Control method of the system subject to constraint on power spectral density of control input

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Funding

This work was not supported by any funding agencies.

Competing interests

The authors declare having no potential conflict of interest.

Author contributions

Conceptualization: Ri Kyong Hyok; Methodology: Ri Kyong Hyok & Kim Kwang Sok; Numerical simulation: Ri Kyong Hyok, Kim Ryong Pom & Kim Kwang Sok; Writing and editing: Ri Kyong Hyok & Kim Ryong Pom.

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Abstract: In this paper, we present a control method for the control plant subject to the constraint on the power spectral density of the control input. The constraint condition on the power spectral density is defined using the periodogram. This constraint is represented as a quadratic inequality on the combination of the control inputs of the adjacent several time steps. To constrain the power spectral density of the control input, the finite impulse response filter is connected to the input channel of control plant and the constrained model predictive controller is used. The filter is designed so that when it receives the amplitude-constrained signal its output signal satisfies the constraint on the power spectral density. The model predictive controller with the amplitude-constrained output controls the augmented plant which consists of the control plant and the filter. Numerical simulations verify the effectiveness of the proposed method.

Keywords: model predictive control; constraint; power spectral density; periodogram; optimization; discrete-time system

1. Introduction

In practice the control systems are operated within a variety of constraints. The actuators have their own physical limits, and sometimes, the control engineers constrain on the control input intentionally for the safe operation of the control system (Gayadeen & Duncan, 2016).

As the interests in the constrained control have been increased, many constraint-handling control methods have been studied. For the discrete-time linear system subject to the constraints on the control input, the stabilizing dynamic controller has been studied based on the polynomial approach (Henrion, Tarbouriech, & Kucera, 2001). The stabilizing linear state feedback has been studied for the system subject to the constraints on the amplitude and the incremental rate of the control input (Mesquine, Tadeo, & Benzaouia, 2004). For the system with the time-varying constraints on inputs, states and outputs, the linear control framework has been developed and the step response characteristics such as the steady state error, settling time and overshoot have been considered (Aangenent, Heemels, Molengraft, Henrion, & Steinbuch, 2012). To handle the constraints on the state and the control input, the reference governors have been studied, in which the reference commands designed for the unconstrained closed-loop system has been modified (Garone, Cairano, & Kolmanovsky, 2017). The control system with the asymmetric time-varying constraints on the amplitude of the control input has been studied (Yuan & Wu, 2015). The nonlinear control system with the constraints on the states and the inputs has been studied (Wang, Wu, Wang, & Zhao, 2019; Wu & Xie, 2019).

The model predictive control has been widely used to control the system under the constraints (Darby & Nikolaou, 2012; Qin & Badgwell, 2003; Rawlings, Mayne, & Diehl, 2019). The discrete-time model predictive controller finds the control input sequence optimizing the cost function subject to the constraints, predicting the behaviour of the plant in the prediction horizon. And then, it outputs the first step of the obtained control input sequence to the control plant. Such processes are repeated at each sampling period, receding horizon by one step. The model predictive controller with the constraint on the amplitude and the incremental rate of the control input has been studied (Wang, 2009). Many researchers have studied the stability of the constrained model predictive control system, and used the additional terminal constraint and terminal cost to guarantee the stability of the closed loop control system (Limon, Alamo, & Camacho, 2005; Limon, Alvarado, Alamo, & Camacho, 2008; Mayne, Rawlings, Rao, & Scokaert, 2000; Muller, Rojas, & Goodwin, 2012; Rawlings, Mayne, & Diehl, 2019).

Besides the amplitude constraint (Hongjiu, Peng, Zhiwei, & Changchun, 2014; Huihui & Yuxin, 2015; Zhou, Shumin, Yaqin, & Engang, 2014) and the incremental rate constraint (Shuyou, Ting, Fang, Hong, & Yunfeng, 2017) on the control input, the constraint on the actuator's power has been considered in the control system design (Gawthrop, Wagg, Neild, & Wang, 2013). The research has been motivated by the fact of that, when the actuator's power is negative, the control system is in the operation of the semi-active mode. The constraints on the actuator's power are represented as the quadratic constraints which depend on the state and the control input. The researchers have controlled the unit mass-spring system, in which the actuator's power which is the product of the control force and the velocity of the mass has been constrained smaller than the prescribed value.

The control inputs, which the actuators can respond to, have the limited power, owing to the limited ability of the actuator. And, the power of the control input can be distributed differently across frequencies. The power spectral density (PSD) of the signal represents the distribution of power across frequencies (Castanie, 2011), and therefore, there exists the limit on the PSD of the permissible control input. Hence, it is required to consider the constraint on the PSD of the control input in the control system design There are some other situations that the control system using the PSD-constrained control input is desirable. For instance, the control plant could be designed to use the control input signal of which power is concentrated on the specific frequency range other than noise frequency band. In some airplane systems the modulated signals are used to make the system less susceptible to the low frequency noises (Golnaraghi, 2010). Also, if the control system is constructed to use the high-frequency control input signal, the control plant can be controlled by wireless, without the modulation.

In the control field, the research associated with PSD have been conducted. For instance, some researchers have studied the generation method of the signal having the prescribed PSD (Muller, Rojas, & Goodwin, 2012).

To our best knowledge, the constraint on the PSD of the control input has not been considered in the control system design.

The aim of our work is to study the control method for the control plant subject to the constraint on the PSD of the control input.

In this paper, the inequality constraint condition on the PSD is defined using the periodogram. To constrain the PSD of the control input, we use the finite impulse response (FIR) filter and the constrained model predictive controller which is connected to filter. The filter which drives the control plant, is designed so that, when the amplitude-constrained signal generated by model predictive controller is input to the filter, the filter output signal satisfies the constraint on the PSD.

The paper is structured as follows. In section 2, we introduce the model predictive control system with the amplitude-constrained control input, and formulate the constraint on the PSD of the control input. In section 3 we present the method to control the plant using the PSD-constrained control input. In Section 4, the numerical simulations illustrate the effectiveness of the proposed control method. In Section 5, we present conclusion.

Notations: $\mathbf{R}^n$ denotes the real n-dimensional space. $\mathbf{R}^{m\times n}$ denotes the set of real $m\times n$ matrices. The transpose of a matrix M is denoted by MT. Given a symmetric and positive semidefinite matrix $T\in\mathbf{R}^{n\times n}$, for a vector $x\in\mathbf{R}^n$, the weighted norm of x is defined as $\|x\|_T := \sqrt{x^T T x}$. For $x\in\mathbf{R}^n$, $y\in\mathbf{R}^n$, $x > y$ means that $x_i > y_i$, $\forall i$. Other inequality symbols are used similarly.

2. Preliminaries

In this section we introduce the model predictive control with the constraint on the amplitude of the control input, and formulate the constraint on PSD of the control input.

2.1. Model predictive control with the constraint on the amplitude of the control input (Mayne, Rawlings, Rao, & Scokaert, 2000)

Consider the control plant represented by the state equation

$$x(k+1) = Ax(k) + Bu(k), \tag{1}$$

where $x(k) \in \mathbf{R}^n$ and $u(k) \in \mathbf{R}^m$ are the state and control input vector at the time step k respectively. Assume that (A, B) is controllable and all the states are measured.

And, assume that the control inputs are required to satisfy $u(k) \in \mathbf{U}$ for all $k \geq 0$, where

$$\mathbf{U} := \left\{ u \mid -u^{\max} \leq u \leq u^{\max} \right\}. \tag{2}$$

Given a vector v, denote the 2-norm of it as $\|v\|_T = \sqrt{v^T T v}$, where T is the weight matrix.

At the time step k, given the state $x(k)$, the model predictive controller solves the constrained optimization problem $\mathbf{P}(x(k))$:

$$V^*(x(k)) := \min_{\hat{\mathbf{u}}(k)} \frac{1}{2} \sum_{i=0}^{N-1} \left(\|\hat{x}(k+i \mid k)\|_Q^2 + \|\hat{u}(k+i \mid k)\|_R^2 \right) + \frac{1}{2} \|\hat{x}(k+N \mid k)\|_P^2 \tag{3}$$

subject to

$$\hat{x}(k \mid k) = x(k), \tag{4}$$

$$\hat{x}(k+i+1\,|\,k) = A\hat{x}(k+i\,|\,k) + B\hat{u}(k+i\,|\,k),\ i = 0,\cdots,N-1, \tag{5}$$

$$\hat{u}(k+i\,|\,k) \in \mathbf{U},\ i = 0,\cdots,N-1, \tag{6}$$

$$\hat{x}(k+N\,|\,k) \in \mathbf{X}_f \tag{7}$$

to get the optimal control input prediction sequence of length N,

$$\hat{\mathbf{u}}^*(k) = \left[\hat{u}^*(k\,|\,k),\ldots,\hat{u}^*(k+N-1\,|\,k)\right], \tag{8}$$

then input $\hat{u}^*(k|k)$ as the control value to the control plant at the time step k as follows.

$$u(k) = \hat{u}^*(k|k). \tag{9}$$

In (3), the matrices Q and R are symmetric and positive definite; and the matrix P is the solution of the matrix algebraic Riccati equation

$$P = (A+BK)^T P(A+BK) + Q + K^T RK, \tag{10}$$

where

$$K = -\left(B^T PB + R\right)^{-1} B^T PA^T. \tag{11}$$

The terminal state constraint set $\mathbf{X}_f$ is represented as

$$\mathbf{X}_f = \left\{x \,|\, \|x\|_P^2 < \delta\right\}, \tag{12}$$

where δ is calculated by solving the constrained optimization problem:

$$\delta := \min_{x} \|x\|_P^2 \tag{13}$$

subject to
$$Kx \notin \mathbf{U} \cdot \tag{14}$$

Model predictive controller repeats this process at every time step, receding the prediction horizon.

Remark 2.1: Since the solution matrix of the Riccati equation P is symmetric and positive definite; and the control input constraint set $\mathbf{U}$ is represented as (2), we can use the constraint

$$Kx > u^{\max} \tag{15}$$

instead of the constraint (14) in the optimization problem to solve the parameter δ which is used to determine the terminal state constraint set, and in this case the optimal function value is not changed.

The following theorem proves the stability of this model predictive control system.

Theorem 2. 1 (Mayne, Rawlings, Rao, & Scokaert, 2000): Consider the closed loop control system, in which the control plant (1) is controlled by the control input (9). Assume that, given the initial state $x(0)$, the optimization problem $\mathbf{P}(x(0))$ has the admissible control input $\hat{\mathbf{u}}(0|0)$, which satisfies the constraints (6) and (7). Then, the model predictive controller has the admissible control input for all $k \geq 0$. Furthermore, $x(k)$ converges to 0, when $k \to \infty$.

2.2. Constraint on the PSD of the control input

Given a signal, the PSD of it represents the distribution of power across frequencies.

Definition 2.1 (Castanie, 2011): Given a discrete-time signal sequence $\mathbf{v} = \left[v(0),\cdots,v(M-1)\right]$, which is sampled with the sampling period $T_s\left[s\right]$, the periodogram of the signal sequence $\mathbf{v}$ is defined as

$$\mathbf{P}_{\mathrm{v}}(n) := \frac{1}{M}\left|\sum_{k=0}^{M-1} v(k)e^{\frac{(-2\pi j)\cdot n\cdot k}{M}}\right|^2, \quad n = 0,1,\cdots M-1, \tag{16}$$

where $j = \sqrt{-1}$.

It is well known that $\frac{1}{M}\sum_{k=0}^{M-1} v^2(k) = \frac{1}{M}\sum_{n=0}^{M-1}\mathbf{P}_{\mathrm{v}}(n)$ is satisfied. Since the left-side hand of the above equation is the power of the signal $\mathbf{v}$, $\mathbf{P}_{\mathrm{v}}(n)$ can be said as the power component of the signal $\mathbf{v}$ at the frequencies $\frac{n}{MT_s}\left[Hz\right]$.

In this paper, at the time step k and the frequencies $\frac{n}{LT_s}\left[Hz\right]$, $n = 1,\cdots,L$, the PSD of i^{th} control input u_i is estimated by using the control input sequence of length M

$$\mathbf{u}_i^{k,M} = \left[u_i(k-M+1),\cdots,u_i(k)\right] \tag{17}$$

as follows (Castanie, 2011).

$$S\left(\mathbf{u}_i^{k,M}, n\right) = \frac{1}{M}\left|\sum_{p=0}^{M-1} w(p)u_i(k-M+1+p)e^{\frac{(-2\pi j)np}{L}}\right|^2, n = 0,1,\cdots L-1 \quad . \tag{18}$$

The right-hand side of (18) is the periodogram of the modified version of $\mathbf{u}_i^{k,M}$; $\left[w(0)u(k-M+1),\cdots,w(M-1)u(k),\overbrace{0,\cdots,0}^{L-M}\right]$ which is obtained from that $\mathbf{u}_i^{k,M}$ is multiplied by the window

$$\mathbf{w} = \left[w(0), w(1), \cdots, w(M-1) \right] \tag{19}$$

and padded with *L*-*M* zeros.

Several kinds of windows are used, for example, Hanning window function of length *M* is as follows (Castanie, 2011).

$$w(n) = 0.5\left(1-\cos\left(2\pi\frac{n}{M-1}\right)\right), 0 \le n \le M-1 . \tag{20}$$

Lemma 2.1: The PSD of the control input sequence $\mathbf{u}_i^{k,M}$, $S\left(\mathbf{u}_i^{k,M}, n\right)$ is the weighted norm of $\mathbf{u}_i^{k,M}$. That is,

$$S\left(\mathbf{u}_i^{k,M}, n\right) = \left\|\mathbf{u}_i^{k,M}\right\|_{H(n)}^2, n = 0, 1, \cdots, L\text{-}1 , \tag{21}$$

where

$$H(n) = \frac{1}{M}\left(H_{\mathrm{R}}(n)\cdot H_{\mathrm{R}}^T(n) + H_{\mathrm{I}}(n)\cdot H_{\mathrm{I}}^T(n)\right), \tag{22}$$

with

$$H_{\mathrm{R}}(n) = \left[w(0)\cos\left(\frac{-2\pi\cdot n\cdot 0}{L}\right), \cdots, w(M-1)\cos\left(\frac{-2\pi\cdot n\cdot(M-1)}{L}\right)\right]^T,$$

$$H_{\mathrm{I}}(n) = \left[w(0)\sin\left(\frac{-2\pi\cdot n\cdot 0}{L}\right), \cdots, w(M-1)\sin\left(\frac{-2\pi\cdot n\cdot(M-1)}{L}\right)\right]^T.$$

Proof: Note that

$$\begin{aligned} S\left(\mathbf{u}_i^{k,M},n\right) &= \frac{1}{M}\left|\sum_{p=0}^{M-1} w(p)u_i\left(k-M+1+p\right)\left(\cos\left(\frac{-2\pi np}{M}\right)+j\sin\left(\frac{-2\pi np}{M}\right)\right)\right|^2 \\ &= \frac{1}{M}\left|\mathbf{u}_i^{k,M}H_{\mathrm{R}}(n)+j\mathbf{u}_i^{k,M}H_{\mathrm{I}}(n)\right|^2 = \left\|\mathbf{u}_i^{k,M}\right\|_{H(n)}^2 \end{aligned}.$$

Therefore, we have (21).

Definition 2.2: Given a PSD bound $\overline{\mathbf{S}}=\left[\overline{S}(0),\cdots\overline{S}(L\text{-}1)\right]^T$, define the set $\mathbf{T}\left(\overline{\mathbf{S}}\right)$ as follows.

$$\mathbf{T}\left(\overline{\mathbf{S}}\right):=\left\{\mathbf{v}\mid S\left(\mathbf{v},n\right)\le\overline{S}\left(n\right),n=0,\cdots,L\text{-}1,\mathbf{v}\in\mathbf{R}^M\right\}. \tag{23}$$

Given a PSD bound of the control input u_i: $\overline{\mathbf{S}}_i=\left[\overline{S}_i(0),\cdots\overline{S}_i(L\text{-}1)\right]^T$, if

$$\mathbf{u}_i^{k,M}\in\mathbf{T}\left(\overline{\mathbf{S}}_i\right),\ \forall k \tag{24}$$

is satisfied, then the control input u_i is said to satisfy the constraint on the PSD of $\overline{\mathbf{S}}_i$.

Remark 2.2: The constraint on the amplitude of the control input depends on the control value at each time step and the constraint on the incremental rate depends on the control values at the two adjacent time steps. The constraint on the PSD of the control input depends on the control values at the many adjacent time steps. By this meaning, the constraint on the PSD of the control input can be regarded as more general constraint than the one on the amplitude or the incremental rate.

In general, while the plant is controlled, the spectrum of control input can be varied rapidly and intensively, therefore, a large value of *M* would not be preferred. And we can adjust the frequency resolution of the PSD, changing the value of *L*.

3. Control method using the PSD-constrained control input

Consider the control plant represented by (1). And assume that the i^{th} control input channel of the control plant is not allowed to receive the signal of which PSD is larger than $\bar{\mathbf{S}}_i$. Such control plant should be controlled by using the control input of which PSD is smaller than $\bar{\mathbf{S}}_i$.

In this section we discuss on the control problem for such control plant using the control input of which PSD is constrained by $\bar{\mathbf{S}}_i = \left[\bar{S}_i(0), \cdots \bar{S}_i(L\text{-}1)\right]^T$.

The model predictive controller solves the constrained optimization problem to calculate the control input. Therefore, we can arrange the constraint on the PSD of the control input in the constraint part of the optimization problem of the model predictive controller explicitly.

But, since the constraint on the PSD of the control input is represented as the quadratic inequality constraints on the control input sequence (Lemma 2.1), the admissible control input set satisfying the constraints is not generally convex set. Therefore, it is difficult to get the optimal solution and to determine the terminal state cost function and terminal state constraint set to guarantee the stability of the control system.

If the amplitude of the control input u_i is constrained as $-\bar{u}_i \le u_i(k) \le \bar{u}_i$ for all $k \ge 0$, the PSD of u_i will be implicitly bounded as follows.

$$S\left(\mathbf{u}_i^{k,M}, n\right) \le S^*\left(\bar{u}_i, n\right),\ n = 0, \cdots, L-1, \tag{25}$$

where $S^*\left(\bar{u}_i, n\right)$ is determined by the constrained optimization problem:

$$S^*\left(\bar{u}_i, n\right) := \max_{\mathbf{v}=\left[v(0),\cdots,v(M-1)\right]} \|\mathbf{v}\|_{H(n)}^2, \tag{26}$$

subject to

$$-\overline{u}_i \le v(l) \le \overline{u}_i, l = 0, \cdots, M-1. \quad (27)$$

Therefore, the PSD constraint can be implicitly satisfied by constraining the amplitude of the control input. But the PSD of the control input should not be much more constrained than the required one, $\overline{\mathbf{S}}_i$.

If we constrain the amplitude of the control input as $-\overline{u}_i^* \le u_i(k) \le \overline{u}_i^*$ for all $k \ge 0$, where $\overline{u}_i^*$ is determined by the constrained optimization problem

$$\overline{u}_i^* := \max \ \overline{u}_i \quad (28)$$

subject to

$$S^*(\overline{u}_i, n) \le \overline{S}_i(n), \ n = 0, \cdots, L-1, \quad (29)$$

then $S(\mathbf{u}_i^{k,M}, n) \le S^*(\overline{u}_i^*, n) \le \overline{S}_i(n)$, $n = 0, \cdots, L-1$ are satisfied and the control input u_i satisfies the PSD constraint.

However, in this case, the difference between $S^*(\overline{u}_i^*, n)$ and $\overline{S}_i(n)$ can be very large at some value of n , which means that the power of the control input will be much more constrained than the required one, at some frequencies.

This problem can be resolved, if we connect the FIR filter to the control plant serially to constitute the augmented control plant and constrain the amplitude of the input of this augmented plant using model predictive controller, as shown in Figure 1.

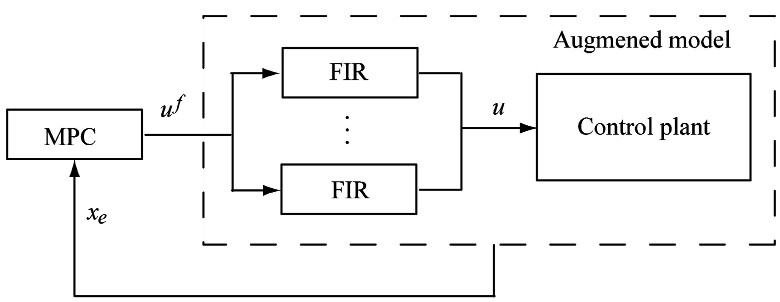

Figure 1. Control scheme for the control plant subject to the constraint on the PSD of control input *u*.

When the amplitude of filter input is constrained, the maximum PSD of the filter output depends on the filter coefficients. Therefore, assuring that the filter receives the amplitude-constrained signal from the model predictive controller, we adjust the filter coefficients to increase the maximum PSD of the filter output as much as possible, while satisfying the required PSD bound constraint.

In the FIR filter connected to the i^{th} input channel, which has the order of N_i^f, the filter output u_i is calculated using the filter input u_i^f and the filter coefficient vector $\mathbf{h}_i = \left[h_i(0), h_i(1), \cdots, h_i\left(N_i^f\right) \right]$ as follows.

$$u_i(k) = h_i(0)u_i^f(k) + h_i(1)u_i^f(k-1) + \cdots + h_i\left(N_i^f\right)u_i^f\left(k - N_i^f\right) \tag{30}$$

We use the finite impulse response filter rather than the infinite impulse response filter so that the control input is determined by using the filter input signal sequence, without considering the control input values at the previous time steps.

If we use the linear phase FIR filter, which has the order of even number and whose coefficients satisfy the symmetric property

$$h_i(l) = h_i\left(N_i^f - l\right), 0 \le l \le N_i^f, \tag{31}$$

the maximum PSD of the filter output can be shaped by tuning the $N_i^f/2+1$ number of filter coefficients.

The control input sequence $\mathbf{u}_i^{k,M}$ can be written as

$$\left[\mathbf{u}_i^{k,M}\right]^T = \begin{bmatrix} u_i(k-M+1) \\ u_i(k-M+2) \\ \vdots \\ u_i(k) \end{bmatrix} = F(\mathbf{h}_i) \begin{bmatrix} u_i^f\left(k-M-N_i^f+1\right) \\ u_i^f\left(k-M-N_i^f+2\right) \\ \vdots \\ u_i^f(k) \end{bmatrix}, \tag{32}$$

where

$$F(\mathbf{h}_i) = \begin{bmatrix} h_i\left(N_i^f\right) & h_i\left(N_i^f-1\right) & \cdots & h_i(0) & 0 & \cdots & \cdots & 0 \\ 0 & h_i\left(N_i^f\right) & h_i\left(N_i^f-1\right) & \cdots & h_i(0) & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & 0 & h_i\left(N_i^f\right) & h_i\left(N_i^f-1\right) & \cdots & h_i(0) \end{bmatrix}$$

(33)

If the amplitude of the filter input is constrained as $-1 \le u_i^f(k) \le 1$ for all $k \ge 0$, the PSD of the control input u_i will be bounded as

$$S\left(\mathbf{u}_i^{k,M}, n\right) \le S^{f*}(\mathbf{h}_i, n), \forall k, \tag{34}$$

where $S^{f*}(\mathbf{h}_i, n)$ is determined by the constrained optimization problem

$$S^{f*}(\mathbf{h}_i, n) := \max_{\mathbf{v}=\left[v(0),\cdots,v\left(M+N_f-1\right)\right]} \|\mathbf{v}\|^2_{F^T(\mathbf{h}_i)H(n)F(\mathbf{h}_i)} \tag{35}$$

subject to

$$-1 \le v(l) \le 1, \; l = 0, \cdots, M+N_f-1. \tag{36}$$

The method to control the control plant (1) using the control input of which PSD is constrained by $\bar{S}_i(n)$, $i = 1, \cdots, m$, $n = 0, \cdots, L-1$ is as follows.

Method 3.1: For $i=1,\cdots,m$, design the linear phase FIR filters of which coefficients vectors are the optimal solutions of the constrained optimization problems

$$\mathbf{h}_i^* := \arg\max_{\mathbf{h}_i} \sum_{n=0}^{L-1}\left(10\log_{10} S^{f*}\left(\mathbf{h}_i,n\right)-10\log_{10}\bar{S}_i\left(n\right)\right)^2, \quad (37)$$

subject to

$$S^{f*}\left(\mathbf{h}_i,n\right)<\bar{S}_i\left(n\right),\ n=0,\cdots,L-1, \quad (38)$$

$$h_i\left(l\right)=h_i\left(N_i^f-l\right),0\le l\le N_i^f, \quad (39)$$

and connect the designed filters to the corresponding control input channels of the control plant respectively, to constitute the augmented plant. And then, control the augmented plant using the model predictive controller 2.1., in which $u^{\max}=1$.

Note that the stability of the closed loop control system is guaranteed by Theorem 2.1 and PSDs in the cost function (37) are represented in dB.

The following method explains how to get the state equation of the augmented plant, which is used for the model predictive controller to predict the future behaviour.

Method 3.2: For $i=1\ldots m$, represent the transfer function model of i^{th} filter, $G_i^f\left(z\right)=h_i^*\left(0\right)+h_i^*\left(1\right)z^{-1}\cdots+h_i^*\left(N_i^f\right)z^{-N_i^f}$ as the state space model

$$\begin{aligned} x^{f,i}\left(k+1\right)&=A_i^f x^{f,i}\left(k\right)+B_i^f u_i^f\left(k\right)\\ u_i\left(k\right)&=C_i^f x^{f,i}\left(k\right)+D_i^f u_i^f\left(k\right) \end{aligned},$$

where $A_i^f=\begin{bmatrix}0&0&0&\cdots&0\\1&0&0&\cdots&0\\0&1&0&\cdots&0\\\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&\cdots&1&0\end{bmatrix}$, $B_i^f=\begin{bmatrix}1\\0\\0\\\vdots\\0\end{bmatrix}$, $C_i^f=\begin{bmatrix}h_i^*(1)&h_i^*(2)&\cdots&h_i^*\left(N_i^f\right)\end{bmatrix}$ and

$D_i^f=h_i^*(0)$.

Then, construct the state space model of the filter bank as

$$\begin{aligned}x^f(k+1)&=A^f x^f(k)+B^f u^f(k)\\u(k)&=C^f x^f(k)+D^f u^f(k)\end{aligned},$$

where $A^f=\begin{bmatrix}A_1^f&\cdots&0\\\vdots&\ddots&\vdots\\0&\cdots&A_m^f\end{bmatrix}$, $B^f=\begin{bmatrix}B_1^f&\cdots&0\\\vdots&\ddots&\vdots\\0&\cdots&B_m^f\end{bmatrix}$, $C^f=\begin{bmatrix}C_1^f&\cdots&0\\\vdots&\ddots&\vdots\\0&\cdots&C_m^f\end{bmatrix}$,

$D^f=\begin{bmatrix}D_1^f&\cdots&0\\\vdots&\ddots&\vdots\\0&\cdots&D_m^f\end{bmatrix}$ and $x^f=\begin{bmatrix}x^{f,1}\\\vdots\\x^{f,m}\end{bmatrix}$.

Finally, we have the state equation of the augmented control plant

$$x_e(k+1)=A_e x_e(k)+B_e u^f(k), \tag{40}$$

where $A_e=\begin{bmatrix}A&BC^f\\0&A^f\end{bmatrix}$, $B_e=\begin{bmatrix}BD^f\\B^f\end{bmatrix}$ and $x_e=\begin{bmatrix}x\\x^f\end{bmatrix}$.

4. Numerical Simulation

Example 1: We apply the proposed method to the control plant represented as the following discrete-time state equation

$$x(k+1)=Ax(k)+Bu(k),$$

where

$$A = \begin{bmatrix} -0.2184 & -1.092 & -0.2539 & -0.3157 \\ 0.8621 & -0.1914 & -0.1922 & -0.353 \\ 0.2409 & 0.2231 & 0.9759 & -0.05281 \\ 0.0721 & 0.1227 & 0.7954 & 0.9888 \end{bmatrix}, \quad B = \begin{bmatrix} 0.1078 \\ 0.1205 \\ 0.01802 \\ 0.00381 \end{bmatrix}.$$

The sampling period is $T_s = 0.1[\mathrm{s}]$ and assume that all the states are measured.

Actually, this model is the discrete-time state space version of the continuous transfer function $G(s) = \dfrac{3000}{(s^2 + 0.3s + 10)(s^2 + 0.2s + 300)}$.

At each time step, the control input sequence of the length $M = 7$ is multiplied by the Hanning window sequence $\mathbf{w} = [0, 0.25, 0.75, 1, 0.75, 0.25, 0]$ and padded with zeros so that the signal sequence of length $L = 22$ is constructed. Then, the periodogram is calculated to get the PSD of the control input. Assume that $u_f(k) = 0$ for all $k < 0$. The prediction horizon length of the model predictive controller is $N = 20$.

At first, we will simulate the control system without the constraint on the control input. In this case the terminal state constraint is also removed. The weight matrices of the cost function are $Q = \mathrm{diag}\{1,1,1,1\}$, $R = 0.001$, respectively.

For the initial state of the control plant $x(0) = [0.5, 0, 0.5, 0]^T$, the simulation results are shown in Figure 2, where (a) shows the state responses of the control plant; (b) shows the response of the control input u; (c) shows the PSD of the control input in dB; and (d) is the side view of (c). In the constrained case the maximum PSD of the control input is about 5~10[dB].

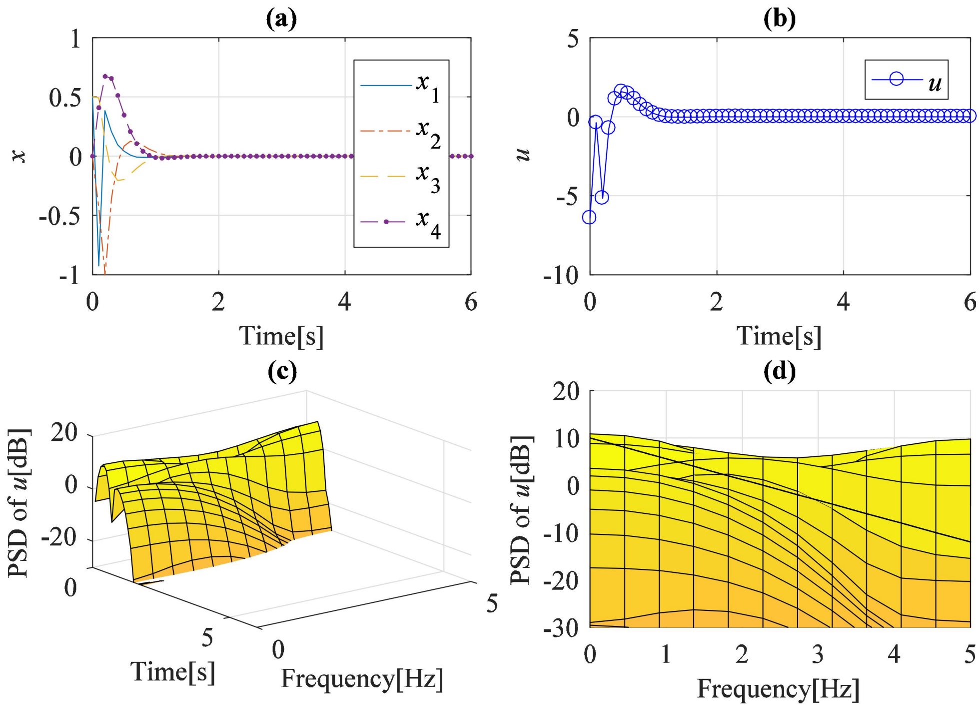

Figure 2. Simulation results of Example 1(the unconstrained case). (a) states of control plant *x*, (b) control input *u*, (c) PSD of control input *u*, and (d) PSD of control input *u* (side view of (c)).

Next, we simulate the control system using the PSD constrained control input. The required PSD constraint bound of the control input is

$\bar{\mathbf{S}}[\mathrm{dB}] = [10, 8, 6, 4, 2, 0, -2, -4, -6, -8, -10, -12, -10, -8, -6, -4, -2, 0, 2, 4, 6, 8]$.

The optimization problem (37-39) is solved by using Global Optimization Toolbox of MATLAB. For $N^f = 4$, the 4th order filter coefficient vector is obtained as $\mathbf{h}^* = [0.0824, 0.5327, 0.9062, 0.5327, 0.0824]$. And, for $N^f = 0$, the 0th order filter coefficient is obtained as $h^* = 0.2215$. In fact, using 0th order filter is corresponding to the case of using the control input whose amplitude is constrained smaller than h^*, without using filter.

For $N^f = 4$ and $N^f = 0$, the maximum PSDs of the filter output $S^{f^*}(\mathbf{h}^*, n)[\mathrm{dB}]$ are shown together in Figure 3.

Figure 3 shows that $S^{f*}(\mathbf{h}^*, n)$ are constrained smaller than $\overline{\mathbf{S}}$ in both cases. It can be seen that, when using the higher order filter, the control plant can be controlled by using the more powerful control input.

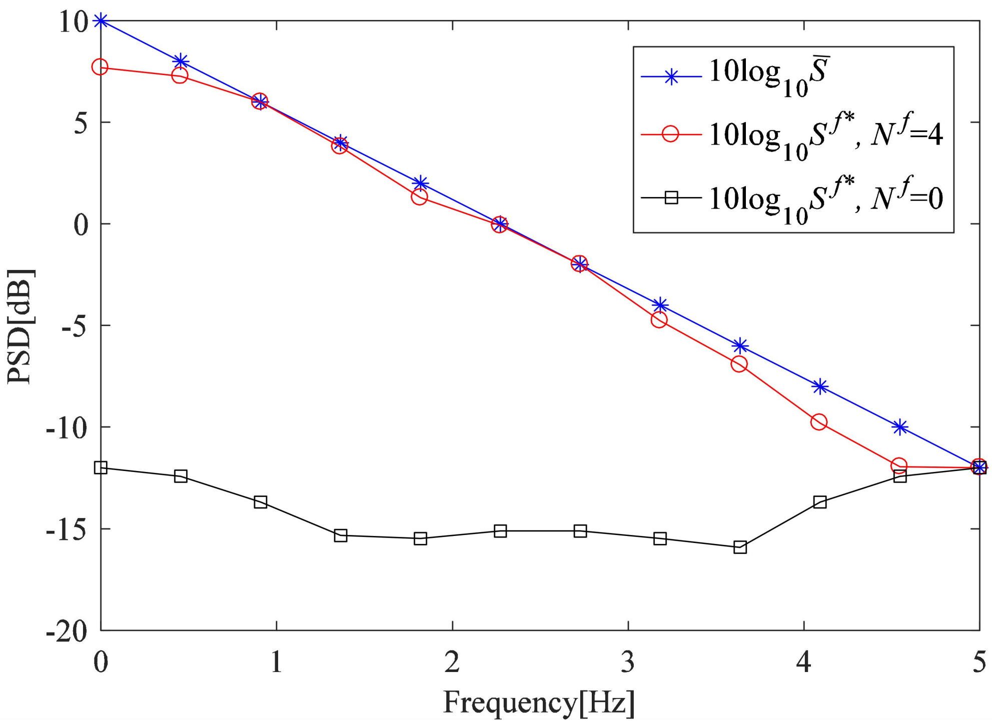

Figure 3. PSD Constraint bound $\overline{S}(n)[\text{dB}]$, and maximum PSD of the control input *u*: $S^{f*}(\mathbf{h}_i, n)[\text{dB}]$ when $N^f = 4$ and $N^f = 0$.

We simulate the control system using the 4th order filter. We use the weight matrices of for the augmented model: $Q = \text{diag}\{1,1,1,1,0.001,0.001,0.001,0.001\}$ and $R = 0.001$. For $u^{\max} = 1$, we have δ=0.0542.

For $x(0) = [0.5, 0, 0.5, 0]^T$, the simulation results are shown in Figure 4, in the same way as Figure 2, but Figure 4 (b) shows the filter input u^f as well as the control input u. In Figure 4, (b) shows that the amplitude of the filter input u^f is constrained smaller than a unit value, and (d) shows that the PSD of the control input is bounded under $\overline{\mathbf{S}}[\text{dB}]$ which is plotted as the thick line.

Comparing Figure 4 with Figure 2, the settling time in Figure 4 (the constrained case) is longer than the one in Figure 2 (the unconstrained case), which is reasonable.

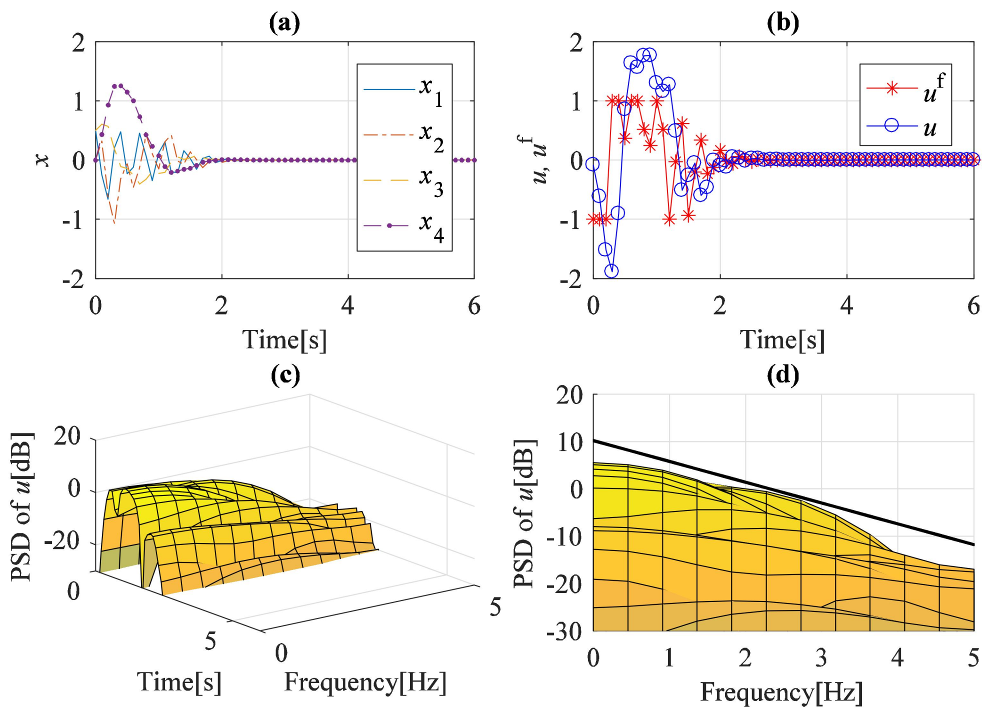

Figure 4. Simulation results of Example 1 (the constrained case). (a) states of control plant x, (b) control input u and filter input u^f, (c) PSD of control input u, and (d) PSD of control input u (side view of (c)) with $\overline{\mathbf{S}}[\mathrm{dB}]$(thick line).

Example 2: We apply the proposed method to the control plant

$$x(k+1) = Ax(k) + Bu(k),$$

where

$$A = \begin{bmatrix} -0.2524 & -1.0623 & -0.3657 & -0.2959 \\ 0.8080 & -0.1716 & -0.3277 & -0.3400 \\ 0.2321 & 0.2252 & 0.9551 & -0.0514 \\ 0.0702 & 0.1231 & 0.7910 & 0.9891 \end{bmatrix}, \quad B = \begin{bmatrix} 0.1010 \\ 0.1161 \\ 0.0175 \\ 0.0037 \end{bmatrix}.$$

This model is the discrete-time state space version of the continuous transfer function $G(s)=\frac{3000}{(s^2+0.8s+10)(s^2+0.8s+300)}$.

Simulation conditions are the same as in Example 1, except that the required PSD constraint bound of the control input is

$$\overline{\mathbf{S}}[\text{dB}]=[10,8,6,4,2,0,-2,-4,-6,-8,-10,-12,-10,-8,-6,-4,-2,\ 0,2,4,6,8].$$

In this case the power of control input will be concentrated on the high frequency range.

The 4th order filter coefficient vector is obtained as $\mathbf{h}=[-0.0801,\ 0.5324,\ -0.9048,\ 0.5324,\ -0.0801]$. For $u^{\max}=1$, we have δ=0.0662.

For $x(0)=[0,1.8,0,0]^T$, the simulation results are shown in Figure 5 where (d) shows that the PSD of the control input is bounded under $\overline{\mathbf{S}}[\text{dB}]$ which is plotted as the thick line and the power of the control input is concentrated on the high frequency range. Figure 5 (a) and (b) show that, when using the control input of which power is more constrained on the low frequency range and concentrated on the high frequency range, the responses are somewhat oscillating.

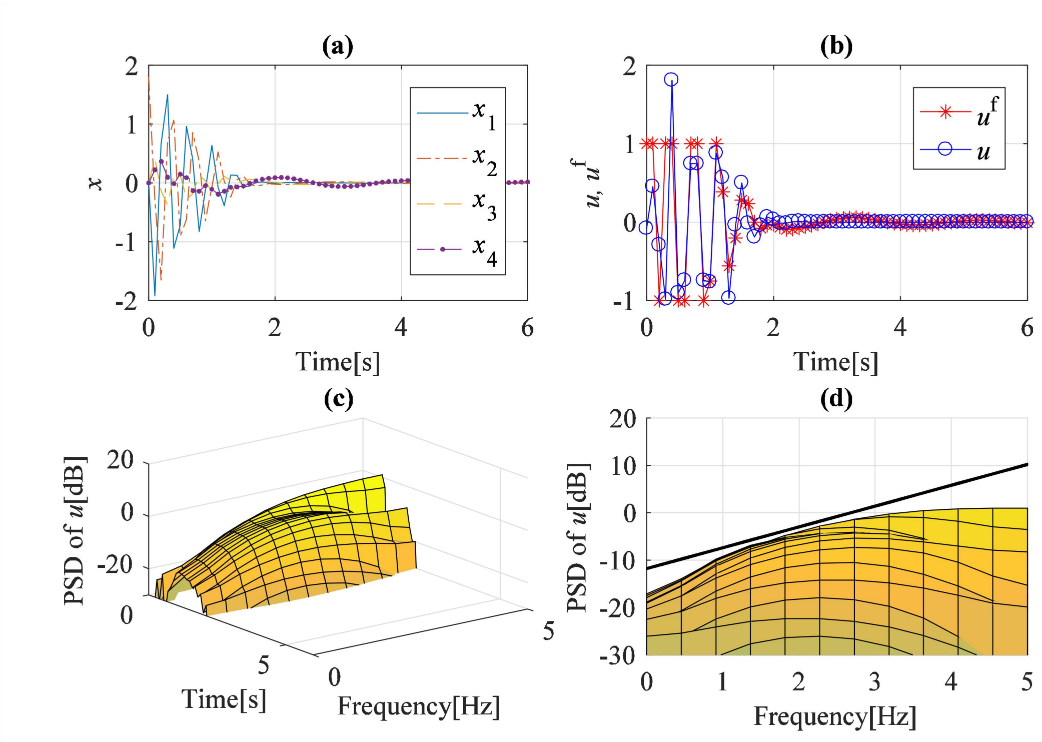

Figure 5. Simulation results of Example 2. (a) states of control plant x, (b) control input u and filter input u^f, (c) PSD of control input u, and (d) PSD of control input u (side view of (c)) with $\bar{S}[\mathrm{dB}]$(thick line).

5. CONCLUSION

In this paper we present the control method of the system subject to the constraint on the PSD of the control input. The PSD constraint condition is given by the quadratic inequality on the combination of the control inputs of the adjacent several time steps. We use the combination of the FIR filter and the model predictive controller with the amplitude-constrained output. The FIR filter is optimally designed so that, when the amplitude-constrained signal is input, the PSD of the filter output is smaller than the prescribed value. The proposed method can be used to control the plant using the control input of which PSD is constrained.

Future work is to apply the proposed method to the real physical system.

Another problem to be investigated is to analyze the control performance degradation when the PSD of the control input is constrained.

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