The various representative values representing between the two figures, and how to create these representative values.

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Abstract: There are countless means that are neither arithmetic nor geometric means, and to satisfy the mean, $f(x,y)$ must be a one-to-one correspondence to a bivariate function $f(x,x)$

Keywords: Average, Arithmetic Average, Geometric Average

Introduction: As an instructor, I encountered the harmonic mean at the corner of the textbook while explaining the relationship between the arithmetic mean and the geometric mean. It is interesting that there is more way to express two values as one value, so I study whether there are other representative values.

Main body:

problem setting - investigate whether the representative values of two figures exist outside of the arithmetic mean, geometric mean, and harmonic mean and reveal what are the conditions for becoming representative values

Theoretical background - The discussion of the study has been developed for two figures, and here we only deal with representative values of the two figures.

It is basically the same as a binary function by receiving two numbers and expressing them as one number.

Method: The arithmetic mean is like $\frac{a+b}{2}$ and the geometric mean is like $\sqrt{ab}$. Basically, it works as a binary operator. However, $\frac{a}{b}$ is far from the intuitive mean and is limited to operators for which the "law of exchange" holds, and operators that produce representative values are denoted as bivariate functions for simplicity of notation.

However, in addition to $f(a,b)=\frac{a+b}{2}$, there are countless examples of establishing the laws of exchange such as $a^b + b^a$. It also goes far from the purpose of representing the two values. Therefore, with the idea of $f(a,b) = \frac{a+b}{2}$, we need a calculation that returns it to around the middle. I denote this as $g^{-1}(x)$ when $f(x,x) = g(x)$.

For example, it is $g^{-1}\left(f(a,b)\right) = \frac{a+b}{2}$
because it is $f(a+b) = a+b$, $f(a,a) = g(a) = 2a, g^{-1}(a) = \frac{a}{2}$.

Or

$$f(a,b) = ab, f(a,a) = g(a) = a^2, g^{-1}(a) = \sqrt{a}\ (a \geq 0)\ \rightarrow\ g^{-1}\left(f(a,b)\right) = \sqrt{ab}\ (a,b\ \geq 0)$$

This shows that $f(x,y) = f(y,x)$ for the bivariate function f, and that a representative value can be made only when an inverse function of g exists for $g(x)(= f(x,x))$.

But just like in the case of $f(a,b) = a^b + b^a, g(a) = 2a^a$, Representative values of 4 and 6 can be obtained by adjusting the domain of g.

$$g^{-1}\left(f(a,b)\right) = g^{-1}\left(f(4,6)\right) = g^{-1}(4^6 + 6^4) = g^{-1}(5392) \approx 4.9432$$

Discussion - Means for 3 or more variables lack understanding.

Since it is a function of a variable, it can be understood as a spatial coordinate, but I did not learn spatial coordinates.

Conclusion: Let $f(a,a)$ be $g(a)$ for a bivariate function f that satisfies $f(a,b) = f(b,a)$ for a real number a, b.

When the $a < g^{-1}(f(a,b)) < b$ expression is satisfied for an appropriate range of g that has an inverse function, $g^{-1}(f(a,b))$ is a representative value of a and b.

*If $f(a,b)$ is the greatest common divisor of a and b, $f(a,a) = a$, so setting $g(x)$ to x does have $g^{-1}(f(a,b))$ but does not satisfy $a < g^{-1}(f(a,b)) < b$. Therefore, not all $g^{-1}(f(a,b))$ are representative values.