Evidence of Simple Harmonic Motion in the Setting of Elementary Electric Charge: A Review of Historical Data and Implications for Future Research

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Abstract

In many early studies of the value of elementary electric charge (e), experimentalists identified what appeared to be substantial discrepancies among results. Some discrepancies were explained; some were not. This investigation provides evidence suggesting that some discrepancies among experimental findings for the value of e may have been the result of the electromagnetic field between interacting particles behaving somewhat like a spring undergoing simple harmonic motion. Here, the standard value of e is associated with the equilibrium position of the field and the other values are associated with displacement from that position. This would have led to higher values of e in some cases and lower values in others, consistent with the experimental findings. Implications for the spatiotemporal nature of electrons in materials, fine-structure constant, and Landau pole are discussed.

Keywords:
Electron, elementary electric charge, electromagnetism, simple harmonic motion, fine-structure constant, electromagnetic coupling constant, running coupling, Standard Model, particle physics, quantum physics, quantum mechanics, quantum electrodynamics, quantum theory, Landau pole

About the Author:
Lamont Williams is a science writer in the Washington, DC area. For more than 25 years, he has overseen the production of science-based communications for organizations in the public and private sectors. He is author of the book, “The Greatest Source of Energy—A New Theory of Time,” which provides a roadmap for combining general relativity and quantum mechanics.

Introduction

The elementary electric charge (e) of electrons is their most distinguishing characteristic. The value of e today ultimately arises from experimentation and has been defined to be precisely 1.602176634 x 10^{-19} C based on a special least-squares adjustment of the value of e and several other constants performed by the Committee on Data for Science and Technology (CODATA).[1,2,3,4]

The effort to establish a definition of e was part of the work of moving the international system of units (SI) away from an artefact-based system to one based on the values of unchanging atomic properties and several fundamental constants, with the value of e being among them.[4] By 2016, the relative standard uncertainty of e had fallen from 2.2 x 10^{-8} to 6.1 x 10^{-9}.[2] This created confidence in establishing an experimental "cutoff" for the constant and using it to set the definition. The least-squares adjustment was based on all relevant data available on or before July 1, 2017.[3] As such, today's definition of e has a lineage stretching back to the first measurements of the value of the constant to modern times.

In many early studies, experimentalists identified what appeared to be discrepant results concerning the value of e.[5,6,7,8,9] Some discrepancies were determined to be due to experimental factors, but this was not the case in all instances. Over time, efforts for determining the value of e and several other fundamental physical constants focused on identifying the "most probable" value.[10] As noted by Feltin and Piquemal, "The most probable value was the one observed most frequently and which was most consistent with a set of … other constants." [10] In identifying the most probable value of e, discrepancies that plagued some experiments were largely avoided, but left unexplained.

This study provides evidence suggesting that some unexplained discrepancies between experimental results for the value of e might have been the result of the electromagnetic field between interacting particles behaving like a spring undergoing simple harmonic motion (SHM), where the standard, or most probable, e value represents the equilibrium position of the field and other values represent displacement from that position.

The concept of a spring-like state to the electromagnetic field and the value of e changing along the lines of SHM have implications concerning the spatiotemporal behavior of electrons in materials, as well as the fundamental basis of the fine-structure constant (α), and the nature of the Landau pole, as discussed below.

Discrepancies Among Experimentally Determined Values of Elementary Electric Charge

The most famous early measurements of the value of e were performed by Robert A. Millikan through his oil-drop procedure in the early 1900s.[10,11,12,13,14,15,16] Millikan achieved a value of about 4.774 ± 0.005 x 10^{-10} esu, or 1.592 x 10^{-19} C, determined by taking the esu value and dividing it by 10 times the speed of light (299,792,458 m/s).[15] However, it was found that he used too low of a value for air viscosity (approximately 1823 x 10^{-7} P).[10,14] The variation in the air viscosity value in oil-drop experiments was the foundation of the first great discrepancy among experimental results concerning e. Over time, new measurements and calculations were made by many others, using higher values for air viscosity, between 1829 and 1835 x 10^{-7} P.[8,17]

In 1936, Bond applied a higher air viscosity value than Millikan did to Millikan's data and data from an oil-drop study by Bäcklin and Flemberg (viscosity value, 1834.7 ± 0.8 x 10^{-7} P).[18,19] Rather than finding the data from the two studies aligning, he found two values with non-overlapping margins of error: 4.816 ± 0.005 x 10^{-10} esu (Millikan) and 4.800 ± 0.005 x 10^{-10} esu (Bäcklin and Flemberg). Later studies would suggest that Bond's results were too high, with the Millikan e value likely being closer to 4.803 x 10^{-10} esu.[8,17] The Bäcklin and Flemberg result is likely between 4.781 and 4.7941 ± 0.0089 x 10^{-10} esu, with the average of these two results being 4.788 ± 0.004 x 10^{-10} esu.[8,17]

Over time, reliance on oil-drop experiments began to fall in favor of x-ray methods.[20] Determinations of the value of e by x-ray methods around the early- to mid-1930s produced results largely in the range of 4.800 to 4.810 x 10^{-10} esu, with an average of about 4.805 x 10^{-10} esu.[6,20] However, in 1937, DuMond and Bollman found a result suggestive of a lower value of 4.768 or 4.785 x 10^{-10} esu.[21]

Birge in 1936 compared direct x-ray studies against indirect approaches for determining the value of e, involving experiments measuring the ratio of the Planck constant over e (h/e) and e over the mass of the electron (e/m).[22] He found a sharp discrepancy between the results. He noted that the discrepancy between the numbers is beyond experimental uncertainties, commenting that "one may adopt 4.8029 ± 0.0005 as the best direct determination of e, and the discrepancy with the indirect value 4.7824 ± 0.0015 is then seen to be more than ten times the sum of the stated probable errors."[22]

Finally, in 1939, Dunnington completed the work, "The Atomic Constants: A Revaluation and an Analysis of the Discrepancy," an expansive investigation in which a number of experimental studies of the value of e were evaluated.[8] His study is arguably the most rigorous study showing inconsistencies among experimental values of the constant before efforts more seriously began leaning toward finding the most probable value of e. As discussed below, Dunnington's analysis identified three major value groupings. The present study shows that the values he identified can be replicated by way of a specific, basic equation that begins to reveal a pattern consistent with SHM among e values when charge is translated into distance quantities associated with the electromagnetic field.

Dunnington's Study and the Emergence of a Simple Harmonic Motion Pattern Among Experimental Results for Elementary Electric Charge

Dunnington rigorously analyzed the results of 11 experimental studies.[8] He recalculated the results from the studies by using "a consistent set of auxiliary constants," where the auxiliary constants included such values as the speed of light and gas constant. He then analyzed the results by way of a least-squares adjustment and Birge-Bond diagrams, using the latter to graphically examine the data. Table 1 lists the types of studies he evaluated and the value of e he determined per study.

TABLE 1. Types of studies evaluated in Dunnington's analysis to determine the value of elementary electric charge, and value obtained through his recalculation.[8]

Data Point[a]	Type of Study	e Value[b]
"Group B"		
1	Ruled grating	4.8025±0.0004
2	Oil drop	4.8036±0.0048
3	Limit of continuous x-rays	4.8026±0.0014
4	Ionization and excitation	4.8090±0.0045
5	Radiation constant c_2	4.8145±0.0101
6	Stefan-Boltzmann constant	4.8168±0.0046
"Group A"		
7	Electron diffraction (voltage)	4.7964±0.0019
8	Electron diffraction (velocity)	4.7972±0.0026
9	Compton effect	4.7956±0.0020
10	Specific charge	4.7963±0.0002
11	X-ray photoelectrons	4.7953±0.0006
Mean Value: 4.8025±0.0004 x 10^{-10} esu		

[a] Same point designation as in original work.

[b] Each value x 10^{-10} esu.

By using the least squares method, he obtained a value of 4.8025±0.0004 x 10^{-10} esu, (~1.602 x 10^{-19} C), consistent with today's value. This was also the value of the mean of the results. However, the Birge-Bond diagrams provided more insight into how the values related to one another. By using this method, Dunnington found that the experimental values generally fell into two groups, which he called Group A, with a value of 4.796 x 10^{-10} esu (~1.600 x 10^{-19} C), and Group B, with a value of 4.803 x 10^{-10} esu (~1.602 x 10^{-19} C).

While it is now known that Group B’s value is more consistent with today’s value, it is interesting that Dunnington noted that he had the most confidence in the correctness of Group A’s value, calling that group’s data “structurally strong” and of “good consistency.”[8]

Dunnington remarked that “[t]he discrepancy between these two groups seems to be well beyond experimental uncertainties,” and considered his finding of two values of *e* to be, in his words, “peculiar and perhaps significant.” He speculated on the cause of the discrepancy but could not definitively identify the source.

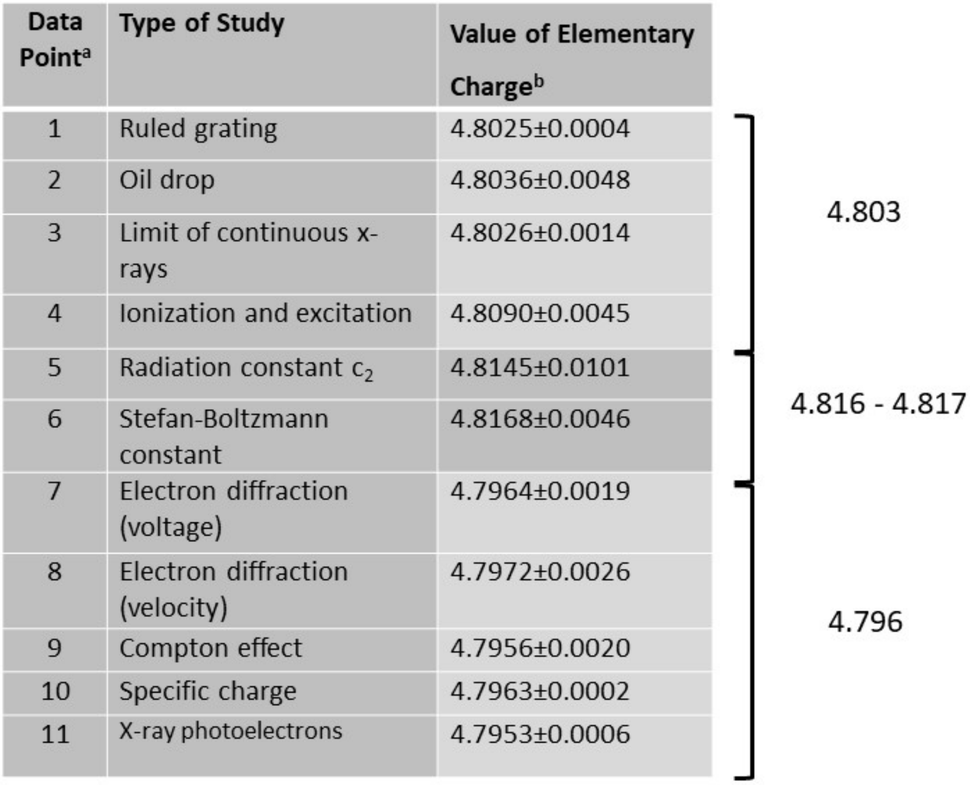
Dunnington, on further examination of his data, noted that the discrepancy between Groups A and B was not the only one. He indicated that the difference between data points 5 and 6 versus the other Group B points constituted a “second discrepancy”.

Dunnington stated that he considered omitting data points 5 and 6, but ultimately did not, noting that doing so would have made only a negligible difference in the ultimate results obtained. Data point 5’s value of $4.8145 \pm 0.0101 \times 10^{-10}$ esu is ambiguous, because its margins of error suggest that it could belong with points 1 through 4 of Group B. However, data point 6 at its lowest would be 4.8122×10^{-10} esu, still higher than those points. Of this data point, Dunnington noted the following:

> A peculiar feature of this point is that the elimination in recent years of the error from absorption in the moisture and CO_2 of

> the air has raised the point from a position of good consistency with the other points.[8]

Here, he is stating that improvement in experimental techniques caused this point to move further away from the other points, rather than to be more consistent with them at a lower value level. The average of points 5 and 6 is about 4.816 x 10^{-10} esu, while point 6, the more unambiguous data point, suggests 4.817 x 10^{-10} esu to four significant digits, leading to three value groupings (Figure 1).



Data Point[a]	Type of Study	Value of Elementary Charge[b]
1	Ruled grating	4.8025±0.0004
2	Oil drop	4.8036±0.0048
3	Limit of continuous x-rays	4.8026±0.0014
4	Ionization and excitation	4.8090±0.0045
5	Radiation constant c_2	4.8145±0.0101
6	Stefan-Boltzmann constant	4.8168±0.0046
7	Electron diffraction (voltage)	4.7964±0.0019
8	Electron diffraction (velocity)	4.7972±0.0026
9	Compton effect	4.7956±0.0020
10	Specific charge	4.7963±0.0002
11	X-ray photoelectrons	4.7953±0.0006

Note: Points 7-11, Group A; points 1-6, Group B, in reference [8].

[a] Same point designation as in original work.

[b] Each value x 10^{-10} esu.

Fig. 1. Results from Dunnington's study revisited—see Table 1 of present work—showing three groups of discrepant values identified in the investigation: the two

principal groups represented by the values of 4.803 and 4.796 x 10^{-10} esu and a possible third category in the range of 4.816 to 4.817 x 10^{-10} esu.[8]

Importantly, the e value categories Dunnington identified do not differ from one another by random amounts. The value 4.803 x 10^{-10} esu differs from 4.796 x 10^{-10} esu by a factor of about 1.0015, and 4.817 x 10^{-10} esu differs from 4.803 x 10^{-10} esu by a factor of about $(1.0015)^2$:

$$\frac{4.803\ x\ 10^{-10} esu}{4.796\ x\ 10^{-10} esu} \approx 1.0015 \qquad (1)$$

$$\frac{4.817\ x\ 10^{-10} esu}{4.803\ x\ 10^{-10} esu} \approx (1.0015)^2 \qquad (2)$$

As discussed below, the experimental results identified by Dunnington can be calculated through a simple "cascading" procedure involving a basic equation for α and the successive application of the approximately 1.0015 ratio value. The basic equation allows for the translation of the e values into dimensionless distance quantities associated with the electromagnetic field. This, in turn, begins to reveal the SHM pattern among the experimental e value results.

Calculations Replicating Dunnington's Results

The standard equation for α, based on its relationship with other physical constants, is as follows:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} \qquad (3)$$

where,

$\alpha = 0.007297\ 352\ 5693(11)$,

$e = 1.602\ 1766\ 34 \times 10^{-19}$ C,

$\varepsilon_0 = 8.854\ 187\ 8128 \times 10^{-12}$ F m^{-1},

$\hbar = 1.054\ 571\ 817 \times 10^{-34}$ J s,

$c = 299\ 792\ 458$ m/s.

As noted above, Dunnington's value of 4.803×10^{-10} esu is equivalent to today's value of e, at about ~1.602×10^{-19} C, and is also equivalent to his 4.796×10^{-10} esu (~1.600×10^{-19} C) value times approximately 1.0015. As such, the standard value of e in equation (3) can be replaced by those two factors:

$$\alpha \approx \frac{[(\sim 1.600 \text{ x } 10^{-19} \text{ C})(\sim 1.0015)]^2}{4\pi\varepsilon_0\hbar c}. \tag{4}$$

Performing these steps allows the equation to be reduced. Separating 4π for clarity leads to:

$$\alpha \approx \left(\frac{1}{4\pi}\right)\frac{[(\sim 1.600 \text{ x } 10^{-19} \text{ C})(\sim 1.0015)]^2}{\varepsilon_0\hbar c}, \tag{5}$$

with the expression on the far right containing all of the physical constants. That expression can be distilled down to a simple set of mathematical constants as follows, where the symbol e-bar (ē) is for Euler's number, at approximately 2.718, to distinguish it from e for elementary electric charge, and where 1.0015 is carried through as a critical factor of the reduced equation, providing clarity regarding the mathematical constants that appear to ultimately be present:

$$\frac{[(\sim 1.600 \text{ x } 10^{-19}\text{ C})(\sim 1.0015)]^2}{\varepsilon_0 \hbar c} \approx \left(\frac{1}{4\bar{e}}\right)\left(\frac{1}{\sim 1.0015}\right)^2 . \tag{6}$$

Thus,

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \approx \left(\frac{1}{4\pi}\right)\left(\frac{1}{4\bar{e}}\right)\left(\frac{1}{\sim 1.0015}\right)^2 \approx 0.007297. \tag{7}$$

Using the precise, known value of α—at 0.0072973525693(11)—and solving for a more precise value than 1.0015 leads to 1.00146359514.

By itself, the value of 4.796 x 10^{-10} esu (~1.600 x 10^{-19} C) replaced for e in equation (3) leads to the following expression associated with a smaller value for α:

$$\frac{(\sim 1.600 \text{ x } 10^{-19}\text{ C})^2}{4\pi\varepsilon_0 \hbar c} = \left(\frac{1}{4\pi}\right)\left(\frac{1}{4\bar{e}}\right)\left(\frac{1}{\sim 1.0015}\right)^4 = 0.007275 \tag{8}$$

This can also be seen in equation (6), although without the 4π factor. Reverse calculating using the precise value of 1.00146359514 from the standard value of α leads to 4.79618503946 x 10^{-10} esu, consistent with Dunnington's study.

By itself, the value of 4.817 x 10^{-10} esu (~1.607 x 10^{-19} C) replaced for e in equation (3) leads to the following expression associated with a higher value for α:

$$\frac{(\sim 1.607 \text{ x } 10^{-19}\text{ C})^2}{4\pi\varepsilon_0 \hbar c} = \left(\frac{1}{4\pi}\right)\left(\frac{1}{4\bar{e}}\right)(1.0015)^2 = 0.007341 \tag{9}$$

Reverse calculating using the precise value of 1.00146359514 leads to 4.81727489572 x 10^{-10} esu, also consistent with Dunnington's study.

Figure 2 shows the above as a cascade of values beginning with 4.796 x 10^{-10} esu, with the successive application of about $(1.0015)^2$ to the basic equation, leading to Dunnington's other results.

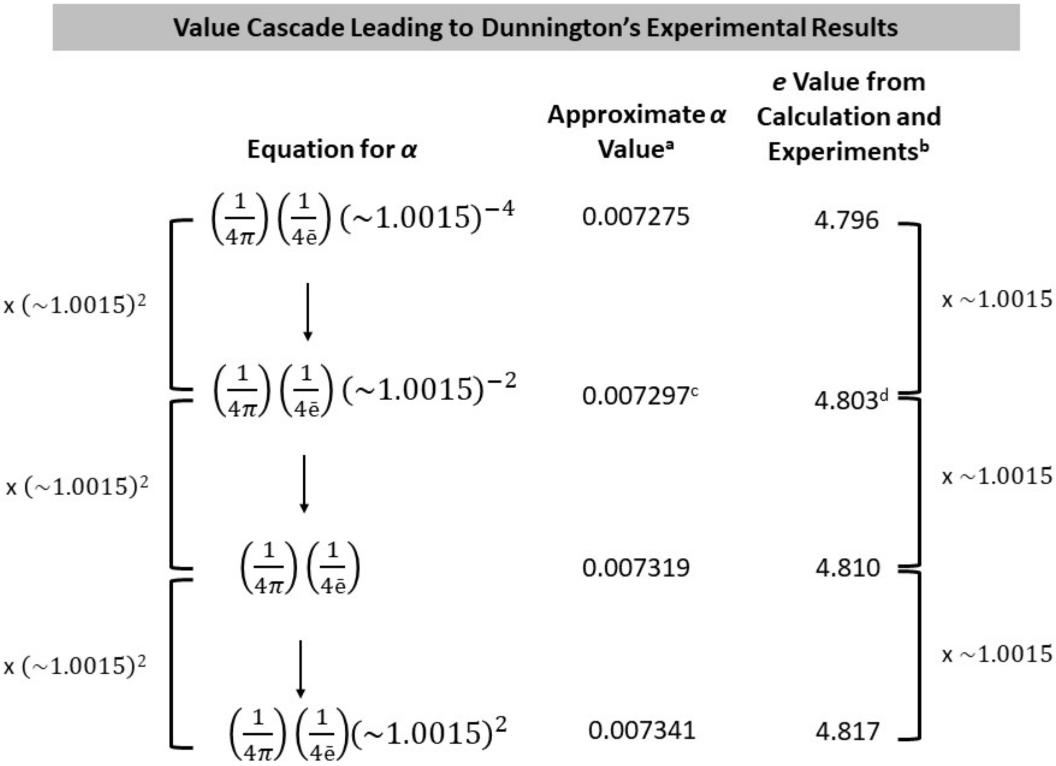
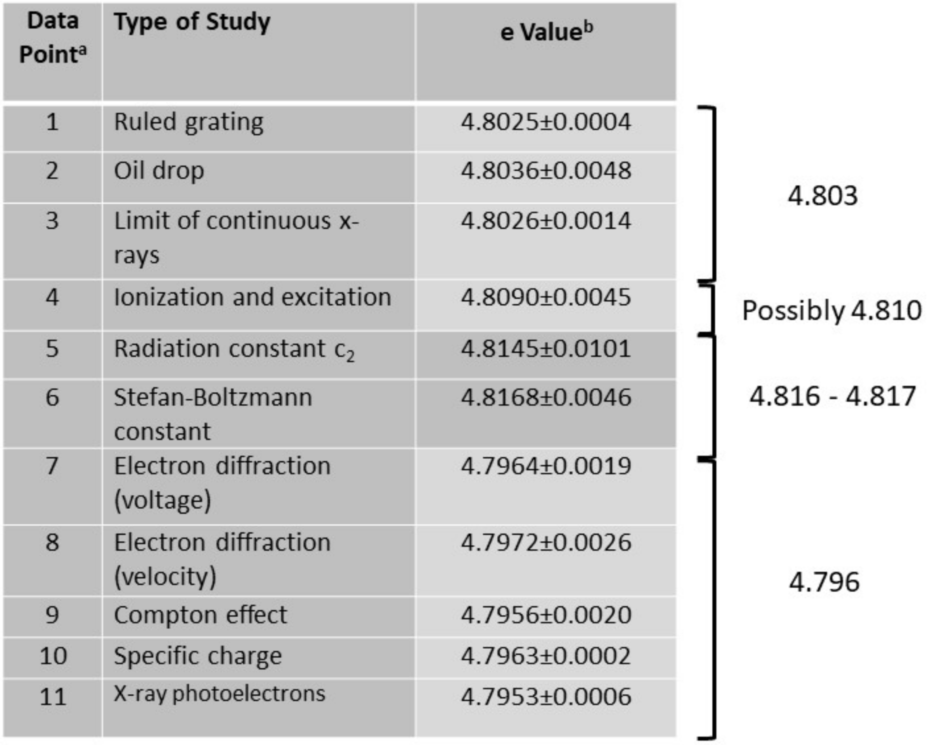


Fig. 2. Dunnington's experimental results shown among a cascade of values calculable by way of a basic equation for α and the successive application of a factor of about $(1.0015)^2$ to the basic equation—see equations (1) and (2). a) Values of α inserted into equation (3) to determine the associated values of e, b) each value x 10^{-10} esu, c) standard α value, d) esu value equivalent to today's value of e of 1.602176634 x 10^{-19} C.[8]

Although Dunnington did not highlight the intermediate value of 4.810 x 10^{-10} esu—which arises naturally through the cascade—among his results is the value of 4.8090 ± 0.0045 x 10^{-10} esu, which could ultimately be the 4.810 x 10^{-10} esu value (Figure 3). However, that is not definitive given the large error margins associated with the number.



Data Point[a]	Type of Study	e Value[b]
1	Ruled grating	4.8025±0.0004
2	Oil drop	4.8036±0.0048
3	Limit of continuous x-rays	4.8026±0.0014
4	Ionization and excitation	4.8090±0.0045
5	Radiation constant c_2	4.8145±0.0101
6	Stefan-Boltzmann constant	4.8168±0.0046
7	Electron diffraction (voltage)	4.7964±0.0019
8	Electron diffraction (velocity)	4.7972±0.0026
9	Compton effect	4.7956±0.0020
10	Specific charge	4.7963±0.0002
11	X-ray photoelectrons	4.7953±0.0006

[a] Same point designation as in original work.

[b] Each value x 10^{-10} esu.

Fig. 3. The 4.810 x 10^{-10} esu value that emerges in the value cascade shown in Figure 2 might also have been captured within Dunnington's study.[8]

Interestingly, the vast majority of the other data points well reflect the categorial value before error margins are even considered. If this holds true for data point 4, then indeed it might ultimately relate to the 4.810 x 10^{-10} esu value. This value has arisen in the context of some x-ray studies, as well.[6]

Dunnington's Results Translated to Physical Distance Associated With the Electromagnetic Field

Many models of electromagnetic interactions have been put forth, particularly to demonstrate the recoil between two electrons, stemming from the repulsive force caused by the exchange of virtual photons. As a photon can exist as a particle or wave, and electromagnetic radiation can be described using classical waves with particular wavelengths and photon energies, one of the simplest ways to depict an electromagnetic interaction, from a classical perspective, is by way of two electrons each exchanging a single photon modeled as a classical wave of arbitrary frequency. Figure 4 shows such a model, with each photon represented as a wave 2π radians in length.

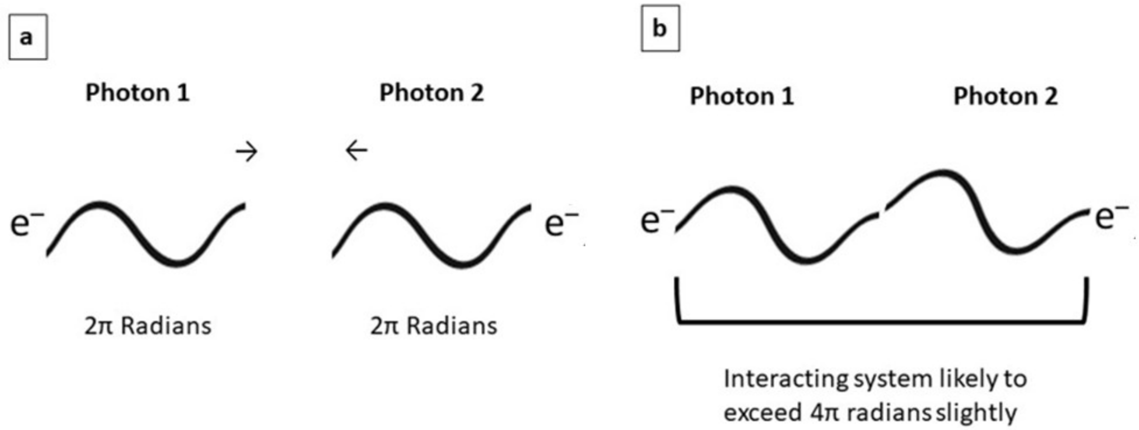


Fig. 4. (a) In this model, each of two electrons emits and subsequently absorbs one photon, with the photons depicted as classical waves—with each wave being 2π radians in length. (b) In the setting of vacuum fluctuations, the combined system would likely exceed 4π radians, if only slightly, as such fluctuations would prevent perfect geometric alignment.

Although the nature of the electromagnetic field is ultimately probabilistic and does not have an inherent classical quality to it, the field and the phenomena it causes can nonetheless be described classically. All electromagnetic interactions can be described by way of classical waves traveling within the fields. However, the quantum nature of the field ultimately bestows no “realness” to the waves, just to the actions attributed to them.

If the model is set within the context of the fluctuations of spacetime, of the vacuum, the two photons should not be considered to have perfect geometric alignment, leading to exactly 4π radians in the combined system. The more appropriate scenario would be for the system to have a value slightly larger than 4π (Figure 4b) due to the quantum fluctuations preventing perfect geometric conditions. The gap between the two 2π-valued waves would represent a correction on the total 4π value, making it 4π-plus.

As the model implies, the greater the gap (increasing the size of the 4π-like value), the weaker would be the strength of the interaction (and vice versa). As such, the 4π-like number would be inversely proportional to the value of α. The 4π-like value represents an overall threshold that must be crossed for the full interaction strength (highest α value) to be attained. Another way of viewing this is that each photon must be fully absorbed for the full interaction strength to be realized. Thus, the more this threshold is overcome, the greater the interaction strength, leading to an inverse relationship between α and the 4π-plus value:

$$\alpha \propto \frac{1}{4\pi +} \tag{10}$$

Their inverse relationship and the fact that 4π is associated with a correction on its value are consistent with the basic equation for α discussed above, where

$$\alpha \approx \left(\frac{1}{4\pi}\right)\left(\frac{1}{4\bar{e}}\right)\left(\frac{1}{\sim 1.0015}\right)^2$$

(from equation [7]).

As shown above, the existence of the approximately 1.0015 factor is evident from the experimental data, as the e values attained through research efforts differ from one another by this value or its square—see equations (1) and (2) and Figure 2. Ultimately, the value might embody quantum corrections from the vacuum, translated to a modification of the 4π value in the model, which again represents two photons traveling through the vacuum in the course of the electromagnetic interaction. In this way, it would not be dissimilar to the approximately 1.001159652 factor that embodies the quantum corrections affecting the electron *g* factor, changing it from 2 to 2 times 1.001159652, or about 2.0023.[1] The $(1.0015)^2$ value applied specifically to the 4π value in equation (7) leads to the slightly larger value of about 4.012π, consistent with the scenario in Figure 4. Using the more precise factor of 1.00146359514, from the known value of α, leads to 4.01171732957π:

$$\alpha = \left(\frac{1}{4.01171732957\pi}\right)\left(\frac{1}{4\bar{e}}\right). \tag{11}$$

From a purely qualitative perspective, there is nothing to suggest that the approximately 4.012π value would necessarily be fixed or static, changing due to continuous fluctuations of the vacuum and electromagnetic field. Such fluctuations would lead to a different 4π-like value at different times, sometimes perhaps larger and sometimes smaller, making the quantity a variable—in particular, a dimensionless length variable of the field, designated as x below:

$$\alpha = \left(\frac{1}{x}\right)\left(\frac{1}{4\bar{e}}\right), \tag{12}$$

where the corresponding e value is attained by placing the α value into equation (3). In this way, the field in Figure 4 would appear somewhat like an oscillating spring (Figure 5).

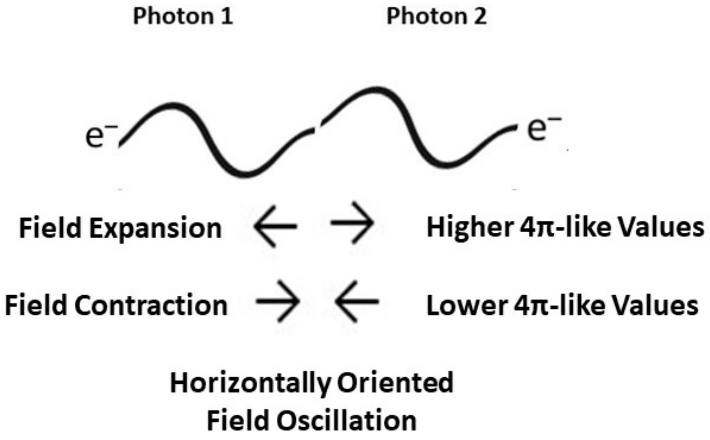


Fig. 5. The electromagnetic field between two interacting elementary charged particles may oscillate horizontally like a spring leading to larger or smaller 4π-like values in comparison to Figure 4.

Table 2 translates the e values in Figure 2 into dimensionless 4π-like length quantities. Smaller e values are associated with larger 4π-like values, whereas larger e values are associated with smaller 4π-like values. This is consistent with the fact that the 4π-like number represents an energy barrier or threshold that must be surmounted for full interaction strength to be realized. The greater the value, the larger the barrier and the smaller or weaker is the value of e and α.

TABLE 2. Elementary electric charge values translated into dimensionless, pi-related length quantities of the electromagnetic field between two interacting particles.

e Value[a]	**Associated Field Length in Units of π**	
4.796	4.012π + 0.012π	4.024π
4.803[b]	4.012π	4.012π
4.810	4.012π – 0.012π	4π
4.817	4.012π – 2(0.012π)	3.988π

Note: All values approximate.

a. Each value x 10^{-10} esu.
b. Associated with the standard value of e and α.

As shown in Table 2, the sizes of the 4π-like values are not random: Three of the four values appear to pivot by discrete, consistent amounts around the 4.012π value related to 4.803 x 10^{-10} esu. The 4.796 x 10^{-10} esu value corresponds to 4.012π + 0.012π, whereas the 4.810 x 10^{-10} esu value corresponds to 4.012π – 0.012π, and 4.817 x 10^{-10} esu corresponds to 4.012π – 2(0.012π). Thus, from the standpoint of the field operating like a spring, the e value of 4.803 x 10^{-10} esu and its corresponding length value of 4.012π appear to represent the equilibrium position of the field. This is consistent with the fact that these numbers are associated with the standard value of α—as in equation (11). A SHM state becomes visible by way of the other e and length values, representing discrete levels of displacement from the equilibrium position in opposite directions (Figure 6).

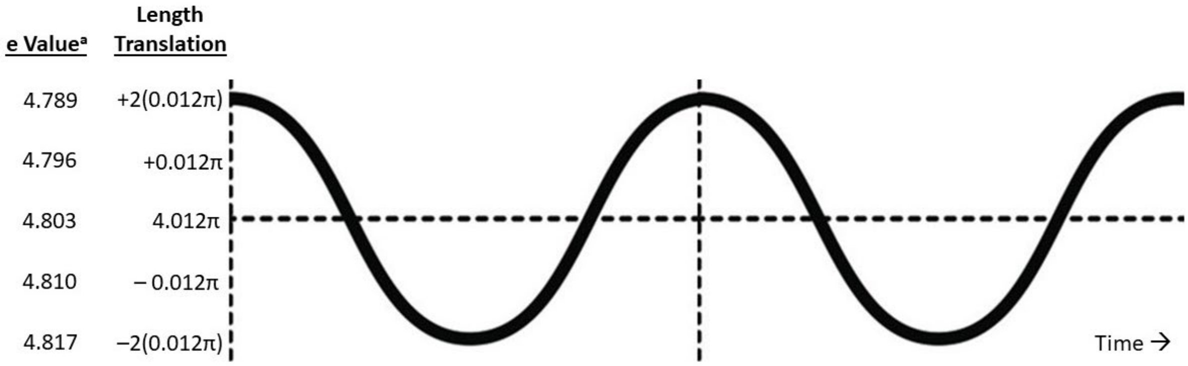


Figure 6. The e value of 4.803 x 10^{-10} esu and its corresponding length value of 4.012π appear to represent the equilibrium position of the field, with the other e and length values representing discrete levels of displacement from the equilibrium position—with 4.796 x 10^{-10} esu corresponding to 4.012π + 0.012π, 4.810 x 10^{-10} esu corresponding to 4.012π – 0.012π, and 4.817 x 10^{-10} esu corresponding to 4.012π – 2(0.012π)—leading to a SHM-associated wave pattern. a, each value x 10^{-10} esu.

While the electromagnetic interaction between two particles in a given material might allow for sustained SHM over a continuous stretch of time, any measurement would, of course, break the process or capture only a single moment in time. Here, the SHM would become evident over multiple measurements of the value of e.

In the context of past experiments, any given measurement—obtained at any given time, by any given technique—may have captured a “snapshot” or “still frame” of the field in a particular extended or compressed state. This state would be associated with a particular value of e, with a given set of experiments possibly capturing e values within the full amplitude range of the field. The SHM of the field then becomes visible by way of the “still frames” being placed in order of field length and viewed collectively, as if in animation over a single stretch of time, with the understanding that the cycle would continuously repeat itself in an undisturbed substance.

The 4.789 x 10^{-10} esu value shown in Figure 6 was not captured by Dunnington. However, as discussed below, Dunnington’s range of values may be just a subset of a greater range of values still reflective of the electromagnetic field undergoing SHM, with 4.803 x 10^{-10} esu and 4.012π still at the equilibrium point.

Thus, the seemingly discrepant values of e can be viewed through the lens of SHM when charge is translated into distance quantities. This view also removes the semblance of the experimental results being a random set of values. From the SHM perspective, it is clear why different e values would have been encountered and why again the concept of a “most probable” value for the constant would be needed and appropriate, being consistent with the state of equilibrium within the system and thus the average of all the values. Often in studies of the value of e, such as in Dunnington’s work, 4.803 x 10^{-10} esu, or a value close to this, was indeed stated to be an average value among several different results.[8,17]

Additional Experimental Elementary Electric Charge Values Consistent With Simple Harmonic Motion of the Electromagnetic Field

In his 1935 report, Robinson gave an overview of prominent x-ray results at the time, as shown in Table 3.

TABLE 3. Prominent elementary electric charge values determined by x-ray experiments as highlighted in [6].

e Value[a]	Study
4.793±0.0144	Bäcklin, 1929 [23]
4.798	Shiba, 1932 (as reported in [6])
4.8036±0.0005	Bearden, 1935 [24]
4.805	Bäcklin, 1935 [25]
4.806	Söderman, 1935 [26]
4.806±0.003	Bearden, 1931 [27]
4.810±0.002	Compton, 1929 [28]
4.8162	Cork, 1930 [29]
4.821	Cork, 1930 (as reported in [6])

Mean: Approximately 4.807 ± 0.002 x 10^{-10} esu.

a. Each value x 10^{-10} esu.

This brief analysis by Robinson shows results similar to that of Dunnington: The two low values of 4.793 ± 0.0144 x 10^{-10} esu and 4.798 x 10^{-10} esu are fairly consistent with the 4.796 x 10^{-10} esu value—indeed 4.796 ± 0.007 x 10^{-10} esu is the average of the two. The 4.810 x 10^{-10} esu value is present, as is 4.816 x 10^{-10} esu. The mean of the above results is about 4.807 ± 0.002 x 10^{-10} esu, close to the 4.803 x 10^{-10} esu value.

The lowest value of e reported most often in some other studies is 4.768 x 10^{-10} esu. Sometimes this was in relation to too low of a viscosity value used in oil-drop experiments, but even the x-ray study by DuMond and Bollman identified this value.[21] The highest value of e often reported is from a study by Ishida et al.[30] While in their published paper, they reported a value of 4.806 x 10^{-10} esu, this was with a particularly low viscosity value. A reevaluation of their finding by Robinson suggested a result of approximately 4.835 x 10^{-10} esu.[17] A reevaluation by Dunnington suggested a value of about 4.8453 ± 0.0043 x 10^{-10} esu.[8] The average of these two is about 4.840 ± 0.002 x 10^{-10} esu. The key issue here is that, while there appears to be a large discrepancy between the values of 4.768 x 10^{-10} esu and 4.840 ± 0.002 x 10^{-10} esu, when viewed from the SHM picture, they are actually consistent with one another, as they constitute the same displacement from the equilibrium position, but in opposite directions.

As shown in Figure 7, 4.768 x 10^{-10} esu corresponds to 4.012π + 5(0.012π), and 4.839 x 10^{-10} esu corresponds to 4.012π – 5(0.012π). Viewed from this perspective, the experimental results of 4.768 x 10^{-10} esu and 4.835 to 4.8453 ± 0.0043 x 10^{-10} esu are actually not unusual and do not represent discrepancies. This is in sharp contrast to the general sentiment in the past that Ishida's value, in particular, represented an extreme outlier and was sometimes excluded from analyses.[8,31] Indeed, at 4.804 ± 0.001 x 10^{-10} esu, the average of just 4.768 x 10^{-10} esu and 4.840 ± 0.002 x 10^{-10} esu by themselves, closely matches the equilibrium value, suggesting again that they represent the maximum displacement from the equilibrium point in opposite directions.

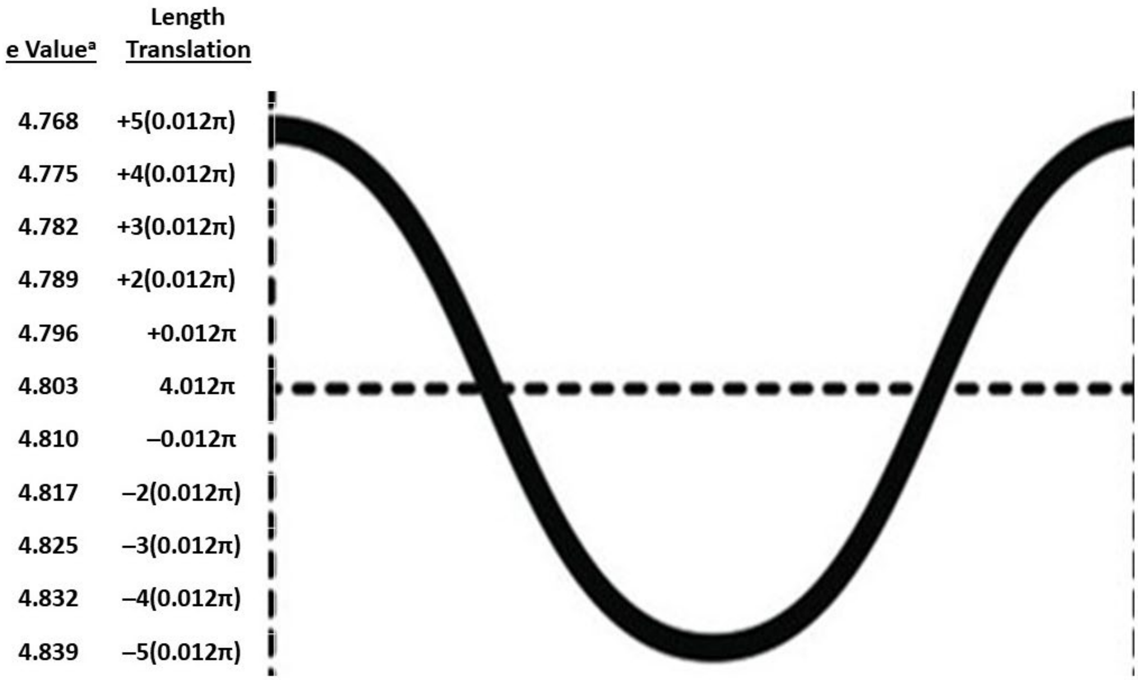


Fig. 7. Extreme values of 4.768 and 4.839 x 10^{-10} esu shown within the context of a SHM picture, where their associated length values are taken as the maximum displacement, or amplitude, on either side of the equilibrium position of 4.803 x 10^{-10} esu. Sub-amplitude values are shown corresponding to prominent e value results from multiple past experiments—see Table 4. All values approximate; e values calculated by way of equations (3) and (12). a, each value x 10^{-10} esu.

Also shown in Figure 7 and Table 4 are various sub-amplitude values and their associated e values, showing good agreement with prominent experimental results from across multiple studies. Of course, not all experimental results will necessarily follow the discrete pattern of sub-amplitude length and e values, as there are often exceptions to many naturally occurring phenomena. However, the prominence of the discreteness of the values further suggests a connection to quantum phenomena.

TABLE 4. Values of elementary electric charge from Figure 7 compared with examples of experiments with similar results.

e Value From Figure 7[a]	e Value From Experimentation[a]	Study, Date, and Method
4.768	4.768[b]	DuMond, 1937, determination of h/e through x-ray methods [21]
4.775	4.774±0.007	Wadlund, 1928, x-ray methods [32]
4.782	4.7824±0.0015	Birge, 1936, determination of h/e and e/m through various methods, such as x-ray, Compton shift, and energy of photoelectrically ejected electrons [22]
4.789	4.785[b]	DuMond, 1937, x-ray [21]
4.796	4.796	Dunnington, 1939, a reexamination of experimental data from electron diffraction (voltage), electron diffraction (velocity), Compton effect, specific charge, x-ray photoelectrons [8]
4.803	4.8029±0.0005	Dunnington, 1939, mean value, from a reexamination of experimental data from ruled grating, oil drop, limit of continuous x-rays, ionization and excitation, radiation constant c_2, Stefan-Boltzmann constant, electron diffraction (voltage), electron diffraction (velocity), Compton effect, specific charge, x-ray photoelectrons [8]
	4.8036±0.0005	Bearden, 1935, x-ray [24]
	4.803, 4.805	Robinson, 1935, 1937, mean, x-ray [6,20]
	4.805	Robinson, 1938, mean, oil drop [17]
	4.8059±0.0052	Millikan, 1917, oil drop, as recalculated by Dunnington, 1939 [8,14]
4.810	4.8090±0.0045[c]	Dunnington, 1939, reexamination of ionization and excitation studies [8]
	4.810±0.002	Compton, 1929, x-ray [28]
	4.8137±0.0030	Hopper and Laby, 1929, oil drop, as recalculated by Birge, 1945 [31,33]
4.817	4.8162	Cork, 1930, x-ray [29]

	4.816 – 4.817	Dunnington, 1939, a reexamination of experimental data involving radiation constant c_2, Stefan-Boltzmann constant [8]
4.825	4.825±0.005	Bearden, 1929, x-ray [34]
4.832	—	—
4.839	4.840±0.002 (Avg.)	
	4.835	Ishida, 1937, oil drop, as recalculated by Robinson, 1945 [17,30]
	4.8453±0.0043	Ishida, 1937, oil drop, as recalculated by Dunnington, 1939 [8,30]

a. Each value x 10^{-10} esu.
b. Values of 4.768 x 10^{-10} esu and 4.785 x 10^{-10} esu presented in study as "either/or." [21]
c. Value ambiguous due to error margins.

Importantly, while adjustments such as using a different air viscosity value in oil-drop experiments would have removed discrepancies between some experimental results, no amount of improvement in experimental technique would have eliminated all of the variation—as some appears related to natural field dynamics. This would help to explain why some of the "discrepancies" among measured e values appeared so intractable in years past, and why again, efforts had to lean toward identifying a most probable value, as opposed to a single, consistently found value.

Also, it must be acknowledged that many varying e values may have gone unreported and unpublished, perhaps because they seemed unusually high or low. Millikan, for example, had data concerning 25 oil drops that yielded an e value of about 4.789 ± 0.007 x 10^{-10} esu, based on a calculation by Franklin.[35,36] Given his 4.774 x 10^{-10} esu value is now considered to be closer to 4.803 x 10^{-10} esu, the 4.789 x 10^{-10} esu value would likely be closer to 4.817 x 10^{-10} esu, which is one of the values Dunnington contemplated excluding from his own analysis.[8] Fortunately, many results were published despite their being very different compared with other published values and despite perhaps not meeting expectations.

Implications

Should future research confirm the findings above, the information has important implications for the spatiotemporal character of electrons in materials, as their positions would likely change in space and time with slightly shifting values of e, even at technically zero energy. The correction on the 4π value suggests quantum phenomena and may offer a step closer to finding a fundamental formula for α and a resolution to the Landau pole problem.

Spatiotemporal Character of Electrons

Understanding the nature and behavior of electrons continues to be of fundamental importance. Indeed, the 2023 Nobel Prize in Physics was awarded to Pierre Agostini, Ferenc Krausz, and Anne L'Huillier for their groundbreaking work concerning "experimental methods that generate attosecond pulses of light for the study of electron dynamics in matter."[37] Their work has the potential for giving unprecedented access to the spatiotemporal characteristics of electrons in materials, and as such, has the potential to greatly advance multiple areas, from physics to medicine to various technological fields. As the strength of e would have a direct effect on the position of electrons in materials in space and time, knowledge of any SHM-like field dynamics altering the value of e might provide important information for better understanding the structure and function of atoms and molecules in the setting of attosecond research or other studies.

Fine-Structure Constant

The approximately 1.0015 value does appear to be related to e based on experimental data—where experimental values of e differ from one another by this value or a power of the number (equations [1] and [2] and Figure 2). Also, when applied to the standard equation for α, it leads to that equation being reducible to mathematical constants, as shown in equations (6) and (7). If it indeed represents the effects of quantum fluctuations from the vacuum affecting the value of e, it opens up an area for further exploration. This suggests also that a true fundamental formula for α may exist—consisting of the mathematical constants of equation (7) together with perhaps a quantum mechanical description leading to a precise version of the 1.0015 value and ultimately the 4.012π value as well.

As the amplitude of the SHM-associated wave corresponding to the experimental e values appears to change in discrete increments of about 0.012π (Figure 7, Table 4), a quantum mechanical description might also help explain why and when a specific e value might arise in a material. It might also explain how the frequency and amplitude of the wave might be manipulated, perhaps giving the power to control some spatiotemporal aspects of electrons in the material and possibly even allow some quantum-based signaling.

Landau Pole

The Landau pole is the point at which the value of α reaches infinity but does so at a finite energy level. This is regarded as a problem in the Standard Model (SM) because one would expect α to reach infinity with nothing less than an infinite amount of energy.[38]

Of the Landau pole, Pirogov and Zenin note the following:

> Technically, we can hoped [*sic*] to solve it by an improvement of the perturbative series or by the development of strong coupling methods; but more probably it has a physical origin, and it could be solved eventually by a more complete theory which should effectively result in a physical cutoff.[39]

The simplicity of the model in Figure 4 in describing the interaction between two elementary charged particles should not detract from its utility. Indeed, without the model, the SHM picture identified above may not have been apparent, as it is difficult to observe the pattern from the e values alone. The question then is, How far does the utility of the model go? As stated above, the more the 4π-like barrier is overcome, the higher becomes the value of α. But what does it suggest about an infinite α value and the energy of the electrons at that interaction-strength level, consistent with the Landau pole?

The values of e in Table 4 are all essentially at zero energy. Higher e values would be attained by reductions in the 4π-like value brought about by adding energy to the electrons, such as in a particle accelerator. As two electrons are slammed together, they would penetrate further into the 4π-like field, which would overlap with the cloud of virtual particles surrounding the electrons, as the cloud would exist in the same intervening space. As such, overcoming the 4π-like barrier is the same as drilling past the virtual particle cloud and encountering more of the intrinsic charge of the electron and hence a rising e and α value.

The values of e and α have been measured to rise to the following at an energy level corresponding to the Z boson mass (91.188 GeV): e to approximately 1.654 x 10^{-19} C, or 4.959 x 10^{-10} esu; α^{-1} to approximately 128.5±1.8(stat) ±0.7(sys.).[40,41,42,43,44]

The 128.5 value is equivalent to a reduction of the 4π-plus value by approximately 0.25π:

$$\alpha = \frac{1}{(4.01171732957\pi - 0.25\pi)(4\bar{e})} = \frac{1}{128.5} \qquad (13)$$

When the full 4.01171732957π value is mitigated, it would lead to an infinite α value at a finite energy, consistent with the Landau pole:

$$\alpha = \frac{1}{(0)(4\bar{e})} = \infty \qquad (14)$$

(Landau pole reached: α value infinite at finite energy level)

The finite amount of energy is attributable to the simple fact that 4.01171732957π is a finite number—only a finite amount of energy would be needed to overcome this finite barrier. However, according to the model, the Landau pole would be unobservable because, at the moment it would be created, the field would vanish. That is, with no 4π-like field, there is no interaction, and with no interaction, there is no observable interaction strength, infinite or otherwise. The infinite value of α at high energy thus appears to be mathematical only, not observable.

As such, the 4.01171732957π value in equation (11) would indeed represent a physical cutoff in the context of the model in Figure 4, as the entirety of the field, physically, is represented by that number. Given the power of the model in helping to reveal the SHM characteristic of the field, consistent with various experimental data, consideration should be given to the possibility that it might help resolve the Landau pole problem as well.

Conclusion

Throughout most of the 1930s, much concern existed over the apparent discrepancies among experimental values of e. The key finding of the present study is evidence of there being a connection between experimental e value results and SHM associated with the electromagnetic field between interacting particles. The SHM pattern would not be particularly apparent without the translation of the e values into distance quantities. This study shows that many of the experimental values of e—from smaller numbers such as 4.768 x 10^{-10} esu to higher values such as 4.840 x 10^{-10} esu—are related to one another within the context of the SHM picture. This eliminates the semblance of randomness among the experimental results, helps explain why so many different values of e were attained, and sheds light on why the 4.803 x 10^{-10} esu value, again equivalent to today's value, would stand out as the most probable value, being associated with the equilibrium position. Further study into this phenomenon might assist in better understanding the spatiotemporal dynamics of electrons in materials and additional elementary particle characteristics.

Data Availability

All data generated or analyzed during this study are included in this paper, the referenced articles, and the article, "CODATA recommended values of the fundamental physical constants: 2018" (Tiesinga, E., Mohr, P. J., Newell, D. B. & Taylor, B. N. *Rev. Mod. Phys.* **93,** 025010 [2021]. https://doi.org/10.1103/RevModPhys.93.025010; https://physics.nist.gov/cuu/Constants).

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