$ each time; The e 's cluster facet that create G_x^+ return to its position each $2T_L$, but in the direction of B_u , $2G_x^+$ s are thrown each $2T_L$ (created by different facet); Thus, with average cluster orientation, 2 same/opposite B_u e 's positioned along B_u attract/repel twice stronger than 2 opposite/same B_u e 's positioned perpendicular to B_u ;

■ e & e^+ with same B_u have exactly opposite L_w & thus the same average cluster orientation; Therefore they behave the same in B ; **■** As X^+ (& Y^+) of e/e^+ return to its position after $2T_L$ s; $n_o=n_{odd}=1,3,5..$; & $n_e=n_{even}=2,4,6..$; represent different phase; Quantum spin state consist of: $L_q (\rightarrow B_u, L_w, \omega_w)$ & phase (n_o/n_e) but only $L_q(\omega_w)$ effect B & E ;



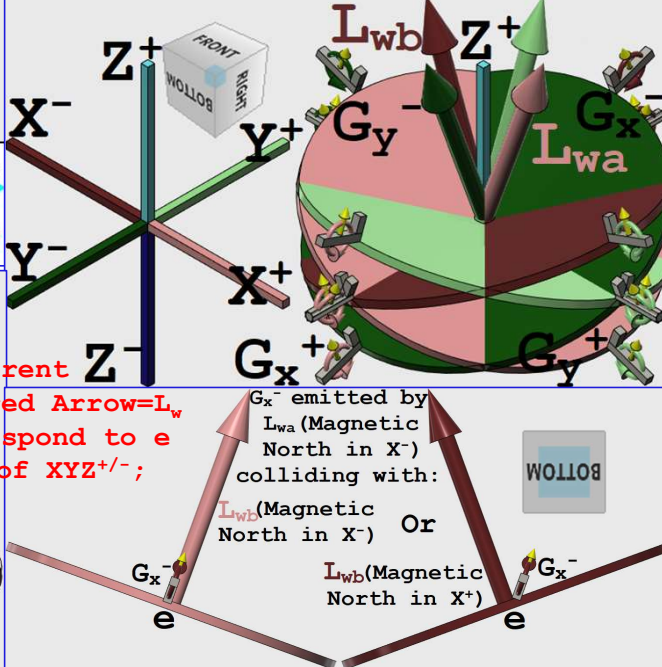
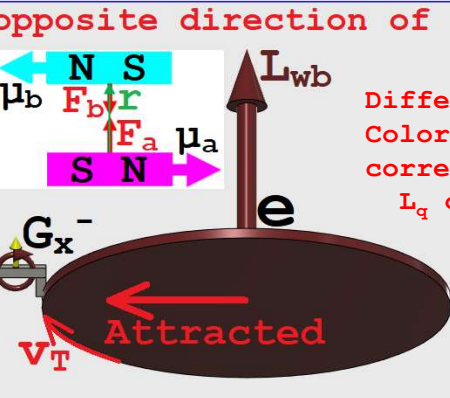
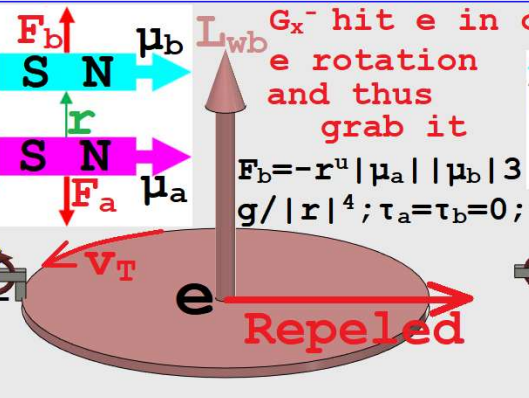
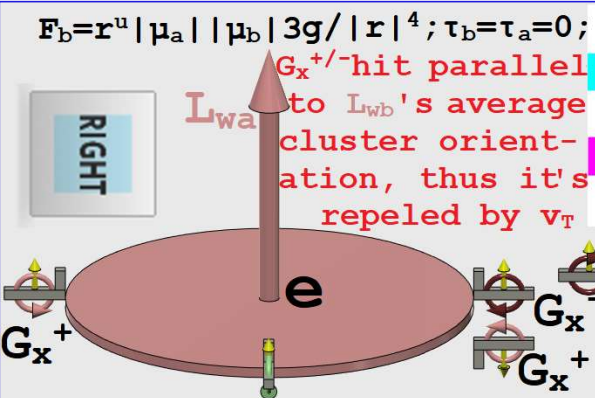
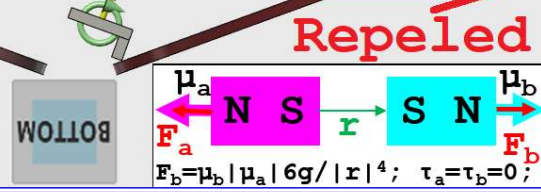
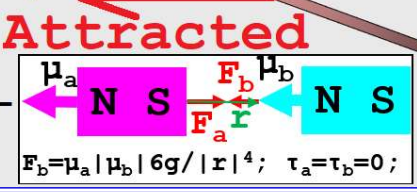
Plane perpendicular to $L_w =$ Average Cluster orientation

Twice $G_{xy}^+/-$ in direction of $\pm B_u$; More G_y^+ hit the closer side (geometric dilution)

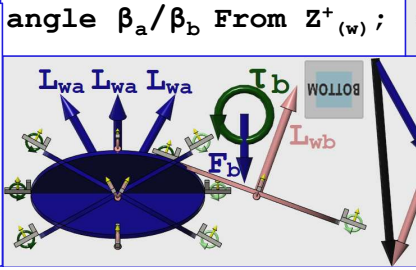
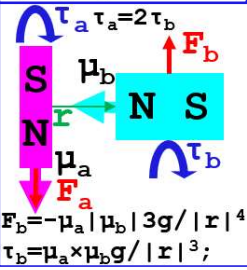
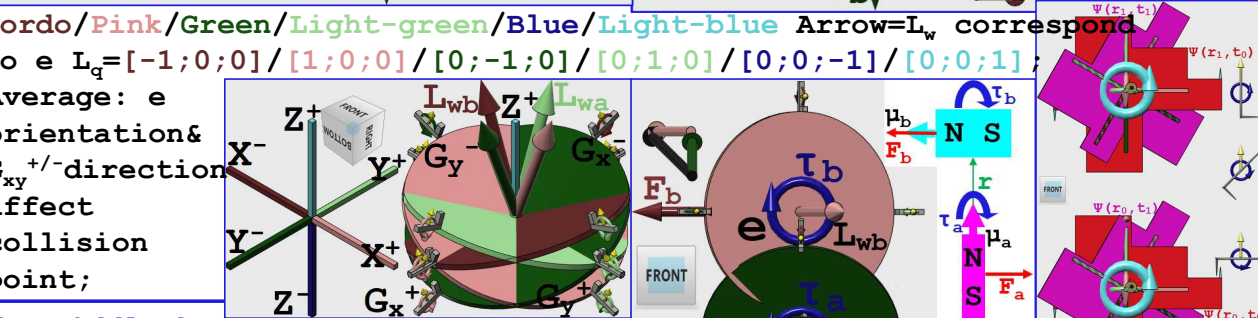
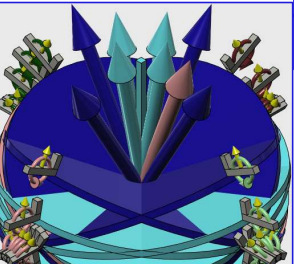
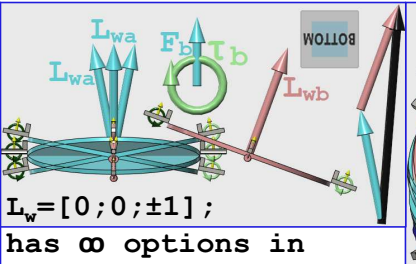
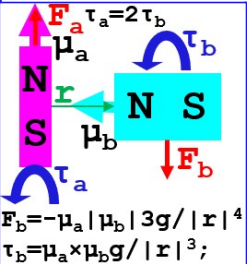
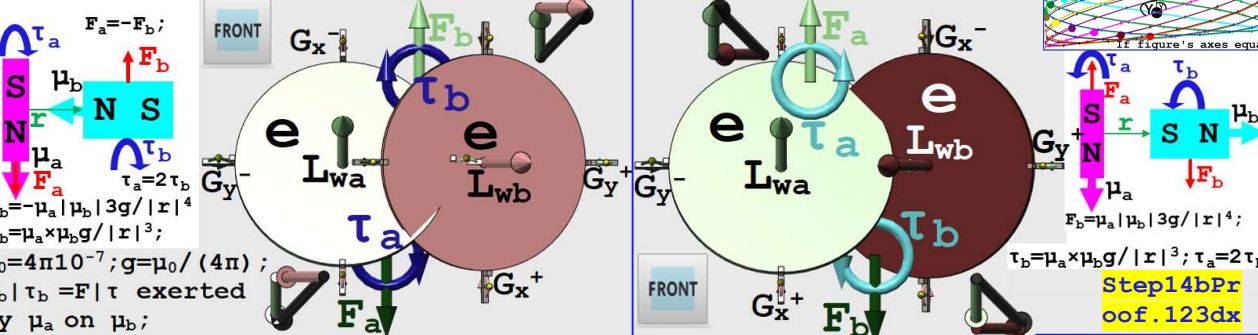
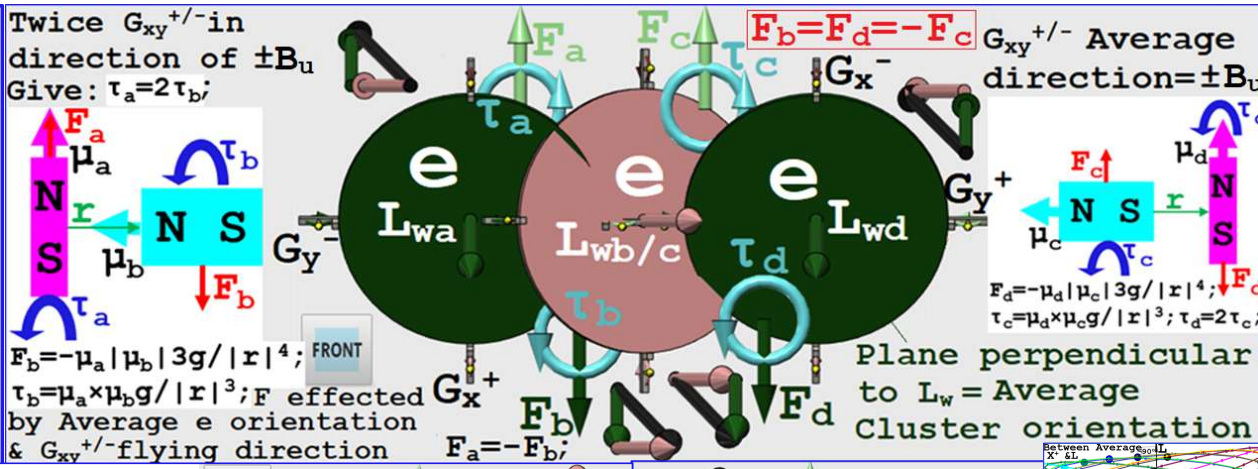


e 's L_w & $B_u | \mu_a$ in opposite direction

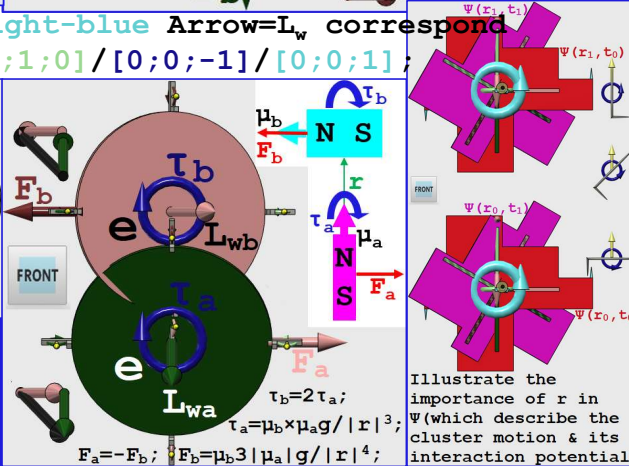
$\mu_0 = 4\pi$
 $g = \mu_0 / (4\pi)$;
 $F_b | \tau_b = F | \tau$ exerted by μ_a on μ_b



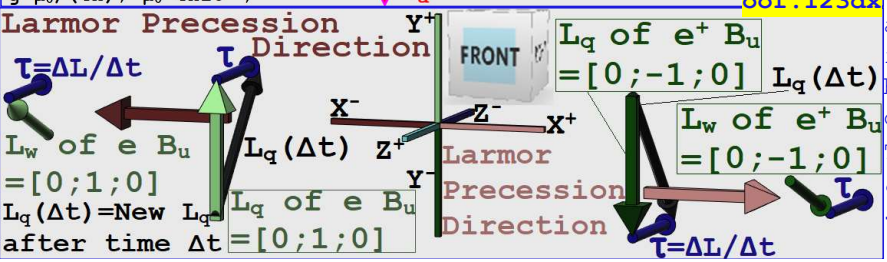
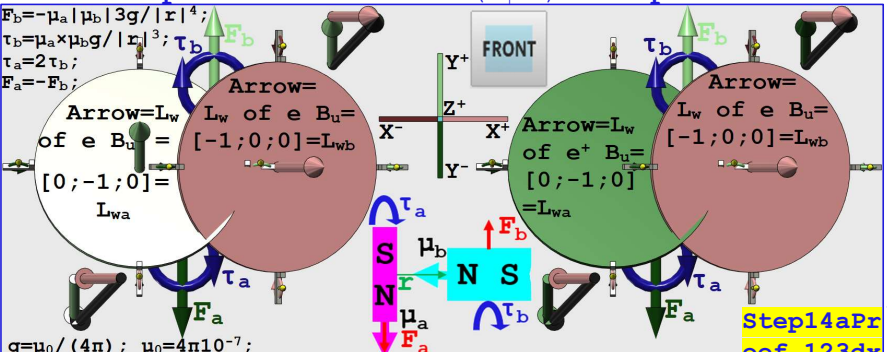
e Quantum spin state: $|+n\rangle = [e^{-i\phi/2} \cos(\theta/2); e^{i\phi/2} \sin(\theta/2)]$; **corespond to:** $L_q(\theta, \phi)$ {for universe (unknown) constant $I_1, \beta_a, \beta_b; \beta = \beta_a + \theta(\beta_b - \beta_a) / \pi; L_w = |L_w| [\sin\beta \cos\phi; \sin\beta \sin\phi; \cos\beta]$; $I_3 \approx 2I_1; \omega_0 = |L_w| [\sin\beta \cos\phi / I_1; \sin\beta \sin\phi / I_1; \cos\beta / I_3]$ } & **n_e phase** ($X^+ & Y^+$ of e return to its position) $| -n \rangle = [-e^{-i\phi/2} \sin(\theta/2); e^{i\phi/2} \cos(\theta/2)]$; **corespond to:** $-L_q(\theta, \phi)$ & n_e phase;
■ $|+/-n\rangle$ derived from L operator, derived from P & r operator, derived from $\Psi(r, t) = A \exp(i(p \cdot r - Et + \phi\hbar) / \hbar)$;
 Satisfy 3D wave equation: $\partial^2 f / \partial t^2 = |\mathbf{v}|^2 \Delta f$; $\{\Delta f = \partial^2 f / \partial x^2 + \partial^2 f / \partial y^2 + \partial^2 f / \partial z^2\}$ & **travel (Ψ shape) along p with $|\mathbf{v}| = E / |p|$;**
 & **Satisfy: photon: $E = hf$; & quantum particle: $\lambda = h / |p|$** {so $E / |p| = hf\lambda / h = f\lambda = |\mathbf{v}|$ } Ψ also satisfy **Simple Harmonic Oscillator equation:** $\partial^2 f / \partial t^2 = -C^2 f$; $C = E / \hbar = 2\pi f$; [rad/s];
 Which is also **satisfied by the ω_{r1} & ω_{r2} of cluster with $I_{r1} = I_{r2}$** $\{d^2\omega_{r1} / dt^2 = -C^2\omega_{r1}; d^2\omega_{r2} / dt^2 = -C^2\omega_{r2}; C = \omega_{r03} (I_{r1} - I_{r3}) / I_{r1}$;
 Real solution: $\omega_r = [\omega_{01} \cos(Ct) + \omega_{02} \sin(Ct); \omega_{02} \cos(Ct) - \omega_{01} \sin(Ct); \omega_{03}]$; & **Complex: $\omega_r = [\omega_{r01} \exp(iCt); \omega_{r02} \exp(iCt); \omega_{r03}]$**
 $\Psi = A \exp(i\mathbf{r} \cdot \mathbf{n}) \exp(-iEt / \hbar)$; $\mathbf{n} = 2(p \cdot \mathbf{r} + \phi\hbar) / \hbar$; **If $n = \text{odd} | \text{even}$ than $\exp(i\mathbf{r} \cdot \mathbf{n}) = -1 | +1$; Thus Ψ describe cluster motion & phase (n_o / n_e ; interaction potential); $C = E / \hbar = \sim \omega_{r03}$;**



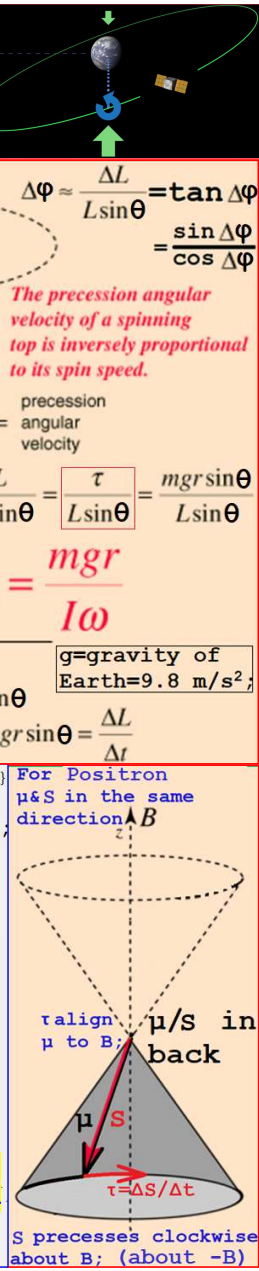
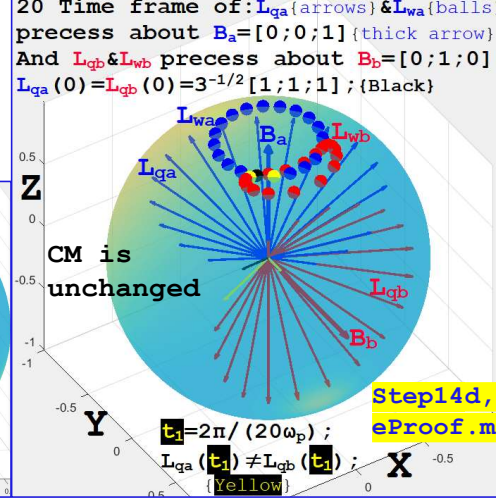
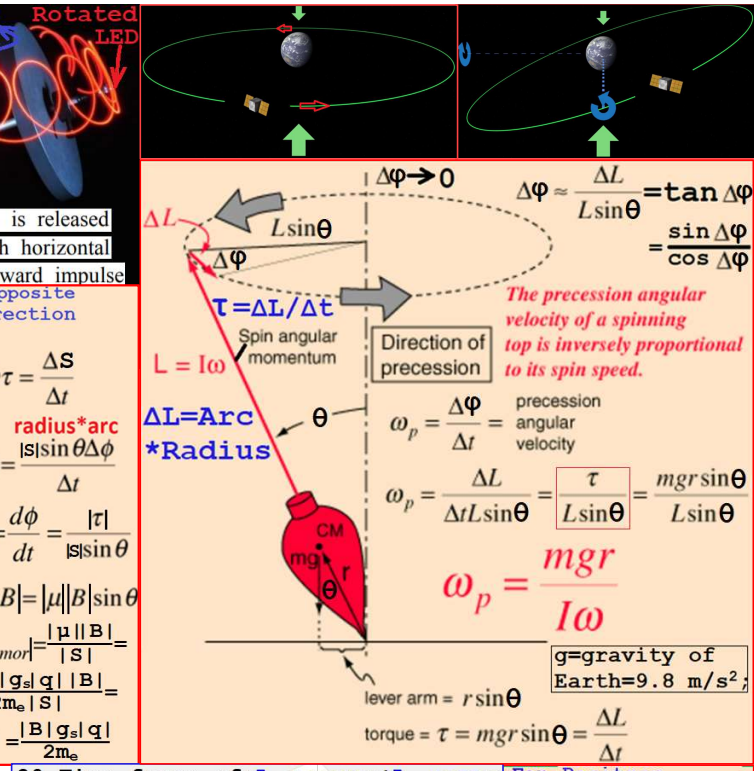
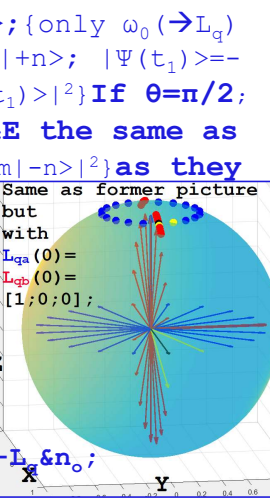
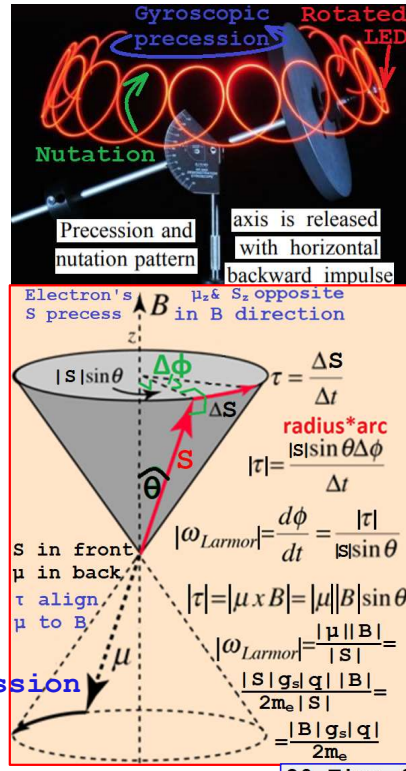
If e stand in the middle between 2 same B_u e's (homogeneous B) it feels τ but no F; But if e stand in inhomogeneous B (e.g. near 1e) it feels F & changing τ , which cause it's B_u to align with B or -B;



■ e^- & e^+ with same B_u feel the same τ in external homogeneous $B(B_h)$; but because they have opposite L_w , their L_q precess about B_h in opposite direction (B & $-B$); this is τ induced precession (Gyroscopic/Larmor precession), in contrast to the τ free precession (nutration) that described in page 32; so e^- & e^+ in B_h have both τ free & τ induced precession; ■ In quantum mechanics L_q is called S ; L_q can precess at the same angle for infinite time; An interaction with another particle is required in order to change the precession angle { τ perpendicular to L_q (effective L_w) at all times} ■ Total S & μ cannot be measured but for e^- : $S_z = \hbar/2$; ($\hbar = 1.054571817646157 \times 10^{-34}$ Js); $\mu_z = -1.760859630234 \times 10^{11} S_z = \gamma S_z = S_z g_s q / (2m_e)$; [J/T] $\mu = S g_s q / (2m_e)$; In $B_h = B = [0; 0; B_3]$; S precess about B with $\omega_L = -\gamma|B|$; $S_{xy} = S_{xy}(0) e^{-i\gamma|B|t}$; $\{S = S_z + S_{xy}$; $\tau = \mu \times B = \gamma S \times B = \gamma|B| [S_y; -S_x; 0] = S'$; $S'_z = 0$; since $S' = \gamma S \times B =$ Perpendicular to B ; $S' = S'_z + S'_{xy} = S'_{xy}$; we can write $S_{xy} = S_x + iS_y$; So $S'_{xy} = S' = \gamma|B| (S_y - iS_x) = -i\gamma|B| S_{xy}$ ■ If $|+n\rangle$ particle is in $B_h = B = [0; 0; B_3]$; its quantum spin state change with time: $|\Psi(t)\rangle = [\exp(-i(\varphi - B_3 \gamma t)/2) \cos(\theta/2); \exp(i(\varphi - B_3 \gamma t)/2) \sin(\theta/2)]$; e^-/e^+ L_q precess about $B/-B$ every: $t_1 = |2\pi / (B_3 \gamma)| = 2\pi / \omega_L$; s; but while L_q return to its value after 1 precession circle (t_1 s) the spin state return to its value only after 2 precession circles ($2t_1$ s); Thus $t_1 = n_0 T_L$; But in B & E



only after 2 precession circles ($2t_1$ s); Thus $t_1 = n_0 T_L$; But in B & E $|\Psi(t_1)\rangle$ act as $|+n\rangle$; {only $\omega_0 (\rightarrow L_q)$ effect E & B ; $|\Psi(2t_1)\rangle = |+n\rangle$; $|\Psi(t_1)\rangle = -|+n\rangle$; $|\langle m | +n \rangle|^2 = |\langle m | \Psi(t_1) \rangle|^2$ If $\theta = \pi/2$; $|\Psi(t_1/2)\rangle$ act in B & E the same as $|-n\rangle$ { $|\langle m | \Psi(t_1/2) \rangle|^2 = |\langle m | -n \rangle|^2$ } as they have the same L_q ; {but $|\Psi(t)\rangle \neq |-n\rangle$ for any t ; Because $t = t_1/2$ is needed to get $-L_q$ but $t = n_e T_L$; needed to get n_e & $t_1/2 = n_e T_L / 2 \neq n_e T_L$; $n_0 \neq 2n_e = n_e$ } $|+n\rangle \rightarrow L_q \& n_e$; $|-n\rangle \rightarrow -L_q \& n_e$; $-|+n\rangle \rightarrow L_q \& n_0$; $-|-n\rangle \rightarrow -L_q \& n_0$;



Orientation & L_w of e that generate B_e . Generate both G_x^+ & G_x^- at Y^- ; but v of G_x^- (v_G) is more opposite to the tangential v (v_T) at the most powerful collision point (so it collide more strongly).

Square Arrow $B_e = [1; 0; 0]$

Circular Arrow = Average $e L_w$

Hexagonal Arrow $= F_m$

Average e Orientation

The most powerful collision point is dictated by first encounter due to v ; & it's more powerful in v direction

15- E ($G_z^{+/-}$) interacts with e charge (ω_3), but not spin ($\omega_{1/2}$); B ($G_{xy}^{+/-}$) interacts with e spin ($\omega_{1/2}$), but not charge (ω_3); But if the e move in a static B it experiences an effective E , this is Lorentz force $= F_m = qv \times B_e =$ Magnetic force on q [C] moving (v [m/s]) through External Magnetic field (B_e [Tesla]); All Lorentz force situations can be explained by simple collision resolution see [Step15Proof.123dx](#)

Orientation & L_w of e that generate $B_e = [-1; 0; 0]$

Average $e^+ L_w$ Arrow is into the page

B_e Generate both G_x^+ & G_x^- at Y^- ; But G_x^- participate at the most powerful collision point as its v_G is more opposite to v_T there

16-Biot Savart law:
 $B_i = 10^{-7} qv \times r^u / |r|^2$;
 Moving (v [m/s]) q [C] generates internal Magnetic Field (B_i [T]) at r [m];

This is the point at which we want to find B .

Magnetic field of the moving point charge

Point charge q

Wall

Flor

corner

Velocity of the charged particle

Average $e^+ L_w$ Arrow is into the page

B_e Generate both G_y^+ & G_y^- at Z^- ; but v of G_y^+ (v_G) is more opposite to v (so it collide more strongly)

Average $e^+ L_w$ Arrow is into the page

Average $e^+ L_w$ & F_m Arrows are into the page

Average e^+ Orientation

Most powerful collision point is dictated by first encounter due to v in its direction; Colliding edge of the moving e^+ ;

Average $e^+ L_w$ is into the page

Average $e^+ L_w$ & F_m Arrows are into the page

Average e^+ Orientation

Here G_x^+ participate at the most powerful collision point as its v_G is more opposite to v_T there;

Due to r, v & e^+ L_w most $G_{xy}^{+/-}$ at r are G_x^- with v_G that match $B_e = [-1; 0; 0]$

Average $e^+ L_w$ & v are into the page

Orientation & L_w of e that generate $B_e = [-1; 0; 0]$

Average e^+ Orientation

Due to r, v & e^+ L_w most $G_{xy}^{+/-}$ at r are G_x^+ with v_G that match $B_e = [1; 0; 0]$

Average $e^+ L_w$ is into the page

Orientation & L_w of e that generate $B_e = [1; 0; 0]$

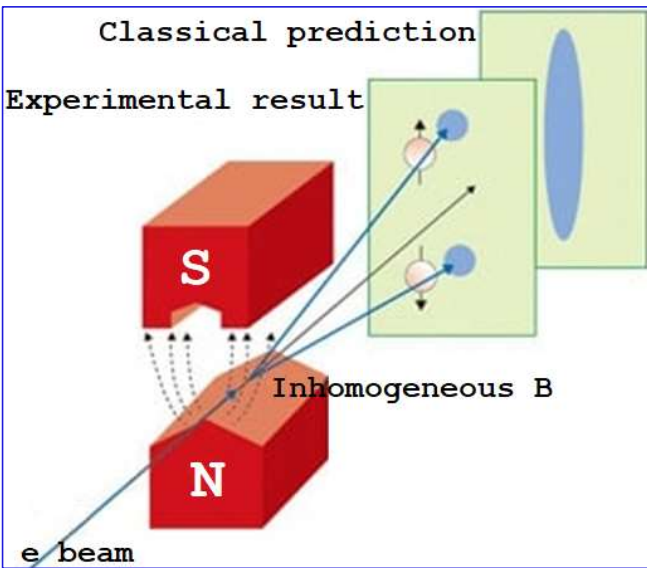
Average e^+ Orientation

Average $e L_w$ Arrow is into the page

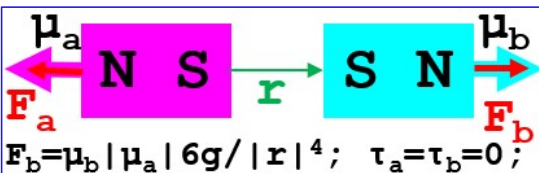
Average $e^+ L_w$ & F_m Arrows are into the page

Average e^+ Orientation

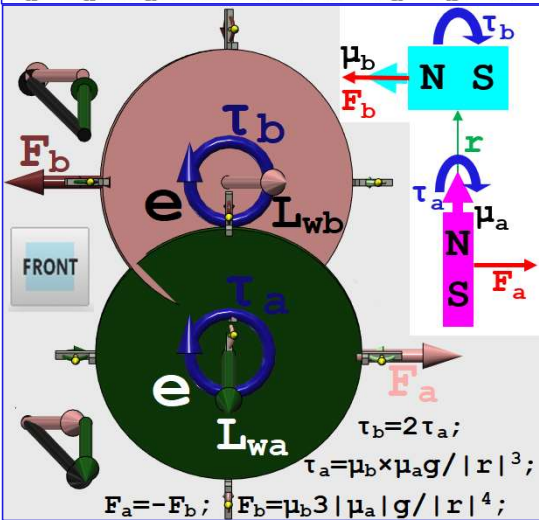
Average $e^+ L_w$ & v Arrows are into the page



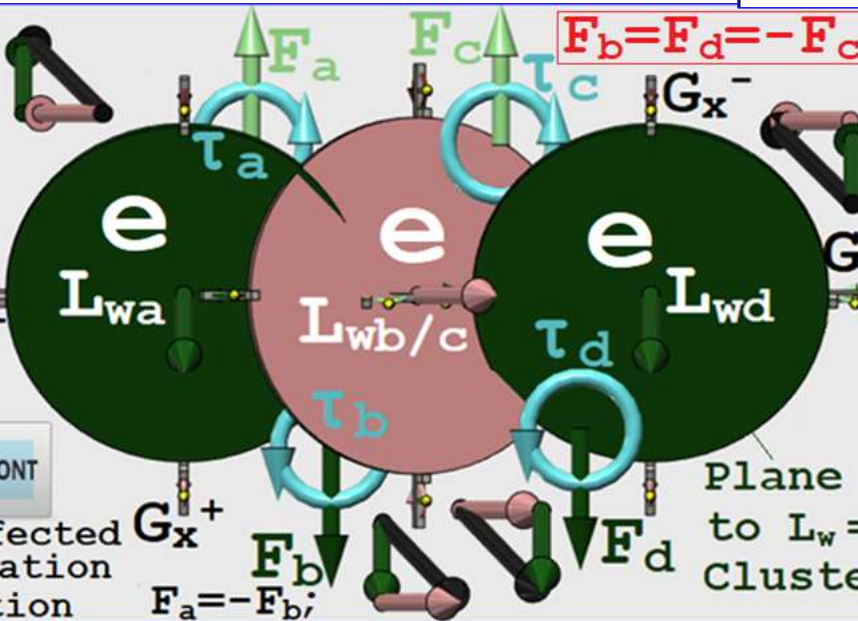
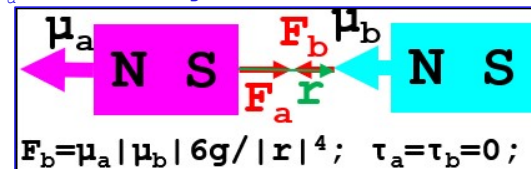
17-Stern Gerlach: If we fire e through inhomogeneous B ($|B_{North}| > |B_{South}|$; near 1e), the $G_{xy}^{+/-}$ emitted from B collide with e & cause it to feel F & changing $\tau/-\tau$ (due to F; $+/-\tau$ depend on the collision point), which cause the B_u of e to align with $-B/B$; {if it's homogeneous B (e in the middle between 2 same B_u) there is no F & the τ is constant, so e's S precess about B_u (Larmor precession)} When its B_u align with $-B/B$, τ is no longer exerted on the e, & it is attracted upward/downward (with same magnitude & shifted to the left due to Lorentz F); Thus beam of e's in inhomogeneous B create 2 distinct parts & not continuous smear; The probability that the B_u of e will be align with B is p; p depend on $G_{xy}^{+/-}$ collision point, which depends in what amount B_u goes in the direction of $B = B_u \bullet B = \cos(\alpha)$; $\{\alpha = \text{angle between } B_u \& B\}$ but $-1 \leq \cos(\alpha) \leq 1$; converting to probability range ($0 \rightarrow 1$; $0 \leq 1 + \cos(\alpha) \leq 2$) give: $0 \leq \frac{1 + \cos(\alpha)}{2} \leq 1$; & by trigonometry identity: $\frac{1 + \cos(\alpha)}{2} = \cos^2(\alpha/2)$; so $p = \cos^2(\alpha/2)$; & the probability to get e with B_u in $-B$ direction is $1-p$; {(number of e's with B_u in B) > (number of e's with B_u in $-B$) by very tiny amount (due to neighbor e's B_u); thus magnet that contain many e's always align with B (& not $-B$)}



Twice $G_{xy}^{+/-}$ in direction of $\pm B_u$
Give: $\tau_a = 2\tau_b$



$F_b = -\mu_a |\mu_b| 3g / |r|^4$;
 $\tau_b = \mu_a \times \mu_b g / |r|^3$; F effected by Average e orientation & $G_{xy}^{+/-}$ flying direction



$G_{xy}^{+/-}$ Average direction = $\pm B_u$

$F_d = -\mu_d |\mu_c| 3g / |r|^4$;
 $\tau_c = \mu_d \times \mu_c g / |r|^3$;
 $\tau_d = 2\tau_c$;

Plane perpendicular to $L_w =$ Average Cluster orientation

18-For Homogenous $B=[B_1;0;B_3]$; The probability to get $|-Z\rangle$ from time evolve $|+Z\rangle=|<-Z|+Z(t)\rangle|^2=p(+\rightarrow-)=B_1^2/|B|^2\sin^2(t|B|\gamma/2)$; $\gamma=g_sq/(2m_e)$; The evolution is due to Larmor precession about B ;

For Homogenous $B=[0;0;B_3]$:

$$|<-Z|+Z(t)\rangle|^2=0; |<+Z|+Z(t)\rangle|^2=1;$$

$L_w=[0;0;+|L_w|]$; remain $[0;0;+|L_w|]$; during the precession;

$$|<-X|+Z(t)\rangle|^2=\frac{1}{2}; |<+X|+Z(t)\rangle|^2=\frac{1}{2};$$

During precession $L_w=[0;0;+|L_w|]$ become closer to $[+|L_w|;0;0]$; half of the time & half of the time to $[-|L_w|;0;0]$;

$$|<-Y|+Z(t)\rangle|^2=\frac{1}{2}; |<+Y|+Z(t)\rangle|^2=\frac{1}{2};$$

like X ;

For Homogenous $B=[B_1;0;0]$:

$$|<-X|+Z(t)\rangle|^2=\frac{1}{2}; |<+X|+Z(t)\rangle|^2=\frac{1}{2};$$

L_{wx} remain 0 during the precession; Thus equal probability to flip to $|+X\rangle$ or $|-X\rangle$;

$$|<-Z|+Z(t)\rangle|^2=\sin^2(tB_1\gamma/2)=\frac{1}{2}-\frac{1}{2}\cos(tB_1\gamma);$$

{If $t=2\pi/(|B|\gamma)$; $p(+\rightarrow-)=0$;

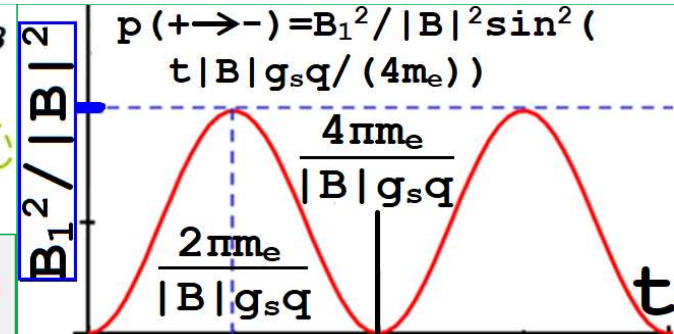
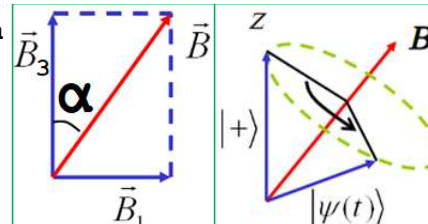
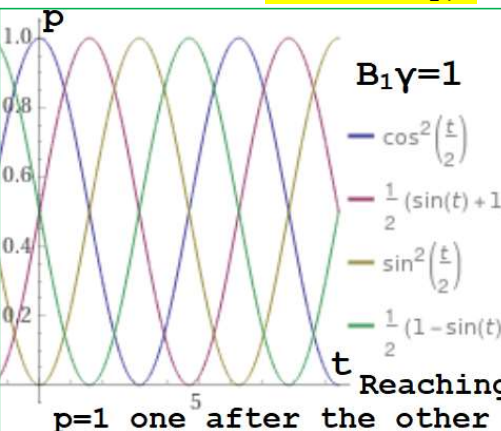
If $t=\pi/(|B|\gamma)$; $p(+\rightarrow-)=B_1^2/|B|^2=\sin^2(\alpha)=\cos^2((\pi-2\alpha)/2)$ };

L_{wz} changed periodically during the precession; Thus Each $\pi/(|B|\gamma)$ s it become closer to $[0;0;-|L_w|]$ & each $2\pi/(|B|\gamma)$ s it become closer to $[0;0;+|L_w|]$; similarly:

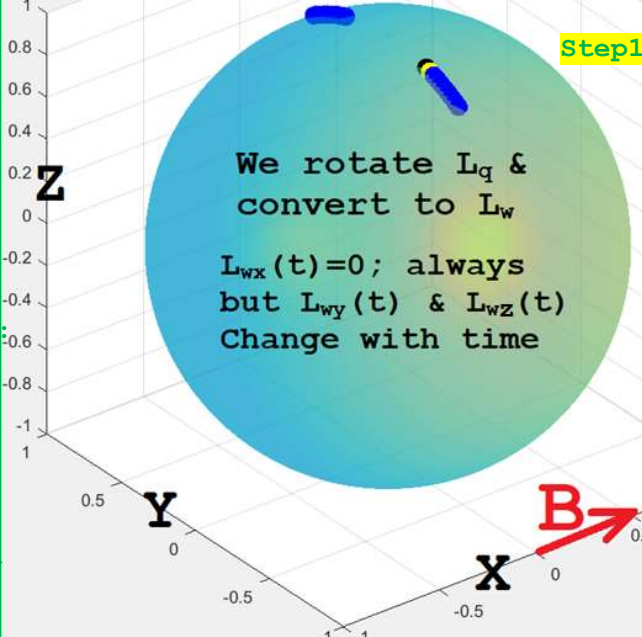
$$|<+Z|+Z(t)\rangle|^2=\cos^2(tB_1\gamma/2)=\frac{1}{2}+\frac{1}{2}\cos(tB_1\gamma);$$

$|<-Y|+Z(t)\rangle|^2=\frac{1}{2}+\frac{1}{2}\sin(tB_1\gamma)$; The act of separating $|<+D\rangle$ and $|<-D\rangle$ create

precession about D , which make distance to $+A$ or $-A$ equal, where A is perpendicular to D ; Thus:



20 Time frames of e's L_w =Blue balls; precess about $B=[|B|;0;0]$ $L_w(0)=[0;0;|L_w|]$ =Black ball; $t_1=2\pi/(20\omega_p)$; $L_w(t_1)$ =Yellow ball



Balls=e possible L_w values

$\propto L_w$ values correspond to:

$$L_w=[0;0;+|L_w|] \text{ \& } L_w=[0;0;-|L_w|]$$

β_a & β_b from Z ;

Time Evolved: $|+X\rangle$, $|-Z\rangle$, $|+Z\rangle$

While 1 L_w value correspond to rest L_w ; e.g.

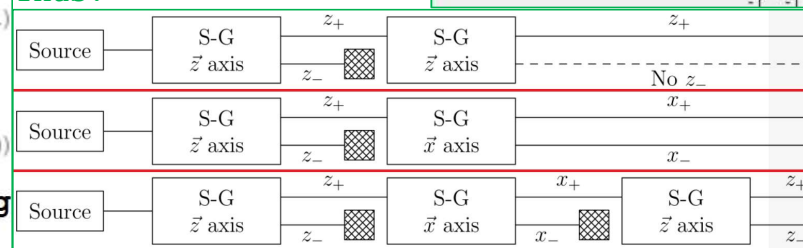
$$\text{If e in } B=[0;0;|B|] \text{ } L_w=[|L_w|;0;0] \text{ \& } L_w=[0;-|L_w|;0]$$

L_w rotate about B (see same coloured balls)But while $L_w=[|L_w|;0;0]$ changes during this rotation

don't

$$L_w=[0;0;+|L_w|] \text{ \& } L_w=[0;0;-|L_w|]$$

don't



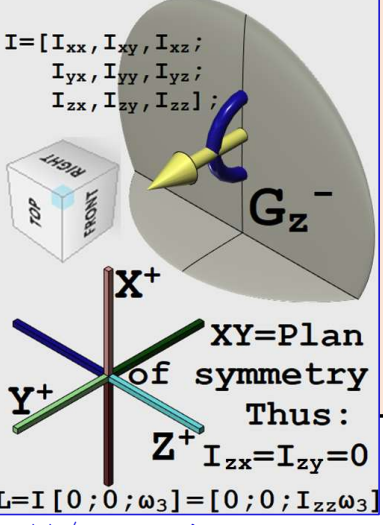
19-G particle is created by a powerful collision of 2 non-existing spheres; & thus, having a maximally thin oblate spheroid shape. When it collides with e, it bends, such that it has plan of symmetry. & the normal of this plan is its ω_0 ; Because this plan of symmetry remains plan of symmetry during all of its motion, its ω always align with its L_w (products of inertia in ω_0 direction are 0 & remain 0; I_{zz} never changed; $L=I\omega$) & $|L|=I_{03}\omega_{03}=\hbar$; This formed bended G is G_z^- ; j_e =biggest magnitude of collision impulse that cause no deformation when e^+ collide with G; In our universe:

$j_e=M_G(c+v_{G0})=\text{constnat}$; $\{M_G=G \text{ mass}; v_G=\text{speed of G}\}$ Thus the speed of G that j_e cause= c ; $\{\text{Because if: } G=A=\text{Ellipsoid}[s_1=s_3=r; s_2=0]; M_G=M_A; G_z^+=Ac; e^+=B=\text{ellipsoid}[s_1=s_2=R; s_3]; V_B=\omega_A=[0;0;0]; V_A=[0;-v_G;0]; \omega_B=[0;0;-w]; \text{No orientation}; n=[0;-1;0]; T_A=[r;0;0]; T_B=[-R;0;0]; j_e=(v_G+Rw)(\check{e}+1)M_A M_B/(3.5M_A+6M_B); V_{Ac}=[0; j_e/M_A - v_G; 0]; |V_{Ac}|=(M_A(c+v_G))/M_A - v_G=c\}$

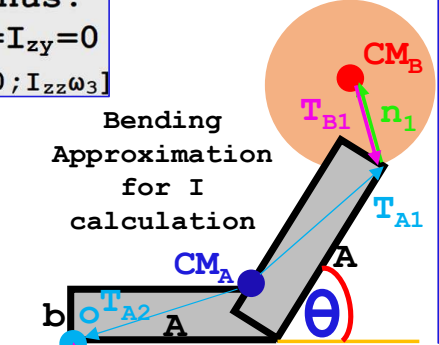
j_d =bigger($j_d > j_e$) collision impulse magnitude that cause deformation; $j_d=|J_d|$; n_d =unit normal of the last contact point of G & e^+ ; G_z^+ is emitted at n_d direction; The amount of J_d that goes in the direction of $n_d=J_d \cdot n_d=j_d \cos\theta=\text{constant}=j_e$; $\{\theta=\text{angle between } J_d \& n_d\}$ Because as long as the impulse is stronger than j_e it continue to bend the G particle & it won't be its last contact point. Therefore the speed of the emitted $G_{x/y/z}^{\pm}$ is always c , regardless of e^+ or G velocity $\{\text{slower G } v, \text{ require different: } s_2, j, n, I_A, T_A, T_B\}$; Larger j_d increase the ω_{03} of the formed G_z^+ but also increase its deformation, & its bending angle (θ), which reduces I_{03} ; such that $|L|=I_{03}\omega_{03}=\hbar$; remain constant for any G_z^+ $\{\text{with any } \theta=\text{acos}(j_e/j_d)\}$

G_z^+ deformation demonstration: By approximating G into a box $[2A, b, c]$ $\{A=500; b=1; c=1000; p=0.001; [kg/m^3] |L|=10^8\}$ & the deformation into θ , we can evaluate G_z^+ shape (θ) as function of j_d/j_e ; by θ we find I_{03} & than $\omega_{A3}=\omega_{03}=|L|/I_{03}=\hbar/I_{03}$; $\{\omega_{03}$ needed to conserve $|L|=\hbar\}$ We can see that as j_d increases: θ° & ω_{A3} increase; Furthermore, we can see that G_z^+ with bigger ω_{A3} cause more powerful subsequent collision $\{\text{bigger } |J_{out}|\}$; even though its I values smaller; only the longer T_A collide with the ball, as rotation is faster than velocity;

Other deformed shapes (with same volume & mass as G_z^+) can require any $|\omega_{A3}|$ to preserve $|L|$; $\{\text{e.g. cube}[1;1;1000000] \text{ require } |\omega_{A3}|=600,000; \text{ to preserve } |L|\}$;

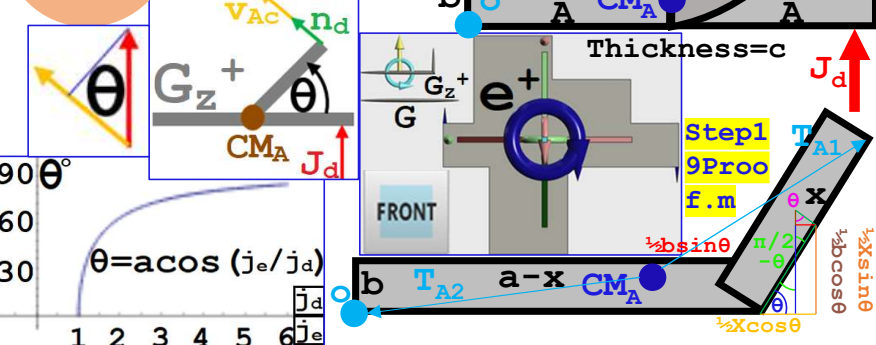


Collision response (No deformation):
 $(\omega, I$ in rotating frame, rest in world frame)
 $I^{-1} = [5/(s_2^2+s_3^2), 0, 0; 0, 5/(s_1^2+s_3^2), 0; 0, 0, 5/(s_2^2+s_1^2)]/M$;
 R =Transform from rotating to world frame;
 n =unit normal; For elastic|plastic:
 $\check{e}=1|0$; $Q_A=R_A^{-1}(T_A Xn)$; $Q_B=R_B^{-1}(T_B Xn)$;
 $j=(\check{e}+1)\{(V_A-V_B) \cdot n + \omega_A \cdot Q_A - \omega_B \cdot Q_B\} / \{M_A^{-1} + M_B^{-1} + I_A^{-1} Q_A \cdot Q_A + I_B^{-1} Q_B \cdot Q_B\}$;
 $\omega_{Ac} = \omega_A - j I_A^{-1} Q_A$;
 $\omega_{Bc} = \omega_B + j I_B^{-1} Q_B$;
 $V_{Ac} = V_A - j n / M_A$;
 $V_{Bc} = V_B + j n / M_B$;
 If collide: j =positive; So n dictates V_{Ac}
 $J = j n = \int F_{A \rightarrow B} dt; [t_1 \rightarrow t_2]$ but in deformation $T_B(t_1) \neq T_B(t_2)$; & $I_B(t_1) \neq I_B(t_2)$;

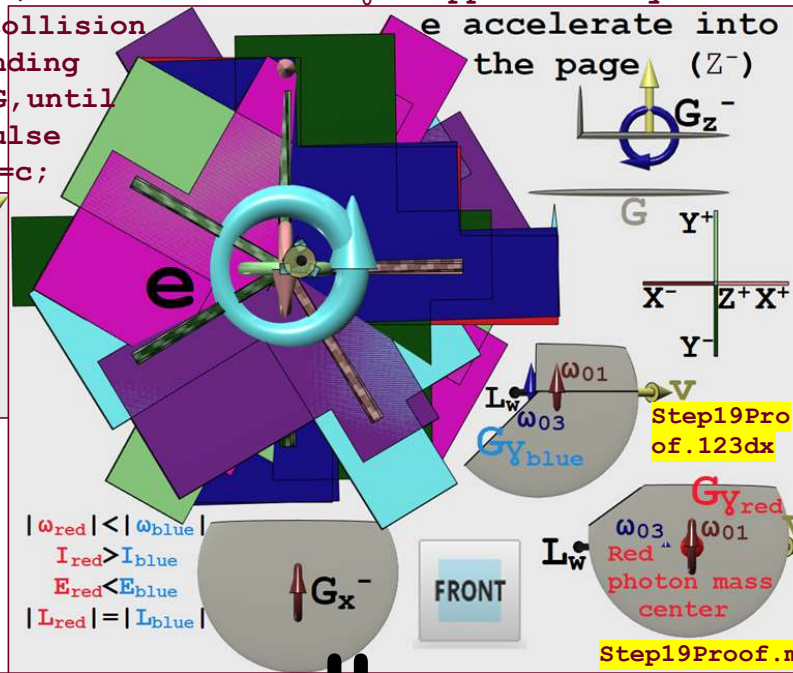
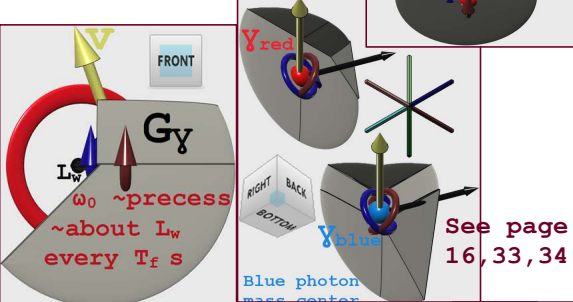


j_d/j_e	θ°	ω_{A3}	$ T_{A2} $	$ J_{out} $
1	0	1.20	500	1,195
1.02	11	1.21	498	1,200
1.06	19	1.23	495	1,209
1.14	29	1.26	488	1,224
1.28	39	1.31	479	1,249
1.5	48	1.37	468	1,279
2	60	1.48	451	1,328
3	71	1.60	433	1,382
5	78.5	1.75	414	1,444
100	89.4	1.91	396	1,511
1000	89.9			

For different j_d calculate: $\theta^\circ, I; \omega_{A3}$ needed to conserve $|L|$; & J_{out} in subsequent collision where n is perpendicular to T_A ;



Accelerated (a) e/e⁺ collide with G twice, creating what we call photon (G_y). The stronger the collisions, the larger the G_y's: |ω| & its deformation; but larger deformation has smaller values of I, thus |L|=|Iω|=ħ; remain constant; while E_r=½ω₀•Iω; increase with |ω₀|; T_f=1/f=time taken for ω₀ to approximately return to its position=2π/ω_p=2πdt/dφ; Collision impulse bigger than the non-bending impulse will continue to bend G, until it reaches the non-bending impulse value, which cause G_y/G_{xyz}^{+/-} |v|=c; Regardless of G/e/e⁺ v; W_x=∫Fdx=∫Fr dφ [φ=0 → 2π]=∫Frω_pdt [dt=0 → T_f]=ω_p∫r dt=ω_pL=ω_pħ=E_r=hf=½ω₀•Iω;



The electric **E** and magnetic **B** parts of the electromagnetic field produced by a moving charge q on field point \mathbf{x} at time t are

$$\mathbf{E}(\mathbf{x}, t) = \frac{q}{4\pi\epsilon_0} \left[\frac{(1 - \beta^2)(\mathbf{n} - \beta)}{(1 - \beta \cdot \mathbf{n})^3 R^2} \right] + \frac{q}{4\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} \left[\frac{\mathbf{n} \times [(\mathbf{n} - \beta) \times \dot{\beta}]}{(1 - \beta \cdot \mathbf{n})^3 R} \right]$$

$$\mathbf{B}(\mathbf{x}, t) = \frac{1}{c} \mathbf{n} \times \mathbf{E}(\mathbf{x}, t)$$

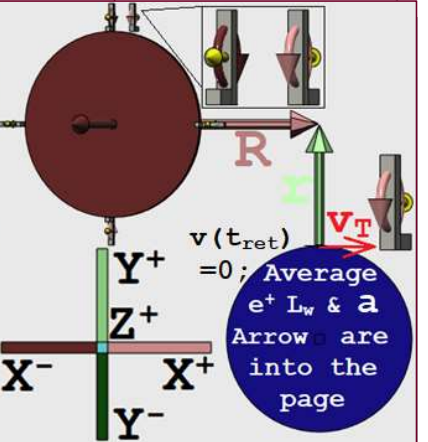
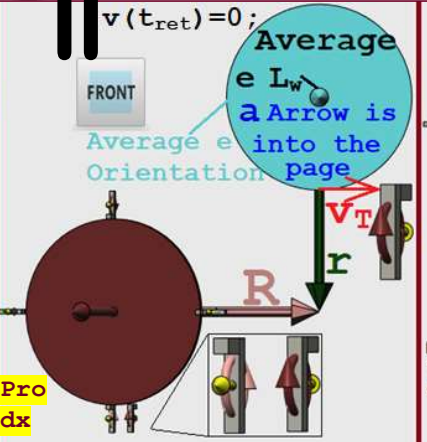
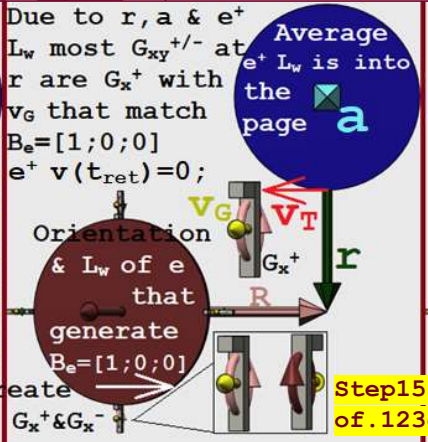
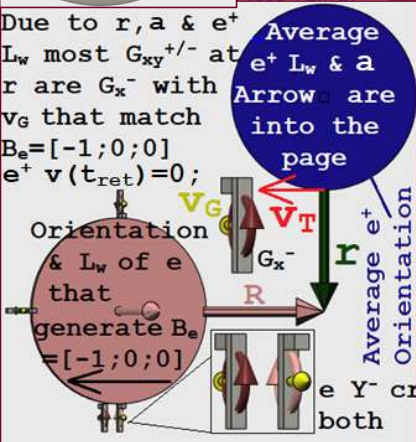
$$\mathbf{n} = \frac{\mathbf{R}}{\|\mathbf{R}\|}$$

$$\beta = \frac{\mathbf{v}(t_{\text{ret}})}{c}$$

$$\dot{\beta} = \frac{\dot{\mathbf{v}}(t_{\text{ret}})}{c}$$

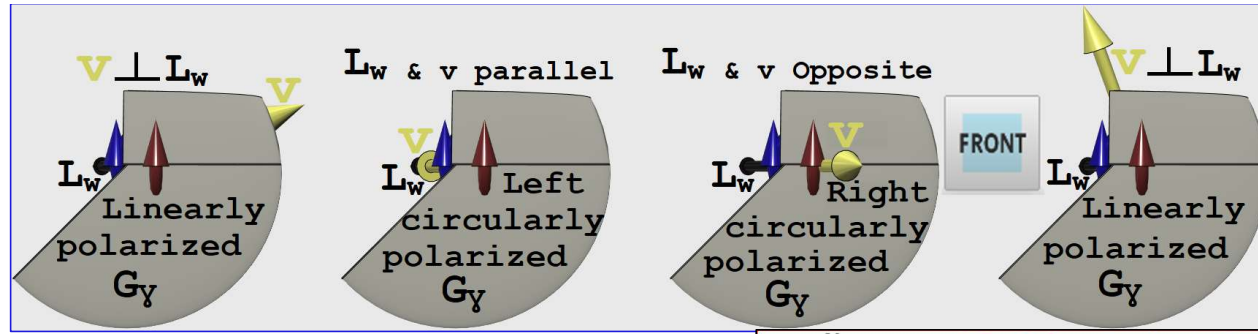
Liénard-Wiechert
 Light propagation direction = $\mathbf{E} \times \mathbf{B}$
 $\phi, \mathbf{r} = \text{present}$
 $\mathbf{R}, \mathbf{n} = \text{retarded}$
 $t_{\text{ret}} = t - |\mathbf{R}|/c$
 $\mu_0 = 4\pi \cdot 10^{-7}$

■ $\mathbf{E}(\mathbf{x}, t)$ 2nd term proportional to $1/R$ (& not $1/R^2$) Thus it fall slower with distance & represent light; ■ If there is no acceleration: $\mathbf{E}(\mathbf{x}, t)$ is in \mathbf{r} direction {line $q\mathbf{q} = \mathbf{v}t = \mathbf{v}|\mathbf{R}|/c = \beta|\mathbf{R}|$; & $\mathbf{R} = \mathbf{n}|\mathbf{R}|$;

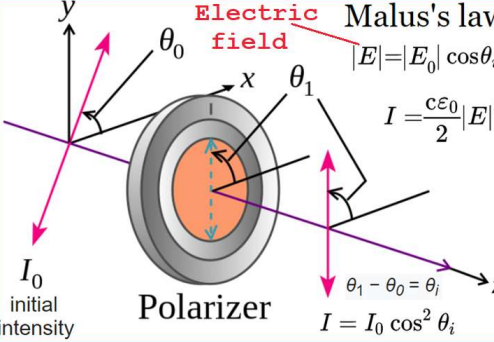


by similar triangles: $(\mathbf{n} - \beta)$ parallel to \mathbf{r} & $\mathbf{B}(\mathbf{x}, t) = 10^{-7} q \mathbf{v} \times \mathbf{n} / (1 - \beta^2) / ((1 - \beta \cdot \mathbf{n})^3 |\mathbf{R}|^2) \approx 10^{-7} q \mathbf{v} \times \mathbf{r}^u / |\mathbf{r}|^2$; {if $v \ll c; \beta = v/c \approx 0; (\mathbf{n} - \beta) \approx \mathbf{n} \approx \mathbf{r}^u; \mathbf{R} \approx \mathbf{r}$; so it's Relativistic correction of biot savart law} ■ If motion starts: $\mathbf{v}(t_{\text{ret}}) = 0$; & $\mathbf{a}(t_{\text{ret}}) \neq 0$; $\mathbf{B}(\mathbf{x}, t) = \mathbf{a} \times \mathbf{n} 10^{-7} q / (c|\mathbf{R}|) \approx 10^{-7} q \mathbf{a} \times \mathbf{r}^u / (c|\mathbf{r}|)$; $\{\mathbf{E}(\mathbf{x}, t) = nq / (4\pi\epsilon_0 |\mathbf{R}|^2) + \mathbf{n} \times [\mathbf{n} \times \dot{\beta}'] q \mu_0^{1/2} / (4\pi\epsilon_0^{1/2} |\mathbf{R}|)\}$; $\mathbf{B}(\mathbf{x}, t) = 10^{-7} \mathbf{n} \times (\mathbf{n} \times [\mathbf{n} \times \mathbf{a}]) q / (c|\mathbf{R}|)$

20-Right/Left circularly polarized photon= G_γ that its L_w & v are opposite/parallel; Their $\omega \sim$ precess around $-v/v$; create rotational effect when collide into a target (many G_γ adds up the same rotational effect); Linearly polarized photon= G_γ that its L_w & v are perpendicular, its $\omega \sim$ precess perpendicular to v , thus its total effect is canceled out upon many collisions (because each G_γ collide into different point at target), & we say mistakenly that it has no angular momentum;



Elliptically polarized photon= G_γ that its L_w & v are not perpendicular and not parallel/opposite; Some G_γ 's require e^+ (or other positively charge particle) to create them; Polarizer=Long sheets of molecules that are capable of moving in only 2 specific direction (Q&-Q), if G_γ 's L_w is parallel to Q/-Q, than G_γ 's precessing ω push the polarizer in its unmovable direction, & the polarizer

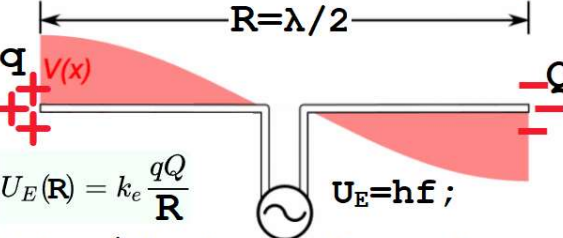


Circularly polarized light can be generated using helical antenna



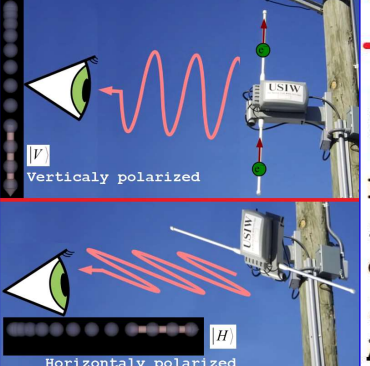
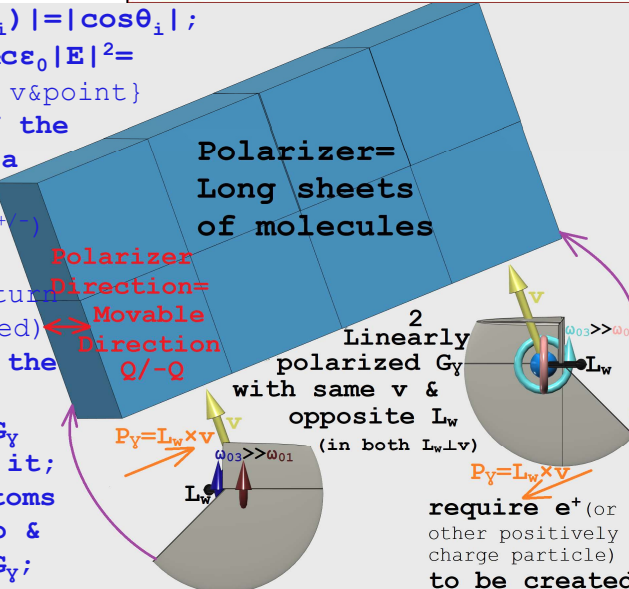
won't vibrate & won't emit G_γ (G_γ doesn't pass); If G_γ 's L_w is perpendicular to Q, than G_γ 's precessing ω push the polarizer in its movable direction, & the polarizer will vibrate & will emit G_γ (G_γ pass); Thus G_γ passing probability = $|Q \times L_w^u| = |\sin \alpha|$; (α = angle between L_w & Q; $-1 \leq \sin \alpha \leq 1$; but -1 & 1 are solutions so use $||$) What we call linear photon polarization = $P_\gamma = L_w^u \times v^u$; ($v^u = v/|v|$) => $L_w^u = v^u \times P_\gamma$; Thus G_γ pass probability = $|Q \times (v^u \times P_\gamma)| = |v^u (Q \cdot P_\gamma) - P_\gamma (Q \cdot v^u)| = |v^u (Q \cdot P_\gamma)| = |v^u (\cos \theta_i)| = |\cos \theta_i|$; {Q & P_γ perpendicular; $a \times (b \times c) = b(a \cdot c) - c(a \cdot b)$ } $|E| = |E_0| \cos \theta_i$; $I_0 = \frac{1}{2} c \epsilon_0 |E_0|^2$; $I = \frac{1}{2} c \epsilon_0 |E|^2 = I_0 \cos^2 \theta_i$; The emitted G_γ from the polarizer has $P_\gamma = Q$; {follow collision v & point}

Antenna emit/receive λG_γ



Extra/Shortage of e pile up in both ends creating electric potential energy U_E
 $\lambda = c/f = ch/U_E = chR/(k_e qQ)$
 AC tuned to: $qQ = ch/(2k_e)$

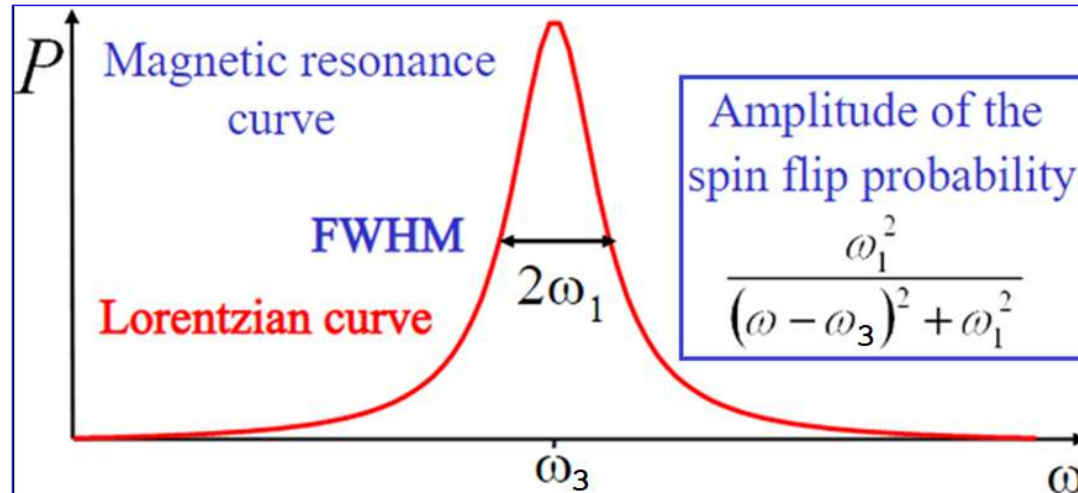
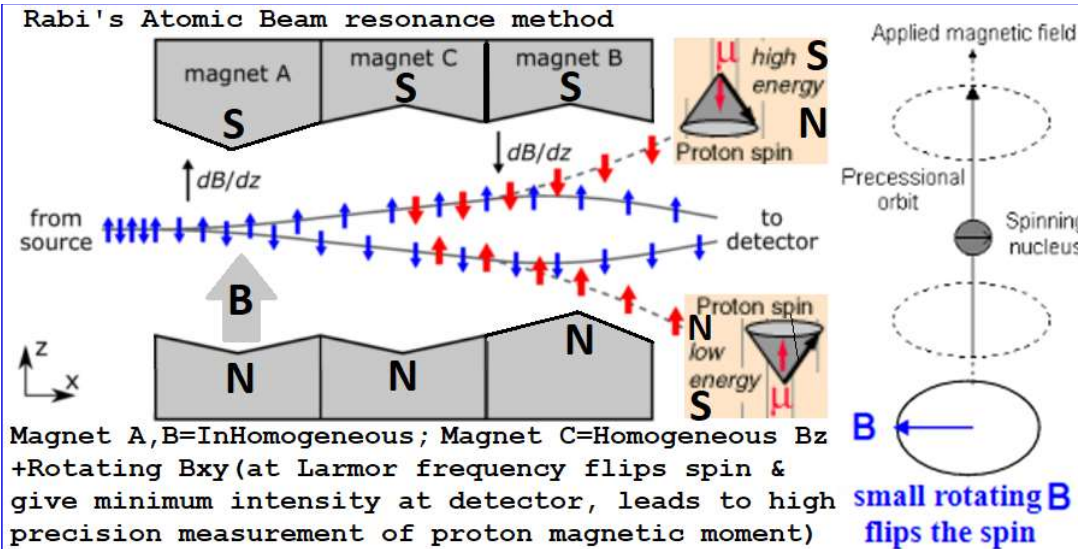
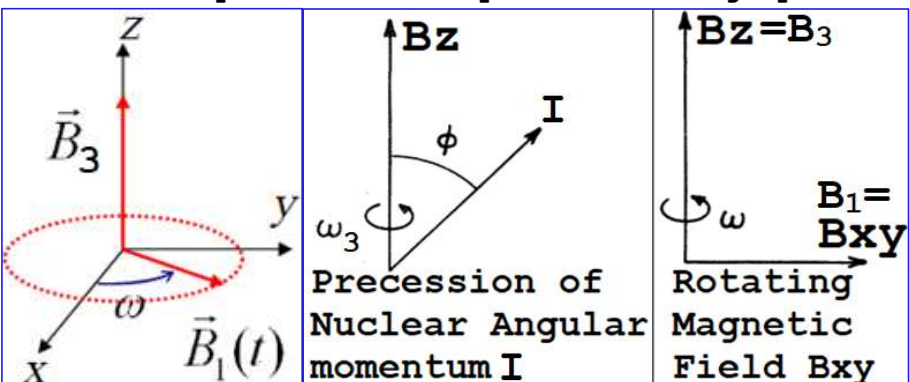
No light
 $P_\gamma = L_w \times v$
 Polarization by scattering
 $B(x, t) \sim 10^{-7} q a \times r^u / (c |r|)$
 Linearly polarized



■ The bigger the G_γ the slower it travel in a material; as e returning time (by $G_\gamma^{+/-}$) increased (when e return G_γ emitted)
 ■ The denser the material the slower G_γ travel in it; as more atoms absorb & emit G_γ ;

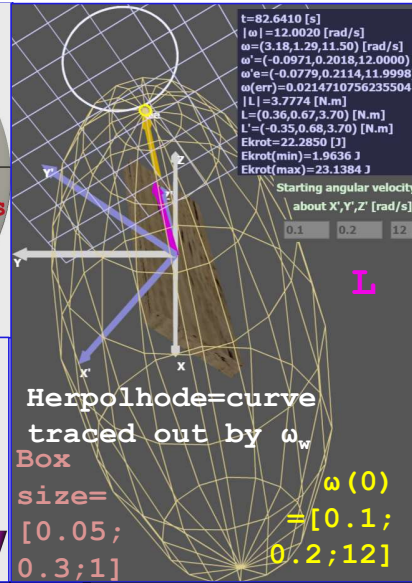
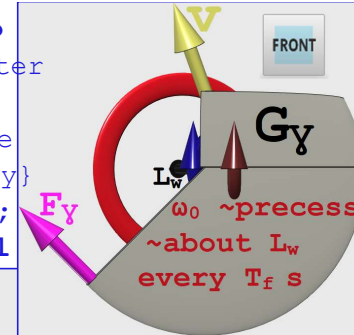
Linearly polarized G_γ with same v & opposite L_w (in both $L_w \perp v$) require e^+ (or other positively charge particle) to be created

21-Rabi cycle: For Homogenous B_1 rotating about Homogenous B_3 :
 $B = [B_1 \cos(\omega t); B_1 \sin(\omega t); B_3]$; $\omega_1 = -B_1 \gamma$; $\omega_3 = -B_3 \gamma$; $\gamma = g_s q / (2m_e)$; $p(+ \rightarrow -) =$
Spin flip probability = probability to get $|-Z\rangle$ from time evolve $|+Z\rangle = \frac{\omega_1^2}{(\omega_1^2 + (\omega - \omega_3)^2)} \sin^2(\frac{1}{2}t(\omega_1^2 + (\omega - \omega_3)^2)^{1/2})$; If $\omega \rightarrow \omega_3$; $p(+ \rightarrow -) =$
 $\sin^2(t \frac{1}{2} \omega_1)$; making the amplitude of spin flip probability to 1;
and the time taken for this flip $= t = \pi / \omega_1 = 2\pi m_e / (B_1 g_s q)$; $\{p(+ \rightarrow -) =$
 $\sin^2(\pi/2) = 1\}$ e in B_3 will have larmor precession with $|\omega_L| = B_3 \gamma = \omega_3$;
rad/s; If we rotate magnetic field perpendicular to B_3 at $|\omega_L|$
rad/s $G_{xyz}^{+/-}$ will always hit e face in opposite direction of its
face motion, creating stronger force that flip its internal
magnetic field direction (opposite L_q but not opposite L_w ; see $L_q \rightarrow L_w$
transformation); The time taken for this flip increase with m_e &
decrease with B_1 & q ; While ω_3 is rad/s; f is (number of occurrences
of repeating event)/s. thus $\omega_3 = 2\pi f$; and if we fire circularly
polarized photon with $f = \omega_3 / (2\pi)$; at right angle to B_3 it will
also flip the electron; G_y with $f = \omega_3 / (2\pi)$; has period $= T_f = 1/f = 2\pi$
 $/ \omega_3 = 4\pi m_e / (B_3 g_s q) = n_0 T_L = T_w =$ time taken for G_y 's ω_0 to ~return to its
position (it never return exactly) same as the larmor precession
period of e; Thus all G_y s will also collide with e in opposite
direction of its motion, creating strong force that flip e;
■ $E(G_z^{+/-})$ interacts with e charge (ω_3), but not spin ($\omega_{1/2}$); $B(G_{xy}^{+/-})$
interacts with e spin ($\omega_{1/2}$), but not charge (ω_3); In homogenous B,
spin can precess at same angle for infinite time. Interaction
with other particle is required to change precession angle.



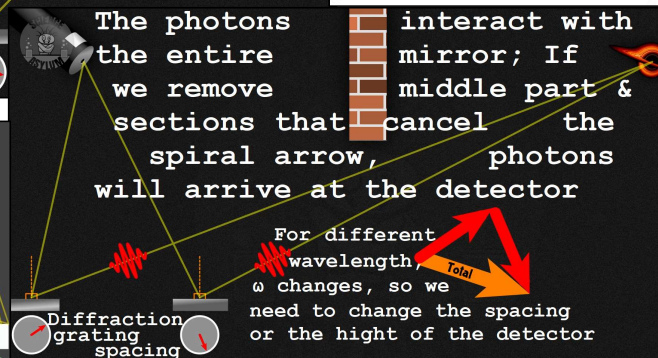
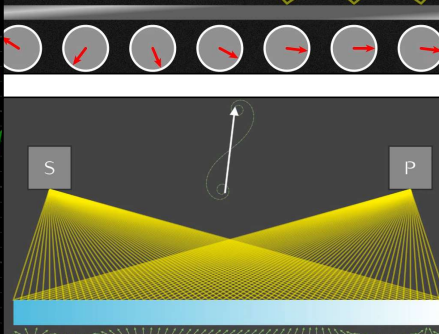
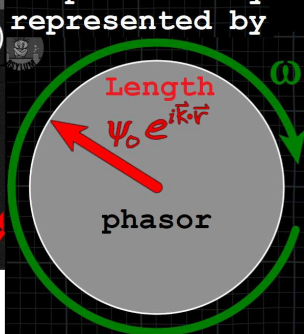
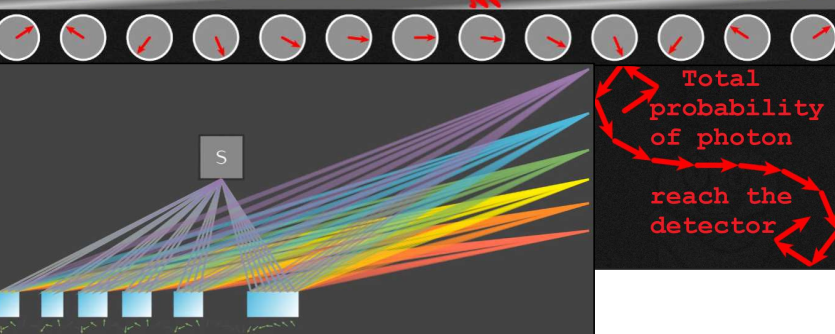
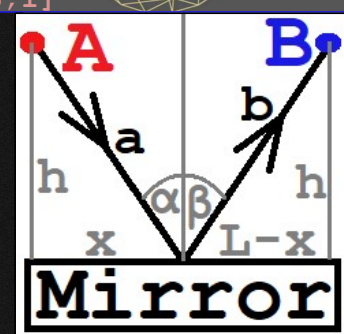
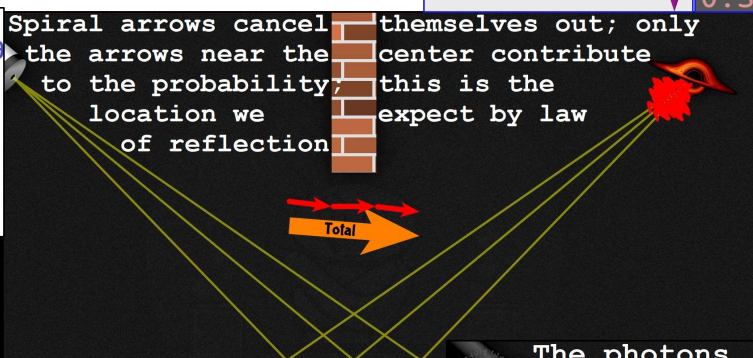
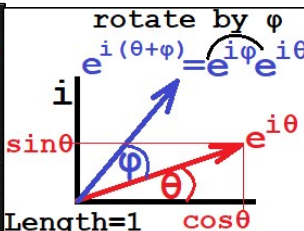
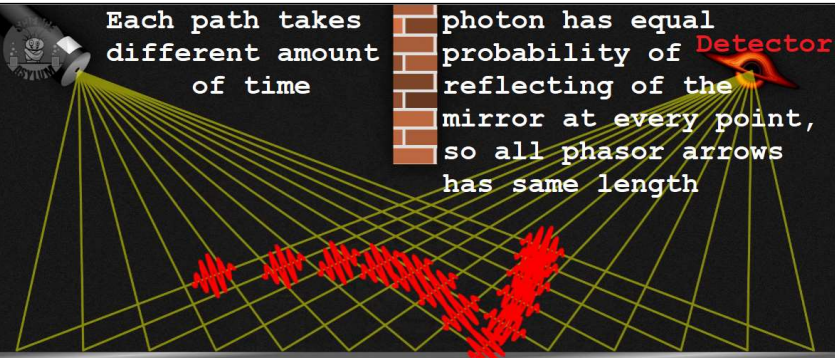
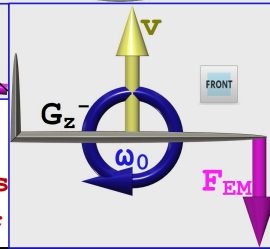
22-The total effect of many colliding G_V s can be roughly calculated by summing up all their F_V =collision normal vector from G_V point that is farthest from G_V CM(Center of mass; this point collide first);If G_{Vn} arrive to target after different time dt_n than its F_{Vn} can be ~calculated by rotating F_V about L_w by: $2\pi dt_n/T_f$ rad; $\{T_f=1/f$ =time taken for G_V 's ω_0 to ~return to its position}Or by $2\pi f dt_n=\omega dt_n$;rad; $\{\omega$ =angular frequency}

■ While G_V 's ω_0 is roughly ~rotated about G_V 's L_w , $G_{xyz}^{+/-}$'s ω_0 is parallel to its L_w ; thus the total effect of many $G_{xyz}^{+/-}$ can be precisely calculated by summing up all their F_{EM} (collision normal vector from $G_{xyz}^{+/-}$ point that is farthest from $G_{xyz}^{+/-}$ CM); If $G_{xyzn}^{+/-}$ arrive to the target after different time dt_n than its F_{EMn} can be calculated by rotating F_{EM} about L_w by: $dt_n|\omega_0|$ rad;

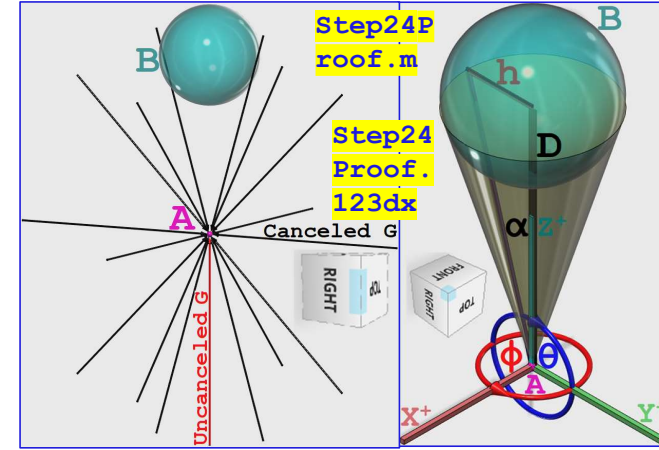


23-Mirror:1) Wrong explanation:Fermat's principle:If a beam of Light travels from point A to B, it does so along the fastest path possible;=> $\alpha=\beta$ {syms h L x real; $a=(x^2+h^2)^{(1/2)}$; $b=((L-x)^2+h^2)^{(1/2)}$; $f=a+b$; $xs=solve(diff(f,x,2)=0,x)$ % $x=L/2=>\alpha=\beta$ }

2) Correct explanation: G_V can be reflected from any mirror's e, but the force it exerts on the detector is canceled out by step 22, unless it obey the law of reflection ($\alpha=\beta$);

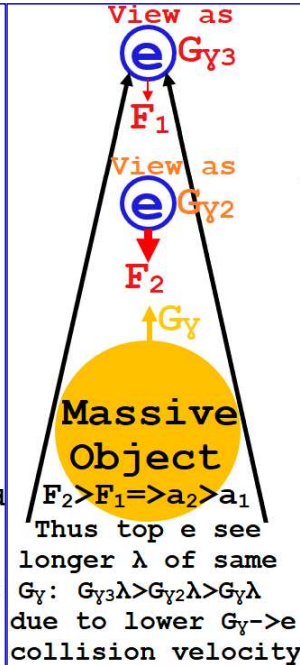
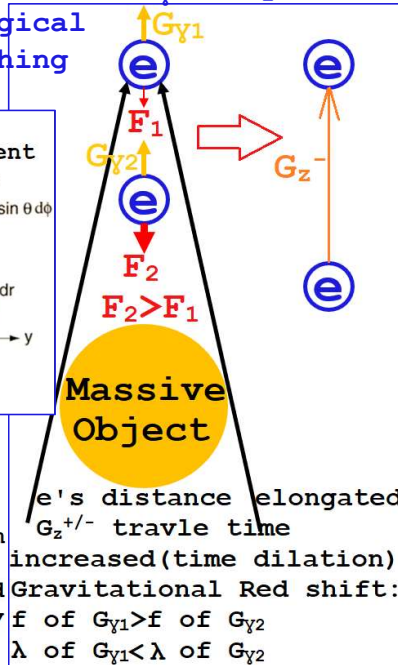
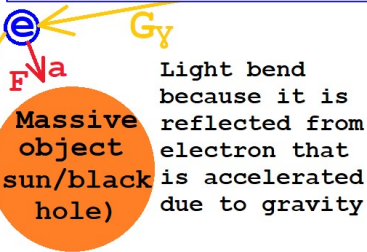
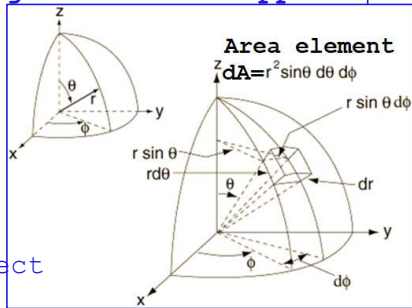


24-Gravity: In the universe, everywhere & every time NE-shape with any velocity can be created; Thus any object'll feel collision forces from all directions, that on average cancel each other out. But if 2 objects stand close to each other, they will feel less collision forces from the side that in between them. Thus a small object A (radius r) will not feel collision forces from a cone angle α that its base originate from the big object B (radius h), which is at distance D; Thus $\alpha = \text{atan}(h/D)$; Force = Area * Pressure; The effective pressure for any prevented collision at θ, ϕ inside the cone is $k \cos(\theta)$; $k = \text{constant}$; $\cos(\theta) = \text{effective direction}$ (as collision directions that are not parallel to D are cancel out) So we can calculate the cone prevented force = force exerted from the opposite direction = $F = \int \int k \cos(\theta) r^2 \sin(\theta) d\phi d\theta$; [first integrate by ϕ from 0 to 2π ; & then by θ from 0 to α] {Area element = $r^2 \sin(\theta) d\phi d\theta$ } $F = k \pi h^2 r^2 / (D^2 + h^2) \approx k \pi h^2 r^2 / D^2$; { $D \gg h$ } so it obey the Inverse-square law like: Newton Gravitational force: $F = G m_1 m_2 / D^2$;



All bodies fall with the same acceleration regardless of their mass, because the more massive the body the more fundamental particles (e.g. e) it contain, & each fundamental particles get a prevented collision cone that cause it to attract with $F \approx k \pi h^2 r^2 / D^2$; Where r^2 is an expression to its mass; {sphere surface area = $4\pi r^2$ } ■ gravitational redshift is caused by the movement of the e that create the G_y or by the e in the receiver; The changes in e & G/G_y collision impulse is increased with gravity; ■ Cosmological redshift doesn't caused by stretching G_y , but by long series of doppler Redshifts;

{ G_y is absorbed & re-emitted between many stars until it reach us, each move with respect to each other due to the expanding universe & its e emit a more redshifted G_y }



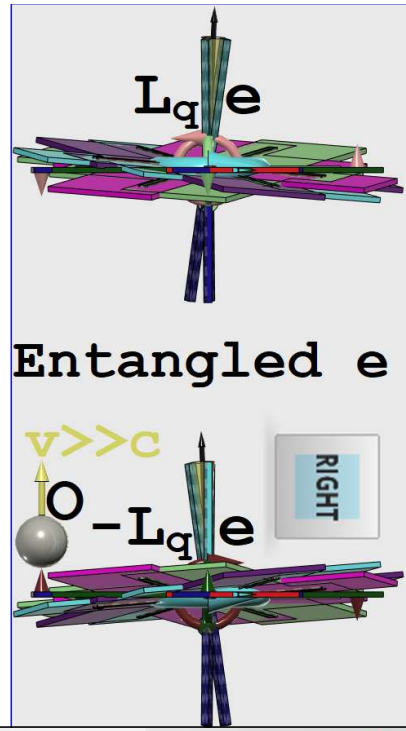
non-rotating massive spherically symmetric object.

$$t_0 = t_f \sqrt{1 - \frac{2GM}{rc^2}} = t_f \sqrt{1 - \frac{r_s}{r}} = t_f \sqrt{1 - \frac{v_e^2}{c^2}} < t_f$$

- t_0 is the proper time between two events for an observer close to the massive sphere
- t_f = time between events for observer at large distance (clock at infinite distance from the massive sphere would tick at one second per second, while closer clocks would tick at less than that rate)
- G is the gravitational constant,
- M is the mass of the object creating the gravitational field,
- r is the radial coordinate of the observer within the gravitational field
- $r_s = 2GM/c^2$ = Schwarzschild radius = the radius of the event horizon within which nothing (including light) can escape the black hole
- $v_e = \sqrt{\frac{2GM}{r}}$ is the escape velocity
- $\lambda_1/\lambda_2 = [(1 - r_s/R_1) / (1 - r_s/R_2)]^{1/2}$
- λ_∞ is the wavelength of the light as measured by the observer at infinity,
- λ_e is the wavelength measured at the source of emission, and
- R_e is the radius at which the photon is emitted.

$$\frac{\lambda_\infty}{\lambda_e} = \left(1 - \frac{r_s}{R_e}\right)^{-1/2}$$

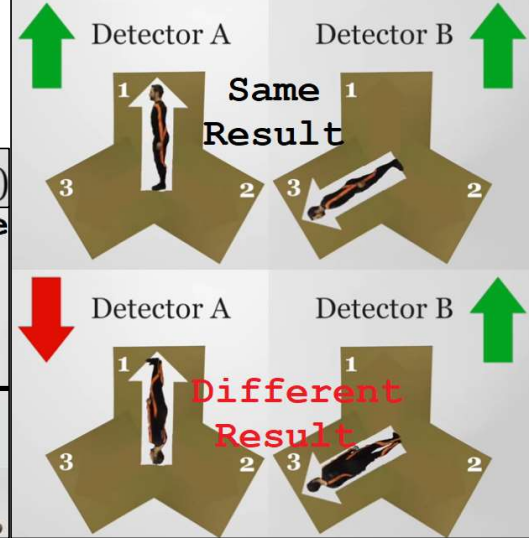
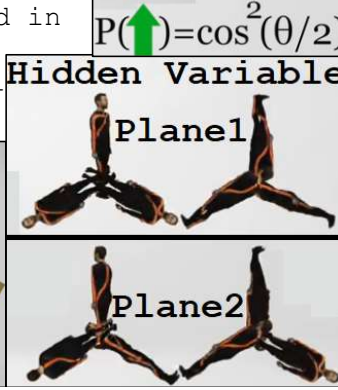
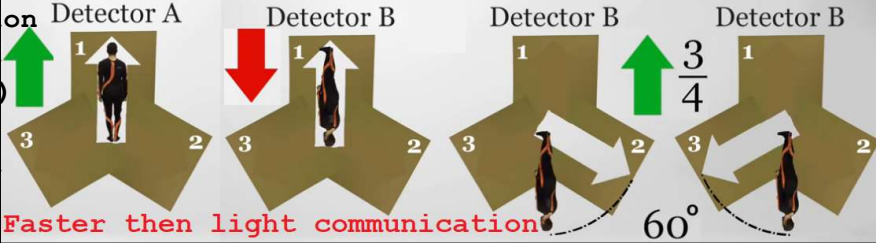
25-Entanglement: If e/e+ collide with G it bend & rotate it, transform it into $G_y/G_{xyz}^{+/-}$ with $v=c$; always, because the bending reduce the collision energy responsible for v; But in the universe there are also small clusters(O) that are not capable of being deformed {O can be a ~spherical cluster composed of Gs}; Thus if e/e+ collide with O it emit it with $v \gg c$ {no deformation so no energy loss}; This O can be thrown back& forth between 2 opposite $L_q e/e^+$ creating what we call entanglement{changing e A L_q change O collision point, which change O collision point into e B, which transform e B L_q to be opposite of e A L_q again}; Entangled photons are created when the e in their transmitter is entangled to e in their receiver/polarizer;



Bell experiment proof that there is no hidden variable: pair of entangled e's send to 2 spin detectors, each measure spin in 1 of 3 directions (120° apart) selected randomly & independently of each other. We recorded whether the measured spin are the same (uu, dd) or different (ud, du). repeating this over and over, randomly varying the measurement directions independently.

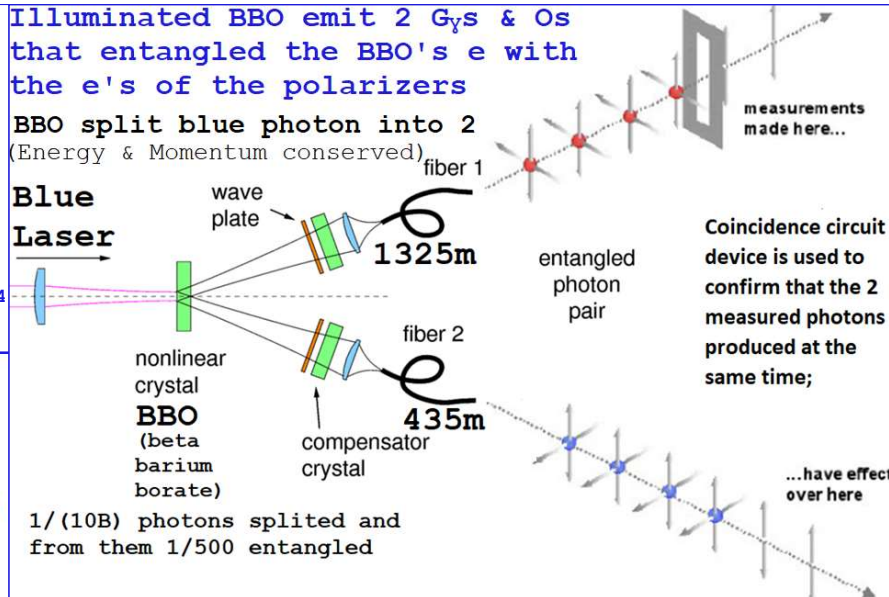
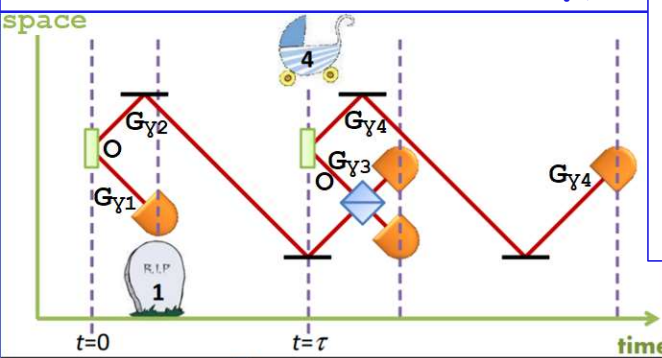
■ If there are hidden variables, each particle has a plane (how to be measured in each direction (1,2,3)); plane1 (uuu, ddd), plane2 (udu, dud) rest equivalent. So probability to get different result for plane1 is 1 & for plane2 is $1/3 * 2/3 + 1/3 * 1/3 + 1/3 * 2/3 = 5/9$; so overall $P > 5/9$ for different results; ■ If there is faster than light communication

Probability for same result = $3 * (1/3 * 0 + 1/3 * 2/3 * \cos^2(60/2)) = 1/2$; P for different results = $1/2$; & that is what we get in the experiment;

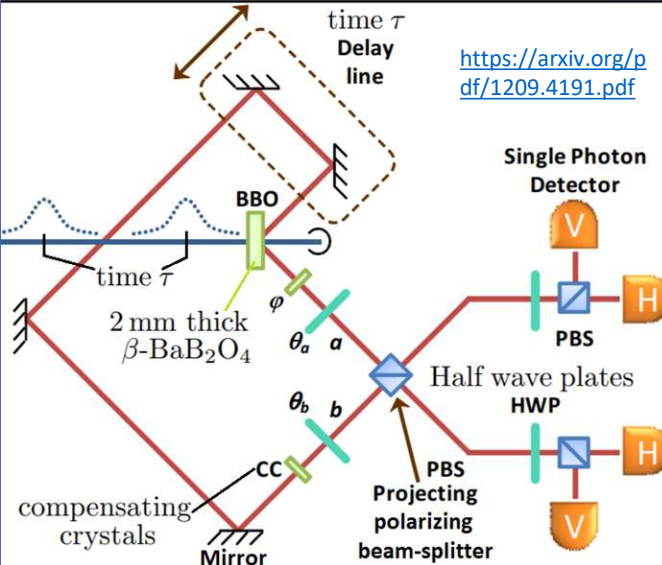
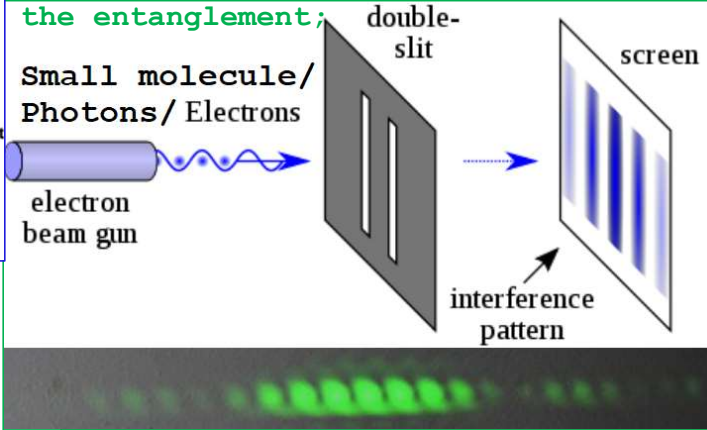


Universe properties
involving many particles
(Less certain explanations)

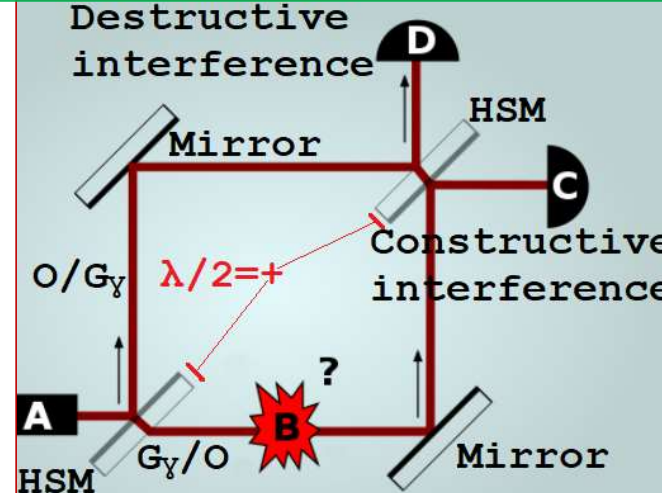
26-Photons 1 & 4 exhibit quantum correlations (entanglement), despite that they have never coexisted;
Explanation: β -BaB₂O₄ (BBO) emit G_y s that fly at $v=c$; & Os that fly at $v \gg c$; O steer the e in the beam splitter (BS) & the e in BBO to be correlated, thus when BBO emit $G_{y3/4}$ they will be correlated with $G_{y1/2}$;



28-Double slit: e/ G_y collide with e in the edge of the narrow slits, this e emit e/ G_y & O; O steer the e in the screen & entangled them with the e in the slits' edges. Thus next emitted e/ G_y will thrown to the entangled es in the screen (which were chosen by G_y 's cT_f or e's vT_L); Any measurement device before/after the slit interfere with the entanglement;

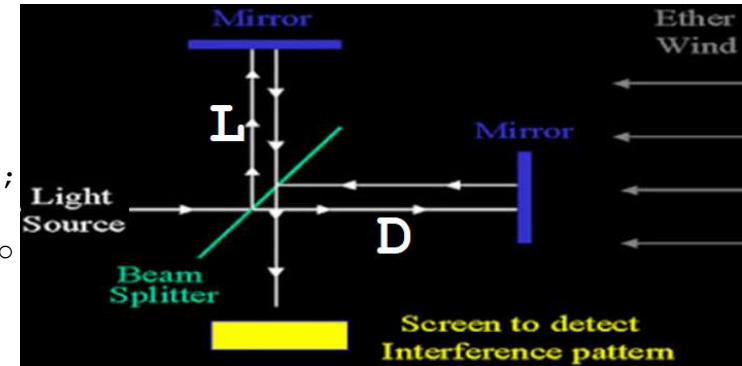


27-Elitzur-Vaidman bomb tester: QM Wrong
explanation: Bad bomb=photon pass through without absorbance; photon go in both pathways; C click; D not (destructive interference); Good bomb=explode upon photon absorbance; photon chose way, if pass through bomb it explodes, if chose upper path bomb not explode & D or C click; so if D click we know the bomb good without explode it (25% chance)
Correct Explanation: Half-silvered mirror (HSM) eject G_y in one direction & O in the other; O steer the e in mirror & e in upper HSM to be correlated; If there is bad bomb (or no bomb) G_y hit the e in the upper HSM & C click (D won't click as e in D path get O that destructively interfere with G_y); If there is a good bomb, G_y explode it, or O get absorb by the bomb without exploding it (O is a sphere & is not rotated) & G_y go & click D/C;



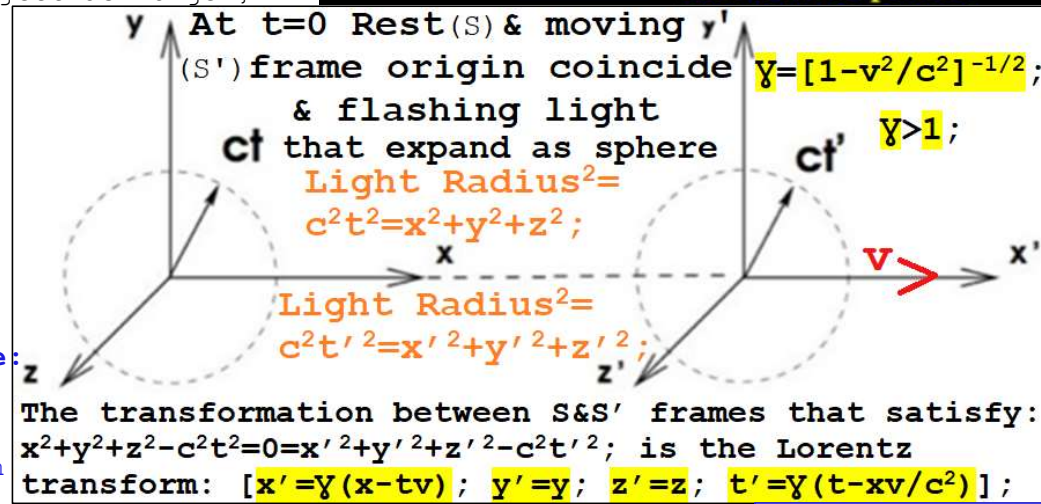
29-Michelson-Morley, Kennedy-Thorndike & Ives-Stilwell experiments explanation:

1) Less probable explanation that give correct result: Einstein's relativity principle: There is no absolute rest: Each observer can claim that he is at rest & the Light Source (LS) is moving, & because light's $|v|=c$ regardless of LS movement, each observer'll measure light's $|v|=c$; regardless of its/LS v ; =>Lorentz transform; =>Length contraction [$L_0=YL$ =Length of the object as measured by observer at rest with respect to the object], Time dilation [$T_0=T/Y$ =The time interval between 2 events (light go between mirrors) as measured by observer at rest with respect to the clock] {if 2 objects move with respect to each other, each claim that it is at rest & measure: the other length as shorter & the time interval between 2 events in the other object as longer} =>



Relativistic Doppler Effect [In motion direction: $\lambda_{\text{observer}} = \lambda_{\text{source}} ((c+v)/(c-v))^{1/2}$; & Perpendicular direction $\lambda_{\text{observer}} = \lambda_{\text{source}} Y$]; => Doppler & length contraction explain these experiments; consequence: There is no universal truth in the universe: The universe doesn't exist;

2) More probable explanation: The result of these experiments is consequence of 2 processes: a) λ ($\lambda=c/f$) of G_y result from e & G collision velocity; and G_y & e collision velocity depend also on λ of G_y ; => Doppler equations; b) If in stationary atom the distance between 2 vertical e's & 2 horizontal e's is equal ($L=D$) than G_z^- travel time in these cases must remain equal even if the atom move:

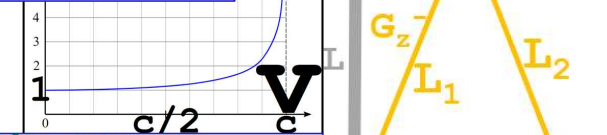


$dt_{r1} = dt_{ud}$; to achieve this the atom is contracted along the direction of motion such that: $D \rightarrow D/Y$; {atomic e's really move in direction of motion, such that $dt_{r1} = dt_{ud}$; as e rotates & must collide with G_z^- the same as stationary atom to conserve the atomic structure;

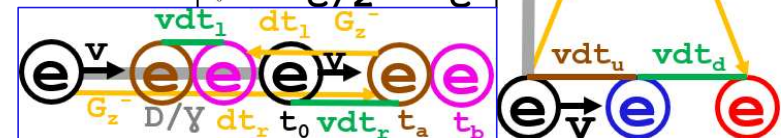
$dx_r = D + v dt_r$; $dt_r = (D + v dt_r) / c$; => $dt_r = D / (c - v)$; $dx_r = Dc / (c - v)$; $dx_1 = D - v dt_1$; $dt_1 = (D - v dt_1) / c$; => $dt_1 = D / (c + v)$; $dx_1 = Dc / (c + v)$; $dt_{r1} = dt_r + dt_1 = Y^2 2D / c$; $dx_{r1} = dx_r + dx_1 = Y^2 2D$;



$dt_u = dt_d = \frac{1}{2} dt_{ud}$; $L_t = L_1 = L_2 = [L^2 + (\frac{1}{2} v dt_{ud})^2]^{1/2}$; $dt_{ud} = 2L_t / c$; => $dt_{ud} = Y 2L / c$; $L_t = [L^2 + (v Y L / c)^2]^{1/2}$; $dx_{ud} = 2L_t = Y 2L$; M-M (1887): $L = D = 11$; $v = 3 \times 10^4$; $c = 3 \times 10^8$; $dx = dx_{r1} - dx_{ud} = 2Y(YD - L) = 1.1 \times 10^{-7}$; light $\lambda = 5 \times 10^{-7}$; should give destructive interference, but didn't; $D \rightarrow D/Y$; solve it; K-T (1932): $D \neq L$; $D \rightarrow D/Y$; $dx_r \rightarrow Dc / (Y(c - v))$; $dx_1 \rightarrow Dc / (Y(c + v))$; Fringe shift = FS = $(dx_r + dx_1 - 2L_t) / \lambda_s \neq 2(D - L) / \lambda_s$; [no movement result]; Solution: $\lambda_u = \lambda_d = \lambda_s Y$; $\lambda_r = \lambda_s [(c + v) / (c - v)]^{1/2}$; { t_1 e, red bottom e, t_a e move away from the light, thus feel weaker collision force (longer λ)}; $\lambda_1 = \lambda_s [(c - v) / (c + v)]^{1/2}$; {left magenta e move toward light, thus feel stronger collision force (shorter λ)}



FS = $dx_r / \lambda_r + dx_1 / \lambda_1 - L_t / \lambda_u - L_t / \lambda_d = 2(D - L) / \lambda_s$; I-S (1938) can be explained by doppler; so real length contraction of e & doppler explain all 3 experiments;



30-stellar aberration explanation: **1)** Less probable explanation that give correct result: Einstein's relativity principle;=>Lorentz transform;=>relativistic velocity addition;=>stellar aberration; **2)** More probable explanation: For rest frame light goes along c m ruler in 1s; ■ In the direction of motion, in moving frame the ruler shrink to c/γ m, but we can't know that (as we compare it with the unchanged dy & when we switch the scale ruler it shrink also), so light suppose to pass this length in $1/\gamma$ s, but we measure $1/\gamma$ s as $1/\gamma^2$ { $G_z^{+/-}$ travel time between 2 e's represent time unit duration; stationary $dt_{ud}=2L/c$; $\rightarrow dt_{ud}=\gamma 2L/c$; in a moving clock; so the number of time units in a period shrink for a moving clock (explain Muon decay in the atmosphere)} so we measure the speed of light as $c/(1/\gamma^2)=c\gamma^2$; ■ In direction perpendicular to motion direction, in the moving frame, the ruler doesn't shrink, but 1s still measured as $1/\gamma$, so we measure the speed of light as $c/(1/\gamma)=c\gamma$; ■ Thus, in stellar aberration, when we add the v of earth; $u_y'=c\sin\theta \rightarrow \gamma c\sin\theta$;

$$\tan(\phi) = \frac{u_y}{u_x + v} = \frac{\sin(\theta)}{v/c + \cos(\theta)} \neq \tan(\theta)$$

Relativistic velocity addition:

$$u'_x = (u_x + v)/(1 + u_x v/c^2)$$

$$u'_y = u_y/\gamma(1 + u_x v/c^2)$$

$$\tan(\phi) = \frac{u'_y}{u'_x} = \frac{u_y}{\gamma(u_x + v)} = \frac{\sin(\theta)}{\gamma(v/c + \cos(\theta))}$$

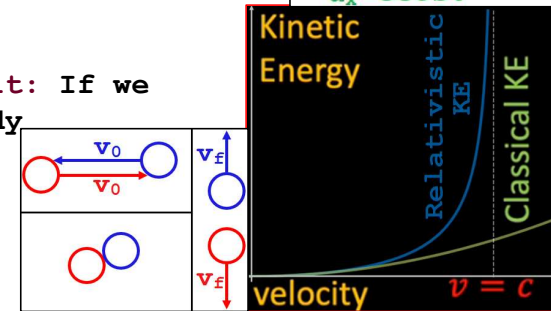
Rest Frame
Moving Frame
Earth
v
 $u_x = c \cos \theta$

and $u'_x = (v + c \cos \theta) \rightarrow \gamma^2 (v + c \cos \theta)$; Thus: $\tan \phi = u'_y / u'_x = \gamma c \sin \theta / (\gamma^2 (v + c \cos \theta)) = \sin \theta / (\gamma (v/c + \cos \theta))$;

31: $E^2 = (\gamma m v)^2 c^2 + m^2 c^4$; explanation:

1) Less probable explanation that give correct result: If we analyze eccentric impact collision from a moving body frame, using relativistic velocity addition, we'll see that momentum is not conserved; Thus we change definition of momentum to $\gamma m v$ & now momentum always conserved; integration of $(\gamma m v)'$ Give E equation;

2) More probable explanation: In direction perpendicular to motion direction, in a moving frame length doesn't change & $v \rightarrow \gamma v$ so $p = mv \rightarrow \gamma m v$; integrate $(\gamma m v)'$ give E;



Relativistic Energy

$$KE = W = \int_{x_1}^{x_2} F dx \xrightarrow{\text{momentum is defined as } p = mv} KE = W = \int_{x_1}^{x_2} \left(\frac{dp}{dt}\right) dx$$

if we are in the rest frame of the moving object:

$$p = m \frac{dx}{dt_0} = m \frac{dx}{dt} \left(\frac{dt}{dt_0}\right) = \gamma m \frac{dx}{dt} = \gamma m v = \text{Relativistic Momentum}$$

Chain Rule $\frac{dt}{dt_0} = \frac{d}{dt_0}(\gamma t_0) = \gamma$

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\frac{dp}{dt} = \frac{d}{dt} \left(\frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} \right) = m \frac{d}{dt} \left(v \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \right)$$

$$\frac{d}{dt} (A(t)B(t)) = A \left(\frac{dB}{dt}\right) + \left(\frac{dA}{dt}\right) B = m \frac{dv}{dt} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} + mv \frac{d}{dt} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

$$\frac{d}{dt} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = -\frac{1}{2} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} \times \frac{d}{dt} \left(-\frac{v^2}{c^2}\right) = \frac{v}{c^2} \frac{dv}{dt} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}}$$

$$\frac{dp}{dt} = m \frac{dv}{dt} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} + \frac{mv^2}{c^2} \frac{dv}{dt} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}}$$

$$= m \frac{dv}{dt} \left[\frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}} + \frac{\frac{v^2}{c^2}}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \right] = m \frac{dv}{dt} \left[\frac{1 - \frac{v^2}{c^2} + \frac{v^2}{c^2}}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \right] = \gamma^3 m \frac{dv}{dt}$$

$$KE = \int_{x_1}^{x_2} \left(\frac{m}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \right) \left(\frac{dv}{dt}\right) dx$$

$\left(\frac{dv}{dt}\right) dx \rightarrow dv \left(\frac{dx}{dt}\right)$
 At x_1 $v = 0$
 At x_2 $v = v$

$$KE = \int_0^v \left(\frac{m \cancel{v}}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \right) dv = \left[\frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \right]_0^v = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2$$

The mass of an object is a fundamental invariant of the theory of relativity, something that all observers can agree on

$$E^2 = \frac{m^2 c^4}{1 - \frac{v^2}{c^2}} = \frac{m^2 c^4}{1 - \frac{v^2}{c^2}} + \frac{m^2 c^2 v^2}{1 - \frac{v^2}{c^2}} - \frac{m^2 c^2 v^2}{1 - \frac{v^2}{c^2}}$$

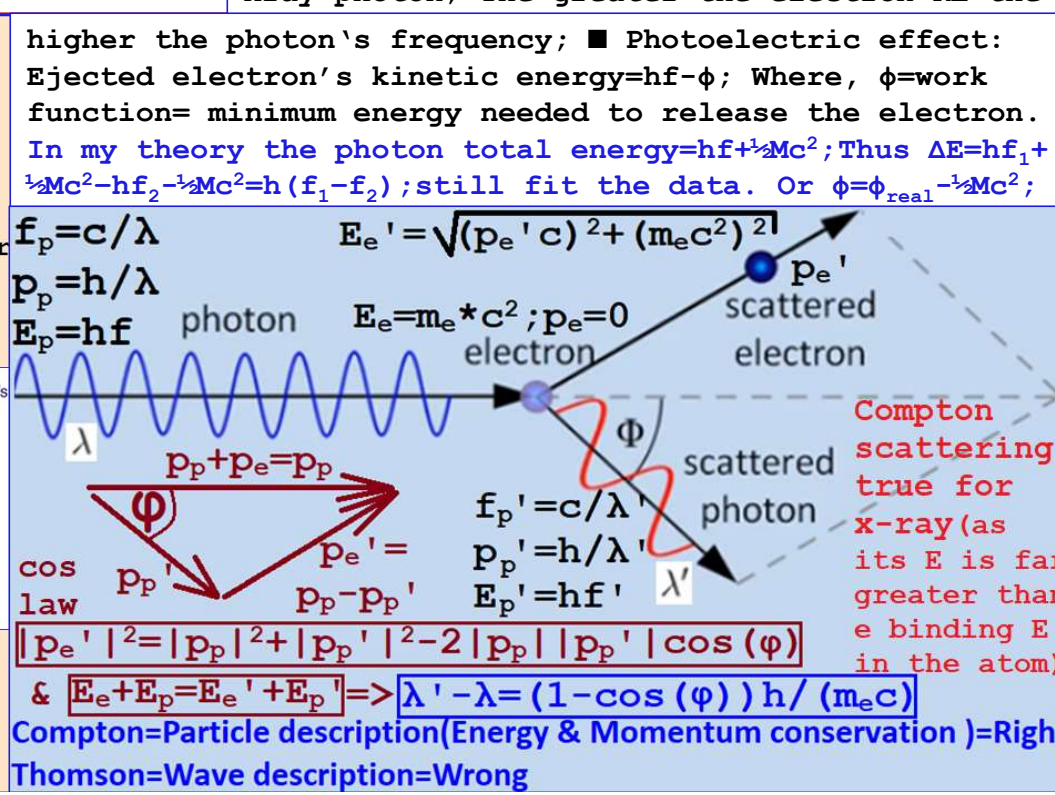
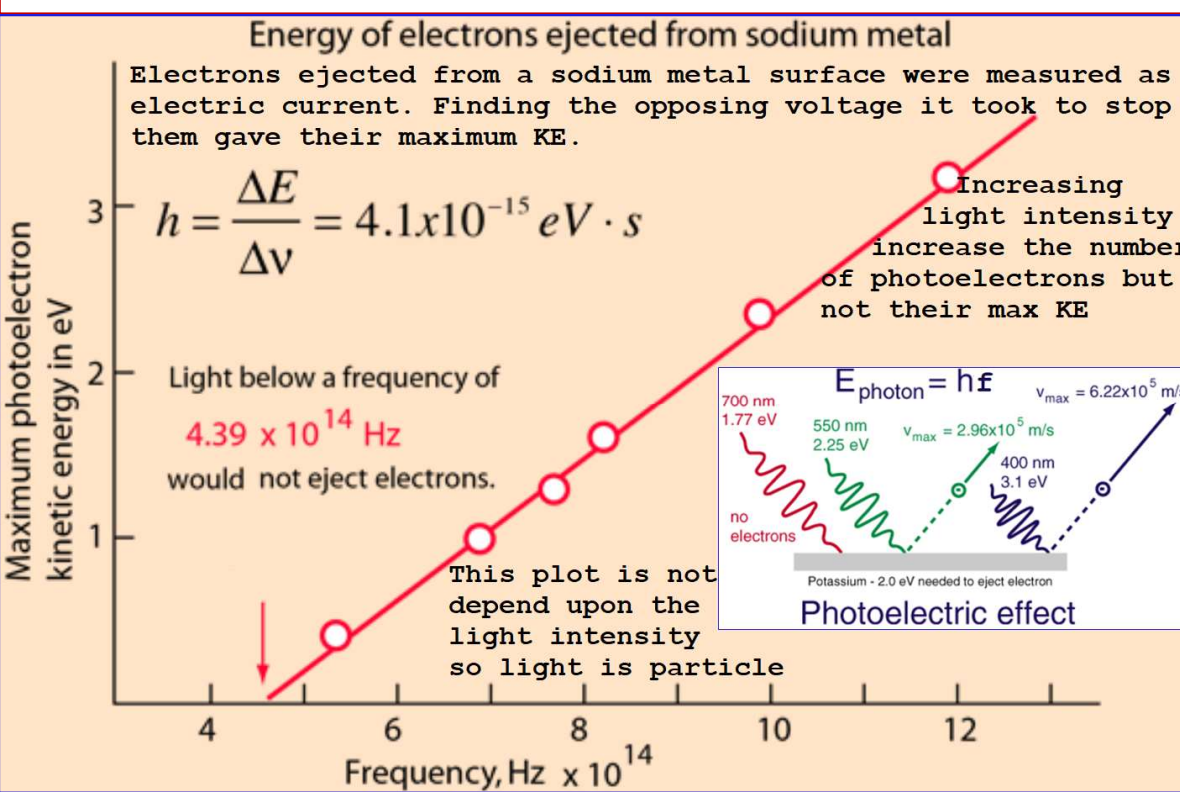
0

$$= \left(\frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} \right)^2 c^2 + \frac{m^2 c^4 - m^2 c^2 v^2}{1 - \frac{v^2}{c^2}}$$

$$= p^2 c^2 + \frac{m^2 c^4 \left(1 - \frac{v^2}{c^2}\right)}{1 - \frac{v^2}{c^2}} = p^2 c^2 + m^2 c^4 = E^2$$

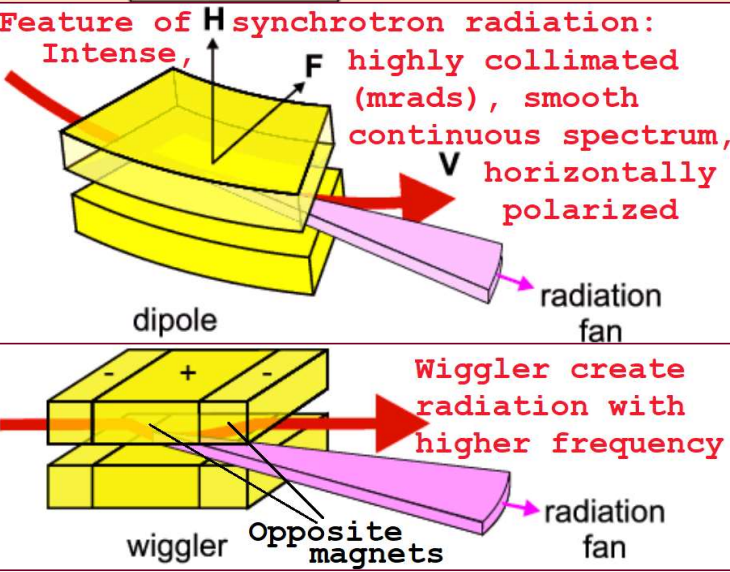
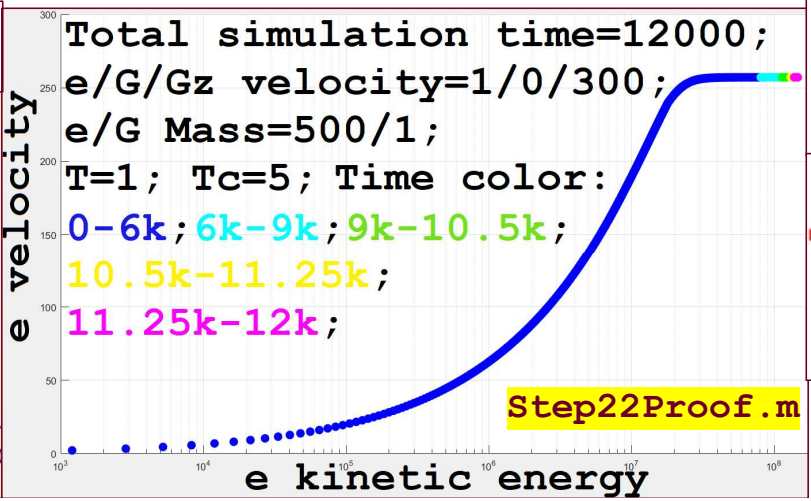
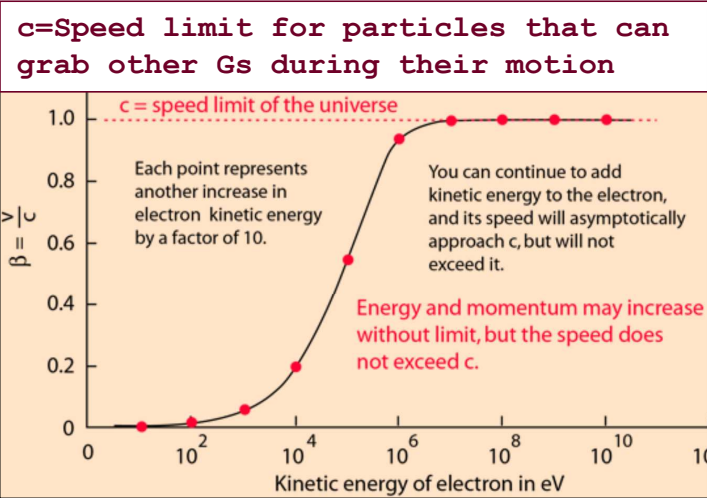
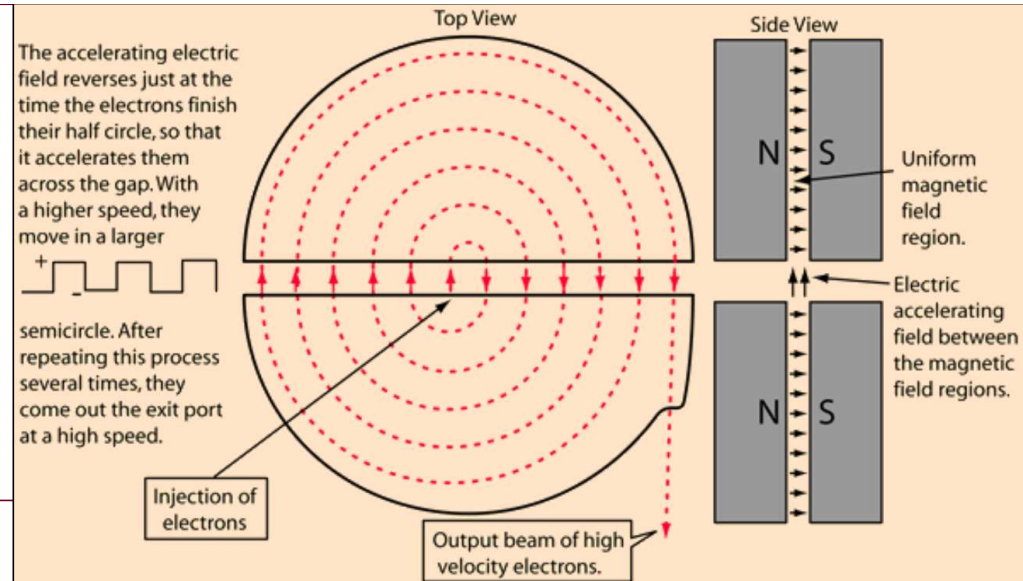
So all experimental foundation of special & general relativity can be explain by: **Time dilation** {result from increased $G_z^{+/-}$ travel time between 2 e's in: moving atom, or atom that is in gravitational field (due to distance elongation)}, **Length contraction** {result because in atom $G_z^{+/-}$ continuously collide between e's until a stable structure achieved; thus $G_z^{+/-}$ travel time between 2 vertical e's & 2 horizontal e's each D meter apart when the atom at rest, must remain equal even if the atom move}; **Relativistic Doppler effect** {result because λ of G_γ caused by e velocity when collide with G; & G_γ & e collision velocity depend on e's v & G_γ 's λ } and the randomly creating & flying shapes (Gs) in the universe {result: in G grabbing by a fast moving e/e+; and in the gravitational force}; Without accepting Einstein's relativity principles that claim that there is no absolute rest and thus no absolute truth, and this can only happen if the universe doesn't exist; & without accepting Einstein's equivalence principle;

32-Compton scattering: e detached from the atom after the collision: correct e & G_γ collision result achieved when assuming e & G_γ are particles & make sure Energy & Momentum conserved; In my theory the photon total energy = $hf + \frac{1}{2}Mc^2$; Thus in: $E_e + E_p = E_e' + E_p'$; $\frac{1}{2}Mc^2$ can be reduced & the derivation still work. In Rayleigh scattering the e remain bound to the atom after the collision thus $G_z^{+/-}$ also participate in the collision; Bremsstrahlung: Firing electron with high KE into piece of metal yield predominantly Xray photon; The greater the electron KE the



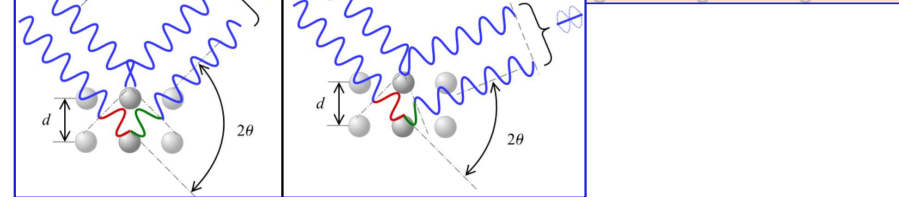
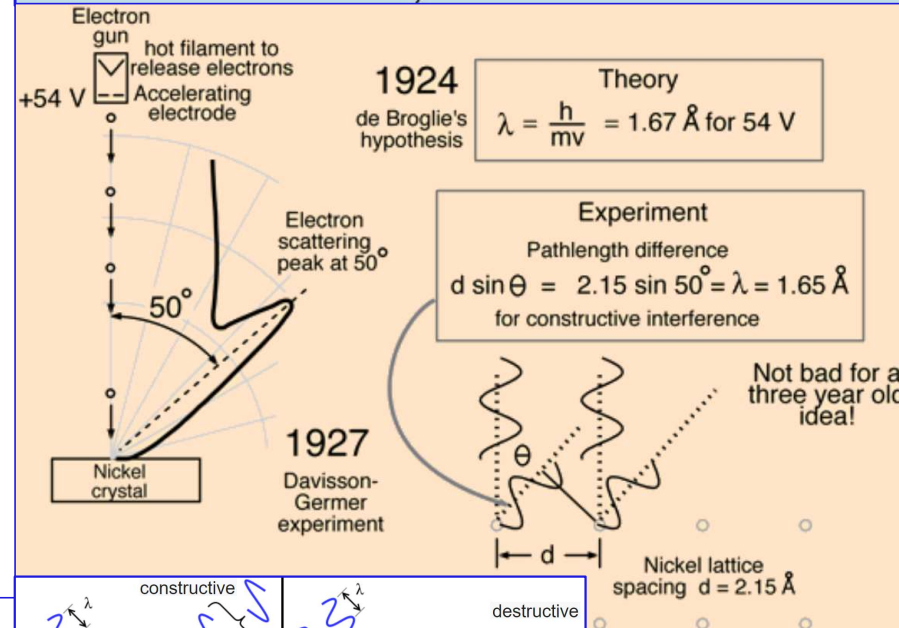
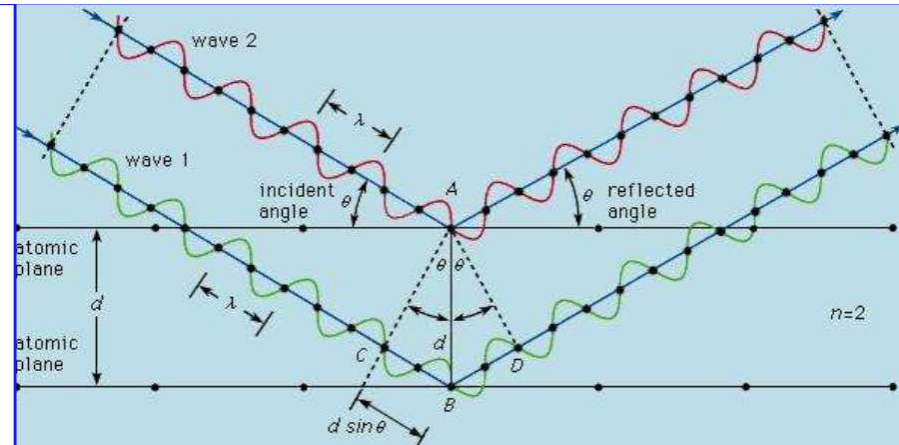
33-Cyclotron high velocity electron failure: cyclotron circular motion period= $T=2\pi r/v=2\pi m/(qB)$; {Centripetal Force= $mv^2/r=vqB$ }; is independent of v only for $v \ll c$; for high speed $m=m_0\gamma$; $\{\gamma=[1-v^2/c^2]^{-1/2}$; $\gamma > 1$; m_0 =rest mass} Because each T, G_z^- collide with e & accelerate it; & each Tc/g ; $\{g=1/(1-ev^2/Gzv^2)^{1/2}$; Tc =constant derived from G density in the universe; $Gzv/ev=G_z^-/e$ velocity} e collide with G & grab it (the faster e move, the more G it grab); so in high speed the mass of e is increased, some colliding G are transformed into G_γ ;

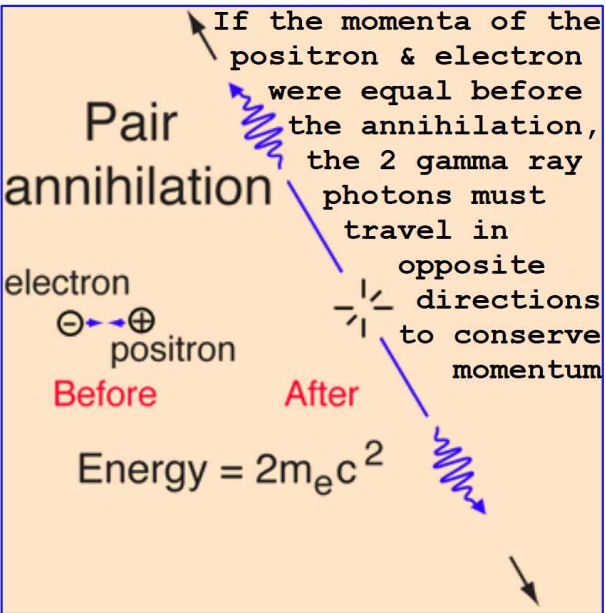
In deceleration e release G as G_γ ; {synchrotron solve this by increasing $|B|$ as e speeds up; cyclotron can't work for e as it gain high speed immediately, due to it's small mass ($v=rqB/m$)}; e/e^+ collide with G but G_γ/G_{xyz}^{\pm} usually don't collide with G , because G_γ/G_{xyz}^{\pm} are very small compared to e/e^+ ; so they miss the G s almost always;



34-While the multiple steps creating e/e^+ give them a specific structure [$I_1=I_2\sim I_3/2$, ω_w & Z^+ Return to ω_0 & Z^+_0 each $T_L=2\pi I_1/|L_w|s$; $|\omega_{03}| \gg |\omega_{01/2}|$: X^+ ~return to X^+_0 each $2T_L$] (by Intermediate & Major axis theorem and by numerous collisions & centrifugal force) G_V is creating by only 2 collisions and has no unique shape, however, as we can see from page 13, its ω_w ~return to ω_0 each T_f s; For both G_V & e/e^+ ω_w period dictates the B interaction; While the spin state period indicate a more precise period that is still common to all e/e^+ (G_V with same f); Thus for e/e^+ the spin state period is $2T_L$ (take into account ω_w & X^+ period; & called spin $\frac{1}{2}$ particle) but for G_V s with same f it remain T_f as each G_V originate from different G shape and has different G_V shape, even though they all have the same f ; (spin 1 particle) Yet all $e/e^+/G_V$ have period (T) and thus $f=1/T$; If they move they have: $\lambda=v/f=vT$; For G_V : $\lambda=c/f=cT_f=h/p_R$; $\{E^2=(\gamma m v)^2 c^2+m^2 c^4$; $\gamma=[1-v^2/c^2]^{-1/2}$; $p_R=\gamma m v$; $m \rightarrow 0$; $E=cp_R=hf=hc/\lambda\}$

■ **Davisson Germer experiment:** Electrons released from tungsten filament accelerated using $54V=KE/q$ battery into nickel chloride crystal ($d=2.15 \times 10^{-10}$), the scattered electrons have maximum intensity at $\theta=50^\circ$;
1) Less probable explanation: de Broglie's hypothesis: electron is wave: $\lambda=h/p=h/(mv)=1.67 \times 10^{-10}$; $\{v=(2 \cdot m \cdot KE)^{1/2}/m=(2 \cdot m \cdot V \cdot q)^{1/2}/m=4.36 \times 10^6 \text{m/s}$; $m=9.109 \times 10^{-31}$; $V=54$; $q=1.602 \times 10^{-19}$; $h=6.626 \times 10^{-34}\}$ the peak is due to constructive interference of wave: $\lambda=d \sin \theta/n=1.65 \times 10^{-10}/n$; $n=\text{integer}$;
2) More probable explanation: For e/e^+ : $T=T_L=2\pi I_1/|L_w|=8\pi^2 m s_2^2/(5h)$; $\{|L_w|=h/(4\pi)$; $I_1=m(s_2^2+s_3^2)/5$; $s_3 \rightarrow 0$; T_L is common to all e/e^+ with any angular momentum} Thus, if es, with same L_w , that fly with v , arrive at the detector with the same ω_w , we see larger signal; for this to happen, the extra travelling time= $d \sin(\theta)/v=nT$; $n=\text{integer}$; {indicate that $T=1.89 \times 10^{-17}/n$; & $s_2=2.95 \times 10^{-11}/n^{1/2}$ }; Because all e/e^+ are same mass asymmetric clusters [$I_1=I_2\sim I_3/2$; $|\omega_{03}| \gg |\omega_{01/2}|$] & their Z^+ align each T_L ;

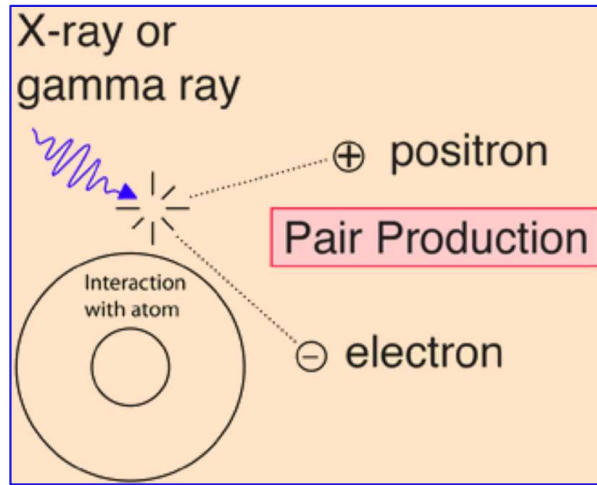




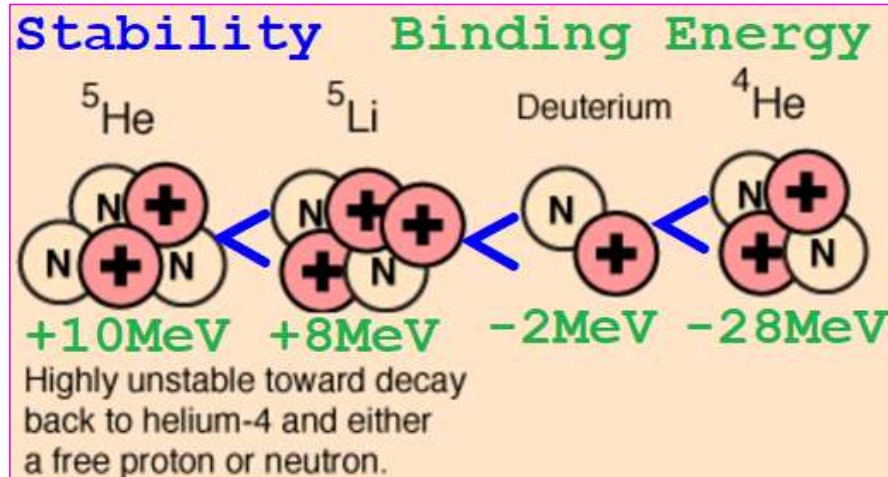
35-Annihilation: Explanation: e^- collide with e^+ , they become attached together and stop rotating, while releasing 2 γ 's that carry their rotational energy; γ 's $f = m_e c^2 / h$;

■ Pair production: Electron's rest mass energy is 0.511 MeV, so for photon energy above 1.022 MeV, pair production is possible {conserve energy & momentum, but quantum numbers of produced particles sum to 0. probability increases as \sim [nearby atomic number] 2 & with photon energy (dominant mode of interaction for X & gamma-rays)}

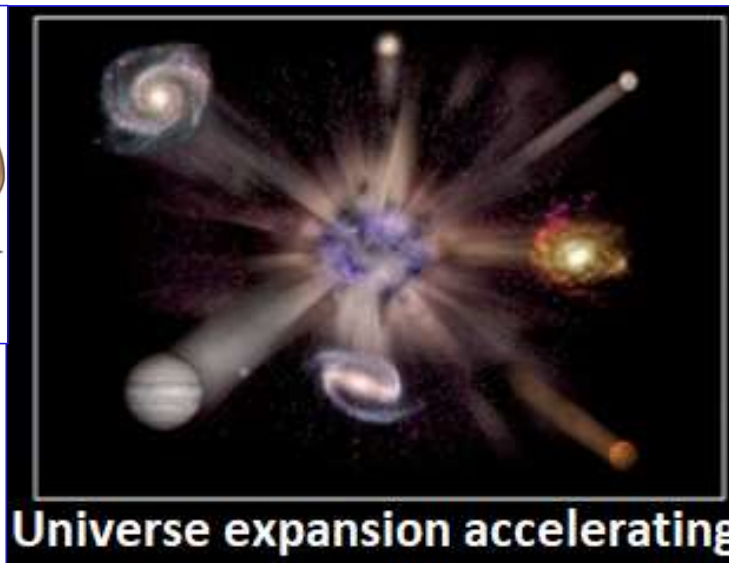
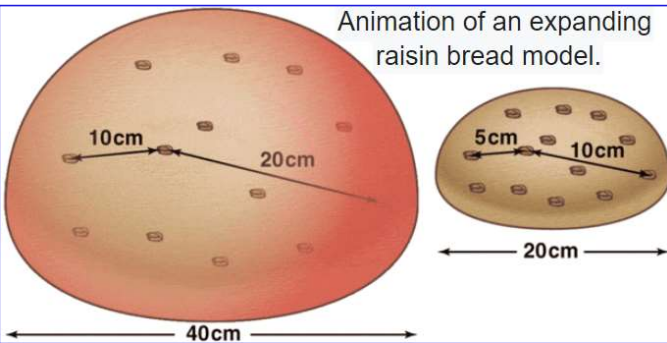
Explanation: γ collide with annihilated e^-e^+ cluster detach them & rotate them again; γ 's $f = 2m_e c^2 / h$;



36. [mass of nucleus] - [mass of its nucleons] = $\Delta m < 0$ always, but different for different elements. $\Delta m c^2 =$ nuclear binding energy that holds the nucleons together {Nucleus is made up of nucleons (protons, neutrons); while Energy to release e from H atom = 13.6 eV; (ionization E); the Energy to break apart p & n from alpha particle (2p+2n) = 28 MeV $V = c^2 [2m_p + 2m_n - m_{\alpha}]$; $m_n = 939.5$; $m_p = 938$; $m_{\alpha} = 3727$; [MeV/c 2] binding E}



37- The expansion of the universe appears to be accelerating. The velocity of galaxy away from earth [km/s] = The distance of the galaxy from earth [km] * 26×10^{-19} . {we have red shift from everything so or we are moving away from everything or the universe is expanding and its expanding faster and faster with acceleration}. **Explanation: In the universe, every-where & time NE-space/3D-spaces is being created;**



$$1 \text{ Mpc} = 3.08567758 \times 10^{22} \text{ meters}$$

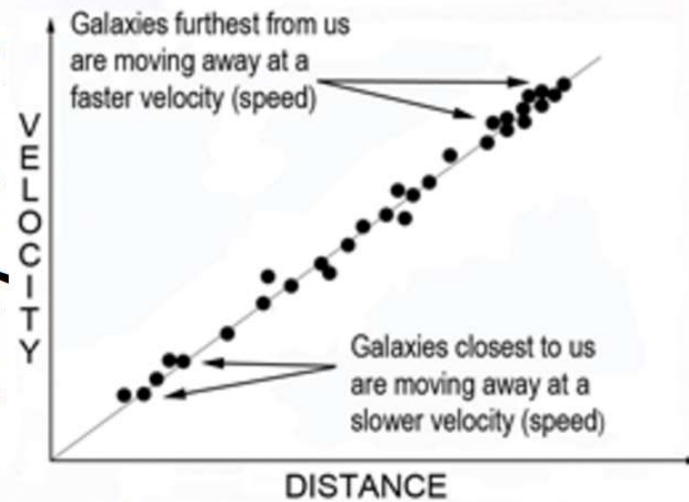
$$V = Hd$$

recessional velocity (km/s) distance (Mpc)

- H is the Hubble constant
- H = 80 km/s/Mpc

$$\text{VELOCITY} = \text{HUBBLE CONSTANT} \times \text{DISTANCE}$$

How fast it's moving away from us



How far away the galaxy is

Wild guess

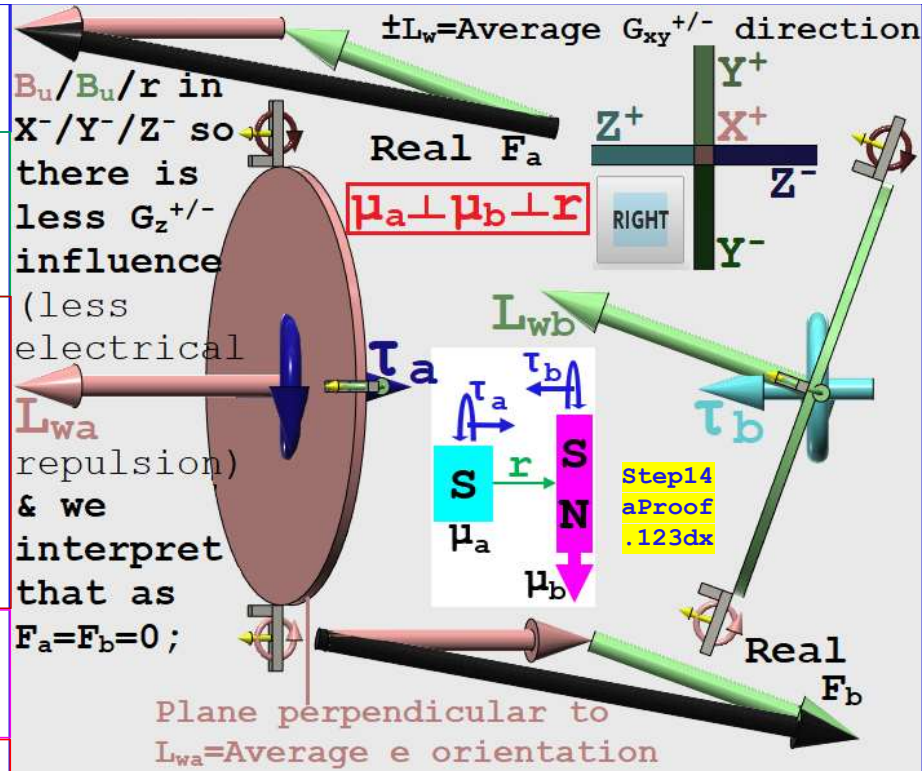
38-u|d Quark is probably like e+|e cluster but with more G components & $L_3=2/3$ of e+'s L_3 | $L_3=1/3$ of e-'s L_3 ; Anti u|d Quark is probably like e|e+ cluster but with more G components & $L_3=2/3$ of e-'s L_3 | $L_3=1/3$ of e+'s L_3 ;

39-Color charge is probably the particle's B_u ; where Red/Green/Blue are $X^+/Y^+/Z^+$ B_u direction & AntiRed/AntiGreen/AntiBlue are $X^-/Y^-/Z^-$ B_u ; While $X^+&X^-$ B_u particles attracts $X^+&Y^+&Z^+$ B_u or $X^-&Y^-&Z^-$ B_u particles impose no B force on each other & allow a delicate attraction dance by opposite L_3 ;

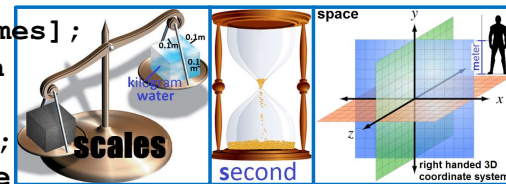
40-While Helicity= $\text{sign}(L \cdot p)$; {+=RH; -=LH}, Chirality is probably a function of $p \cdot r$ & thus it dictate if the particle hit the target such that its L contribute positively to increase the collision impact; The target is hit by the particle p & by the particle rotational motion, which cause greater collision force that release sub-cluster particle as what we call weak force; ■ Neutrino is probably like e cluster but with less G component & with initial orientation that flip it 90° about X axis; thus its major rotation is about the world Y axis and its $L_3 \rightarrow 0$;

41-In atom the e & the positive nucleus play give and take with $G_{xyz}^{+/-}$; Thus number of $G_{xyz}^{+/-}$ rotations must be integer number in equilibrium. The equilibrium can reach in many ways, lead to different energy level.

42-Superconductor stay locked in space, when put near magnet; Explanation: If the superconductor move, each e in it is moving in B, so it feel force, so the e's in the superconductor move in circle, this circular motion & B now opposing the superconductor's movement;

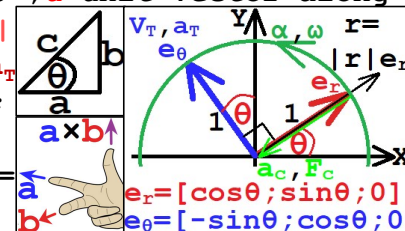


Appendix

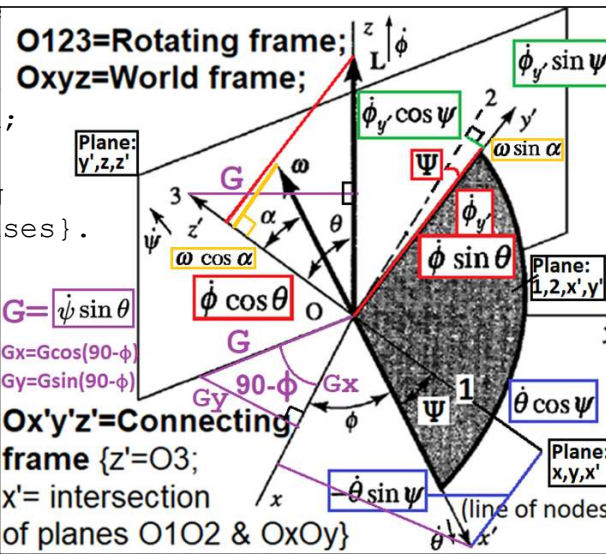
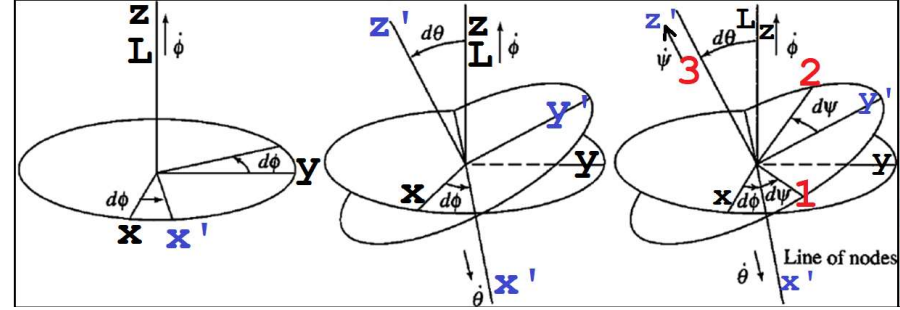
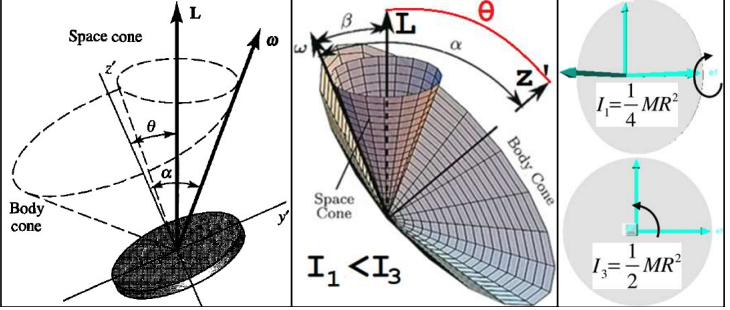


Green=Pure definition; Quantity=a=1+1..+1[a times]; a+b=a+1+1..+1[b times]; a-b=a-1-1..-1[b times]; a*b=ab=a+a..+a[b times]; a/b=c>a-b-b..-b[c times]=0; a^b=a*a..*a[b times]; function=f(x)=return 1 output(f(x)) for each input(x); ()=Do first; Derivative=f'(x)=(f(x+h)-f(x))/h; h→0; Integral=∫f(t)dt=dt(f(t₁)+f(t₁+dt)+f(t₁+2dt)..+f(t₂-dt)); dt→0; vector=[x;y;z]; a•b=a_xb_x+a_yb_y+a_zb_z; |a|=(a•a)^{1/2};

Blue=Measurements: Mass=m=The quantity of Kilograms that balance an object in scales; Time=t=The quantity of Seconds (Sand grains) that pass; Space=The quantity of Meters in every direction; Red=Dressed definition; Each is [f_x(t); f_y(t); f_z(t)]; Position=r; Velocity=v=r'; Acceleration=a=v'; Linear Momentum=p=m*v; Force=F=p'; {if m=m(t)=m(0); F=m*a; 2nd newton law} Impulse=J=∫Fdt=∫p'(t)dt=p(t₂)-p(t₁); {Thus if F=0:p(t₂)=p(t₁)=>p conserved} Rigid body=continuous distribution of point masses(m_i) with neglected deformation; Center of mass=CM=1/M*∑m_ir_i; M=∑m_i; CM of Rigid body follow trajectory of point mass; Π=Circle circumference/(2*Radius); Angle=θ; [rad=1/(2π) circle]; sin(θ)=b/c; cos(θ)=a/c; Free Rigid body rotates about its CM; Any rotation can be described by 1 axis & 1 angle; Rotation & Translation can be analyzed separately; If r is point in rigid body that rotates about z; r=|r|*[cosθ; sinθ; 0]=|r|e_r; {|r|=constant; x=x(t)} r'=|r|*θ'*[-sinθ; cosθ; 0]=|r|θ'e_θ=Tangential velocity=V_T; Angular velocity ω; {e_θ&e_r perpendicular} a×b=[a_yb_z-a_zb_y; a_zb_x-a_xb_z; a_xb_y-a_yb_x]; ω=u*θ'; u=unit vector along axis of rotation(right hand indicates+direction) V_T=ω×r; ω=(r×V_T)/|r|²; r''=V_T'=-|r|θ''e_r+|r|θ'θ'e_θ=-|r|θ'²e_r+|r|θ|α|e_θ; Centripetal Acceleration=a_c=ω×V_T; Angular Acceleration α=ω'; Tangential Acceleration=a_T=α×r; If m(t)=m(0): Centripetal Force=F_c=m*a_c; Matrix*v=[a,b,c;d,e,f;g,h,i]*[x;y;z]=[xa+yb+zc; xd+ye+zf; xg+yh+zi]; if M*v=λv; eigenvector; eigenvalue (quantity) Angular momentum=L=r×p; For rigid body L about CM=L_{CM}=∑[r_i×p_i]=∑[r_i×V_{Ti}m_i]=∑[r_i×(ω×r_i)m_i]=I*ω; Inertia tensor=I(t)=[I_{xx}, I_{xy}, I_{xz}; I_{yx}, I_{yy}, I_{yz}; I_{zx}, I_{zy}, I_{zz}]; I_{xx}=∑(r_{iy}²+r_{iz}²)m_i; I_{yy}=∑(r_{ix}²+r_{iz}²)m_i; I_{zz}=∑(r_{ix}²+r_{iy}²)m_i; {always+; measure mass distribution about an axis} I_{xy}=I_{yx}=-∑r_{ix}*r_{iy}m_i; I_{xz}=I_{zx}=-∑r_{ix}r_{iz}m_i; I_{zy}=I_{yz}=-∑r_{iz}r_{iy}m_i; {+/-/0; measures of symmetry; If xy is plane of symmetry: I_{zx}=I_{zy}=0; If xy&zy are plane of symmetry: I_{xy}=I_{xz}=I_{zy}=0}; Torque=τ=L'=(r×p)'=r'×p+r×p'=V_T×mV_T+r×F=r×F=m*r×a; For rigid body τ=∑m_ir_i×a_i=∑m_i[r_i×(a_{Ti}+a_{ci})] =∑m_i[r_i×(α×r_i+ω×V_{Ti})] =∑m_i[r_i×(α×r_i+ω×(ω×r_i))] =Iα+ω×Iω; Angular Impulse=K=∫τdt=∫L'(t)dt=L(t₂)-L(t₁); {Thus if τ=0:L(t₂)=L(t₁)=>L Conserved; I(t₁)ω(t₁)=I(t₂)ω(t₂); but usually I(t₁)≠I(t₂); &ω(t₁)≠ω(t₂)} Until here, component of each vector were taken along world frame_w; but I(t)=I_w=unknown; If r=frame in CM at principal axes rotate with body; &its basis vectors=v_i=v_i(t); than I_wv₁=I_{r1}v₁; I_wv₂=I_{r2}v₂; I_wv₃=I_{r3}v₃; I_{ri}=constants{I_w, v_i change together with time; v_i&I_{ri}=eigen vector& value of I_w} & ω=ω_w=ω_{w1}[1; 0; 0]+ω_{w2}[0; 1; 0]+ω_{w3}[0; 0; 1]=ω_{r1}v₁+ω_{r2}v₂+ω_{r3}v₃=Rω_r; (ω_r=ω_r(t)=component of ω_w taken along r; R=R(t)=[v₁, v₂, v₃]=rotation matrix from r to w) &L_{CM,w}=I_wω_w=I_w(ω_{r1}v₁+ω_{r2}v₂+ω_{r3}v₃)=ω_{r1}I_{r1}v₁+ω_{r2}I_{r2}v₂+ω_{r3}I_{r3}v₃=RI_rω_r=I_wω_r; I_r=[I_{r1}, 0, 0; 0, I_{r2}, 0; 0, 0, I_{r3}]; I_w=RI_rR⁻¹; If r=body point; r_r=r_r(t)=r_r(CM); r_w=Rr_r; r_w'_r=components of r_w' taken along r; (r_w'_r≠0; r_r'=0) R⁻¹=Inverse R=R^T; ω×W=[0, -ω₃, ω₂; ω₃, 0, -ω₁; -ω₂, ω₁, 0]; r_w'_r=R^Tr_w'=R^T(ω_w×r_w)=R^Tω_w×R^Tr_w=ω_r×r_r=W_rr_r=R^T(Rr_r)'=R^TR'r_r; L_r=R^TL_w=R^T(I_wω_w)=R^T(RI_rR^Tω_w)=I_rω_r; τ_w=L_w'=(RL_r)'=(RI_rω_r)'=RI_rω_r' +R'I_rω_r; (I_r'=0) τ_r=R^Tτ_w=R^T(RI_rω_r' +R'I_rω_r)=R^TRI_rω_r' +R^TR'I_rω_r=I_rω_r' +ω_r×I_rω_r; {τ_w=L_w'; τ_r≠L_r'} If τ_w=0; we can find R& ω_r by solving: I_rω_r'=I_rω_r×ω_r; & R'=R[0, -ω_{r3}, ω_{r2}; ω_{r3}, 0, -ω_{r1}; -ω_{r2}, ω_{r1}, 0]; {I in vector R from CM=I_r=I_{CM}+M[R₂²+R₃², -R₁R₂, -R₁R₃; -R₁R₂, R₁²+R₃², -R₂R₃; -R₁R₃, -R₂R₃, R₁²+R₂²]; I of 2 bodies v&u from system CM=I_v+I_u} Translation velocity=V_t; Work on rigid body=∑{∫F_i(r_i(t))•r_i'(t)dt}=∑{∫m_ia_i•v_idt}=∑{½m_i∫(|v_i|²)'dt}=∑{½m_i∫(|V_t+V_{Ti}|²)'dt}=∑{½m_i∫(|V_t+ω×r_i|²)'dt}=½ω(t₂)•I(t₂)ω(t₂)-½ω(t₁)•I(t₁)ω(t₁)+½M|V_t(t₂)|²-½M|V_t(t₁)|²=KE_{rot}(t₂)-KE_{rot}(t₁)+KE_{tr}(t₂)-KE_{tr}(t₁)=ΔKE; {F_i(r_i(t))=∇KE_i(t); V_{Ti}=V_t; if F_i=0:KE_{tr}(t₂)+KE_{rot}(t₂)=KE_{rot}(t₁)+KE_{tr}(t₁)=>E conservation; V_t, (I&ω) in w or r frame (½ω•Iω=½|ω||Iω|cosα); ∑{∫F_i(r_i(t))•dr_i}=>ΔPE} In a&b collision (F_{ia}>b=-F_{ib}>a): W_a=-W_b;



Feynman's Wobbling Plate: For body: I_{r1}, I_{r2}, I_{r3} ; **If $\tau_w(t)=0$:** then $\tau_r(t)=0$; & by $\tau_r(t)=I_r \alpha_r(t) + \omega_r(t) \times I_r \omega_r(t)$; we get: $I_{r1} d\omega_{r1}(t)/dt + (I_{r3} - I_{r2}) \omega_{r2}(t) \omega_{r3}(t) = 0$; $I_{r2} d\omega_{r2}(t)/dt + (I_{r1} - I_{r3}) \omega_{r1}(t) \omega_{r3}(t) = 0$; $I_{r3} d\omega_{r3}(t)/dt + (I_{r2} - I_{r1}) \omega_{r2}(t) \omega_{r1}(t) = 0$; **If $I_{r1} = I_{r2}$:** $d\omega_{r3}(t)/dt = 0$; so $\omega_{r3} = \text{constant}$; by $C = \omega_{r3} (I_{r1} - I_{r3}) / I_{r1}$; we get: $d\omega_{r1}(t)/dt = C * \omega_{r2}(t)$; $d\omega_{r2}(t)/dt = -C * \omega_{r1}(t)$; derivate: $d^2\omega_{r1}(t)/dt^2 = C * d\omega_{r2}(t)/dt = -C^2 * \omega_{r1}(t)$; \Rightarrow **Simple harmonic oscillator**; $d^2\omega_{r2}(t)/dt^2 = -C * d\omega_{r1}(t)/dt = -C^2 * \omega_{r2}(t)$; $\omega_{r1}(t) = A * \sin(C * t)$; $\omega_{r2}(t) = A * \cos(C * t)$; $A = (|\omega_r(0)|^2 - \omega_{r3}(0)^2)^{1/2}$; {ensure $|\omega_r(0)| = (A^2 + \omega_{r3}(0)^2)^{1/2}$ } **If we define: World frame $[x, y, z]$ such that z align with L_w ; Connecting frame $[x', y', z']$ which is the result of rotation about z (by ϕ' rad/s), and then about x' (by θ' rad/s); Thus z, z', y' on same plane; Rotating frame $[1, 2, 3]$ which is the result of rotating the connecting frame about z' (by Ψ' rad/s); $\omega(t) = \phi'(t) \rightarrow \theta'(t) \rightarrow \Psi'(t)$; but $\omega_3 \neq \Psi'$; ϕ, θ, Ψ are not orthogonal & used to separate spin from precession; **If 3=body symmetry axis (circular body):** then x', y', z' are also principal axes; $I_3 = I_z$; & $I_1 = I_2 = I_x = I_y$; $L_z = |L_w| \cos(\theta(t)) = I_z \omega_z = I_3 \omega_3$; because $\omega_3 = \omega_{r3} = \text{constant}$; $\{d\omega_{r3}(t)/dt = 0\}$ $\theta(t) = \arccos(I_3 \omega_3 / |L_w|) = \text{constant}$; thus $\theta'(t) = 0$; $\theta(t) = \theta(0) = \theta$; $|L_w| = I_3 \omega_3 / \cos\theta$; $L_{y'} = |L_w| \sin\theta = I_y \omega_{y'} = I_1 \omega_{y'}$; $\omega_{y'} = |L_w| \sin\theta / I_1 = \phi'(t) * \sin\theta$; $\{\theta' = 0 \& \Psi'$ perpendicular to $\omega_{y'}$, so only ϕ' effect $\omega_{y'}$ \} $\phi'(t) = |L_w| / I_1 = D = \text{constant} = I_3 \omega_3 / (I_1 \cos\theta) = \text{relative precession rate}$; $\omega_x = \theta'(t) = 0$; $\omega_z = \omega_3 = \text{perpendicular to } \omega_1, \omega_2, \omega_{y'}$; so only $\omega_{y'}$ effect ω_1, ω_2 ; $\omega_2 = \omega_{y'} \cos\Psi = \phi'(t) \sin\theta \cos\Psi$; $\omega_1 = \omega_{y'} \sin\Psi = \phi'(t) \sin\theta \sin\Psi$; $\omega_1'(t) = \phi''(t) \sin\theta \sin\Psi + \phi'(t) \theta'(t) * \cos\theta \sin\Psi + \phi'(t) \sin\theta * \Psi'(t) \cos\Psi = 0 + 0 + \phi'(t) \sin\theta \Psi'(t) \cos\Psi$; $I_1 \omega_1' - (I_1 - I_3) \omega_3 \omega_2 = 0$; gives: $I_1 \phi'(t) \sin\theta \Psi'(t) \cos\Psi - (I_1 - I_3) \omega_3 \phi'(t) \sin\theta \cos\Psi = 0$; $\Psi'(t) = \omega_3 * (I_1 - I_3) / I_1 = C = \text{relative spin rate} = \text{constant}$; $\{I_{ri} = I_i$; ω_3 effected by Ψ' & ϕ' ; $\omega_3 = \text{spin} = \Psi'(t) + \phi'(t) \cos\theta$; $\Psi'(t) = L_w \cos\theta (1/I_3 - 1/I_1)\}$ **If the body symmetry axis is initially inclined by an angle θ from L_w (vertical), with an angular velocity ω_3 , then the motion of the plate is given by:** $\phi(t) = D * t = \omega_3 I_3 / (I_1 \cos\theta) * t$; $\theta(t) = \theta(0)$; $\Psi(t) = C * t$; $\{\Psi(0) = \phi(0) = 0\}$ $\Psi(t)$ represent rotation; $\phi(t)$ represent wobbling (nutaton=torque free precession) {if the body has no initial orientation, to find orientation in time t : rotate the body $C * t$ rad about its 3-axis, then rotate the body $D * t$ rad about $L = [I_{r1} \omega_{r1}(0); I_{r2} \omega_{r2}(0); I_{r3} \omega_{r3}(0)]$ (that now is not the world Z axis), L is $\theta = \arccos(I_3 \omega_3 / |L|)$; rad from the body 3-axis; $\phi' = \text{rate that body 3-axis precess about } L [\text{rad/s}]$ not depend on the rotation about the body 3-axis (Ψ), because Ψ don't move the body 3-axis; but Ψ depend on ϕ , each ϕ rotation rotate Ψ one cycle} {Observer in world frame would see the z' -axis (& ω) trace out a cone as it precesses about L ; the body cone (z' center) rolling along the space cone; ω is line where the 2 cones touch; ω also precess about z' }; **If the body symmetry axis has negligible thickness:** $I_{r1} = I_{r2} = \frac{1}{2} MR^2$; $I_{r3} = MR^2 = 2I_{r1}$; ($M = \text{mass}, R = \text{radius}$); $\Psi(t) = -\omega_3 * t$; $\phi(t) = 2 * \omega_3 / \cos\theta * t$; **If $\theta \rightarrow 0$:** $\phi'(t) = 2 * \omega_3$; {wobbling frequency approaches twice the rotation frequency, but the wobbling amplitude also decreases}.**



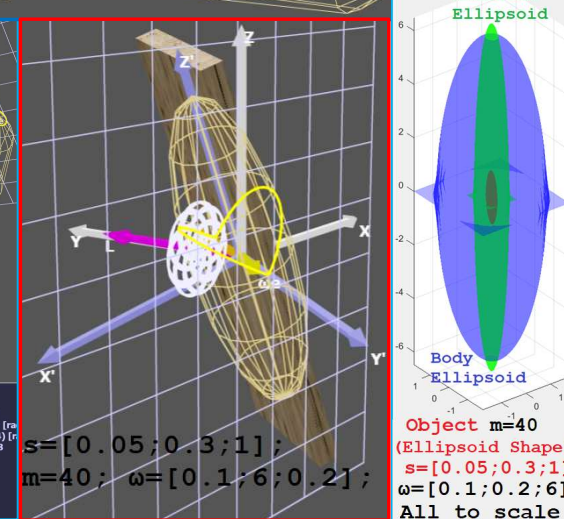
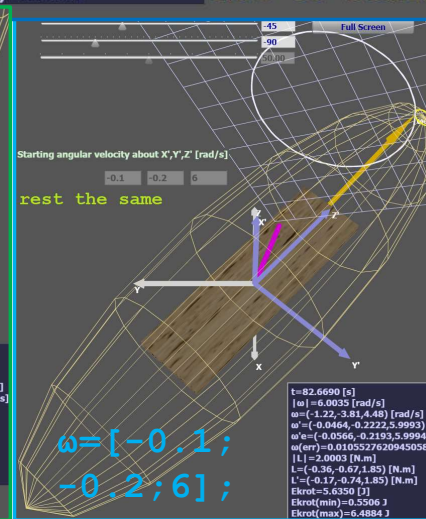
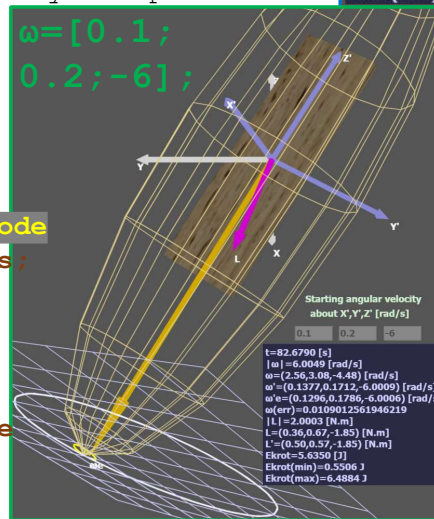
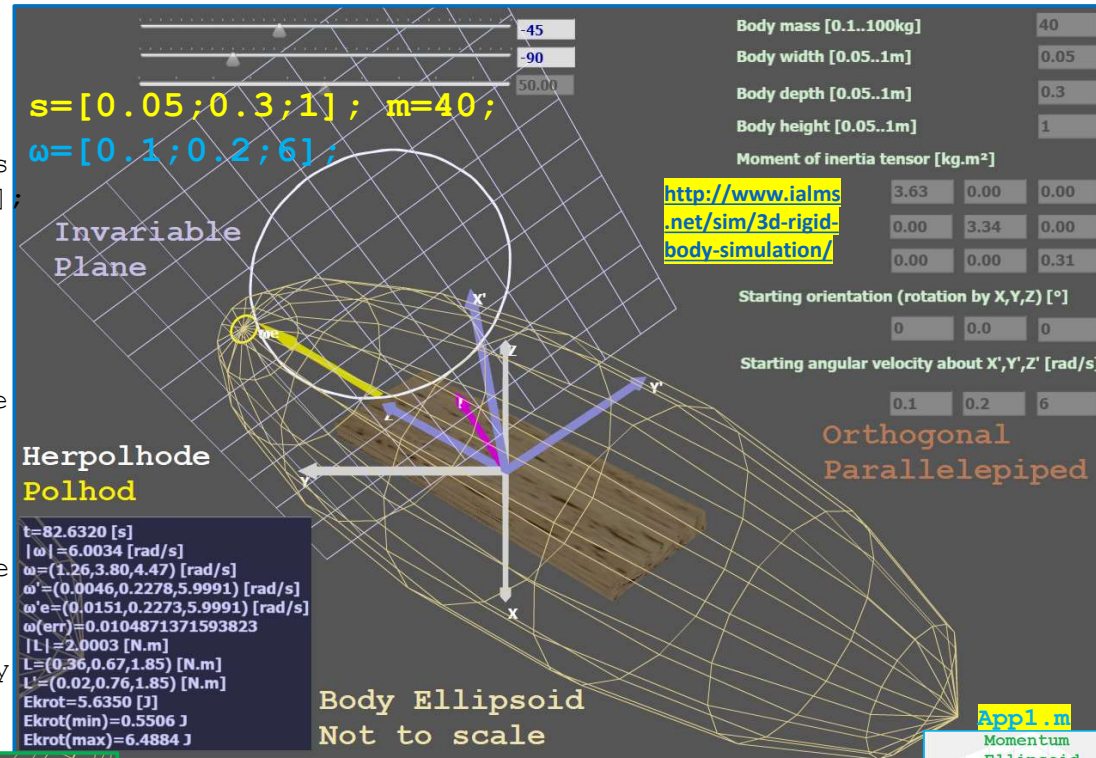
Appendix 1: General Rotational Motion Visualization

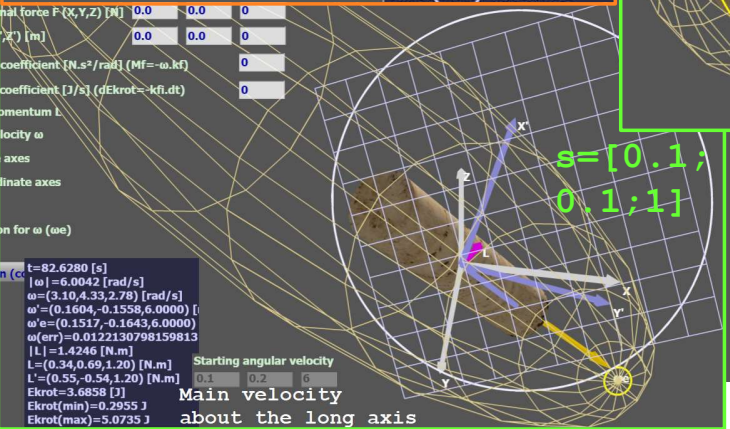
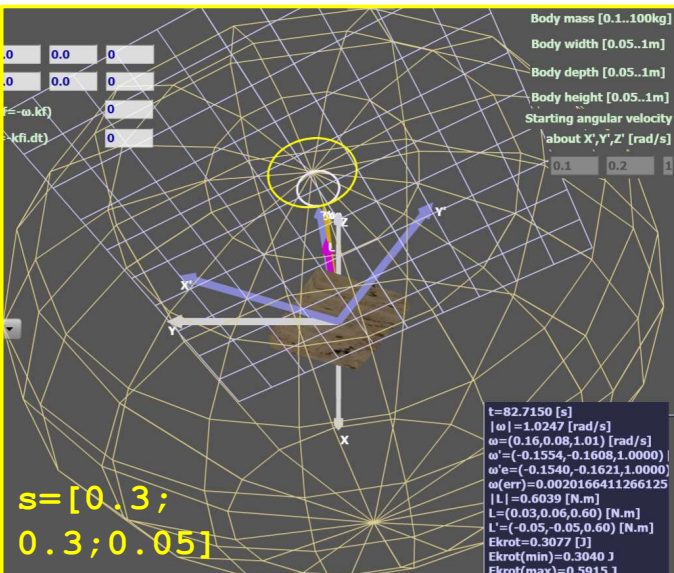
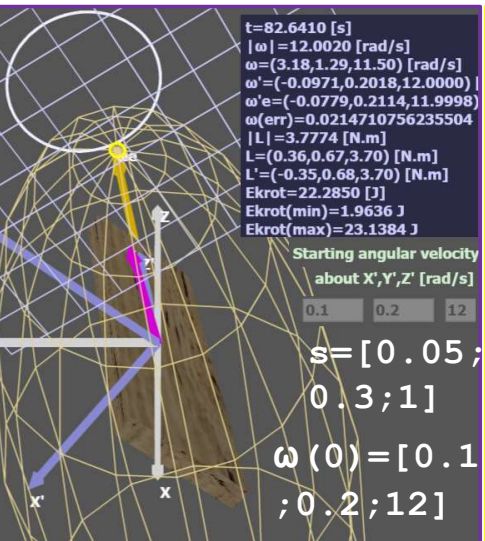
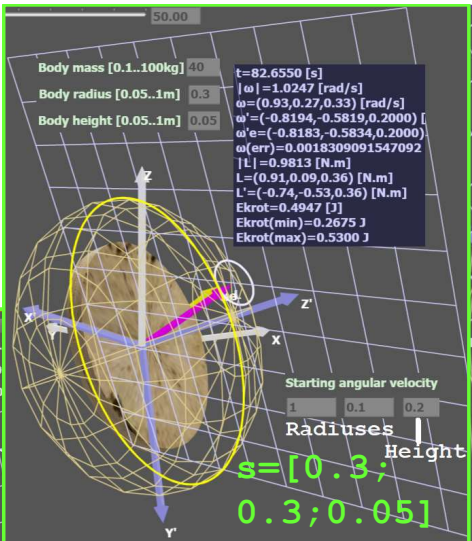
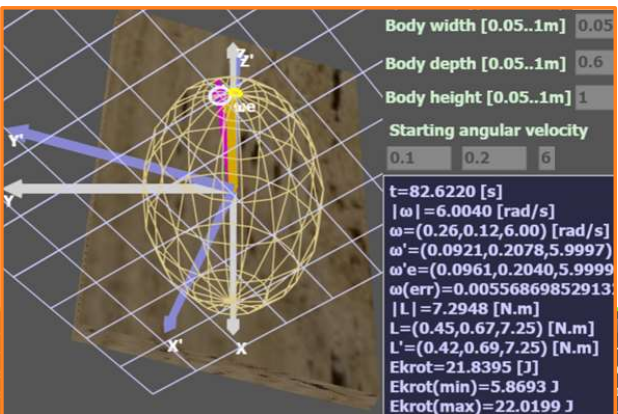
To visualize the rotational motion of a general object: find its principal axes of inertia (axes from center of mass where all products of inertia are 0; possible to find for any object; for ellipsoid its our s_1, s_2, s_3); Align these axes with world coordinates (x, y, z) , the object center of mass will not move and it's on $[0;0;0]$; $\omega = \omega$ at time 0; $m = \text{object mass}$; For ellipsoid object calculate: $I_1 = m \cdot (s_2^2 + s_3^2) / 5$; $I_2 = m \cdot (s_1^2 + s_3^2) / 5$; $I_3 = m \cdot (s_2^2 + s_1^2) / 5$; $T = I_1 \omega_1^2 + I_2 \omega_2^2 + I_3 \omega_3^2$; draw **body ellipsoid**: $1 = x^2 / (T/I_1) + y^2 / (T/I_2) + z^2 / (T/I_3)$; From now on, object orientation will be the same as the orientation of this body ellipsoid; Calculate: $L = (I_1 \omega_1)^2 + (I_2 \omega_2)^2 + (I_3 \omega_3)^2$; draw the **momentum ellipsoid**: $1 = x^2 / (L/I_1^2) + y^2 / (L/I_2^2) + z^2 / (L/I_3^2)$; Draw the **Polhode** on the body ellipsoid, the **Polhode** is the intersection curve of the body ellipsoid & the momentum ellipsoid. The point ω lie on the body ellipsoid, draw a normal to the body ellipsoid at that point (This **normal** = $(2/T) \cdot [I_1 \omega(1); I_2 \omega(2); I_3 \omega(3)]$), draw an **invariable plane** with the same normal at point ω (distance between $[0;0;0]$ & invariable plane = T/\sqrt{L}); To visualize the motion: roll without slip the body ellipsoid on the unchanged invariable plane, such that: the center of mass is unchanged, the curve traced out on the body ellipsoid by the points of contact with the plane is polhod & the curve traced out on the plane by the points of contact with body ellipsoid is the **Herpolhode**. While polhod is closed curve, herpolhode is an open curve meaning that rotation doesn't perfectly repeat itself.

Example: Rotational motion visualization of orthogonal parallelepiped sizes $s = [0.05; 0.3; 1]$; $m = 40$; ($I_{i \text{ orthogonal_parallelepiped}} = I_{i \text{ Ellipsoid}} \cdot 5/12$);

For various ω ; in the **green/blue** cases the **Polhode** finish to roll in the invariable plane in 1.147s; & in 11s the **Herpolhode** finish to create the first almost closed circle in the invariable plane; The **Herpolhode** curve is almost a closed circle but it's never exactly repeat itself.

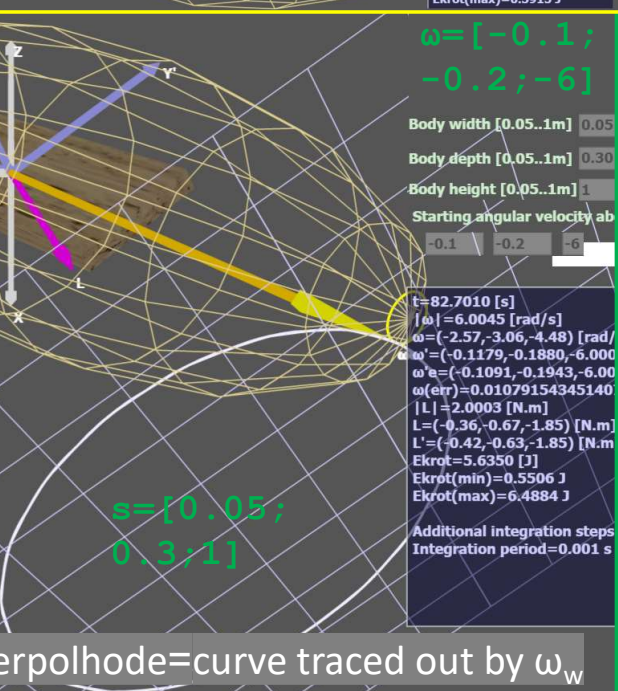
The **red** case show rotation about an intermediate axis, which is unstable, meaning that the direction of motion can vary a lot.



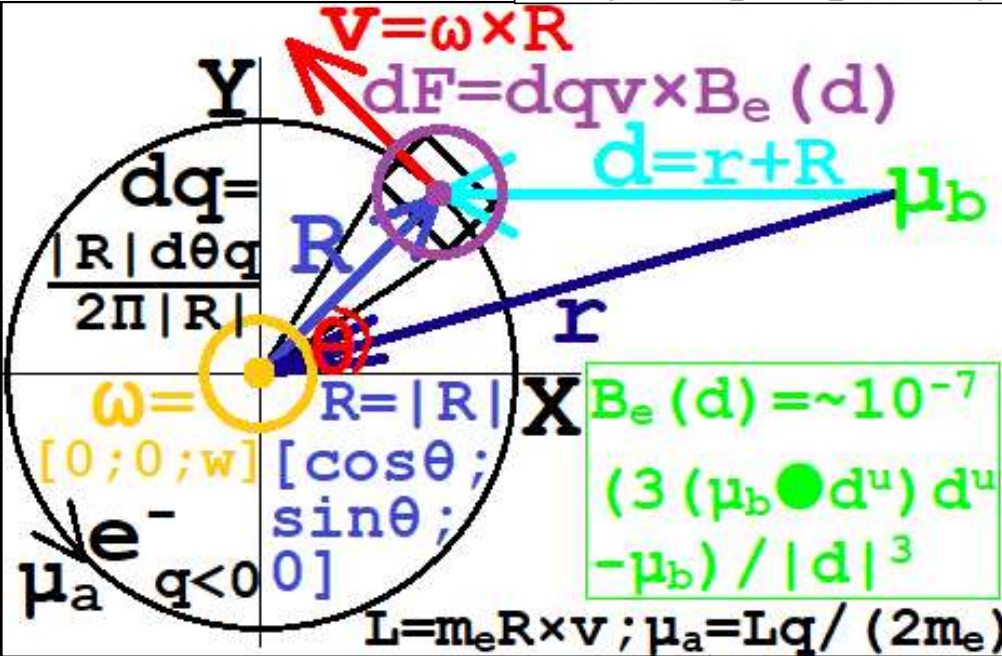
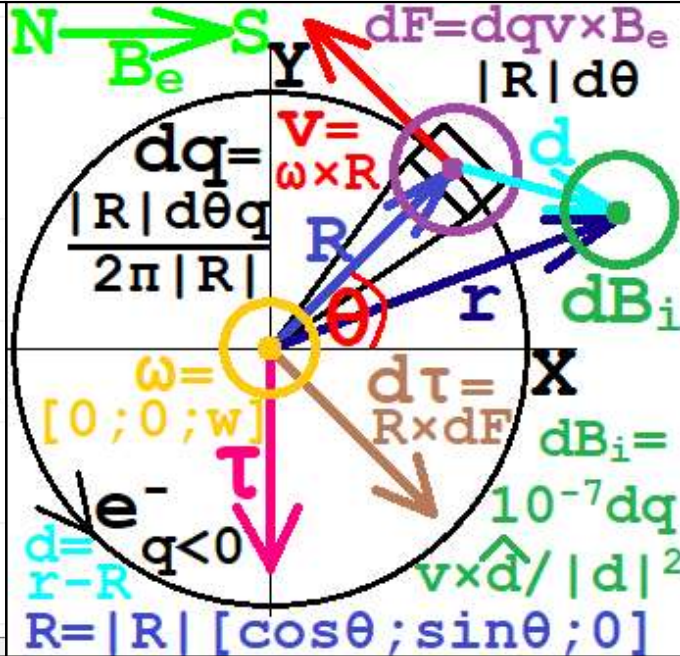
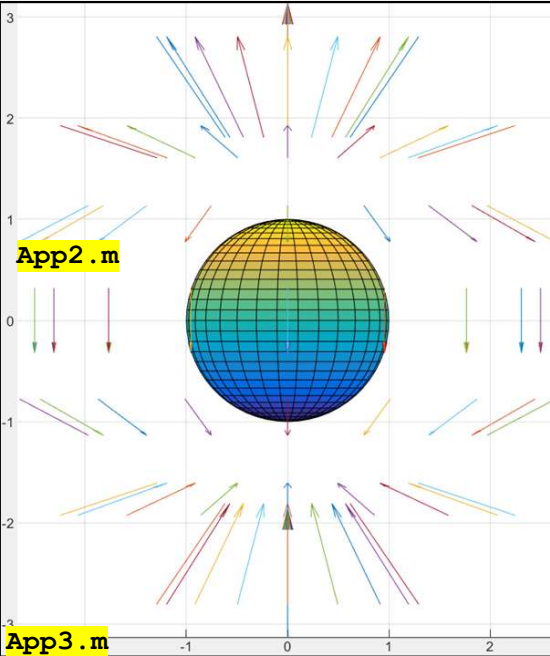
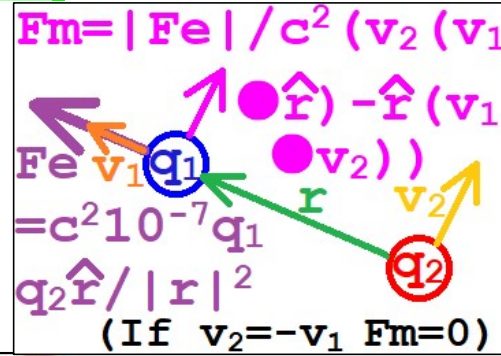


Torque free precession=Rotation=Wobble=
 When Axially Symmetric bodies(football/disks) rotate about axis that is between the axis of symmetry & other axis; for football (s₁=s₂, s₃):
 $r = (s_3^+ \text{ cycle time}) / (s_2^+ \text{ cycle time}) = 5/3$
 Increasing s₂⁺ cycle time increases s₃⁺ cycle time by the same proportion.

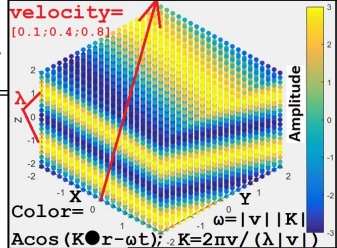
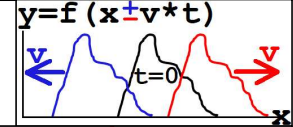
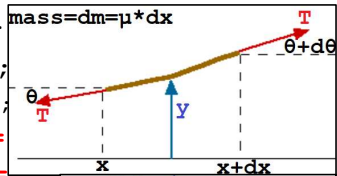
parallelepiped Mass=40; Sizes:	ω	Herpolhode ~cycle (s)	Polhode cycle (s)
[0.05; 0.3; 1]	[0.1; 0.2; 6]	11	1.147
[0.05; 0.3; 1]	[0.1; 0.2; 12]	5.8	0.54
[0.05; 0.3; 1]	[0.1; 0.2; 24]	2.9	0.277
[0.05; 0.6; 1]	[0.1; 0.2; 6]	3.2	1.53
[0.3; 0.3; 1]	[0.1; 0.2; 6]	6.12	1.25
[0.3; 0.3; 0.05]	[0.1; 0.2; 1]	3.3	6.6
Cylinder[0.3; 0.3; 0.05]	[1; 0.1; 0.2]	=5.82real	32
Cylinder[0.1; 0.1; 1]	[0.1; 0.2; 6]	=15.25real	1.1



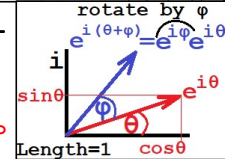
Classical Electro-Magnetic Theory: μ of stationary e/e^+ $\{\mu \leftarrow F_{a,b} \& \tau_{a,b} \leftarrow L_w\}$ can be simulate by q that move in circular path & using: $F_e = q_1 q_2 r^u / (\epsilon_0 4\pi |r|^2)$; $F_m = qv \times B_e$; $B_i = 10^{-7} qv \times r^u / |r|^2$; we can calculate the general equations: $\tau = \mu \times B_e$; {if e (mass= m_e) move in circle ($\omega = [0; 0; w]$; Radius) $L = m_e R \times v$; is in B_e , its circular path feels $\tau = \int dt$ ($\theta = 0$ to 2π) $= \frac{1}{2} w |R|^2 [B_{e2}; -B_{e1}; 0] = \mu \times B_e$; $\mu = Lq / (2m_e)$; $\omega = |\omega| [\sin\theta \cos\phi; \sin\theta \sin\phi; \cos\theta]$; Perpendicular: $R_0 = |R| [\sin(\theta + \pi/2) \cos\phi; \sin(\theta + \pi/2) \sin\phi; \cos(\theta + \pi/2)]$; Rodrigez: $R = R_0 \cos\theta + (\omega^u \times R_0) \sin\theta + \omega^u (\omega^u \cdot R_0) (1 - \cos\theta)$ Thus, for e ($q < 0$) / e^+ ($q > 0$) μ & L antiparallel/parallel; This τ tends to line up μ with B_e , highest/lowest energy configuration is when μ is antiparallel/parallel to B_e ; $\{\tau = \mu \times B_e$; $|\tau| = |\mu| |B_e| \sin\alpha$; $\alpha =$ angle between μ & B_e ; $W_{max} = \int |\mu| |B_e| \sin\alpha$ ($\alpha = \pi$ to 0) $= 2|\mu| |B_e|$ $B_i = \sim 10^{-7} (3(\mu \cdot r^u) r^u - \mu) / |r|^3$; {e orbit generate $B_i = \int dB_i$ ($\theta = 0$ to 2π) at r ; If $r \gg R$ $|B_i| = \sim 10^{-7} |\mu| (1 + 3\cos^2\alpha)^{1/2} / |r|^3$; $\{\alpha =$ between r & μ $|B_i|_{max} = 2|B_i|_{min}$; τ exerted by μ_a on $\mu_b = \tau_b = \mu_0 / (4\pi |r|^3) (3(\mu_a \cdot r^u) \mu_b \times r^u - \mu_b \times \mu_a)$; $\tau_a = \mu_0 / (4\pi |r|^3) (3(\mu_b \cdot r^u) \mu_a \times r^u - \mu_a \times \mu_b)$; Force exerted by μ_a on $\mu_b = F_b = 3\mu_0 / (4\pi |r|^4) (\mu_b (\mu_a \cdot r^u) + \mu_a (\mu_b \cdot r^u) + r^u (\mu_a \cdot \mu_b) - 5r^u (\mu_a \cdot r^u) (\mu_b \cdot r^u)) = \nabla (\mu_b \cdot B_a)$; {The circular path when $r \gg R$ feels $F = \int dF$ ($\theta = 0$ to 2π); $r =$ from a to b ; $\mu_0 = 4\pi 10^{-7}$; $B_a = B_a(r) = 10^{-7} (3(\mu_a \cdot r^u) r^u - \mu_a) / |r|^3$; ∇ on variable r ; & then Placement of r $F_a = -F_b$; If B_e is uniform & doesn't depend on d $F_a = F_b = 0$; $\{W = \int \nabla (\mu_a \cdot B_b(r)) dr = \mu_a \cdot B_b(r_2) - \mu_a \cdot B_b(r_1)\}$ If 2 same μ e's stand along μ : $|F_e| = q_1 q_2 / (\epsilon_0 4\pi |r|^2)$; $|F_m| = |\mu_a| |\mu_b| 6\mu_0 / (4\pi |r|^4)$; $|F_e| / |F_m| = |r|^2 / 3 (cm_e / (\hbar g_s))^2 = |r|^2 / 3 (cm_e / (\hbar g_s))^2 = 4.460352055599433 \times 10^{24} |r|^2$; $\{S_z = \pm \hbar / 2$; $\mu_z = S_z g_s q / (2m_e)$; $\epsilon_0 \mu_0 c^2 = 1$; $\mu_0 = 4\pi 10^{-7}$ $|B_{ip}|_{max} = \sim 2 \times 10^{-7} |\mu| / |r|^3 = 10^{-7} \hbar g_s q / (2m_e |r|^3)$; $|B_{iv}|_{max} = 10^{-7} q |v| / |r|^2$; $|B_{iv}|_{max} / |B_{ip}|_{max} = |v| |r| 2m_e / \hbar g_s = 8627.9872780518003976252304522023 |r| |v|$;



If $y(x,t)$ describe the y component of tiny ($dx \rightarrow 0$) piece of an almost horizontal rope ($\theta \rightarrow 0$) with mass density μ ($dm = \mu * dx$) it must satisfy: $\partial^2 y / \partial t^2 = T_R / \mu * \partial^2 y / \partial x^2$; 1D wave equation (1Dwe); $\{F = ma; F_y = \mu dx \partial^2 y / \partial t^2; T_R = \text{Rope Tension}; \theta \rightarrow 0; \sin(\theta) = \theta; F_y = -T_R \sin(\theta) + T_R \sin(\theta + d\theta) = -T_R \theta + T_R(\theta + d\theta) = T_R d\theta; \tan(\theta) = \partial y / \partial x; \partial^2 y / \partial x^2 = d \tan(\theta) / dx = \theta' / \cos(\theta)^2 = d\theta / dx; \theta \rightarrow 0; \cos(\theta)^2 = 1; dx * \partial^2 y / \partial x^2 = d\theta; F_y = \mu * dx \partial^2 y / \partial t^2 = T_R d\theta = T_R dx * \partial^2 y / \partial x^2\}$ If $w^2 = T_R / \mu * k^2$; Any $f(k * x + w * t + ph)$ satisfy 1Dwe $= \partial^2 y / \partial t^2 = |v|^2 \partial^2 y / \partial x^2$; $\{T_R = ma [kg * m / s^2]; \mu = [kg / m]; (T_R / \mu)^{1/2} = |v| [m / s]; v = -w / k\}$ wavelength $= \lambda$ & period T defined such that $f(x, t) = f(x + \lambda, t) = f(x, t + T)$; frequency $= v = \lambda / T$; Amplitude $= A$; Phase $= \phi$; $\{f(x, t) = A e^{i(k * x + w * t + \phi)}\}$; f shape travel along x with v ; $\lambda = 2\pi / k$; $T = 2\pi / w$ $\Delta f = \partial^2 f / \partial x^2 + \partial^2 f / \partial y^2 + \partial^2 f / \partial z^2$; Any $f = f(k * r - w * t + \phi)$ $\{f$ shape travel along $k\}$ $|v|^2 = c^2 = w^2 / |k|^2$; satisfy $\partial^2 f / \partial t^2 = c^2 * \Delta f$; 3Dwe; If g, f satisfy 3Dwe (same c) $f + g, f * g, af + bg$ also; superposition principle; Thus, if a particle



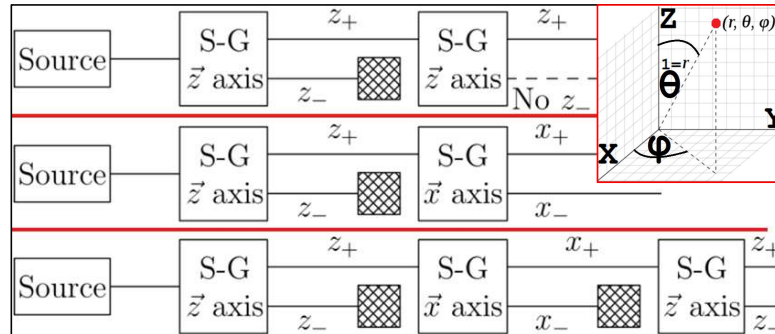
is a wave, it can be analyzed as a linear combination of $\Psi(r, t) = C * e^{i * (K * r - \omega * t)}$; waves with the same $|v|$; where $C = A * e^{i\phi}$; $K = 2\pi v / (\lambda |v|)$; $\omega = |v| |K|$; $|p| = \hbar / \lambda$; $\hbar = \hbar / (2\pi)$; true also for electrons & also for any particle; $E = KE + PE = m * |v|^2 / 2 + U = |p|^2 / (2 * m) + U$; $\{p = mv; PE = U\}$ by frequency $= f = |v| / \lambda$; $E = hf$; $WE = -\hbar^2 / (2m) * \Delta \Psi + U \Psi = i \hbar \partial \Psi / \partial t$; $\{\Delta \Psi = -\Psi |K|^2 = -\Psi (2\pi / \lambda)^2 = -\Psi (|p| / \hbar)^2; \Psi |p|^2 = -\hbar^2 \Delta \Psi; |p|^2 = 2m(E - U); \Psi 2m(E - U) = -\hbar^2 \Delta \Psi; \omega = |v| |K| = |v| 2\pi / \lambda = f 2\pi = 2\pi E / \hbar = E / \hbar; \partial \Psi / \partial t = -i \omega C e^{i(K * r - \omega t)} = -i \omega \Psi = -i \Psi E / \hbar; -\hbar / (i \Psi) \partial \Psi / \partial t = E\}$ Schrodinger equation (s.e); If $z = a + ib$; $z^* = a - ib$; $|z|^2 = a^2 + b^2 = z z^*$; If Ψ is normalized such that $\iiint |\Psi(r, t)|^2 * dr_x * dr_y * dr_z = \int_v |\Psi|^2 * dv = 1$; (each \int from $-\infty$ to ∞) then: $|\Psi(r_1, t_1)|^2 * dv$ can describe the probability that a particle that is measured at time t_1 exists at position r_1 (if the particle is measured at time t it must exist somewhere; $|\Psi|^2 = \text{probability density}$); By s.e: Knowing $\Psi(r, t_0)$ determined Ψ at all times. if Ψ_1, Ψ_2 are solutions, $a * \Psi_1 + b * \Psi_2$ is a solution ($a, b = \text{complex}$); Ψ_1, Ψ_2 equivalent if $\Psi_1 = a * \Psi_2$; $H^0 = -\hbar^2 / (2m) \Delta + U$; $H^0 \Psi = \Psi E = i \hbar \partial \Psi / \partial t$; $K = p / \hbar$; $\Psi = C e^{i(K * r - \omega t)} = C e^{i(K * r)} e^{-i \omega t} = C e^{i(K * r)} e^{-i E t / \hbar} = \Psi(r) U(t) = C e^{i / \hbar (p * r - E t)}$; $\nabla f = [\partial f / \partial x; \partial f / \partial y; \partial f / \partial z]$; $\nabla \Psi = i / \hbar * \Psi * p$; Momentum operator $= P^0 = \hbar / i \nabla = -i \hbar \nabla$; $P^0 \Psi = p \Psi$; Position operator $= r^0$; $r^0 \Psi = r \Psi$; Angular momentum ($r \times p$) operator $= L^0$



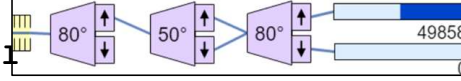
$= -i \hbar r \times \nabla = -i \hbar [y \partial / \partial z - z \partial / \partial y; z \partial / \partial x - x \partial / \partial z; x \partial / \partial y - y \partial / \partial x]$; $\{a \times b = [a_y b_z - a_z b_y; a_z b_x - a_x b_z; a_x b_y - a_y b_x]\}$ $L^0 \Psi = L \Psi$; Kinetic energy ($mv^2 / 2 = p^2 / (2m)$) operator $= K^0 = (-i \hbar * \nabla)^2 / (2m) = -\hbar^2 \nabla^2 / (2m)$; $K^0 \Psi = K \Psi$; Potential energy ($U = U(x, y, z)$) operator $= U^0$; $U^0 \Psi = U \Psi$; Commutator $= [O_1, O_2] = O_1 O_2 f - O_2 O_1 f = 0$; $[X, P_x] = -X i \hbar \partial / \partial x f + i \hbar \partial / \partial x X f = -X i \hbar f' + i \hbar (f + X f') = i \hbar f$; $[L_x, L_y] = i \hbar L_z$; $[L_z, L_x] = i \hbar L_y$; $[L_y, L_z] = i \hbar L_x$; If we can measure 2 things at the same time then their commutators must equal 0 (their operators must be able to act simultaneously on the same state) Thus we can't measure X & p_x, L_x & L_y, L_z & L_x, L_y & L_z at the same time; Heisenberg's uncertainty principle; functions are just infinite dimensional vectors

Dirac notation: Instead of $\Psi(x)$ we write vector with x dimensions (for position $x = \infty$) with values $\Psi(x)$ in each dimension $= |\Psi\rangle = \sum \Psi(x) |x\rangle = [\Psi(-\infty) \dots \Psi(x - dx); \Psi(x); \Psi(x + dx) \dots \Psi(\infty)]$; $|x\rangle = \text{basis vector}$ (e.g: $[1; 0; 0 \dots], [0; 1; 0; \dots] \dots$) while $|a\rangle = \text{Ket Vector} = [a_1; a_2; \dots]$; $\langle a| = \text{Bra Vector} = [a_1^*, a_2^*, \dots] = (|a\rangle)^T$; If $|x\rangle = \text{basis vector}$ then $\Psi(x) = \langle x | \Psi \rangle = 0 * \Psi(-\infty) + \dots + 1 * \Psi(x) + \dots + 0 * \Psi(\infty)$; $\langle \Psi | \Psi \rangle = \Psi^*(-\infty) * \Psi(-\infty) + \dots + \Psi^*(\infty) * \Psi(\infty) = 1$; For any operator O : $O \Psi(x) = g \Psi(x)$; $\hat{O} |\Psi\rangle = g |\Psi\rangle$; $g = \text{eigenvalue}$ {real; result}; $|\Psi\rangle = \text{eigenvector}$ {state of system (2 or more)}; $\hat{O} = \text{Hermitian}$ {Observable; measurable} {Matrix M is Hermitian if $M = (M^*)^T$; Matrix M is Hermitian if its eigenvalues are real} The measured spin along axis (z) $= \pm \hbar, -\hbar$; (eigenvalue) lets define their eigenvectors as $|+Z\rangle = [1; 0], |-Z\rangle = [0; 1]$ & find their operator $\hat{S}_z = [A, B; C, D]$; by $\hat{S}_z [1; 0] = \hbar / 2 [1; 0] = [A; C]$; $\hat{S}_z [0; 1] = -\hbar / 2 [0; 1] = [B; D]$; thus $\hat{S}_z = \frac{1}{2} \hbar \sigma_z$; $\sigma_z = [1, 0; 0, -1]$; Assuming spin operator is like L operator $[\hat{S}_z, \hat{S}_x] = i \hbar \hat{S}_y$; $[\hat{S}_y, \hat{S}_z] = i \hbar \hat{S}_x$; $[\hat{S}_x, \hat{S}_y] = i \hbar \hat{S}_z$; using this we find $\hat{S}_x = \frac{1}{2} \hbar \sigma_x$; $\sigma_x = [0, 1; 1, 0]$; $\hat{S}_y = \frac{1}{2} \hbar \sigma_y$; $\sigma_y = [0, -i; i, 0]$; & their eigenvectors $|+X\rangle = [2^{-0.5}; 2^{-0.5}]$; $|-X\rangle = [2^{-0.5}; -2^{-0.5}]$; $|+Y\rangle = [2^{-0.5}; i * 2^{-0.5}]$; $|-Y\rangle = [2^{-0.5}; -i * 2^{-0.5}]$; $\sigma = [\sigma_x; \sigma_y; \sigma_z]$; $\hat{S} = \frac{1}{2} \hbar \sigma$; $|\hat{S}| = (\hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2)^{1/2} = \frac{1}{2} \hbar 3^{1/2} / 2$; ($I = \text{Identity}$); Position operator has ∞ eigenvectors & ∞ eigenvalues; $\langle \Psi | \hat{O} | \Psi \rangle = \sum P(g_i) g_i = \text{Average value}$ $\langle -Y | \hat{S}_y | -Y \rangle = -\frac{1}{2} \hbar$

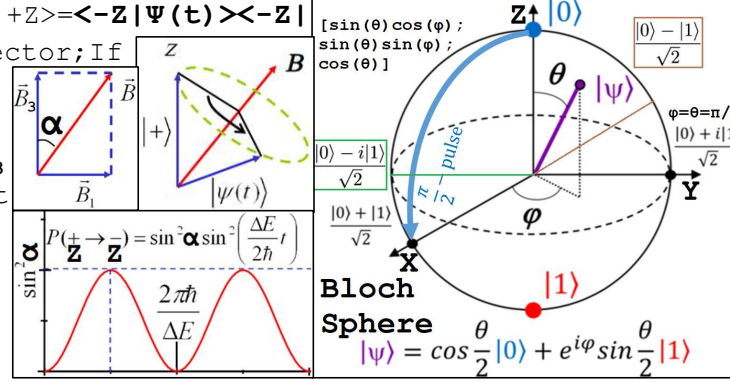
Unit direction $\mathbf{u} = [\sin\theta\cos\phi; \sin\theta\sin\phi; \cos\theta]$; $\hat{S}_n = \mathbf{u} \cdot \hat{S} = \frac{\hbar}{2} [\cos\theta, \sin\theta(\cos\phi - i\sin\phi); \sin\theta(\cos\phi + i\sin\phi), -\cos\theta]$; its eigenvalues $+\frac{1}{2}\hbar, -\frac{1}{2}\hbar$ & eigenvectors $|+n\rangle = [e^{-i\phi/2}\cos(\theta/2); e^{i\phi/2}\sin(\theta/2)]$; $|-n\rangle = [-e^{-i\phi/2}\sin(\theta/2); e^{i\phi/2}\cos(\theta/2)]$; {or $|+m\rangle = [\cos(\theta/2); e^{i\phi}\sin(\theta/2)]$, $|-m\rangle = [-e^{-i\phi}\sin(\theta/2); \cos(\theta/2)]$; are spin up, down point along θ, ϕ {by $[V, D] = \text{eig}(S_n)$ & normalize by $\mathbf{u} = [V(1,2); V(2,2)]$; $\mathbf{nu} = \mathbf{u}/\text{norm}(\mathbf{u})$..} {if there is spin $\frac{1}{2}$ particle in state $|+n\rangle$ probability of finding it in spin up state $|+Z\rangle$ is $\langle +Z | +n \rangle \langle +Z | +n \rangle^* = \cos^2(\theta/2)$ } **If a particle prepared as $|\Psi\rangle$ to see how it will be measured in $|+n\rangle$ we can write it with base change: $|\Psi\rangle = [e^{-i\beta/2}\cos(\alpha/2); e^{i\beta/2}\sin(\alpha/2)] = \mathbf{x} [e^{-i\phi/2}\cos(\theta/2); e^{i\phi/2}\sin(\theta/2)] + \mathbf{y} [-e^{-i\phi/2}\sin(\theta/2); e^{i\phi/2}\cos(\theta/2)]$; & solve for \mathbf{x} ; $\mathbf{x} = \langle +n | \Psi \rangle$; The probability**



that it'll be measured in $|+n\rangle$ is $\mathbf{x}^* \mathbf{x}$; {If $\beta = \phi = 0$; $\mathbf{x}^* \mathbf{x} = \langle +n | \Psi \rangle \langle +n | \Psi \rangle^* = \cos^2((\theta - \alpha)/2)$; $\langle +Y | +X \rangle \langle +Y | +X \rangle^* = \frac{1}{2}$ }
If a particle prepared as $|\Psi\rangle$ & than as $|+n\rangle$ its state is $\mathbf{g} = (\langle +n | \Psi \rangle) | +n \rangle$ & the probability that it'll be measured in $|+m\rangle$ is $\langle +m | \mathbf{g} \rangle \langle +m | \mathbf{g} \rangle^*$; If a particle prepared as $|\Psi\rangle$ & than we use Stern Gerlach machine in direction \mathbf{n} (SG_n) & we take both $|+n\rangle$ & $|-n\rangle$ to SG_m the probability that it'll be measured in $|+m\rangle$ is $\langle +m | \Psi \rangle \langle +m | \Psi \rangle^*$ (mixed state is like we didn't use SG_n); The spin rotation operator for α rad rotation about unit vector $\mathbf{u} = [\cos(\alpha/2) - iu_3\sin(\alpha/2), -\sin(\alpha/2)(u_2 + iu_1), -\sin(\alpha/2)(u_1 - iu_2), \cos(\alpha/2) + iu_3\sin(\alpha/2)] = \exp(-i\alpha/2 \boldsymbol{\sigma} \cdot \mathbf{u})$; { $e^{\mathbf{x}} = \mathbf{x}^0 + \mathbf{x}^1 + \frac{1}{2}\mathbf{x}^2 + \dots + \mathbf{x}^n/n!$; $(i\boldsymbol{\alpha} \cdot \mathbf{S}/\hbar)^n = \text{matrix multiplication}$; $\mathbf{x}^0 = \mathbf{I}$ } **In weak \mathbf{B} : $H^0 = |\mathbf{P}^0|^2 / (2m) + U + \mu_B (L^0 / \hbar + \boldsymbol{\sigma}_g / 2) \cdot \mathbf{B}$; { $\mathbf{P}^0 = -i\hbar\nabla$; $|\mathbf{P}^0|^2 = -\hbar^2\Delta$; $\mu_B = |q|\hbar / (2m_e)$; $[J/T] \gamma = g_s q / (2m_e)$; $\mu_B = |q|\hbar / (2m_e c)$; [erg/G (CGS)]} **Focusing on spin contribution alone: $H^0 = -\boldsymbol{\mu} \cdot \mathbf{B} = -\gamma \hat{S} \cdot \mathbf{B} = -\gamma \hbar \boldsymbol{\sigma} \cdot \mathbf{B} = -\frac{1}{2}\gamma \hbar (\sigma_1 B_1 + \sigma_2 B_2 + \sigma_3 B_3) = \frac{1}{2}\hbar [\omega_3, -i\omega_2 + \omega_1; \omega_1 + i\omega_2, -\omega_3]$; $\omega_1 = -B_1\gamma$; $\omega_2 = -B_2\gamma$; $\omega_3 = -B_3\gamma$; its eigenvalues: $E_p = \frac{1}{2}\hbar(\omega_1^2 + \omega_2^2 + \omega_3^2)^{1/2}$; $E_m = -E_p$; represent electron spin up, down energy; {if $\alpha = \text{between } \mu \& B$; $E = \int |\mu| |B| \sin\alpha = -|\mu| |B| \cos\alpha = -\boldsymbol{\mu} \cdot \mathbf{B}$; Electron spin up/down is in same/opposite direction to \mathbf{B} ; for electron \mathbf{S} & $\boldsymbol{\mu}$ opposite, thus electron spin up has μ opposite to \mathbf{B} & positive energy ($E = -|\mu| |B|$)} **$\mathbf{U}(t) = \text{Time evolution operator} = \exp(-iHt/\hbar) = e^{i\gamma \mathbf{S} \cdot \mathbf{B} t / \hbar} = \text{Rotation operator}$ (with $\alpha = -\gamma |B| t$; $\mathbf{u} = \mathbf{B} / |B|$); For 2π rotation about any axis $|+n\rangle \rightarrow -|+n\rangle$; For $|+n\rangle \rightarrow |+n\rangle$ we need 4π rotation {spin 1/2 particle requires 2π & π ; No experiment has yet been verify the 2π rotation predictions}******



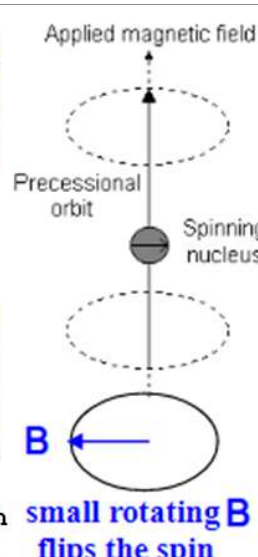
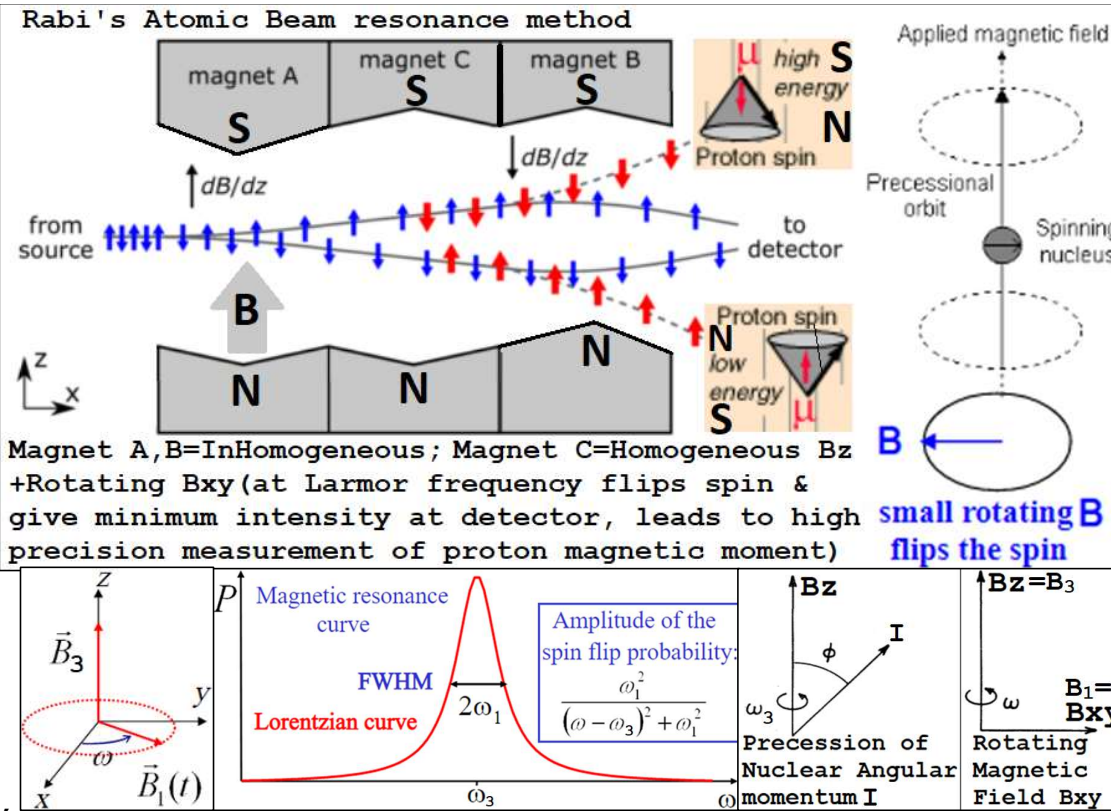
■ For $\mathbf{B} = [B_1; 0; B_3]$; $H = \frac{1}{2}\hbar [\omega_3, \omega_1; \omega_1, -\omega_3]$; $\sin\alpha = B_1 / |B| = B_1 / (B_1^2 + B_3^2)^{1/2} = \omega_1 / (\omega_1^2 + \omega_3^2)^{1/2}$; $\cos\alpha = \omega_3 / (\omega_1^2 + \omega_3^2)^{1/2}$; $H = \frac{1}{2}\hbar (\omega_1^2 + \omega_3^2)^{1/2} [\cos\alpha, \sin\alpha; \sin\alpha, -\cos\alpha]$; Eigenstate: $|+\lambda\rangle = \cos(\alpha/2) |+Z\rangle + \sin(\alpha/2) | -Z\rangle$; & $E_p = \frac{1}{2}\hbar (\omega_1^2 + \omega_3^2)^{1/2}$; $|-\lambda\rangle = \sin(\alpha/2) |+Z\rangle - \cos(\alpha/2) | -Z\rangle$; & $E_m = -E_p$; Rearrange $|+Z\rangle = \cos(\alpha/2) |+\lambda\rangle + \sin(\alpha/2) |-\lambda\rangle$; $| -Z\rangle = \sin(\alpha/2) |+\lambda\rangle - \cos(\alpha/2) |-\lambda\rangle$; $|\Psi(0)\rangle = | +Z\rangle$; Time evolves: $|\Psi(t)\rangle = \exp(-iE_p t/\hbar) \cos(\alpha/2) |+\lambda\rangle + \exp(-iE_m t/\hbar) \sin(\alpha/2) |-\lambda\rangle$; Spin flip probability $p = \text{probability to get } | -Z\rangle \text{ from time evolve } | +Z\rangle = \langle -Z | \Psi(t) \rangle \langle -Z | \Psi(t) \rangle^* = \sin^2(\alpha) \sin^2((E_p - E_m)t / (2\hbar)) = \omega_1^2 / (\omega_1^2 + \omega_3^2) \sin^2(t(\omega_1^2 + \omega_3^2)^{1/2} / 2)$; {with same basis vector; If $B_1 = 0$; $\omega_1 = 0$; $p = 0$; If $B_3 = 0$; $\omega_3 = 0$; $p = \sin^2(t\omega_1/2)$; If $t = 2\pi\hbar / (E_p - E_m)$; $p = 0$; If $t = \pi\hbar / (E_p - E_m)$; $p = \sin^2(\alpha)$ }
 ■ For $\mathbf{B} = [0; 0; B_3]$; $H = \frac{1}{2}\hbar [\omega_3, 0; 0, -\omega_3]$; eigenstate: $|+\lambda\rangle = | +Z\rangle = [1; 0]$; & $E_p = \frac{1}{2}\hbar\omega_3$; $|-\lambda\rangle = | -Z\rangle = [0; 1]$; & $E_m = -\frac{1}{2}\hbar\omega_3$; If $|\Psi(0)\rangle = | +n\rangle$; Time evolves: $|\Psi(t)\rangle = [\exp(-i(\varphi + \omega_3 t)/2) \cos(\theta/2); \exp(i(\varphi + \omega_3 t)/2) \sin(\theta/2)]$; { $i\hbar\partial\Psi/\partial t = i\hbar[-i\omega_3/2\Psi_1; i\omega_3/2\Psi_2] = H^0\Psi = E\Psi$; if $\Psi = \text{eigenvector of } H^0$ than E^*E^* doesn't change with time (& $\Psi = \text{stationary state}$)} **θ between the spin & \mathbf{B} stays constant while φ increased by $\omega_3 t$ (spin precession frequency is independent of θ); corresponds to Bloch vector precessing around \mathbf{B} with angular frequency (angular velocity) of ω_3 {Bloch vector $= [\sin(\theta)\cos(\phi); \sin(\theta)\sin(\phi); \cos(\theta)]$ correspond to $|+n\rangle$; If $\theta \rightarrow \theta + \pi$; $|+n\rangle \rightarrow | -n\rangle$ }**



$\langle +Z | |\Psi(t)\rangle \langle +Z | |\Psi(t)\rangle = \cos^2(\theta/2)$; $\langle -Z | |\Psi(t)\rangle \langle -Z | |\Psi(t)\rangle = \sin^2(\theta/2)$

■ If B_1 rotating about B_3 : $B = [B_1 \cos(\omega t); B_1 \sin(\omega t); B_3]$; $\omega_1 = -B_1 \gamma$; $\omega_3 = -B_3 \gamma$; $H^0 = \frac{1}{2} \hbar [\omega_3, \omega_1 \exp(-i\omega t); \omega_1 \exp(i\omega t), -\omega_3]$; $\Psi_r = \Psi$ as viewed from the rotating frame; $\Psi = [\Psi_1; \Psi_2] = [\Psi_{r1} \exp(-i\omega t/2); \Psi_{r2} \exp(i\omega t/2)]$; {Rotation operator: $u = [0; 0; 1]; \alpha = \omega t$ } $\Psi_r = [\exp(i\omega t/2), 0; 0, \exp(-i\omega t/2)] \Psi = [\Psi_{r1}; \Psi_{r2}]$; { M^{-1} } Rewrite $i\hbar \partial \Psi / \partial t = H^0 \Psi$; $i\hbar \partial \Psi / \partial t = i\hbar [\partial \Psi_1 / \partial t; \partial \Psi_2 / \partial t] = i\hbar [\partial \Psi_{r1} / \partial t \exp(-i\omega t/2) - i\omega/2 \exp(-i\omega t/2) \Psi_{r1}; \partial \Psi_{r2} / \partial t \exp(i\omega t/2) + i\omega/2 \exp(i\omega t/2) \Psi_{r2}] = H^0 \Psi_r = \frac{1}{2} \hbar [\omega_3 \Psi_{r1} + \omega_1 \exp(-i\omega t) \Psi_{r2}; \omega_1 \exp(i\omega t) \Psi_{r1} - \omega_3 \Psi_{r2}] = \frac{1}{2} \hbar [\omega_3 \Psi_{r1} \exp(-i\omega t/2) + \omega_1 \exp(-i\omega t/2) \Psi_{r2}; \omega_1 \exp(-i\omega t/2) \Psi_{r1} - \omega_3 \Psi_{r2} \exp(i\omega t/2)]$; split equations & Rearrange $i\hbar \partial \Psi_{r1} / \partial t = -\frac{1}{2} \hbar \Psi_{r1} (\omega - \omega_3) + \frac{1}{2} \hbar \omega_1 \Psi_{r2}$; $i\hbar \partial \Psi_{r2} / \partial t = \frac{1}{2} \hbar \omega_1 \Psi_{r1} + \frac{1}{2} \hbar \Psi_{r2} (\omega - \omega_3)$; combine with $\Delta\omega = \omega - \omega_3$; $i\hbar \partial \Psi_r / \partial t = \frac{1}{2} \hbar [-\Delta\omega, \omega_1; \omega_1, \Delta\omega] \Psi_r = H_r^0 \Psi_r$; H_r^0 is time independent; Spin flip probability $p = |\langle -Z | \Psi \rangle|^2 = |\Psi_2|^2 = |\Psi_{r2} \exp(i\omega t/2)|^2 = |\Psi_{r2}|^2 = |\langle -Z | \Psi_r \rangle|^2$; & this was calculated 2 sections ago with $H = \frac{1}{2} \hbar [\omega_3, \omega_1; \omega_1, -\omega_3]$; now we have H_r^0 , so we need to replace former section's $\omega_3 \rightarrow -\Delta\omega$ and we get $p = \omega_1^2 / (\omega_1^2 + \Delta\omega^2) \sin^2(t(\omega_1^2 + \Delta\omega^2)^{1/2} / 2)$; If $\omega \rightarrow \omega_3$; (Resonance condition) $\Delta\omega = 0$; $p = \sin^2(t\omega_1/2)$; $\omega_3 = 2\pi f$; so if we fire photon $f = \omega_3 / (2\pi)$; at right angle to B_3 it will flip the electron at $t = \pi / \omega_1$; this photon has energy $E = hf = 2\pi \hbar f = \hbar \omega_3$; $\{E = -\mu \cdot B = -\gamma \hat{S} \cdot B$; so Electron: spin up $E = -\gamma \frac{1}{2} \hbar B_3 = \frac{1}{2} \hbar \omega_3$; & spin down $E = \gamma \frac{1}{2} \hbar B_3 = -\frac{1}{2} \hbar \omega_3$; $\Delta E = \hbar \omega_3$; $|\omega_L| = |B| \gamma$; Rabi method true for electron/atomic nuclei in liquids & solids;

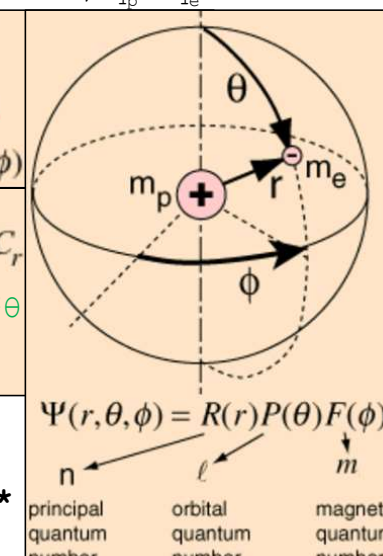
■ Dirac equation: $E = (p^2 c^2 + m^2 c^4)^{1/2}$; so $E = (p^2 + m^2)^{1/2}$; If $\alpha_1 = [0, 0, 0, 0]$; $\alpha_2 = [0, 0, 0, -i; 0, 0, i, 0; 0, -i, 0, 0; i, 0, 0, 0]$; $\alpha_3 = [0, 0, 1, 0; 0, 0, 0, -1; 1, 0, 0, 0; 0, -1, 0, 0]$; $\beta = [1, 0, 0, 0; 0, 1, 0, 0; 0, 0, -1, 0; 0, 0, 0, -1]$; & $I = 4 \times 4$ Identity matrix; $(c(p_1 \alpha_1 + p_2 \alpha_2 + p_3 \alpha_3) + c^2 m \beta)^2 = (c^2(p_1^2 + p_2^2 + p_3^2) + c^4 m^2) I = E^2 I = (EI)^2$; $\sqrt{\quad}$, Replace with Operators ($P^0 = -i\hbar \nabla$; $\partial / \partial x = \partial_x$) & use $\Psi = [\Psi_1; \Psi_2; \Psi_3; \Psi_4]$; we get: $(c(-i\hbar \partial_x \alpha_1 - i\hbar \partial_y \alpha_2 - i\hbar \partial_z \alpha_3) + c^2 m \beta) \Psi = H \Psi = I i \hbar \partial_t \Psi = \text{Dirac equation for free electron}$; expand & rearrange: $[c^2 m \Psi_1 - \hbar c (\partial_y + i \partial_x) \Psi_4 - i \hbar \partial_z \Psi_3 - i \hbar \partial_t \Psi_1; c^2 m \Psi_2 - \hbar c (i \partial_x - \partial_y) \Psi_3 + i \hbar \partial_z \Psi_4 - i \hbar \partial_t \Psi_2; -c^2 m \Psi_3 - \hbar c (\partial_y + i \partial_x) \Psi_2 - i \hbar \partial_z \Psi_1 - i \hbar \partial_t \Psi_3; -c^2 m \Psi_4 - \hbar c (i \partial_x - \partial_y) \Psi_1 + i \hbar \partial_z \Psi_2 - i \hbar \partial_t \Psi_4] = 0 = D_a$; If $\gamma^0 = \beta$; $\gamma^1 = [0, 0, 0, 1; 0, 0, 1, 0; 0, -1, 0, 0; -1, 0, 0, 0]$; $\gamma^2 = [0, 0, 0, -i; 0, 0, i, 0; 0, i, 0, 0; -i, 0, 0, 0]$; $\gamma^3 = [0, 0, 1, 0; 0, 0, 0, -1; -1, 0, 0, 0; 0, 1, 0, 0]$; $\gamma^\mu \partial_\mu = \gamma^0 \partial_t / c + \gamma^1 \partial_x + \gamma^2 \partial_y + \gamma^3 \partial_z$; $i \hbar \gamma^\mu \partial_\mu \Psi - m c \Psi = 0 = D_b$; $\{D_{b1} = D_{a1} / (-c); D_{b2} = D_{a2} / (-c); D_{b3} = D_{a3} / c; D_{b4} = D_{a4} / c\}$ $L_z = -i(x \partial / \partial y - y \partial / \partial x)$; If $\hat{S}_z = \frac{1}{2} [1, 0, 0, 0; 0, -1, 0, 0; 0, 0, 1, 0; 0, 0, 0, -1]$; & $J_z = L_z + \hat{S}_z$; than $[H, J_z] = H J_z - J_z H = 0$; => Total angular momentum (J) is conserved {Observable is constant of motion (does not depend on time) if it commutes with H} so \hat{S}_z correct & because for $p=0$: $i \partial_t \Psi = [m \Psi_1; m \Psi_2; -m \Psi_3; -m \Psi_4]$; Negative energy represent antiparticle (positron); Thus $\Psi = [\text{SpinUp electron}; \text{SpinDown electron}; \text{SpinUp positron}; \text{SpinDown positron}]$; {Thus Quantum mechanics & special relativity gives Dirac equation, Which predict electron spin, antimatter and Hydrogen fine structure line; Shrodinger & Dirac equations predict: Energy level differing by only tiny amounts, & electron probability distributions that are practically indistinguishable}



In **hydrogen atom** electron (mass= m_e ; position vector= x_e) & proton ($m_p; x_p$) orbit each other about a common center of mass; In hydrogen electron exerts on proton force= $F_{e \rightarrow p} = m_p a_p$; & proton exerts on electron force= $F_{p \rightarrow e} = m_e a_e = -F_{e \rightarrow p} = -m_p a_p = m_e a_e$; $a_p = -a_e m_e / m_p$; **The relative position of the electron with respect to the proton= $x_{rel} = x_e - x_p$;** The relative acceleration of the electron with respect to the proton is $a_{rel} = a_e - a_p = d^2 x_e / dt^2 - d^2 x_p / dt^2 = d^2 / dt^2 (x_e - x_p) = d^2 / dt^2 x_{rel} = a_e - a_p = a_e + a_e m_e / m_p = a_e (1 + m_e / m_p) = a_e (m_p + m_e) / m_p = a_e m_e (m_p + m_e) / (m_p m_e) = F_{p \rightarrow e} / \mu$; $\mu = m_e m_p / (m_e + m_p)$; $d^2 / dt^2 x_{rel} = a_{rel} = F_{p \rightarrow e} / \mu$; $r = |x_{rel}|$; For Hydrogen potential energy= $-|F_{p \rightarrow e}| * r = -(k * q_e * q_p / r^2) * r = U = -k * e^2 / r$; $\{k = 8.9875517923 * 10^9; q_p = -q_e = e = 1.602176634 * 10^{-19} C; m_e = 9.1093837015 * 10^{-31} kg; m_p = 1.67262192369 * 10^{-27} kg\}$

$-\hbar^2 / (2m) * \Delta \Psi + U * \Psi = E * \Psi$; $\{ * r^2$; Substitute $\Psi(\theta, \phi, r) = R(r) * P(\theta)$
 $* F(\phi) = R * P * F$; $d\Psi/dr = dR(r)/dr * P(\theta) * F(\phi)$; divide by $R * P * F$

$$\frac{-\hbar^2}{2\mu} \frac{1}{r^2 \sin\theta} \left[\sin\theta \frac{\partial}{\partial r} \left(r^2 \frac{\partial \Psi}{\partial r} \right) + \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial \Psi}{\partial \theta} \right) + \frac{1}{\sin\theta} \frac{\partial^2 \Psi}{\partial \phi^2} \right] + U(r) \Psi(r, \theta, \phi) = E \Psi(r, \theta, \phi)$$



$$\frac{1}{R} \frac{d}{dr} \left[r^2 \frac{dR}{dr} \right] + \frac{2\mu}{\hbar^2} (Er^2 + ke^2/r) + \left[\frac{1}{P \sin\theta} \frac{d}{d\theta} \left[\sin\theta \frac{dP}{d\theta} \right] + \frac{1}{F \sin^2\theta} \frac{d^2 F}{d\phi^2} \right] = 0$$

Δ in spherical coordinates $+U(r)\Psi(r, \theta, \phi) = E \Psi(r, \theta, \phi)$

First 2 terms are function of $r (=a(r))$, So: $a(r) + b(\theta) + c(\phi, \theta) = 0$;
 $a(r) = -b(\theta) - c(\phi, \theta)$; if we vary only r , $b(\theta)$ and $c(\phi, \theta)$ can't vary but the equation must hold so: $a(r) = \text{Constant} 1 = -C_r$; if we vary only

$$\left[\frac{1}{P \sin\theta} \frac{d}{d\theta} \left[\sin\theta \frac{dP}{d\theta} \right] + \frac{1}{F \sin^2\theta} \frac{d^2 F}{d\phi^2} \right] = C_r$$

$$\frac{\sin\theta}{P} \frac{d}{d\theta} \left[\sin\theta \frac{dP}{d\theta} \right] - C_r \sin^2\theta = -\frac{1}{F} \frac{d^2 F}{d\phi^2} = -C_\phi$$

ϕ , left side can't vary but equation must hold so left side=right side=Constant 2= $-C_\phi = m^2$;

if $F(\phi) = F(\phi + 2n\pi)$
 $n = \text{integer}$
 represent same point

$d^2 F / d\phi^2 = F * C_\phi = -F * m^2$; $F(\phi) = A * e^{i * m * \phi}$; $F(\phi) = F(\phi + 2 * \pi)$; {physical constant}
 $A * e^{i * m * \phi} = A * e^{i * m * (\phi + 2 * \pi)} = A * e^{i * m * \phi} * e^{i * m * 2 * \pi}$; $1 = e^{i * m * 2 * \pi} = \cos(m * 2 * \pi) + i * \sin(m * 2 * \pi)$; $m = 0, \pm 1, \pm 2, \pm 3, \dots$; Lets normalized $F(\phi)$ by $\int |F(\phi)|^2 * d\phi = \int |A * (\cos(m * \phi) + i * \sin(m * \phi))|^2 * d\phi = \int A^2 * (\cos^2(m * \phi) + \sin^2(m * \phi)) * d\phi = \int A^2 * d\phi = [A^2 * \phi] = 2\pi * A^2 = 1$; $A = 1 / (2\pi)^{1/2}$; $C_r = -1(1+1)$

$$F_m(\phi) = \frac{1}{\sqrt{2\pi}} e^{im\phi}$$

$$\frac{1}{R} \frac{d}{dr} \left[r^2 \frac{dR}{dr} \right] + \frac{2\mu}{\hbar^2} (Er^2 + ke^2/r) = l(l+1)$$

$$\sin\theta \frac{d}{d\theta} \left(\sin\theta \frac{dP}{d\theta} \right) + [l(l+1) \sin^2\theta - m^2] P = 0$$

$$\frac{d^2 \chi(s)}{ds^2} - \left[\frac{l(l+1)}{s^2} - \frac{n}{s} + \frac{1}{4} \right] \chi(s) = 0$$

$* \hbar^2 R / (2\mu r)$; $X(r) = rR(r)$; $R(r) = X(r) * r^{-1}$; $dR/dr = r^{-1} * dX/dr - X * r^{-2}$; $n^2 = -\mu e^4 k^2 / (2E \hbar^2)$; $E = -\mu e^4 k^2 / (2n^2 \hbar^2)$; $s = 2\mu r e^2 k / (n \hbar^2)$; $r = s \hbar^2 / (2\mu e^2 k)$; $* -\hbar^2 n^2 / (2\mu e^4 k^2)$; $s/r = \text{constant}$; $d^2 X(r) / dr^2 = (s/r)^2 * d^2 X(s) / ds^2$;

Assume $\chi(s) = s^{l+1} L(s) \exp(-s/2)$

$$s \frac{d^2 L}{ds^2} - [s - 2(l+1)] \frac{dL}{ds} + [n - (l+1)] L = 0$$

$$R(r) = s^l L_{n-l-1}^{2l+1}(s) \exp(-s/2)$$

Laguerre polynomials, are solutions of:

$$xy'' + (\alpha + 1 - x)y' + ny = 0.$$

where n is non-negative integer.

$$y = L_n^{(\alpha)}(x) = \frac{x^{-\alpha} e^x}{n!} \frac{d^n}{dx^n} (e^{-x} x^{n+\alpha})$$

Solve by Frobenius to: Legendre polynomial

Normalization constant $P_l^m(x) = (1-x^2)^{|m|/2} \left(\frac{d}{dx} \right)^{|m|} P_l(x)$

$$P_l^m(x) = \frac{1}{2^l l!} \left(\frac{d}{dx} \right)^l (x^2 - 1)^l$$

App4.m $Y_{lm}(\theta, \phi) = \frac{N_{lm}}{\sqrt{2\pi}} e^{im\phi} P_l^m(\cos\theta)$, Normalized constant

$\alpha = 2l + 1$; $L'(\alpha + 1 - s)$; $n_{\text{here}} = n - l - 1 \geq 0$; $n = \text{integer} \geq l + 1$;

wavefunction normalization integral

$$1 = \int_{r=0}^{\infty} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} |R(r)Y(\theta, \phi)|^2 r^2 \sin \theta d\theta d\phi dr$$

$$a_0 = \hbar^2 / (k\mu e^2);$$

$$k = 1 / (4\pi\epsilon_0);$$

$$r = na_0 s / 2;$$

$$s = 2r / (na_0);$$

Normalized hydrogen wave function: $\psi_{nlm}(r, \theta, \phi) = R_{nl}(r) Y_{lm}(\theta, \phi)$

$$Y_{lm}(\theta, \phi) = (-1)^m \left[\frac{(2l+1)(l-m)!}{4\pi(l+m)!} \right]^{1/2} P_l^m(\cos \theta) e^{im\phi}$$

$$P_l^m(x) = (1-x^2)^{|m|/2} \left(\frac{d}{dx} \right)^{|m|} P_l(x)$$

$$a_0 \equiv \frac{4\pi\epsilon_0 \hbar^2}{\mu e^2} = 0.53 \times 10^{-10} \text{ m}$$

$$R_{nl}(r) = - \left[\left(\frac{2}{na_0} \right)^3 \frac{(n-l-1)!}{2n\{(n+l)!\}^3} \right]^{1/2} \left(\frac{2r}{na_0} \right)^l e^{-r/na_0} L_{n+l}^{2l+1}(2r/na_0)$$

$$P_l(x) = \frac{1}{2^l l!} \left(\frac{d}{dx} \right)^l (x^2 - 1)^l$$

$$L_q^p(x) = \frac{d^p}{dx^p} L_q(x)$$

Probability density of finding an electron in hydrogen atom in the n, l, m quantum state is $|\psi_{nlm}|^2 = \psi_{nlm}^*(r, \theta, \phi) \psi_{nlm}(r, \theta, \phi)$, and probability of finding electron in n, l, m state in volume element $d\tau = r^2 dr \sin \theta d\theta d\phi$ is $|\psi_{nlm}|^2 d\tau$.

time-independent Schrödinger equation, ignoring all spin-coupling interactions
reduced mass $\mu = m_e M / (m_e + M)$

$$L_q(x) = e^x \frac{d^q}{dx^q} (e^{-x} x^q)$$

Defined $n^2 = -\mu e^4 k^2 / (2E\hbar^2)$ & than discover that for solution: $n \geq l+1$;
 $n=1, 2, 3, \dots; l < n-1; l=0, 1, 2, 3, \dots, n-1; C_r = -1(l+1); -C_\phi = m^2; m=0, \pm 1, \pm 2, \dots, \pm l$

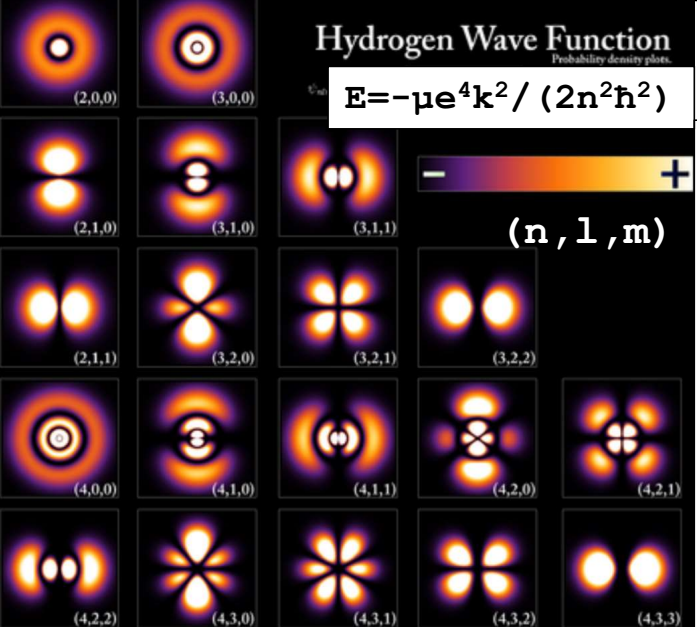
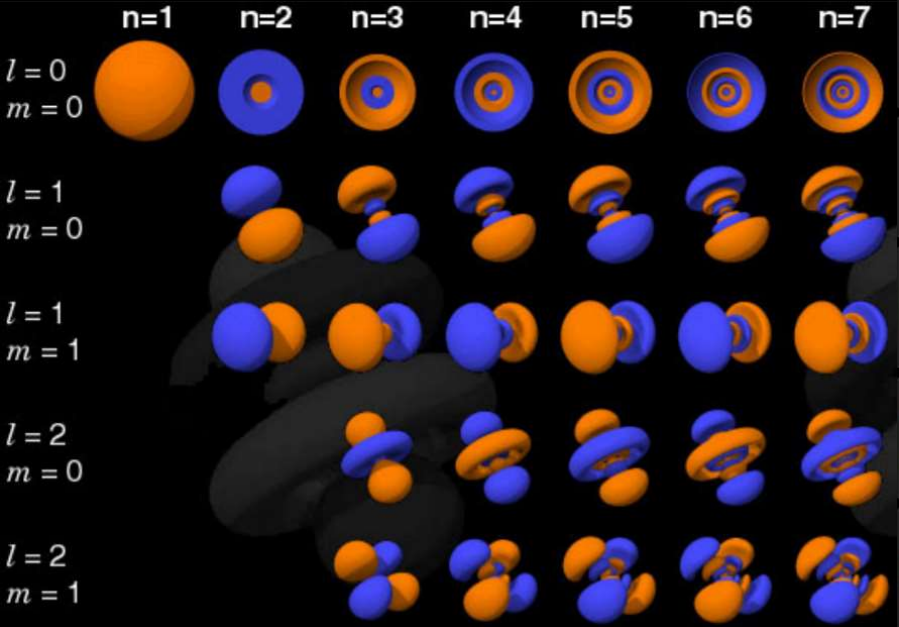
$$\left(-\frac{\hbar^2}{2\mu} \nabla^2 - \frac{e^2}{4\pi\epsilon_0 r} \right) \psi(r, \theta, \phi) = E\psi(r, \theta, \phi)$$

Laplacian in spherical coordinates

$$-\frac{\hbar^2}{2\mu} \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \psi}{\partial \phi^2} \right] - \frac{e^2}{4\pi\epsilon_0 r} \psi = E\psi$$

This is a separable, partial differential equation which can be solved in terms of special functions.

Hamiltonian operator produce specific values for E called energy eigenvalues; $\Psi = \Psi_{n,l,m}; \Psi_i = \Psi_{n,l,m} =$ eigen functions {specific n, l, m }; $H \Psi_i = E_i \Psi_i$



Hydrogen Wave Function

Probability density plots.

$E = -\mu e^4 k^2 / (2n^2 \hbar^2)$

(n, l, m)

```

mu=me*mp/(me+mp); a0=hb^2/(k*mu*e^2); %a0=0.53*10^(-10);
P=(1-x^2)^(abs(m)/2)*diff(1/(factorial(1)*2^1)*diff((x^2-1)^1,x,1),x,abs(m));
L=diff((exp(s)*diff((exp(-s)*s^(n+1)),s,(n+1))),s,(2*1+1));
WF=-(factorial(n-1-1)/(2*n*(factorial(n+1))^3)*(2/(n*a0))^3)^(1/2)*
(2*r/(n*a0))^l*exp(-r/(n*a0))*subs(L,s,(2*r/(n*a0)))*(-1)^m*((2*1+1)*factorial(1-m)/(4*pi*factorial(1+m)))^(1/2)*subs(P,x,cos(T))*exp(i*m*p)
    
```

App4.m

Schrödinger's equation determines e⁻ "wavefunction"

$$\left(-\frac{\hbar^2}{2m_e}\nabla^2 - \frac{ke^2}{r}\right)\psi(r,\theta,\phi) = E\psi(r,\theta,\phi) \Rightarrow \psi_{n,\ell,m_\ell}$$

3 quantum numbers determine e⁻ state

"Principal Quantum Number" $n = 1, 2, 3, \dots$ "SHELL"

$$E_n = -\frac{m_e k^2 e^4}{2\hbar^2} \frac{1}{n^2}$$

Energy For Hydrogen (for other atoms depend also on ℓ, m_ℓ .)

"Orbital Quantum Number" s, p, d, f "SUBSHELL"

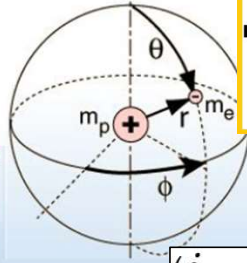
$$|\vec{L}| = L = \sqrt{\ell(\ell+1)}\hbar$$

Magnitude of angular momentum

"Magnetic Quantum Number" $m_\ell = -\ell, \dots, -1, 0, +1, \dots, \ell$

$$L_z = m_\ell \hbar$$

Orientation of angular momentum



f $\ell = 3$

$m_\ell: -3, -2, -1, 0, 1, 2, 3$

d $\ell = 2$

$m_\ell: -2, -1, 0, 1, 2$

p $\ell = 1$

$m_\ell: -1, 0, 1$

$\ell = 0$

$m_\ell: 0$

$$\alpha = \frac{ke^2}{\hbar c} = 0.0072973525693$$

$$KE = \sqrt{p^2 c^2 + m_0^2 c^4} - m_0 c^2$$

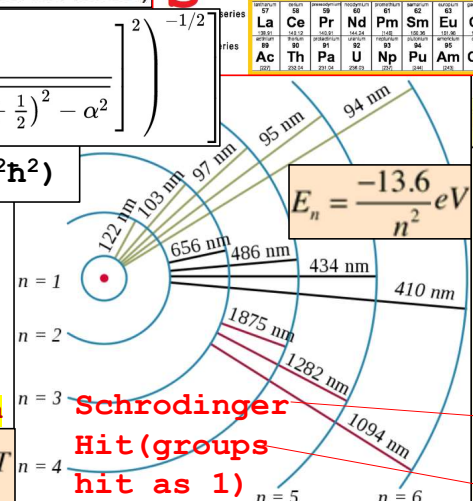
Relativistic

$(j = 1/2 \text{ if } \ell = 0 \text{ and } j = \ell \pm 1/2 \text{ otherwise})$

$$E_{jn} = -m_e c^2 \left[1 - \left(1 + \frac{\alpha^2}{n - j - 1/2 + \sqrt{(j + 1/2)^2 - \alpha^2}} \right)^{-2} \right]^{-1/2}$$

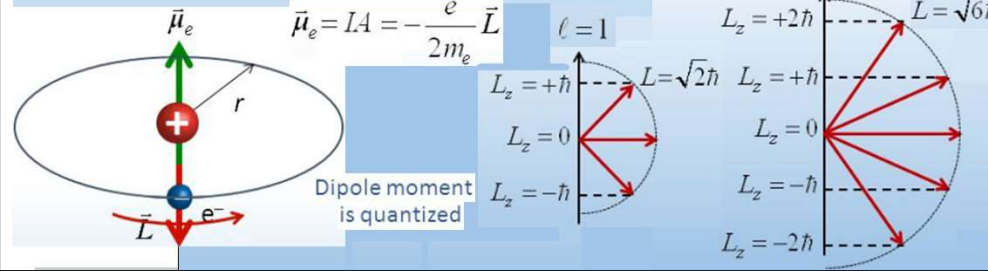
$n_1 = 3; n_2 = 6;$
 $E = -13.6 * (1/n_1^2 - 1/n_2^2);$
 $h = 4.135667696 * 10^{-15};$
 $c = 299792458; \% [eV*s] [m/s]$
 $wl = c * h / E; \% E = h * f; wl = c / f;$

<https://physics.nist.gov/PhysRefData/Handbook/Tables/hydrogentable2.htm>

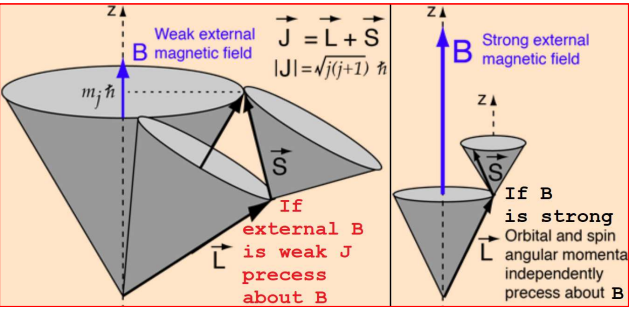


Hydrogen Strong Lines

Intensity	Vacuum Wavelength (Å)
15	926.2256
20	930.7482
30	937.8034
50 P	949.7430
100 P	972.5367
300 P	1025.7222
1000 P	1215.66824
500 P	1215.67364
5	3835.384
6	3889.049
8	3970.072
15	4101.74
30 P	4340.462
30 P	4861.2786
10 P	4861.2870
60 P	4861.3615
90 P	6562.7110
30 P	6562.7248
180 P	6562.8518
5	9545.97
7	10049.4
12	10938.1
20 P	12818.07
40 P, c	18751.01
5	21655.3
8	26251.5
15	40511.6
4	46525.1
6	74578
3	123685



Most probable location (state) for e in atom: For each atom e can be in: Energy level=shell=how far from nucleus= $n=1,2,3\dots$; For each n , e can be in orbital type= $\ell=0,1,2,\dots,(n-1)$; {SubShell s,p,d,f } For each ℓ , e can be in specific orbital= $m_\ell=m=0,\pm 1,\pm 2,\dots,\pm \ell$; For each m_ℓ , e can have spin= $m_s=\pm 1/2$ or $-1/2$; m_s is added because of Stern Gerlach experiment, but we can also get it if in Schrodinger we

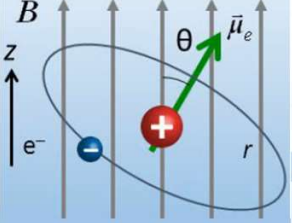


replace $KE = \frac{1}{2}mv^2$; with relativistic KE (Dirac equation); Dirac E_{jn} include fine structure (but exclude Lamb shift and hyperfine structure).

$$\mu_{orbital} = \frac{-e}{2m_e} L$$

$$U(\theta) = -\mu \cdot B = \frac{e}{2m} L_z B = m_\ell \frac{e\hbar}{2m} B$$

Zeeman effect



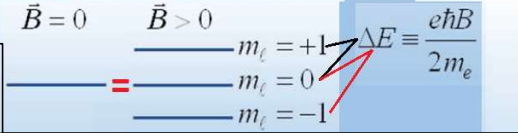
$$U = -\mu_e B \cos \theta = \frac{e\hbar}{2m_e} B m_\ell$$

Calculate the effect of a 1 T B field on the energy of the 2p ($n=2, \ell=1$) level

$$E_{tot} = E_{n=2} - \mu_e B \cos \theta = E_{n=2} + \frac{e\hbar}{2m_e} B m_\ell$$

for H depend only on n

For $\ell=1, m_\ell = -1, 0, +1$



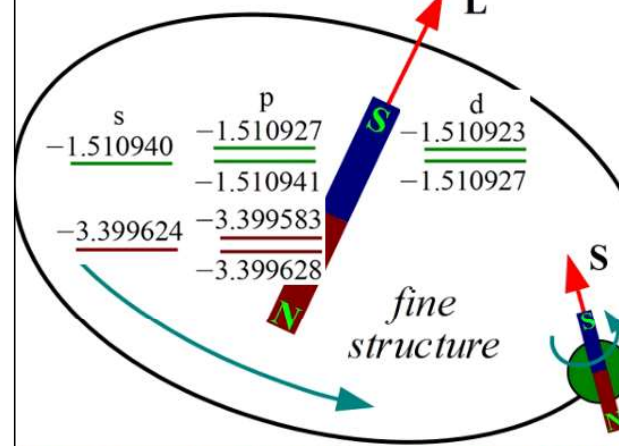
$H = -\hbar^2/(2m_e)\Delta + U$; If B present: $U = -k^*e^2/r + e^*Bz^*Lz/(2m_e)$; $H^*\Psi = E^*\Psi = (E_n + e^*Bz^*\hbar^*m/(2m_e))\Psi$; $E_n = -\mu e^4 k^2/(2n^2 \hbar^2)$; E now depend on 2 quantum number n, m so E split;

Dirac equation:

$E = ((p^*c)^2 + (m^*c^2)^2)^{1/2}$; in natural units $c = \hbar = 1$; so $E = (p^2 + m^2)^{1/2}$; If $Ax = [0, 0, 0, 1; 0, 0, 1, 0; 0, 1, 0, 0; 1, 0, 0, 0]$; $Ay = [0, 0, 0, -i; 0, 0, i, 0; 0, -i, 0, 0; i, 0, 0, 0]$; $Az = [0, 0, 1, 0; 0, 0, 0, -1; 1, 0, 0, 0; 0, -1, 0, 0]$; $B = [1, 0, 0, 0; 0, 1, 0, 0; 0, 0, -1, 0; 0, 0, 0, -1]$; & I = Identity matrix; $(Ax^*Px + Ay^*Py + Az^*Pz + B^*m)^2 = (Px^2 + Py^2 + Pz^2 + m^2) * I = E^2 * I = (E^*I)^2$; so $E^*I = Ax^*Px + Ay^*Py + Az^*Pz + B^*m$; Replace with Operators ($\hbar = 1$; $\partial/\partial X = \partial_x$): $I^*i^*\partial_t\Psi = -i^*Ax^*\partial_x\Psi - i^*Ay^*\partial_y\Psi - i^*Az^*\partial_z\Psi + B^*m^*\Psi$; Dirac equation for free electron; $\Psi = [\Psi_1; \Psi_2; \Psi_3; \Psi_4]$; $i\partial_t\Psi = [-i\partial_x\Psi_4 - \partial_y\Psi_4 - i\partial_z\Psi_3 + m\Psi_1; -i\partial_x\Psi_3 + \partial_y\Psi_3 + i\partial_z\Psi_4 + m\Psi_2; -i\partial_x\Psi_2 - \partial_y\Psi_2 - i\partial_z\Psi_1 - m\Psi_3; -i\partial_x\Psi_1 + \partial_y\Psi_1 + i\partial_z\Psi_2 - m\Psi_4]$; $H = -i^*Ax^*\partial_x - i^*Ay^*\partial_y - i^*Az^*\partial_z + B^*m$; $L_z = -i(x\partial/\partial y - y\partial/\partial x)$; If $\hat{S}_z = \frac{1}{2}[1, 0, 0, 0; 0, -1, 0, 0; 0, 0, 1, 0; 0, 0, 0, -1]$; & $J_z = L_z + \hat{S}_z$; than $[H, J_z] = HJ_z - J_zH = 0$; => Total angular momentum (J) is conserved {Observable is constant of motion (does not depend on time) if it commutes with H} so \hat{S}_z correct and because for $p=0$: $i\partial_t\Psi = [m\Psi_1; m\Psi_2; -m\Psi_3; -m\Psi_4]$; Negative energy represent antiparticle (positron); Thus $\Psi = [\text{SpinUp electron}; \text{SpinDown electron}; \text{SpinUp positron}; \text{SpinDown positron}]$; {Thus Quantum mechanics & special relativity gives Dirac equation, Which predict electron spin, antimatter and Hydrogen fine structure line; Shrodinger & Dirac equations predic: Energy level differing by only tiny amounts, & electron probability distributions that are practically indistinguishable}

Hydrogen atom $i\partial_t\Psi = -i\alpha_x\partial_x\Psi - i\alpha_y\partial_y\Psi - i\alpha_z\partial_z\Psi + m\beta\Psi - \frac{\alpha}{r}\Psi$
 Total angular momentum $J = L + S$ is conserved. $\alpha \approx \frac{1}{137}$ quantum numbers
 Orbital L and spin S angular momenta are not. n, j, m_j

$n = 1, 2, 3, \dots$ "energy"
 $j = \frac{1}{2}, \frac{3}{2}, \dots, n - \frac{1}{2}$ angular momentum
 $m_j = -j, -j+1, \dots, j$ z component of angular momentum



Configuration	J	Level (eV)
2p	1/2	10.19880606470
	3/2	10.19885142904

$$E_{nj} = \frac{m}{\sqrt{1 + \frac{\alpha^2}{\left[n - j - \frac{1}{2} + \sqrt{\left(j + \frac{1}{2}\right)^2 - \alpha^2}\right]^2}}}$$

$$= m - \frac{m\alpha^2}{2n^2} - \frac{m\alpha^4}{2n^4} \left(\frac{n}{j+1/2} - \frac{3}{4} \right) + \dots$$

Schrödinger
 $E_{2,3/2} - E_{2,1/2} = 0.00004535 \text{ eV}$
 $\Delta E_{\text{obs}} = 0.00004536 \text{ eV}$

$1 - \gamma = 0.0000266\dots$ r in units of Bohr radius

hydrogen Ground state wave functions (Schrödinger e^{-r})

"spin up" $|\psi\rangle = A \frac{1}{r^{1-\gamma}} e^{-r} \begin{bmatrix} 1 \\ 0 \\ \frac{i(1-\gamma)}{\alpha} \cos\theta \\ \frac{i(1-\gamma)}{\alpha} \sin\theta e^{i\phi} \end{bmatrix}$

"spin down" $|\psi\rangle = A \frac{1}{r^{1-\gamma}} e^{-r} \begin{bmatrix} 0 \\ 1 \\ \frac{i(1-\gamma)}{\alpha} \sin\theta e^{i\phi} \\ -\frac{i(1-\gamma)}{\alpha} \cos\theta \end{bmatrix}$

Dirac give 2 solutions

$$\alpha = \frac{1}{4\pi\epsilon_0} \frac{e^2}{\hbar c} = \frac{k_e e^2}{\hbar c}$$

Electrons have an intrinsic angular momentum called "spin"

$|\vec{S}| = S = \sqrt{s(s+1)}\hbar$ with $s = 1/2$

"Spin Quantum Number", $m_s = -1/2, +1/2$

$S_z = m_s\hbar$ Orientation of spin

$S_z = +\frac{\hbar}{2}$ $S = \frac{\sqrt{3}}{2}\hbar$

$S_z = -\frac{\hbar}{2}$

Spin also generates magnetic dipole moment

$\vec{\mu}_s = -\frac{e}{2m_e} g\vec{S}$

$U = -\mu_s B \cos\theta = \frac{ge\hbar}{2m_e} B m_s$

$\mu_B = \frac{e\hbar}{2m_e} = 5.788382 \times 10^{-5} \text{ eV/T}$

Spin DOWN (-1/2) Spin UP (+1/2)

$\vec{B} > 0$

$m_s = +1/2$

$m_s = -1/2$

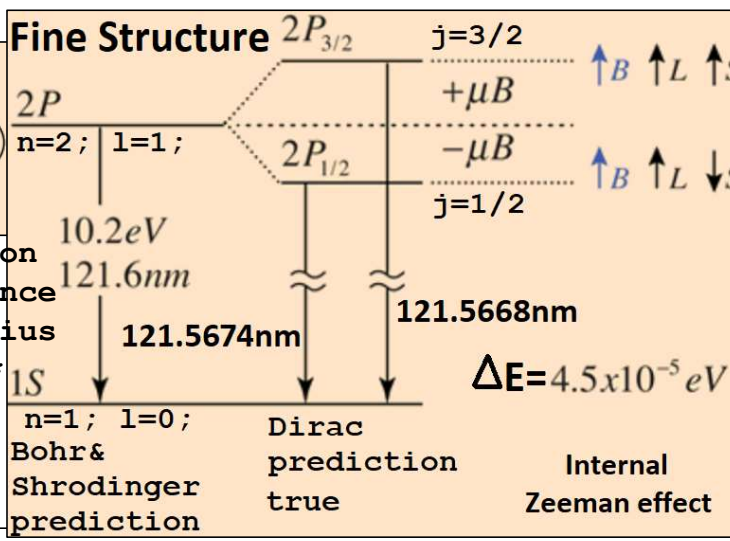
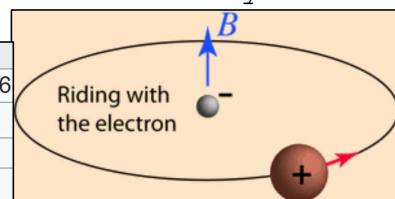
$|\vec{\mu}_s| = \sqrt{s(s+1)}\mu_B g_s$

$\mu_{s,z} = m_s \mu_B g_s = \pm \frac{1}{2} g_s \mu_B$

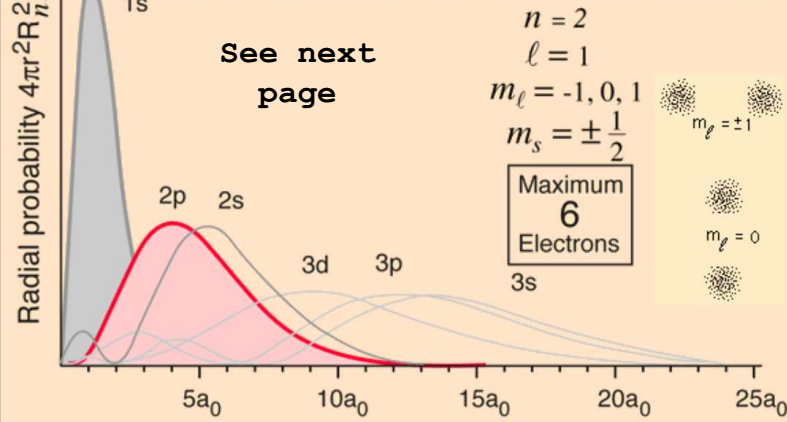
$\Delta E = g_s \mu_B B_0$

Primary spin quantum number $s=0, 1/2, 1, 3/2, 2, 5/2, \dots$; s depends only on the type of the particle & cannot be changed in any way (an elementary particle cannot be made to spin faster or slower; Boson = particle of $s=0, 1, 2, 3, \dots$; Fermion = particle of $s=1/2, 3/2, 5/2, \dots$); but the spin direction can be changed. Spin angular momentum (S) is quantized. Secondary spin quantum number (m_s) defined by $S_z = m_s \hbar$; m_s can take one of $(2s+1)$ value ($m_s = -s, (-s+1), \dots, (s-1), s$); for e $s=1/2$ so $m_s = 1/2, -1/2$; The spin of a charged particle is associated with a magnetic dipole moment with a g -factor differing from 1 (This could only occur classically if the internal charge of the particle were distributed differently from its mass).

Particle	g -factor
electron	$g_e = -2.002\ 319\ 304\ 362\ 56$
muon	$g_\mu = -2.002\ 331\ 8418(13)$
neutron	$g_n = -3.826\ 085\ 45(90)$
proton	$g_p = +5.585\ 694\ 6893(16)$



Hydrogen 2p Radial Probability



If in hydrogen the e circle the proton with radius r , than from the e reference frame, the nucleus circle it with radius r ; so the e feel $B = \mu_0 q v / (4\pi r^2) = \mu_0 q v^* m_e r / (4\pi m_e r^3) = \mu_0 q^* L / (4\pi m_e r^3)$; $L = m_e v r$; for e 2p: $r = 4 a_0$; $l = 1$; $L = \hbar (l(l+1))^{1/2} = \hbar 2^{1/2}$; $B = \mu_0 q^* \hbar 2^{1/2} / (4\pi m_e (4 a_0)^3) = 0.28\text{T}$; close to the correct 0.39T ;

$m_e = 9.1093837015 \times 10^{-31}$; $e = 1.602176634 \times 10^{-19}$; $a_0 = 0.0529 \times 10^{-9}$; $\hbar = (6.62607015 \times 10^{-34}) / (2\pi)$; $\mu_0 = 1.25663706212 \times 10^{-6}$;

$r = 4 a_0$; $L = \hbar 2^{1/2}$; $B = \mu_0 e^* L / (m_e 4\pi r^3) \approx 0.28\text{T}$

n	l	m_l	$\Psi_{n l m_l}(r, \theta, \phi)$
2	1	0	$\frac{1}{4\sqrt{2}\pi a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0} \cos\theta$
2	1	± 1	$\frac{1}{8\sqrt{\pi} a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0} \sin\theta e^{\pm i\phi}$

$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$

B at r , created by q with v at position $[0; 0; 0]$; \hat{r} = unit vector

For electron orbital motion in H atom: μ on any axis $= \mu_B m_l$; $U = \mu_B m_l B$; $\Delta E = U_2 - U_1 = \mu_B B (m_{l2} - m_{l1}) = \mu_B B (1 - 0) = \mu_B B$; as we know $\Delta E = 4.5 \times 10^{-5}$; $B = \Delta E / \mu_B = 0.39\text{T}$ = The magnetic field created by e 's orbital motion;

For electron spin in H atom $U = g \mu_B m_s B$; $\Delta E = U_2 - U_1 = g \mu_B (m_{s2} - m_{s1}) B = g \mu_B (1/2 - (-1/2)) B = g \mu_B 0.39 = 4.5 \times 10^{-5}$;

Hyperfine structure Hydrogen 21cm line

proton magnetic moment = $\mu_I = g_I \mu_N \mathbf{I}$, **proton spin**
nuclear magneton
proton g-factor

There is an energy associated with μ_I in the presence of \mathbf{B}
 Hamiltonian given by: $\hat{H}_D = -\mu_I \cdot \mathbf{B}$.

magnetic field experienced by the **proton**:

is that associated with $\mathbf{B} \equiv \mathbf{B}_{el} = \mathbf{B}_{el}^\ell + \mathbf{B}_{el}^s$

orbital (ℓ) and spin (s) angular momentum of the electrons

$$\mathbf{B}_{el}^\ell = \frac{\mu_0}{4\pi} \frac{-e\mathbf{v} \times \mathbf{r}}{r^3} = -2\mu_B \frac{\mu_0}{4\pi} \frac{1}{r^3} \frac{\mathbf{r} \times m_e \mathbf{v}}{\hbar} = -2\mu_B \frac{\mu_0}{4\pi} \frac{1}{r^3} \ell.$$

electron with spin angular momentum, \mathbf{s} , has a magnetic moment $\mu_s = -g_s \mu_B \mathbf{s}$,

$\mu_B = \frac{e\hbar}{2m_e}$
 $L_z = m\hbar$
 $m_\ell = -\ell, \dots, -1, 0, +1, \dots, \ell$
 $\ell = 0, 1, 2, 3, \dots, n-1$
 $n = 1, 2, 3, \dots$

electron spin g-factor magnetic field of μ_s :

$$\mathbf{B}_{el}^s = \frac{\mu_0}{4\pi r^3} (3(\mu_s \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mu_s) + \frac{2\mu_0}{3} \mu_s \delta^3(\mathbf{r}).$$

"finite distance"

"Fermi contact"

interaction of the nuclear dipole with the field due to electron spin magnetic moments

direct interaction of the nuclear dipole with the electron spin magnetic moments

Include the internal field of the dipole (field at the center of the current loop)

(to describe magnetic moment we make a loop smaller & smaller, while keeping $I \cdot \text{Area}$ constant)

$$dP = \frac{4}{a_0^3} r^2 e^{-2r/a_0} dr$$

$$dr \frac{4}{a_0^3} 2r e^{-2r/a_0} - \frac{4}{a_0^3} \frac{2}{a_0} r^2 e^{-2r/a_0} dr = 0$$

$$2r e^{-2r/a_0} \left[1 - \frac{r}{a_0} \right] = 0 \quad r = a_0$$

Hydrogen's electron mean speed = $c/137$;

```
h=6.62607015*10^-34; c=299792458; mu0=1.25663706212*10^-6;%J*T
me=9.1093837015*10^-31; mp=1.67262192369*10^-27;
e=1.602176634*10^-19; gs=-2.00231930436256; gI=5.5856946893;
hb=h/(2*pi); a=mu0*e^2*c/(2*h); a0=hb/(me*c*a); r=a0;
muB=e*hb/(2*me); muN=e*hb/(2*mp); mus=-gs*muB*1/2; muI=gI*muN*1/2;
v=c/137; Bl=mu0*e*v/(4*pi*r^2);
%l=0;Bl=-2*muB*mu0*1/(4*pi*r^3);%using this give wl=0.48m
Bs1=2*mu0*mus/(4*pi*r^3); Bs2=1/(pi*a0^3)*2*mu0*mus/3;
E=-muI*(Bl+Bsl+Bsl); wl=c*h/E %=0.33m; E=h*f; wl=c/f;
wl_other_calc=12*h^5*mp/(c^5*e^8*gI*mu0^4*me^2) %=0.21094m;
```

total probability of the electron being in a shell at a distance r and

thickness $dr = 4\pi r^2 |\psi_{1s}(r)|^2 dr.$

Sphere surface area = $4\pi r^2$

energy of the Fermi contact = $-\frac{2}{3} \mu_0 \langle \mu_I \cdot \mu_s \rangle |\Psi(0)|^2$

$\Psi(0)$ is the value of the electron wavefunction at the nucleus.

ground state wave function

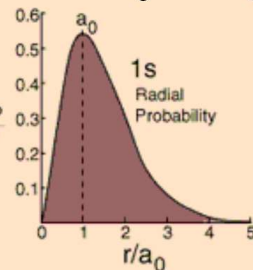
$$\psi_{1s}(r) = \frac{1}{\sqrt{\pi} a_0^{3/2}} e^{-r/a_0}$$

$$|\psi_{1s}(r)|^2 = \frac{1}{\pi a_0^3} e^{-2r/a_0}$$

H_Spectrum.m

Emitted photon precisely measured frequency = 1420405751.7667 Hz; equivalent to vacuum wavelength of 21.1061140542 cm.

The Most Probable Radius Hydrogen Ground State

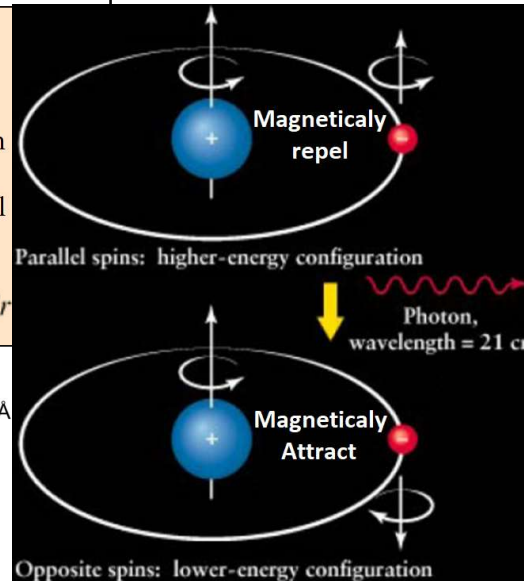
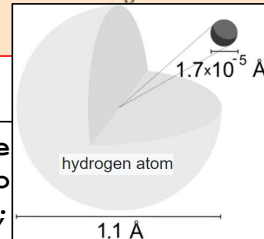


The radial probability density for the hydrogen ground state is obtained by multiplying the square of the wavefunction by a spherical shell volume element.

$$dP = \left[\frac{1}{\sqrt{\pi} a_0^{3/2}} e^{-r/a_0} \right]^2 4\pi r^2 dr = \frac{4}{a_0^3} r^2 e^{-2r/a_0} dr$$

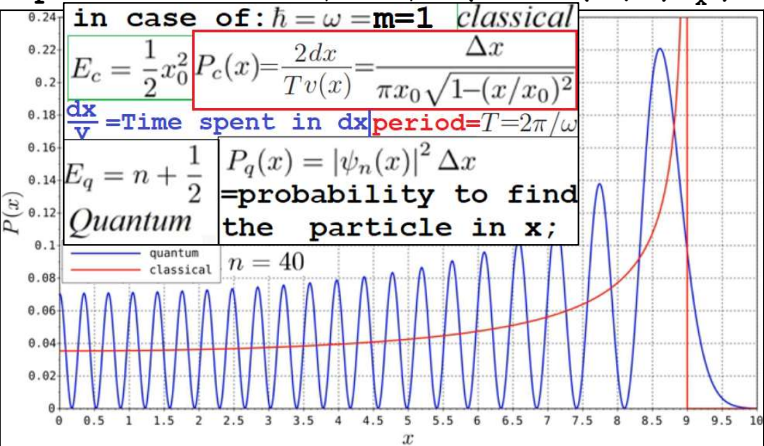
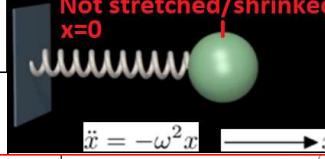
Probability to find e at position $r, \phi, \theta = \Psi\Psi^*$

To find max we take derivative with respect to r & set equal to 0. So most probable radius = a_0 ;



3Dwe: $\partial^2\Psi/\partial t^2=c^2\Delta\Psi$; $\{\Delta=\partial^2/\partial x^2+\partial^2/\partial y^2+\partial^2/\partial z^2\}$ satisfied by any $\Psi=\Psi(\mathbf{k}\cdot\mathbf{r}-\omega t+\phi)$; $\{\Psi$ shape travel along \mathbf{k} with $|\mathbf{v}|=c=\omega/|\mathbf{k}|$ or by $\Psi=\mathbf{q}(t)$ $\mathbf{f}(\mathbf{r})$; if $\mathbf{f}=-\Delta\mathbf{f}c^2/w^2$; $\partial^2\mathbf{q}/\partial t^2=-w^2\mathbf{q}$; $\{\partial^2\Psi/\partial t^2=\mathbf{f}\partial^2\mathbf{q}/\partial t^2=\Delta\Psi c^2=\Delta\mathbf{f}c^2\mathbf{q}\}$ \mathbf{q} =Harmonic oscillator=describe spring length $\mathbf{x}(t)$ =describe $\omega_{r1}(t)$ & $\omega_{r2}(t)$ of a freely rotating body ($\tau=0$) with $\mathbf{I}_{r1}=\mathbf{I}_{r2}, \mathbf{I}_{r3}\{\partial\omega_{r3}(t)/\partial t=0$; so its full description}; If \mathbf{g}, \mathbf{f} satisfy 3Dwe (same c) $\mathbf{f}+\mathbf{g}$, $\mathbf{f}*\mathbf{g}, \mathbf{a}\mathbf{f}+\mathbf{b}\mathbf{g}$ also; Thus, any wave can be analyzed as linear combination of $\Psi(\mathbf{r}, t)=\exp(i(\mathbf{k}\cdot\mathbf{r}-\omega t+\phi))=\mathbf{q}(t)\mathbf{f}(\mathbf{r})=\exp(-i\omega t)\exp(i(\mathbf{k}\cdot\mathbf{r}+\phi))$; $w=\omega$; waves with same $|\mathbf{v}|$; $\mathbf{k}=2\pi\nu/(\lambda|\mathbf{v}|)$; for harmonic oscillator: $\mathbf{x}''=-\omega^2\mathbf{x}$; $\mathbf{x}=\mathbf{x}_0\cos(\omega t-\phi)$; $\mathbf{F}=\mathbf{m}\mathbf{x}'$; $\{\mathbf{F}=-k\mathbf{x}$; $k=m\omega^2\}$; $\mathbf{PE}=\int\mathbf{F}d\mathbf{x}$ (from \mathbf{x} to 0) $=\frac{1}{2}m\omega^2\mathbf{x}^2$; $\mathbf{p}=\mathbf{m}\mathbf{x}'$; $\mathbf{KE}=\frac{1}{2}m\mathbf{x}'^2=\mathbf{p}^2/(2m)$; $\mathbf{E}=\mathbf{KE}+\mathbf{PE}=\frac{1}{2}(m\omega^2\mathbf{x}^2+\mathbf{p}^2/m)=\frac{1}{2}m\omega^2\mathbf{x}_0^2$; replace \mathbf{p}, \mathbf{x} with operators $\mathbf{P}^o=-i\hbar\nabla$; $\mathbf{r}^o=\mathbf{r}$; $\frac{1}{2}(m\omega^2\mathbf{x}^2-(\hbar^2/m)\partial_x^2)\Psi=\mathbf{H}\Psi=\mathbf{E}\Psi$; Find Ψ such that $\mathbf{E}=\text{number}$ & $\int|\Psi_n(\mathbf{x})|^2d\mathbf{x}$ (from $-\infty$ to ∞) $=1$;

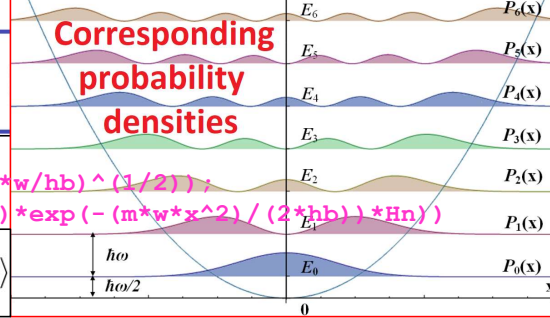
Harmonic oscillator



$$\psi_n(x) = \frac{1}{\sqrt{2^n n!}} \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega x^2}{2\hbar}} H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right) \quad n = 0, 1, 2, \dots$$

$$H_n(z) = (-1)^n e^{z^2} \frac{d^n}{dz^n} (e^{-z^2}) \quad E_n = \hbar\omega\left(n + \frac{1}{2}\right) =$$

```
Matlab: n=2; w=1; hb=1; m=1; syms x z real
Hnf=(-1)^n*exp(z^2)*diff(exp(-z^2),z,n); Hn=subs(Hnf,z,x*(m*w/hb)^(1/2));
p=vpa(simplify((2^n*factorial(n))^(-1/2)*(m*w/(pi*hb))^(1/4)*exp(-(m*w*x^2)/(2*hb))*Hn))
IN1=vpa(int(p*conj(p),x,-inf,inf))
E=(m*w^2*x^2*p-(hb^2/m)*diff(p,x,2))/2/p
xx=0:10/100:10; dx=0.1; Es=hb*w*(n+1/2)
yy=subs(p*conj(p)*dx,x,xx); plot(xx,yy)
```



If $\mathbf{a}^\dagger = (m\omega/(2\hbar))^{1/2}(\mathbf{x} - i\mathbf{p}^o/(m\omega)) = (m\omega/(2\hbar))^{1/2}(\mathbf{x} - \hbar\partial_x/(m\omega)) = \text{Creation}^o$; $\mathbf{a}^\dagger|n\rangle = (n+1)^{1/2}|n+1\rangle$;
 $\mathbf{a} = (m\omega/(2\hbar))^{1/2}(\mathbf{x} + i\mathbf{p}^o/(m\omega)) = (m\omega/(2\hbar))^{1/2}(\mathbf{x} + \hbar\partial_x/(m\omega)) = \text{Annihilation}^o$; $\mathbf{a}|n\rangle = n^{1/2}|n-1\rangle$; $\mathbf{a}|0\rangle = 0$;
 $\mathbf{N} = \mathbf{a}^\dagger\mathbf{a} = \text{Number}^o$; $\mathbf{N}|n\rangle = n|n\rangle$; $\mathbf{H} = \hbar\omega(\mathbf{N} + \frac{1}{2})$;

$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(\mathbf{a}^\dagger + \mathbf{a})$ $\hat{p} = i\sqrt{\frac{\hbar m\omega}{2}}(\mathbf{a}^\dagger - \mathbf{a})$

$\mathbf{a}\mathbf{p} = (m*w/(2*hb))^(1/2)*(x*p-hb*diff(p,x,1)/(m*w))$
 $\mathbf{a}\mathbf{m}\mathbf{p} = (m*w/(2*hb))^(1/2)*(x*p+hb*diff(p,x,1)/(m*w))$
 $\mathbf{N} = ((m*w/(2*hb))^(1/2)*(x*amP-hb*diff(amP,x,1)/(m*w)))^2$

$\{\nabla\cdot\mathbf{A} = \partial_x A_x + \partial_y A_y + \partial_z A_z$; If $\nabla\cdot\mathbf{A}$ is 0/+/- it's source free/source/sink; $\nabla\times\mathbf{A} = [\partial_y A_z - \partial_z A_y, \partial_z A_x - \partial_x A_z, \partial_x A_y - \partial_y A_x]$; direction = max rotation axis; length = amount of rotation; If u/\mathbf{A} = scalar/vector field $\nabla^2\mathbf{A} = [\nabla^2 A_x, \nabla^2 A_y, \nabla^2 A_z]$; $\nabla^2 u = \Delta u$; if ∇^2 at point is 0/+/- the field equal/less/greater than the average of its neighboring values

If $\mathbf{k}=\omega$; \mathbf{A} of EM wave travel in $\mathbf{z}=\mathbf{A}=\mathbf{e}_x b \cos(kz-\omega t+\theta)$
 $\{\partial_t^2 A_x = \nabla^2 A_x\}$ If $\mathbf{q}=\mathbf{b}\cos(\omega t-\theta)$; $\mathbf{A}=\mathbf{e}_x[\mathbf{q}\cos(kz)-\mathbf{p}\sin(kz)/\omega]$;
 $\{\mathbf{p}=\mathbf{q}'$; $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$; $\mathbf{g}=\mathbf{p}\cos(kz)+\omega\mathbf{q}\sin(kz)$; $\mathbf{E}=-\mathbf{e}_x\mathbf{g}$;
 $\mathbf{B}=-\mathbf{e}_y\mathbf{g}$; $\mathbf{p}'=\mathbf{q}''=-\omega^2\mathbf{q}$; $\mathbf{E}=-\partial_t\mathbf{A}$; $\mathbf{B}=\nabla\times\mathbf{A}$ Replace \mathbf{q}, \mathbf{p} with $\mathbf{x}^o, \mathbf{p}^o$;
 $\mathbf{A}^o = \mathbf{e}_x (\hbar/(2\omega))^{1/2} (\mathbf{a}e^{ikz} + \mathbf{a}^\dagger e^{-ikz})$; $\{m=1\}$ $\mathbf{H} = \text{Average (Avg) over Space of } \frac{1}{2}|\mathbf{E}|^2 + \frac{1}{2}|\mathbf{B}|^2 = \frac{1}{2}(\mathbf{p}^2 + \omega^2\mathbf{q}^2)$; $\{\text{Avg}(\sin x) = 0$; $\text{Avg}(\sin^2 x) = \frac{1}{2}$

Maxwell's equations in empty space ($c = 1$)

\mathbf{B} has no sources or sinks & the magnetic flux through any closed surface is 0.

$\nabla \cdot \mathbf{B} = 0 \rightarrow \mathbf{B} = \nabla \times \mathbf{A}$ If \mathbf{A} = vector field: $\nabla \cdot \nabla \times \mathbf{A} \equiv 0$

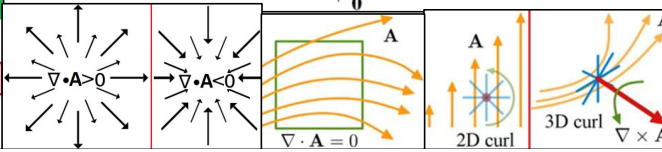
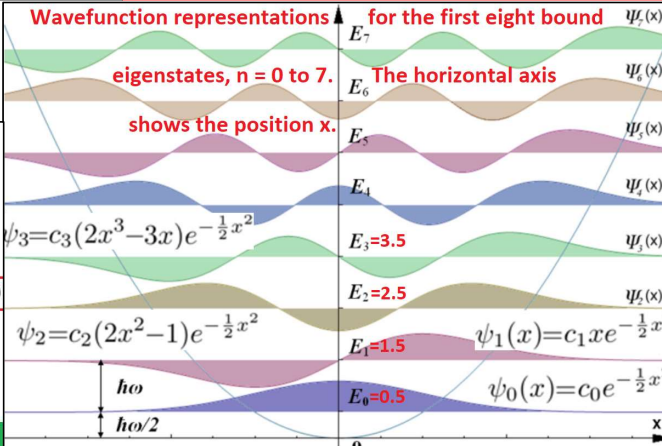
How \mathbf{B} changes with time: $\partial_t \mathbf{B} = -\nabla \times \mathbf{E} \rightarrow \mathbf{E} = -\partial_t \mathbf{A} \quad \nabla \times (\partial_t \mathbf{A} + \mathbf{E}) = 0$

So both \mathbf{E} & \mathbf{B} determined by a single field \mathbf{A} .

$\nabla \cdot \mathbf{E} = 0 \rightarrow \nabla \cdot \mathbf{A} = 0 \quad \partial_x A_x + \partial_y A_y + \partial_z A_z = 0$

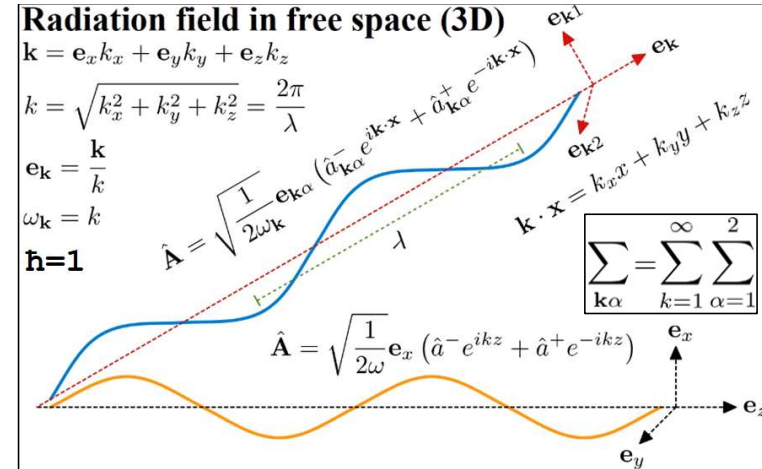
Vector form of wave equation: $\partial_t^2 \mathbf{E} = \nabla \times \mathbf{B} \rightarrow \partial_t^2 \mathbf{A} = \nabla^2 \mathbf{A} \quad \nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

$\partial_t^2 \mathbf{x} = \frac{1}{2} \partial_t^2 A_x = \nabla^2 A_x \quad \partial_t^2 A_y = \nabla^2 A_y \quad \partial_t^2 A_z = \nabla^2 A_z$



Replace q, p in the field energy density H with $x^\circ, p^\circ: H = \frac{1}{2}(p^{\circ 2} + \omega^2 x^{\circ 2}) = \hbar\omega(N^\circ + \frac{1}{2})$;

$\mathbf{A}^\circ = \text{Operator}$ (sum over all possible modes of an expression containing one destruction & one creation operator for the corresponding photons) & the wavefunction is a list of the number of photons present in each electromagnetic mode.



$\hat{\mathbf{A}} = \sum_{\mathbf{k}\alpha} \sqrt{\frac{1}{2\omega_{\mathbf{k}}}} \mathbf{e}_{\mathbf{k}\alpha} (\hat{a}_{\mathbf{k}\alpha}^- e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{a}_{\mathbf{k}\alpha}^+ e^{-i\mathbf{k}\cdot\mathbf{x}})$ **General radiation field**

$\hat{H} = \sum_{\mathbf{k}\alpha} \frac{1}{2} (\hat{p}_{\mathbf{k}\alpha}^2 + \omega_{\mathbf{k}}^2 \hat{q}_{\mathbf{k}\alpha}^2) = \sum_{\mathbf{k}\alpha} \left(\hat{N}_{\mathbf{k}\alpha} + \frac{1}{2} \right) \hbar\omega_{\mathbf{k}}$

$\hat{a}_{\mathbf{k}\alpha}^+$ creates photon in mode $\mathbf{k}\alpha$
 $\hat{a}_{\mathbf{k}\alpha}^-$ destroys photon in mode $\mathbf{k}\alpha$

$\hat{N}_{\mathbf{k}\alpha} = \hat{a}_{\mathbf{k}\alpha}^+ \hat{a}_{\mathbf{k}\alpha}^-$
 Number of photons in mode $\mathbf{k}\alpha$

Wave function of radiation field is a list of the number of photons in each mode $|\psi\rangle = |\dots, n_{\mathbf{k}\alpha}, \dots\rangle$

$\hat{a}_{\mathbf{k}\alpha}^+ |\dots, n_{\mathbf{k}\alpha}, \dots\rangle = \sqrt{n_{\mathbf{k}\alpha} + 1} |\dots, n_{\mathbf{k}\alpha} + 1, \dots\rangle$

modifies only that mode's occupation number (its n)

$\hat{a}_{\mathbf{k}\alpha}^- |\dots, n_{\mathbf{k}\alpha}, \dots\rangle = \sqrt{n_{\mathbf{k}\alpha}} |\dots, n_{\mathbf{k}\alpha} - 1, \dots\rangle$

$\hat{N}_{\mathbf{k}\alpha} |\dots, n_{\mathbf{k}\alpha}, \dots\rangle = n_{\mathbf{k}\alpha} |\dots, n_{\mathbf{k}\alpha}, \dots\rangle$

“Quantum field theory is just quantum mechanics with an infinite number of harmonic oscillators.”

Movie: Jan 17, 2022

[https://www.youtube.com
/watch?v=FPF4nSuvrGA](https://www.youtube.com/watch?v=FPF4nSuvrGA)

Subtitles:

The theory Of Nothing, how everything works and how it was created from nothing (By Guy Abitbol). Before walking through the 25 proven steps of the theory, lets skip the whole document for the good order. This document is attached in the description. Step #1 is the only not rigorously proved step in this theory (due to Gödel's incompleteness theorem). It states that nothing, or shape and space with existence contradiction is always being created (with any relative velocity); For example, if we defined nothing as a 0-dimensional point, then a shape in which a point encloses a line, that encloses a sphere surface, that enclose a spherical volume, is a non-existing shape, because this shape will be always enclosed by nothing. and we can proof that any open shape can be enclosed by something. Similarly, a space that originates from a point can't be created, because we can prove that any space must have origin, and we defined point as nothing. The following steps are proven qualitatively and demonstrated through 8 files of 3 dimensional pictures, and 44 files of MATLAB simulations, that are attached in the description. The certainty of the proofs is definite, because it is based on the pure derivation of classical mechanics as a mathematical theory. (see appendix). Step 2, When 2 high velocity non-existing-Shapes collide, they deformed into 2 Squashed non-existingShapes & move away from each other at lower speed. These squashed non-existing-Shapes will be referred to as G particles. Step 3, If 2G's collide, they bend at the collision point & rotate. Step 4, If 2 bent G's collide, they can be attached to each other & rotate together. Step 5, Further collisions bendings & attachments create cluster. Step 6, As the cluster become larger & larger, any new attachment increases its size by only small amount. But if the new attachment changes the moment of inertia, such that the cluster now rotates about its intermediate moment of inertia, then the cluster changes its orientation dramatically during the motion, even if the intermediate moment of inertia is just slightly bigger then the smallest moment of inertia. by the Intermediate axis theorem. Now, because of the dramatic changes in orientation, further collisions can cause the cluster to reorient and to rotate about its largest moment of inertia, while preserving its angular momentum and losing rotational energy. by the major axis theorem. Thus, over time, any cluster acquires 2 equal moments of inertia, while rotating about its third larger moment of inertia. Step 7, The rotation of any body is governed only by solving the system of equations in the former page. Therefore, bodies with same moment of inertia rotate the same. And this cluster rotate exactly like Feynman's Wobbling Plate, even though it's asymmetric. and the cluster's angular velocity and major axis, both return to their positions each T_{sub-L} seconds. The functions that describe the cluster's angular velocity are simple harmonic oscillators and wave functions. Step 8, Because, the cluster's major axis is the shortest; over time, more stuff collides in its direction, than in the other directions, making the cluster thinner and thinner. Until its large moment of inertia is about twice bigger than the other 2. Step 9, Therefore, over time, any cluster acquires 2 equal moments of inertia, while mainly rotating about its third, twice bigger moment of inertia. and the time taken for the major axis to return to its positions, is 1 half of the time taken to the other 2 axes to return to their former positions. Step 10, When the formed cluster collide with G's, it bent them at the collision point, rotate them and throw them with high speed. creating G-sub-z, which we call Electric field; or G-sub-xy, which we call Magnetic field. Step 11, Over time all the clusters in the universe throw G-sub-z and G-sub-xy on each other. That cause them to attach or detach sub-particles, and to adjust orientation. The equilibrium of these collisions creates types of clusters differing in the number of their sub-particles, or mass. Step 12, Because the clusters throw many G's on each other, some G's increase the magnitude of the clusters' angular momentum, and some decrease it. such that, at equilibrium, this magnitude is equal for all clusters. by the theory of synchronization. And because the major axis period, T_{sub-L} , is inversely proportional to this magnitude, and proportional to the major moment of inertia, the major axes of all same type clusters align with the world Z axis, simultaneously. However, different particles of the same type have different direction of angular momentum, which is restricted to one of 2 spherical zones, which we call positive and negative charge. There is a transformation that transform between any spatial direction to a direction in the spherical zone. This corresponding spatial direction, from now on referred to as L-sub-q, is what quantum mechanics mistakenly taken as the particle angular momentum. What we call magnetic field direction, is equal to L_{sub-q} for a positive charge, and opposite for negative charge. Step 13, We can see that positron create G-sub-z +, and electron create G-sub-z minus. What we call electric field direction at a point, is the flying direction of G-sub-z+ there. Or the opposite flying direction of G-sub-z minus there. The collision of G-sub-z is more powerful when its tangential velocity is in the same direction as its translational velocity. And the collision response is dictated by the collision point. Therefore, because the electron and positron clusters contain many bended edges, we can see that when G-sub-z minus collide with electron it repels it from its source. and when it collides with positron it attracts it. Therefore, Like Charges Repel and Opposite Charges Attract. With a force inversely proportional to the distance squared, because G-sub-z are geometrically diluted in 3-dimensional space. Step 14, The period of the electron's major axis is half of the period of its other axes. Thus 2 different facets are capable of throwing G-sub-x+ in the direction of the electron's magnetic field, but only 1 facet is capable of throwing G-sub-x+ in a specific perpendicular direction. Therefore, the electron's magnetic field is twice stronger in the direction of its primary magnetic field, than in any perpendicular direction. The plane perpendicular to the angular momentum represents the average cluster orientation. Electron and positron with same magnetic field have exactly opposite angular momentum, and thus the same average cluster orientation. Therefore, they behave the same in magnetic field. Taking into account the tangential velocity of the average cluster orientation, and the geometric dilution and rotation of G-sub-xy, we can see that 2 electrons with same magnetic field attract if positioned along their magnetic field and repel if positioned perpendicularly.

Similarly, taking into account also the G-sub-xy flying direction and its collision point, we can prove any force and torque exerted by a magnetic field. While the information about the electron's angular momentum is sufficient to dictate most of its electric and magnetic interactions. As described by the Maxwell equations. A more precise description requires also information about the angular velocity and whether it's in an odd or even round, as real electron period involve 2 rounds of angular velocity about the angular momentum. This odd or even round will be referred to as phase. This more precise description is captured in the quantum spin state, because, it is derived from a wave function, and any wave function satisfy the simple harmonic oscillator equation, which describe our cluster's angular velocity direction and phase. For a moving electron, a more precise description requires also information about its velocity and its distance to the target. In order to determine in what direction of angular velocity and phase the electron will arrive. This information is also incorporated to the wave function in quantum mechanics by the dot product of the particle's momentum, and the target position vector. We can see from the interaction pictures, that electron in external homogeneous magnetic field feels torque but not force. And that electron and positron with same magnetic field feel same torque, but precess in opposite direction about the external magnetic field, because they have opposite angular momentum. This is a torque induced precession, which we call Larmor precession. In contrast to the torque free precession of the electron, that describe previously and create the T-sub-L period. The duration of one round of Larmor precession equal to the product of T-sub-L with an odd integer. Thus, after one round, the angular velocity of the electron returns to its starting point, but its non-major axes complete only half round. Therefore, the electron returns to its starting orientation only after 2 rounds of Larmor precession. Step 15, Charge that move in external magnetic field, feel force perpendicular to their movement and to the external magnetic field in a charge dependent direction. This is what we call Lorentz force, and it is demonstrated by examining the most powerful collision of G-sub-xy into a plane representing the average cluster orientation and rotation of the charge. Step 16, Moving charge generates internal magnetic field, perpendicularly to its movement and to the examined point, in a charge dependent direction. This is what we call Biot Savart Law, and it is demonstrated by examining the collision of the plane representing the average cluster orientation and rotation of the charge, into a G particle due to the charge movement. And the resulting bended particle, that thrown into the examined point. Step 17, In Stern Gerlach experiment we fire electrons through inhomogeneous magnetic field. Because of the movement of the electrons, G-sub-xy, from the magnetic field hit them in various points, and exert a changing torque on them. which align their internal magnetic field, parallel or antiparallel, to the external inhomogeneous magnetic field. As previously demonstrated, after this alignment, torque is no longer being exerted on the electrons, and an equal force push them upward or downward, depending on their internal magnetic field orientation. Creating 2 distinct parts on the screen. The probability that the electron's internal magnetic field will be align parallel to the external magnetic field, depends on G-sub-xy collision point, which depends on what amount the internal and the external magnetic fields go in the same direction, in other words, in their dot product, or in the cosine of the angle between them. However, the cosine range is 1 and -1, and a probability range is 1 and 0. A Simple range conversion and some trigonometric identities show that this probability equal to the square cosine of half the angle. Exactly what we get from quantum mechanics. Step 18, The electron and positron angular momentum is restricted to a spherical zone. Such that, when it aligns parallel or anti-parallel to its major axis, it can take infinite values, but otherwise it is restricted to one value. During Larmor precession, the internal magnetic field and L-sub-q is really precess about the external magnetic field. We can use the previously mentioned transformation to transform between the internal magnetic field to the angular momentum. During a stern Gerlach separation, an indelicate Larmor precession can also contribute to the equal separation pattern observed when applying consecutive perpendicular stern Gerlach apparatus. Step 19, G particle is created by a powerful collision of 2 non-existing spheres; and thus, having a maximally thin oblate spheroid shape. When G particle collides with positron, it bends, such that it has plan of symmetry. where the normal of this plan is its angular velocity. Thus, its angular velocity aligns with its angular momentum. The impulse of collision between positron and G particle, is referred to as non-bending impulse, if it's the biggest collision impulse that doesn't cause G bending. In our universe the non-bending impulse is constant, such that the speed of its emitted G is the speed of light. Any bigger collision impulse will cause G bending, and is referred to as a bending impulse. However, only the impulse at the last contact point, dictates the speed of the emitted G particle. Therefore, any bigger collision impulse will continue to bend G at their contact point. and this point won't be the last contact point until the collision impulse reaches the magnitude of the non-bending impulse. Thus, any collision impulse will emit any bended G at the speed of light. in other words, the speed of the electric and magnetic field particle is always the speed of light, regardless of the velocity of their source. While stationary and uniformly moving charge collide with G only once, an accelerating charge collide with G twice, and thus bends it twice. This twice bended G, is what we call photon. The stronger the collisions, the larger the magnitude of the photon angular velocity and its deformation. But larger deformation has smaller moment of inertia. Thus, the magnitude of the photon angular momentum, which is the product of its moment of inertia with its angular velocity, remains constant for any photon, and equal to the reduced Planck constant. On the contrary, the photon rotational energy, which is half the dot product of its angular momentum with its angular velocity, increases as the magnitude of the photon angular velocity increases. As demonstrated in page 16 and 33, even though each photon has a non-symmetric shape, its angular velocity still undergoes some kind of indelicate precession about its angular momentum.

Let's define T_{sub-f} as the time taken for the photon angular velocity to approximately return to its initial position. Therefore, the photon frequency is $1/T_{sub-f}$. Because the magnitude of the photon angular momentum is constant and equal to the reduced Planck constant, if we use change of variables in the integral that calculate the photon rotational energy, we can show that the photon rotational energy is equal to the product of its frequency and plank constant. Therefore, the total energy of any photon is greater from the known hf by a constant. but in any experiment this constant is reduced, see page 23. If a stationary charge begins an accelerated motion, it generates photon, the magnetic component of the photon, must have a direction perpendicular to the acceleration and to the photon location, in a charge dependent direction. This is a consequence of the Lienard Wiechert equation, which stem directly from Maxwell's equations. This is demonstrated in the following pictures, that examine the collision of the charge with G particle, due to its acceleration, and the resulting photon in its examination point. The electric component of the photon is generated by the second collision, and thus it's perpendicular to its magnetic component, and to its velocity. In the former page I have demonstrated how a bigger collision impulse, between a charge and G particle, increases the amount of G bending, and its angular velocity, while decreasing its moment of inertia, such that, the magnitude of its angular momentum remains constant. And I have also shown that any photon frequency can be obtained by this mechanism. furthermore, I have also demonstrated that a bended G with larger angular velocity will cause more powerful subsequent collision with another electron, rather than a bended G with smaller angular velocity, even though its moment of inertia is smaller, and even though they both have the same magnitude of angular momentum. This explain why only high frequency photons are capable of ejecting electron in the photo electric effect. Moreover, the former page also explains why the speed of any photon is the speed of light, regardless the velocity of its source or its frequency. Because, the shape of the bended G doesn't matter, what is matter, is the impulse exerted on its last contact point. Impulse bigger than the non-bending impulse will continue to bend G, until it reaches the value of the non-bending impulse, which cause emission at the speed of light. Step 20, What we call left and right circularly polarized photons, are photons that their angular momentum is parallel and anti-parallel to their velocity, respectively. Therefore, their precessing angular velocity create rotational effect when hit a target. What we call linearly polarized photon, is photon that its angular momentum is perpendicular to its velocity. We mistakenly say that it has no angular momentum. The other photons are elliptically polarized photons. What we call polarizer is long sheets of molecules, that are capable of moving only parallel or antiparallel to a specific direction, referred to as the polarizer direction. The more the photon angular momentum is perpendicular to the polarizer direction the more probable that it will pass through it. Because this photon will be capable of moving the polarizer upon collision. And the polarizer's moving particles will in turn collide with another G and generate another photon with the same properties and direction. This is because the photon tangential velocity is much greater than its translational velocity. What we call photon polarization direction is a unit direction perpendicular to its velocity and to its angular momentum. Therefore, we can calculate the Malus's law, which is the probability that a photon will pass through a polarizer. By calculating in what amount the photon angular momentum is perpendicular to the polarizer direction, which is their absolute cross product. Thus, using some mathematical identities we arrive at the Malus photon passing probability, which is the square cosine of the angle between the polarizer direction and the photon polarization direction. Similarly, using also the Lienard Wiechert equation, we can prove the properties of: polarization by scattering. Antenna that creates vertical photon polarization direction, orient vertically, such that the moving electron will hit G particle and rotate it with angular momentum perpendicular to its emitted velocity and to its polarization direction. Using potential energy consideration, we can show that the wave length of the created photon, is twice the length of the antenna. Similarly, antenna that create horizontal photon polarization, orient horizontally. And helical antenna can be used to create circular polarized photon. Step 21, The Rabi cycle. The internal magnetic field of an electron will precess in external homogenous magnetic field, such that it returns to its initial position each odd integer multiples of T_{sub-L} , referred to as T_{sub-w} . Therefore, the electron's angular velocity also returns to its position each T_{sub-w} seconds. And as expected, T_{sub-w} is proportional to the electron mass, and inversely proportional to the electron charge, and to the magnetic field magnitude. If we rotate a second external homogenous magnetic field, perpendicularly to the first, such that its direction returns to its initial direction each T_{sub-w} seconds. Then its emitted G_{sub-xy} will always hit the electron face in an opposite direction of its motion, creating a strong force, that flip the electron internal magnetic field, or its L_{sub-q} . As expected, the time taken for this flip, is proportional to the electron mass, and inversely proportional to the electron charge, and to the rotating magnetic field magnitude. As expected, in order to flip the electron, instead of using the rotating magnetic field, we can also fire, circularly polarized photons with T_{sub-f} , that equal to T_{sub-w} , at a direction perpendicular to the first external magnetic field. Step 22, Quantum electro dynamics. Because the photon tangential velocity is much greater than its translational velocity, and because its angular velocity approximately rotates about its angular momentum, there will be a point on the photon that always hit the target first. The normal at this point dictate the photon exerted force direction on the target. The total effect of many colliding photons can be roughly calculated by summing up all these exerted forces directions. Because each photon travel at the speed of light, we can calculate its travel time duration, dt , and then its exerted force direction, by rotating the initial exerted force about the photon angular momentum by the product of dt , and $2\pi/T_{sub-f}$ radians. We can use this technique to prove the law of reflection. But also, to prove diffraction grating, and any other law involving photons.

While the rotating photon approximately return to its orientation each $T\text{-sub-f}$ seconds, the rotating magnetic and electric field particles, exactly return to their orientation, because their angular velocity and angular momentum are parallel. And that is the reason that we were able to predict the electron magnetic moment to a very high accuracy. Step 24, Gravity. In the universe, everywhere and every time non-existing-shapes can be created, with any relative velocity. Therefore, any object will feel collision forces from all directions, which on average cancel each other out. But if 2 objects stand close to each other, they will feel less collision forces from the side that in between them, because each act like a barrier that prevent collisions from far created non-existent shapes. Thus, the amount of these prevented forces of collisions that goes in the direction of these 2 objects is equal to the exerted gravitational force from the other side that each object feels. Therefore, we can calculate the gravitational force, by summing up all these prevented forces that goes in the direction of these 2 objects, using a double integral over the blocked spherical area. We can see that this calculated force, like the Newton Gravitational force, is inversely proportional to the squared distance of these objects. Additionally, this calculated force is proportional to the products of the 2 objects' surface area, which is, an expression to their mass. This explains why anybody falls with the same acceleration, regardless of its mass. As it just cancels out, because acceleration is force over the accelerated mass. Gravitational redshift is caused by the movement of the electron that create the photon or by the movement of the electron in the receiver. In both cases, the change in the collision impulse of this electron, with G-particle or with photon, is increased with gravity. Similarly, massive object bends light. Because the light is reflected from electron, that is accelerated due to gravity. And, gravitational time dilation, is caused by electrons' distance elongation, due to gravitational forces. Step 25, Entanglement. If charge collide with G particle, it bends, rotates and emits it at the speed of light, always. because the deformation reduces the collision energy. However, in the universe, there are also small spherical shaped clusters, that are not capable of being deformed, referred to as O particles. Therefore, if electron collide with O particle, it emits it with a speed much greater than the speed of light. Because there is no energy loss to deformation. See step 29. This O particle can be thrown back and forth between 2 electrons, with opposite internal magnetic field, creating what we call entangled particles. Thus, if we change the internal magnetic field of one electron, the O particle will hit the other electron in a different point and change its internal magnetic field to be again opposite of the first. Entangled photons are created when the electron in their transmitter, is entangled to the electron of their receiver, or polarizer. See appendix.