IS THE ACCELERATED EXPANSION OF THE UNIVERSE RESPONSIBLE FOR THE EMERGENCE OF THE FINE-STRUCTURE CONSTANT (1/137)?

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ABSTRACT

In this contribution, it is shown how the fine-structure constant emerges when two systems interact, one of which, light, perceives the universe as static due to not experiencing the passage of time, while the other, matter, experiences accelerated expansion. In this way, the fine-structure constant can be related to the rate of expansion of the universe; α can thus be expressed through the ratio of the theoretical radius of the universe $R_{uT} \approx$ 13.6-13.8 Gly to the observed radius $R_{uI} \approx$ 46.5 Gly.

$$\alpha = \left(\frac{R_{uT}}{R_{uI}}\right)^4 \approx \left(\frac{13.6 \text{ Gly}}{46.5 \text{ Gly}}\right)^4 \approx \frac{1}{137} \tag{1.21}$$

From the above, it is clear that, since the rate of expansion of the universe has not remained constant over time, the value of the fine-structure constant must also have changed (unless there is over time a proportional variation of the speed of light c), as have other physical constants and quantities. This implies that physical laws have not remained unchanged throughout the history of the universe. Regarding the spatial variation of α, the issue is more complex and falls outside the scope of this work.

1.1 INTRODUCTION

The fine-structure constant, denoted by the Greek letter α, represents the coupling constant of the electromagnetic interaction between light and matter. It was introduced by Arnold Sommerfeld [1] as a correction factor after considering relativistic effects in the calculation of energy levels in the Bohr atomic model. It is a dimensionless quantity with a value of approximately $1/137$ [2]. Since this constant "emerges" when light interacts with matter, we might ask: what was not considered by Sommerfeld? What does this constant really represent? And if the answer lies in the question: what distinguishes light from matter? Light differs from matter in that it maintains a constant speed in all inertial reference frames [3]. This means that, even though the universe is expanding (accelerating), the speed of light remains the same; furthermore, from the perspective of light, since time does not pass [4], the universe is not expanding at all. We thus ask: is the (accelerated) expansion of the universe what makes α emerge? Does this constant then represent the interaction coefficient between two systems, one of which, light, maintains its speed unchanged while propagating through the expanding universe? How can we attempt to demonstrate this hypothesis? A first step is to understand the role of cosmic-scale expansion. Next, we need to examine how these effects manifest on a smaller scale and assess their possible impact on the electromagnetic interaction between light and matter.

1.2 The observable universe

In cosmology, the term "observable universe" refers to the portion of the universe that can be examined by a specific observer [5]. Every point in space has its own observable region. If the universe were static, the horizon of the universe would be located at a distance of approximately 13.6-13.8Gly [6] from the observation point. This is because light could have traveled a maximum distance of cT, where $T \approx$ 13.6-13.8 Gy [6] is the age of the universe. However, since the universe is expanding at an accelerating rate, the actual size of this horizon is larger, estimated at around 46.5 Gly [7]. As the expansion is still ongoing, this means that the boundary of the observable universe continues to move. This boundary represents the maximum distance at which causal contact is possible [8].

1.3 Conversion factors

We thus ask whether the effect of the accelerated expansion of the universe only affects the position and recession velocity of cosmic objects [9] (and thus the value of the radius of the observable universe), or also impacts other types of observations (both on cosmic and smaller scales). To assess the influence of accelerated expansion of the universe (interpreted as the expansion of spacetime [10]) on measurements, we can compare what we observe at the limits of our observable universe (at distances $R_{uI} \approx 46.5\ Gly$) with what we would expect to observe if the spacetime were static (and thus the cosmic horizon would have expanded at a constant speed equal to that of light c). By taking the ratio between these two measurements, we obtain dimensionless coefficients, the conversion factors. We denote the actually observed values (which include a component due to the accelerated expansion of spacetime) by uI, and the (theoretical) values expected at the edge of our observable universe by uT. This way, we obtain two sets of measurements, the only common element being time, 13.6 Gy (for illustrative purposes, we will use the value derived from WMAP data [11], so that the ratio $R_{uI}/R_{uT} \approx 3.42$). Below are derived and listed some of the main conversion factors. Their numerical value is accompanied by the corresponding value in units of $\alpha' \approx 1/137$. The equivalence between α' and the fine-structure constant α will be demonstrated subsequently. Other conversion factors can be deduced from the combination of those listed. It is also possible to obtain unitary values, invariant with respect to the expansion of spacetime.

1) Conversion Factor for Time. The conversion factor for time is defined as the ratio between the measured and theoretical age of the universe; by definition, it equals 1.

$$f[t] = \frac{T_{uI}}{T_{uT}} = \frac{13.6\ \text{Gy}}{13.6\ \text{Gy}} = 1 \tag{1.1}$$

2) Conversion Factor for Lengths. The conversion factor for lengths is determined by taking the ratio between the measured and theoretical radius of the universe.

$$f[l] = \frac{R_{uI}}{(R_{uT} = cT)} = \frac{46.5\ \text{Gly}}{13.6\ \text{Gly}} = \frac{1}{\sqrt[4]{\alpha'}} \approx 3.42 \tag{1.2}$$

3) Conversion Factor for Velocities. The conversion factor for velocities is defined by the ratio between the rate of expansion of the universe and the speed of light c.

$$f[\boldsymbol{v}] = \frac{\boldsymbol{v}_{uI}}{(\boldsymbol{v}_{uT} = c)} = \frac{f[l]}{f[t]} = \frac{1}{\sqrt[4]{\alpha'}} \approx 3.42 \tag{1.3}$$

4) Conversion Factor for Accelerations. The conversion factor for accelerations has the same value as that for lengths, since the conversion factor for time is $f[t] = 1$.

$$f[\boldsymbol{a}] = \frac{\boldsymbol{a}_{uI}}{\boldsymbol{a}_{uT}} = \frac{f[l]}{f^2[t]} = \frac{1}{\sqrt[4]{\alpha'}} \approx 3.42 \tag{1.4}$$

5) Conversion Factor for Curvature. This conversion factor is determined by the ratio between the squares of the theoretical and measured radii of the universe.

$$f[S] = \frac{S_{uI}}{S_{uT}} = \frac{R_{uT}^2}{R_{uI}^2} = \frac{1}{f^2[l]} = \sqrt{\alpha'} \approx \frac{1}{11.7} \tag{1.5}$$

6) Conversion Factor for Gravitational Attraction Force. The conversion factor for gravitational attraction is equal to the inverse of the conversion factor for squared lengths. This is because the conversion factor for the product *Gmm* must necessarily be unitary (invariant), so that the conversion factors for forces, have the same value.

$$f[\boldsymbol{F}_g] = \frac{\boldsymbol{F}_{g\,uI}}{\boldsymbol{F}_{g\,uT}} = \frac{1}{f^2[l]} = \sqrt{\alpha'} \approx \frac{1}{11.7} \tag{1.6}$$

7) Conversion Factor for Energy. The conversion factor for energy can be derived from the conversion factors for force and length.

$$f[E] = \frac{E_{uI}}{E_{uT}} = f[\boldsymbol{F}_g] \cdot f[l] = \sqrt[4]{\alpha'} \approx \frac{1}{3.42} \tag{1.7}$$

8) Conversion Factor for Mass. The conversion factor for mass can be calculated from the conversion factors for energy and velocity.

$$f[m] = \frac{m_{uI}}{m_{uT}} = \frac{f[E]}{f^2[\boldsymbol{v}]} = \sqrt[4]{\alpha'^3} \approx \frac{1}{40} \tag{1.8}$$

9) Conversion Factors for the Gravitational Constant 'G', Permittivity, and Gravitational Permeability. The conversion factor for the gravitational constant 'G' is obtained by combining the conversion factors for force, length, and mass. The conversion factor for gravitational permittivity is simply its inverse.

$$f[G] = \frac{G_{uI}}{G_{uT}} = \frac{f[\boldsymbol{F}_g] \cdot f^2[l]}{f^2[m]} = \frac{1}{\alpha'\sqrt{\alpha'}} \approx 1600 \tag{1.9}$$

$$f[\varepsilon_g] = \frac{\varepsilon_{g\,uI}}{\varepsilon_{g\,uT}} = \frac{1}{f[G]} = \alpha'\sqrt{\alpha'} \approx \frac{1}{1600} \tag{1.10}$$

The conversion factor for gravitational permeability is deduced by combining the conversion factors for velocities and for gravitational permittivity ε_g.

$$f[\mu_g] = \frac{\mu_{g\,uI}}{\mu_{g\,uT}} = \frac{f[\varepsilon_g]}{f^2[\boldsymbol{v}]} = \frac{1}{\alpha'} \approx 137 \tag{1.11}$$

10) Conversion Factor for Coulomb Force. The conversion factor for Coulomb force must be identical to that for gravitational attraction force. This principle applies, in general, to every quantity that has the dimensions of force.

$$f[\boldsymbol{F}_e] = \frac{\boldsymbol{F}_{e\,uI}}{\boldsymbol{F}_{e\,uT}} = \frac{1}{f^2[l]} = \sqrt{\alpha'} \approx \frac{1}{11.7} \tag{1.12}$$

11) Conversion Factor for Electric Charge. The conversion factor for electric charge can be determined by considering that the conversion factor for the electric field $\boldsymbol{E}$ must necessarily be unitary. This because electric fields, unlike gravitational fields, whose variations have led to the postulation of dark energy and matter, do not exhibit comparable anomalies (CMB is homogeneous and isotropic on a large scale) [12].

$$f[q] = \frac{f[\boldsymbol{F}_e]}{f[\boldsymbol{E}]} = f[\boldsymbol{F}_e] = \frac{q_{uI}}{q_{uT}} = \sqrt{\alpha'} \approx \frac{1}{11.7} \tag{1.13}$$

12) Conversion Factors for Coulomb's "Constant", Electric Permittivity, and Magnetic Permeability. The conversion factor for Coulomb's "constant" K is derived from the conversion factors for force, length, and electric charge. The conversion factor for electric permittivity is its inverse.

$$f[K] = \frac{K_{uI}}{K_{uT}} = \frac{f[\boldsymbol{F}_e] \cdot f^2[l]}{f^2[q]} = \frac{1}{\alpha'} \approx 137 \tag{1.14}$$

$$f[\varepsilon_e] = \frac{\varepsilon_{e\,uI}}{\varepsilon_{e\,uT}} = \frac{1}{f[K]} = \alpha' \approx \frac{1}{137} \tag{1.15}$$

The conversion factor for magnetic permeability is deduced by combining the conversion factors for velocity and electric permittivity ε_e.

$$f[\mu_e] = \frac{\mu_{e\,uI}}{\mu_{e\,uT}} = \frac{f[\varepsilon_e]}{f^2[\boldsymbol{v}]} = \frac{1}{\sqrt{\alpha'}} \approx 11.7 \tag{1.16}$$

1.4 Unitary conversion factors

It is important to note that, since Hubble's law [9] is a linear relationship, it is possible to divide the conversion factors for the radius of the observable universe, measured $R_{uI} \approx$ 46.5 Gly and theoretical $R_{uT} \approx$ 13.6-13.8 Gly, in order to obtain two sets of unitary conversion factors (expressed in m^{-1}). These indicate how much the measurements need to be corrected for each meter between the observer (0) and the location of the phenomenon (d), when considering, respectively, an accelerating or a static universe. By taking the ratio of the corresponding unitary conversion factors from the two sets, we once again obtain the same values as in relations (1.1-1.16). In this way, the conversion factors prove to be scale invariants. We can thus use the subscripts uI and uT to generically indicate whether a quantity does or does not contain a component due to the accelerated expansion of the universe.

1.5 Equivalence between α' and the fine-structure costant α

To demonstrate that the previously introduced parameter α' corresponds to the fine-structure constant α, let us consider a generic electromagnetic field, to which a certain energy density is associated. Our goal is to understand how its presence relativistically modifies spacetime. To do this, we use Einstein's field equation [13]; we recall that in the field equation, $G_{\mu\nu}$ represents the Einstein tensor, $T_{\mu\nu}$ the stress-energy tensor, and G the gravitational constant. We have seen that, when propagating in a vacuum, light maintains its constant speed while traversing expanding spacetime. Thus, the quantities to be used to describe the field are of the uT type.

$$G_{\mu\nu(uT)} = \frac{8\pi G_{uT}}{c_{uT}^4} T_{\mu\nu(uT)} \tag{1.17}$$

Let's now suppose that the field is absorbed (in the form of photons) by matter. Even though the field no longer exists as such, it still contributes to the energy of the system of which it is now a part, and therefore to the curvature of that region of spacetime. Consequently, the units must be converted from uT to uI, since matter "perceives" the passage of time and the accelerated expansion of the universe. Now, using the conversion factors (chap.1.3), let's calculate the ratio between the energies:

$$\frac{\int \rho_{E_{uI}}\, dV}{\int \rho_{E_{uT}}\, dV} = f[E] \cdot f^{-3}[l] = \alpha' \tag{1.18}$$

This ratio turns out to be equal to α'. Here, V represents the integration volume measured in an external reference frame, so its value is the same before and after the interaction (it does not depend on the conversion factors). When it is said that the fine-structure constant represents the interaction constant between light and matter, it can refer to the interaction of a photon with a system consisting of a pair of electrons [14] (separated by a distance $r = c/2\pi v$). In this way, α can be defined through the ratio of the electrostatic energy Ke^2/r (where K is the Coulomb constant) to that of the photon hv. If $\rho_{E_0} = \rho_{E_{uT}}$ and $\rho_E = \rho_{E_{uI}}$ are the energy densities of the electromagnetic field, before and after its interaction with matter, we can write:

$$\frac{\int \rho_E\, dV}{\int \rho_{E_0}\, dV} = \frac{Ke^2/r}{hv} = \alpha \tag{1.19}$$

Therefore, the variation predicted by the conversion factors is effectively the same as that observed in the interaction between light and matter. Thus we can write:

$$\alpha' = \alpha \tag{1.20}$$

We can thus invert the relations (1.1-1.16) to derive $\alpha' = \alpha$. In this way, the fine-structure constant turns out to be a coefficient that emerges when two systems interact, one of which, light, "perceives" the universe as static (cosmological redshift affects wavelength and frequency, but not the speed c [15]), while the other, matter, is undergoing accelerated expansion. Therefore, the fine-structure constant can be expressed as a function of the parameters of the observable universe; one of the most striking representations is certainly the one involving the theoretical $R_{uT} \approx$ 13.6-13.8 Gly and measured $R_{uI} \approx$ 46.5 Gly of the observable universe:

$$\alpha = \left(\frac{R_{uT}}{R_{uI}}\right)^4 \approx \left(\frac{13.6\ \text{Gly}}{46.5\ \text{Gly}}\right)^4 \approx \frac{1}{137} \tag{1.21}$$

1.6 THE VARIATION OF α

There has been much discussion about whether the fine-structure constant has remained the same over time. Early tests, based on the observation of spectral lines from distant astronomical objects [16] and on radioactive decay in the natural nuclear reactor at Oklo (Gabon) [17], did not detect significant variations. However, a 2010 study [18] suggested that α might have been different in the past, raising doubts about the universality of physical laws. In 2020, another study [19] proposed that α could vary depending on the direction of observation, calling into question the isotropy of the universe itself. However, since α can be expressed via equation (1.21), unless there was a corresponding variation in the speed of light c, its value must have been different. By using the theoretical and real radii that the universe had at a certain period in the past, it is possible to derive the value of α. Finally, since all other conversion factors (1.1-1.16) are expressed in terms of α, this means that all other constants and quantities also had different values in the past (and so in the future).

1.7 PLANCK CONSTANT

By inverting the relation that defines the fine-structure constant [20] $\alpha = e^2/2\varepsilon_0 hc$, and substituting α with the (1.21), we can express h (Planck's constant) as follows:

$$h = \frac{e^2}{2\varepsilon_0 \alpha c} = \frac{e^2}{2\varepsilon_0 c}\left(\frac{R_{uI}}{R_{uT}}\right)^4 \tag{1.22}$$

Even though the fine-structure constant emerges within quantum theory, it can therefore be expressed solely using “classical” quantities. Indeed, equation (1.22) includes the elementary electric charge e, the vacuum permittivity ε_0, the speed of light in a vacuum c, and the radii R_{uT} and R_{uI} of the observable universe.

SYMBOLS AND UNITS

THEORETICAL AND MEASURED VALUES	uT, uI
CONVERSION FACTORS	$f[\,]$
FINE-STRUCTURE CONSTANT	$\alpha \approx 1/137$
PLANCK’S CONSTANT	$h \approx 6.626 \cdot 10^{-34}\ J \cdot s$
ELEMENTARY CHARGE	$e \approx 1.616 \cdot 10^{-19}\ C$
SPEED OF LIGHT	$c \approx 299\ 792\ 459\ m \cdot s^{-1}$
ELECTRIC PERMETTIVITY	$\varepsilon_0 \approx 8.85 \cdot 10^{-12}\ F \cdot m^{-1}$

CONVERSION FACTORS

TIME	$T_{uI} = 1 \cdot T_{uT}$
LENGTH	$R_{uI} = 1/\sqrt[4]{\alpha} \cdot R_{uT} \approx 3.42 \cdot R_{uT}$
VELOCITY	$\boldsymbol{v}_{uI} = 1/\sqrt[4]{\alpha} \cdot \boldsymbol{v}_{uT} \approx 3.42 \cdot \boldsymbol{v}_{uT}$
ACCELERATION	$\boldsymbol{a}_{uI} = 1/\sqrt[4]{\alpha} \cdot \boldsymbol{a}_{uT} \approx 3.42 \cdot \boldsymbol{a}_{uT}$
CURVATURE	$S_{uI} = \sqrt{\alpha} \cdot S_{uT} \approx 1/11.7 \cdot S_{uT}$
ENERGY	$E_{uI} = \sqrt[4]{\alpha} \cdot E_{uT} \approx 1/3.42 \cdot E_{uT}$
MASS	$m_{uI} = \sqrt[4]{\alpha^3}\, m_{uT} \approx 1/40 \cdot m_{uT}$
GRAVITATIONAL ATTRACTION FORCE	$\boldsymbol{F}_{g\,uI} = \sqrt{\alpha}\, \boldsymbol{F}_{g\,uT} \approx 1/11.7 \cdot \boldsymbol{F}_{g\,uT}$
UNIVERSAL GRAVITATIONAL CONSTANT	$G_{uI} = 1/\alpha\sqrt{\alpha}\, G_{uT} \approx 1600 \cdot G_{uT}$
GRAVITATIONAL PERMETTIVITY	$\varepsilon_{g\,uI} = \alpha\sqrt{\alpha}\, \varepsilon_{g\,uT} \approx 1/1600 \cdot \varepsilon_{g\,uT}$
COGRAVITATIONAL PERMEABILITY	$\mu_{g\,uI} = 1/\alpha\, \mu_{g\,uT} \approx 137 \cdot \mu_{g\,uT}$
ELECTRIC CHARGE	$q_{uI} = \sqrt{\alpha}\, q_{uT} \approx 1/11.7 \cdot q_{uT}$
COULOMB FORCE	$\boldsymbol{F}_{e\,uI} = \sqrt{\alpha}\, \boldsymbol{F}_{e\,uT} \approx 1/11.7 \cdot \boldsymbol{F}_{e\,uT}$
COULOMB CONSTANT	$K_{uI} = 1/\alpha\, K_{uT} \approx 137 \cdot K_{uT}$
ELECTRIC PERMETTIVITY	$\varepsilon_{e\,uI} = \alpha\, \varepsilon_{e\,uT} \approx 1/137 \cdot \varepsilon_{e\,uT}$
MAGNETIC PERMEABILITY	$\mu_{e\,uI} = 1/\sqrt{\alpha}\, \mu_{e\,uT} \approx 11.7 \cdot \mu_{e\,uT}$

SPATIAL INVARIANTS (ON A LARGE SCALE)

THE PRODUCT	Gmm
THE PRODUCT	Kqq
THE RATIO	K/μ_g
THE RATIO	$\boldsymbol{F}_g/\boldsymbol{F}_e$
ELECTRIC FIELD	$\boldsymbol{E}$
FINE-STRUCTURE CONSTANT	α

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