The Pioneer effect, Fermat's last theorem and the gravitational constant G.

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Using the mechanical interpretation of the derivative and analyzing the function y = x^n, we can derive the general form of the equation for Newton's law of universal gravitation. From the general and more precise formulation of the law of gravitation, it follows that the gravitational constant G (if the equation is written in the traditional form) is no longer constant and will increase with increasing distance between interacting bodies.

Sometimes physics helps to more deeply understand purely mathematical problems and vice versa. Let us demonstrate this using the example of Fermat's last theorem.

$$x^n + y^n = z^n$$

Let's assume that the function y = f(t) = x^n describes the distance of a point, for example, a racing car, from the origin at time t (t^n).

Physics provides a formula for calculating distances:

$$S = v(0) * t + (a * t^2)/2$$

v(0) is the initial velocity,

a is the acceleration,

t is the time of movement.

We will demonstrate the calculation for the case of x^2 using the example of the Pythagorean triple (3, 4, 5).

3^2 + 4^2 = 5^2

9 + 16 = 25

25 is the distance that the car traveled from the origin to the point t3 (t = 5). Taking into account Fermat's theorem, let us consider the motion on the segment from the point t1 (t = 3) to the final point t3 (t = 5), through the point t2 (t = 4).

Recall that the speed is the first derivative of the function with respect to the coordinate, and acceleration is the second derivative with respect to the coordinate.

Then for the case n = 2 we have:

y = x^2,

$v = 2x$,

$a = 2$ m/s^2.

That is, the function x^2 describes the uniformly accelerated removal of the car ($a = 2$ m/s^2) from the origin. If we consider the case of the Pythagorean triple (3, 4, 5), we get that the distance that the car will travel from point 3 to point 5 will be equal to 16.

$$S = v(0) * t + (a * t^2)/2 = 6 * 2 + (2 * 2^2)/2 = 16$$

$v(0) = 2x = 2 * 3 = 6$ m/s,

$a = 2$ m/s^2,

t is the time of movement of the car from point 3 to point 5, that is, 2 seconds (5 - 3).

Mathematically, this looks like this:

$y^2 = z^2 - x^2$,

$4^2 = 5^2 - 3^2$,

$16 = 25 – 9$.

For cases $n > 2$, everything is similar.

But, starting with the case of x^3, in addition to speed and acceleration, a jerk also appears - this is the third derivative with respect to the coordinate, or the derivative of acceleration with respect to time.

For $n = 3$ the jerk will be constant $j = 6$.

Then the formula for $n = 3$ will be as follows ($y = x^3$):

$$S = v(0) * t + (a(0) * t^2)/2 + (j * t^3)/6$$

$j = da/dt$,

$v(0)$ is the initial velocity,

$a(0)$ is the initial acceleration.

For $n = 4$, of course, the jerk derivative with respect to time will appear, and the jerk itself will be equal to: $j = 24 * t$.

$$S = v(0) * t + (a(0) * t^2)/2 + (j(0) * t^3)/6 + (w * t^4)/24$$

$j(0)$ is the initial jerk,

v(0), a(0) are, as usual, the initial velocity and initial acceleration,

w = dj/dt.

For n = 4, w is constant and equals w = 24.

This equation can be written for any degree.

For n = 5

$$S = v(0) * t + (a(0) * t^2)/2 + (j(0) * t^3)/6 + (w(0) * t^4)/24 + (u * t^5)/120$$

w(0) is the initial value of this quantity,

u = dw/dt.

For n = 5, u is constant and equals u = 120.

For n = 6

$$S = v(0) * t + (a(0) * t^2)/2 + (j(0) * t^3)/6 + (w(0) * t^4)/24 + (u(0) * t^5)/120 + (k * t^6)/720$$

u(0) is the initial value of this quantity,

k = du/dt.

For n = 6, k is constant and equals 720.

This series can be continued indefinitely. And note that the coefficients in the denominator near a particular quantity "remain" from the value when this quantity first "receives" its constant value (a = 2, j = 6, w = 24, u = 120, k = 720, etc., the next coefficient in the denominator for t^7 will be 5040).

And we clearly see that the motion of a point has a constant acceleration only at x^2.

And at n > 2, the motion of a point will no longer be uniformly accelerated, and this is a fundamental difference from the case of n = 2.

It is also quite obvious why, under the action of a force that varies with distance as F = f (1/r^2), a body will always move with a constant acceleration (a = const) (this is an inverse problem - not the removal of a body, but the approach of a body).

A good example is the acceleration of gravity for gravity. It also follows from the above that a charge under the action of another charge will always move with a certain constant acceleration, which will depend on the initial conditions (meaning the ideal case when one point charge moves towards another, without taking into account the magnetic field and other similar effects). In my opinion, not bad for a simple function y = x^2.

That is, it turns out that gravity can be quite legitimately considered as an ordinary force.

Moreover, based on the above, we can give a mathematical definition of force: if a body approaches another body or moves away from another body according to the law $S = f(x^n) = v(0) * t + (a(0) * t^2)/2 + \ldots$, then the body is acted upon by a force of attraction or repulsion that depends on the distance as $F = f(1/r^n)$.

Considering that our space is three-dimensional, and therefore the attractive/repulsive force will have the following dependence $F = f(1/r^2)$, we can give a general mathematical form for any forces that have this dependence on distance ($1/r^2$):

$$F = a + b/r + c/r^2$$

where a, b, c are constant coefficients,

r is the distance between the interacting bodies.

For gravity, we obtain a formula previously derived in a different way [1]:

$$F = (K1 * M * m)/(r^2) + (K2 * M * m)/r + K3 * M * m$$

Considering the known gravitational effects, it is quite reasonable to assume that K1 is much larger than K2, and K2 is much larger than K3. Then it is easy to explain the effect of the slowing down of the Pioneers, the rotation of stars in galaxies at an almost constant speed, and even the movement of galaxies away from each other. But, for further analysis, we will neglect the third term, which is significant only at intergalactic distances. Then the formula for the force will take the form:

$$F = (K1 * M * m)/(r^2) + (K2 * M * m)/r$$

$F = F1 + F2$,

$F1 = (K1 * M * m)/(r^2)$, $F2 = (K2 * M * m)/r$.

And this means that at small distances (Earth - Moon) the formula will tend to F1, and at large distances (the sizes of galaxies) the formula will tend to F2.

Next, we will write the formula consisting of two terms in the form of Newton's law of gravity:

$$F = (M * m)/(r^2) * (K1 + K2 * r)$$

That is, taking into account the traditional form of Newton's law of gravity, we have obtained that the gravitational constant G is equal to:

$$G = K1 + K2 * r$$

And this means that since Newton's law of gravity is an approximation, the gravitational “constant” G will depend on the distance - with a significant increase in the distance between interacting bodies, the “gravitational constant” G will increase, which will be perceived as the action of an additional force of attraction. The Pioneer effect is exactly this:

“...The effect is detected in the telemetry data collected to calculate the speed and distance traveled by the Pioneers. Taking into account all known forces acting on the cosmic body, an additional, linearly increasing with time, blue shift of the received signal was detected, which is interpreted as a very weak force, not explained by the current model. This force causes a constant acceleration of the device towards the Sun, equal to $(8.74 \pm 1.33) \times 10^{-10}$ m/s²…” [2].

Therefore, accurate measurements of the gravitational constant G will show its increase if the gravitational constant G is measured between bodies located at a significantly greater distance.

At distances on the scale of galaxies, the second term reigns supreme

$$F = (K2 * M * m)/r$$

the effect of which explains the speed of motion of stars in galaxies [1, p. 16]:

$$v = (K2 * M0)^{0.5}$$

where v is the speed of motion of stars in the galaxy,

K2 is the coefficient,

M0 is the mass of the galaxy.

If we move on to intergalactic distances, the general formula will take the following form (since the first two terms are negligible):

$$F = (K1 * M * m)/(r^2) + (K2 * M * m)/r + K3 * M * m \rightarrow F = K3 * M * m$$

$$F = K3 * M * m$$

Based on this formula and taking into account that the third term will have the opposite sign (relative to the other terms), we can derive the law of galaxy recession (the Hubble-Lemaitre law) [1, p. 19]:

$$v = (2 * K3 * M0)^{0.5} * h0^{0.5}$$

where v is the speed of galaxy recession,

h0 is the distance to the galaxy,

M0 is the mass of all matter that is inside a ball of radius h0, that is, this is the mass of the Universe of radius h0.

In the end, I will add that the three coefficients K1, K2 and K3 can be easily calculated using the least squares method. To calculate, three or more experimental measurements of the force of attraction between two bodies of a certain mass (M, m) at different distances must be carried out.

Then the equation

F = (K1 * M * m)/(r^2) + (K2 * M * m)/r + K3 * M * m

will take the following form:

F = a + b/r + c/r^2

And the coefficients of the last equation (a, b, c) can be calculated using a standard method, for example as indicated in the link [3].

Knowing the coefficients (a, b, c) and the mass of the bodies (M, m) it is easy to calculate the true coefficients K1, K2 and K3, since:

c = K1 * M * m,

b = K2 * M * m,

a = K3 * M * m.

In these experiments, the distances between the bodies should differ significantly; restrictions on the minimum and maximum distances will, of course, be imposed by a specific measurement method. To calculate K1 and K2, it is logical to neglect the third term of the equation, which is important only on intergalactic scales (assume that K3 = 0).

With the true K1 and K2 coefficients in hand, the Pioneer effect will be confirmed.

1. Bezverkhniy V. D., Bezverkhniy V. V. Newton's gravity depending on the topology of space. SSRN Electronic Journal, June 29, 2019, p. 9. https://dx.doi.org/10.2139/ssrn.3412216

2. Wikipedia (ru). Pioneer anomaly - Wikipedia (John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Martin Nieto, Slava G. Turyshev. Study of the anomalous acceleration of Pioneer 10 and 11. Physical Review D. 2002. Vol. 65, no. 8. P. 082004).

3. Bezverkhniy V. D. Structure of the Benzene Molecule on the Basis of the Three-Electron Bond. SSRN Electronic Journal, 31 Dec 2018, pp. 12 - 14. https://dx.doi.org/10.2139/ssrn.3065241