The Quark Model of the Electron and the Vacuum Fabric

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Abstract: We propose a quark model for the electron and the vacuum fabric, where the electron is a non-elementary, non-point like exotic tetraquark, and the vacuum fabric is comprised of exotic pion tetraquark tetrahedrons. We assume that electron tetraquarks perform rapid quark flavor exchange reactions with the vacuum pion tetraquark tetrahedrons fabric, which is comprised of the valence quarks and antiquarks, $u , d , \tilde{u} , \tilde{d}$, forming and electron and pion cloud. Motion of the electron tetraquark tetrahedron on the vacuum pion tetraquark tetrahedron fabric is performed by a $u \; and \; d$ quark flavor exchange reactions by tunneling through a double well potential barrier between the electron tetraquark tetrahedron and the vacuum pion tetraquark tetrahedron sites that transform the electron tetraquark tetrahedrons into pion tetraquark tetrahedrons and vice versa. We assume that the quark flavor exchanges occur with the extremely high zitterbewegung frequency and hence a single electron cannot be isolated in the electron and pion fabric cloud. The electron charge polarizes the vacuum pion tetraquark tetrahedron electric dipoles forming together a dense and polarized sphere around the electron. Lattice QCD computation may prove the quark model for the electron and the vacuum fabric and may allow calculating the mass of the proposed pion and electron exotic tetraquarks.

Keywords: QED, QCD, Antimatter, Quantum vacuum, Lattice QCD, Exotic tetraquarks.

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1. Problems with the Quantum Electrodynamics Theory

Schrodinger found that a gaussian wavepacket remains coherent with a harmonic potential and suggested that a wavepacket should describe the electron[1]. However, in free space, the width of the electron gaussian wavepacket grows linearly with time[2]. If an electron wavepacket is initially localized in a region of an atomic dimension of 10^{-10} meters, the width of the wavepacket doubles in about 10^{-16} seconds and after about a milli-second the wave packet width grows to about a kilometer, which is not a reasonable description for the electron.

Dirac proposed a relativistic wave equation and found a new set of negative energy antimatter states[3]. Dirac thought that since the positive electron solutions would decay to the negative energy states there must be an infinite number of invisible electrons that occupy the negative states and prevent the electron decay according to Pauli exclusion principle[4]. Dirac assumed that electrons are not point like particles and proposed a spherical shell electron model[5]. Oppenheimer showed that the vacuum polarization and the electron self-energy closed loop are not small corrections that diverge. Dirac and Feynman did not like the renormalization scheme that succeeded to cancel the infinities. Dirac thought that electrons interact strongly with the vacuum electron-positron virtual pairs and hence are never bare in contrast to Feynman's diagrams, where bare electrons propagate in free space in zero-order of the perturbatubation[6-7]. Dirac thought that better understanding of the vacuum structure is needed for quantum electrodynamics theory[8].

2. The Addressed Questions

The questions we address in this paper are:

1. Are electrons non-elementary, non-point like particles comprised of exotic tetraquarks?
2. Is the quantum vacuum filled with pion tetraquark tetrahedrons fabric?

3. Does electron motion in the vacuum occurs via rapid u and d quark flavor exchanges via gluons in a first electron tetraquark state, and $\tilde{u}$ and $\tilde{d}$ antiquark flavor exchanges in a second electron tetraquark state?

3. The Vacuum Pion Tetraquark Tetrahedron Fabric

We assume that the quantum vacuum is filled with pion tetraquark tetrahedrons fabric[9-13]. We note that the vacuum pion tetraquark tetrahedrons are not ordinary matter particles since they are composed of 50% antiquarks that annihilate the other 50% quarks, however, we assume that the exotic pion tetraquarks condense and remain with a small mass and internal rotation and vibration energies in the vacuum fabric. We assume that in each site in the vacuum fabric there is a single tetraquark tetrahedron, $u\tilde{d}d\tilde{u}$, composed of the two valence quarks, d and u, and their antiquark pairs, $\tilde{d}$ and $\tilde{u}$. Two pion tetraquark tetrahedron enantiomers may exist obtained by exchanging the positions of two quarks at the vertices as shown below in line with Weyl spinors and the chiral symmetry of the QCD vacuum ground state[14-20].

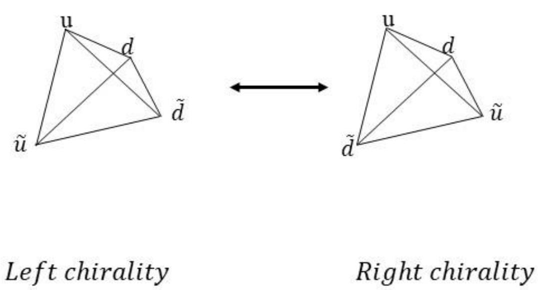


Figure 1 illustrates two exotic pion tetraquark tetrahedron enantiomers, where the $\tilde{u}$ and $\tilde{d}$ antiquarks exchanged positions.

We assume that in the vicinity of a massive body, the pion tetraquark tetrahedron fabric may have higher density and a spherical symmetry according to the gravitational or electrical field. In extreme gravitational field, in the vicinity of a black hole for example, the pion tetraquark tetrahedron fabric cell size may become extremely small, less than a femtometer, and far away from any galaxy in cosmic voids[21-23], the pion tetraquark tetrahedron vacuum fabric may be extremely diluted with cell size of tens and more Compton lengths.

4. The Electron and Pion Tetraquark Tetrahedron Double Well Potential Model

We assume that electrons are non-elementary, non-point like particles comprised of exotic tetraquarks. A first tetraquark state may be $\widetilde{u}\, du\widetilde{u}$, and a second tetraquark state may be $\widetilde{u}\, dd\widetilde{d}$ [9-

[13]. Transforming an electron to a pion tetraquark tetrahedron on the vacuum fabric occurs by quark flavor exchanges between two fabric sites. A pion tetraquark tetrahedron is transformed by the quark flavor exchanges to an electron tetraquark tetrahedron and vice versa by gluon exchanges in a two tetraquark scattering reaction. Since the quark flavor exchanges reactions are symmetric where reactants and products are identical, a double well potential model[24] may be used to represent the reaction like in mmonia molecule inversion[25]. The motion of the electron tetraquark tetrahedron on the pion tetraquark tetrahedron fabric may occur via tunneling through the double well potential barriers. The $u\ and\ d$ quark flavors are exchanged as illustrated in figure 2 below for the first electron exotic tetraquark state (L).

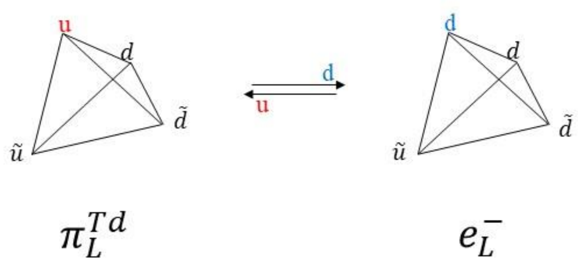


Figure 2 illustrates an electron tetraquark tetrahedron and a pion tetraquark tetrahedron exchanging quark flavors (u and d).

The pion and electron exotic tetraquarks scattering reaction on two fabric sites i and j is

$$\tilde{u}d\tilde{d}u\ (\pi^{Td})_i + \tilde{u}d\tilde{d}d\ (e^L)_j \rightarrow \tilde{u}d\tilde{d}d\ (e^L)_i + \tilde{u}d\tilde{d}u\ (\pi^{Td})_j \qquad (1)$$

In the case of the second electron exotic tetraquark state (R), the $\tilde{u}$ *and* $\tilde{d}$ antiquarks are exchanged according to the following scattering reaction

$$\tilde{u}d\tilde{d}u\ (\pi^{Td})_i + \tilde{u}d\tilde{u}u\ (e^R)_j \rightarrow \tilde{u}d\tilde{u}u\ (e^R)_i + \tilde{u}d\tilde{u}u\ (\pi^{Td})_j \qquad (2)$$

The double well potential Hamiltonian used to model the symmetric quark exchange reactions is[24]

$$\hat{H} = \frac{\hat{p}^2}{2m_e} + m_e\,\lambda\,(\hat{x}^2 - a^2)^2 \qquad (3)$$

Where m_e is the electron rest mass, 2 a is the distance between pion tetrahedron sites and the coupling parameter λ is determined by the potential barrier height, $V_0 = m_e\,\lambda\,a^4$, which is taken to be $V_0 = \hbar\boldsymbol{\omega_z} = 2m_ec^2$, where the frequency $\boldsymbol{\omega_z} = \frac{2m_ec^2}{\hbar}$ is Dirac's free space trembling motion frequency, the zitterbewegung[26-27].

Figure 3 below illustrates the double well potential model for the electron tetraquark tetrahedron and the pion tetraquark tetrahedron quark flavor exchange reaction in adjacent fabric sites i and j. We assume that the electron motion on the vacuum fabric is via quantum tunneling by gluon exchange through the potential barrier V_0 where the double well potential exists between adjacent sites in the vacuum fabric. The barrier height $V_0 = 2m_ec^2$ is twice the electron rest mass energy which is the threshold for electron-positron pair production. Note that the electron tetraquarks on both sides of the double well are identical and hence the electron state (a $\widetilde{u}\,dd\widetilde{d}$ or a $\widetilde{u}\,du\widetilde{u}$) is conserved, the two electron exotic tetraquark tetrahedron states are not mixed by the rapid quark flavor exchanges.

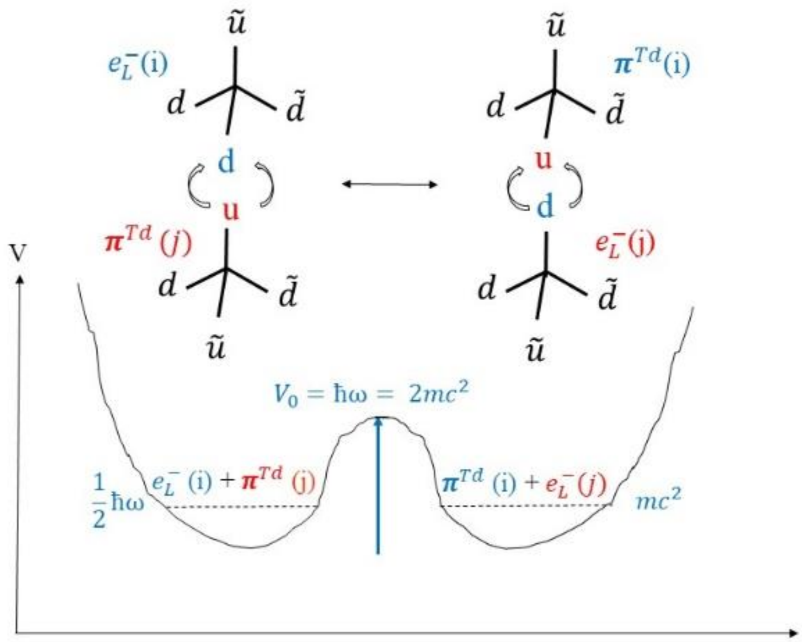


Figure 3 illustrates the double well potential model for the electron tetraquark tetrahedron and pion tetraquark tetrahedron quark flavor exchange reactions by gluons in adjacent sites i and j.

The double well symmetric ground state and the antisymmetric first excited state energies and wavefunctions are calculated numerically by diagonalizing the Hamiltonian (Eq. 3) using a Fourier plane wave basis set. The tunneling time, $T_{tunneling}$, from the left to the right potential well is an inverse function of the energy split between the first anti-symmetric state E_a and the symmetric ground state E_s. With the parameters above, E_a=1.0463 $\hbar\boldsymbol{\omega}$ which is just above the potential well barrier and $E_s = 0.7004\,\hbar\omega$ is in the well**.** The tunneling time is extremely fast, $5.849 * 10^{-21}$seconds.

$$T_{tunneling} = \frac{\pi\hbar}{E_a - E_s} = 5.849 * 10^{-21} \quad \text{seconds} \tag{4}$$

A superposition of the symmetric and antisymmetric eigenstates is taken as the initial state $\psi_{t=0}$ describing an electron wavepacket located in the left well initially.

$$\psi_0 = \frac{1}{\sqrt{2}}\left(\psi_s + \mathrm{j}\ \psi_a\right) \tag{5}$$

After a half period the electron wavepacket tunnels to the right well

$$\psi_{T_p/2} = \frac{1}{\sqrt{2}}\left(\psi_s\ e^{-iE_s t} + \mathrm{j}\ \psi_a e^{-iE_a t}\right) \tag{6}$$

The electron wavepacket continues oscillating between the two wells with a period of $T_p = \frac{2\pi\hbar}{E_a - E_s}$.

The electron velocity may be calculated by dividing the distance between the wells, $2a$, by the tunneling time.

$$v_e = \frac{2a}{T_{tunneling}} = \frac{2a\ (E_a - E_s)}{\pi\hbar} = 0.44\ c\ \ [\frac{m}{sec}] \tag{7}$$

Note that the electron velocity here is about half of the speed of light and on the time scale of the free space trembling motion frequency, the zitterbewegung, similar to electron semi-classical models[26-27].

5. The Electron and Pion Tetraquark Tetrahedron Cloud

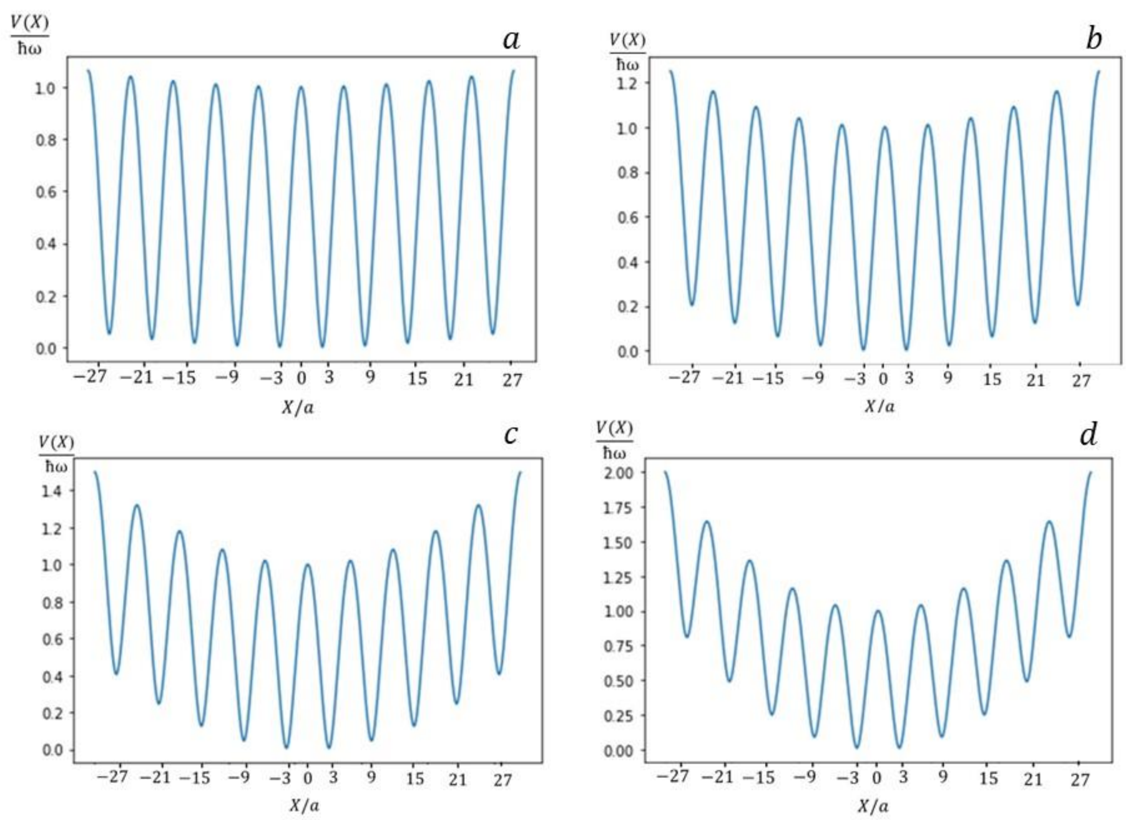
Next, we extend the single electron and pion tetraquark tetrahedron double well potential model to the vacuum fabric and propose that the electron with the vacuum pion tetraquark tetrahedrons form a dense and polarized sphere, where in the center of the sphere the double well potential model length a may be extremely small below the Compton length. Away from the cloud center, the distance between pion tetraquark tetrahedron may increase and the pion tetraquark tetrahedron density is reduced. After about few Compton lengths, the distance between pion tetraquark tetrahedron is such that the quark exchange reactions stop, the electron tunneling is exponentially decreased and the electron is trapped in the vacuum fabric cloud by the lack of quark exchange reactions. Accordingly, the electron is confined by the vacuum pion tetraquark tetrahedron fabric.

The following table summarizes results with increasing distance between the two potential wells, $2a$, $4a$ and $6\ a$ keeping the potential barrier at the same value, $V_0 =\ 2m_e c^2$, by changing the value of the coupling parameter λ, $\lambda = \frac{2\ c^2}{a^4}$, $\lambda = \frac{2\ c^2}{16\ a^4}$ and $\lambda = \frac{2\ c^2}{81\ a^4}$.

Distance between the two wells	E_a/ħω	E_s/ħω	$T_{tunneling}$(sec)
$2\ a$	1.0463	0.7004	5.849510^{-21}
$4\ a$	0.4741	0.4502	8.457710^{-20}
$6\ a$	0.318497	0.31698	1.34040^{-18}

The electron tunneling time between the two wells is reduced significantly with the growth of the distance between the wells. With the $6\ a$ distance the tunneling is about 229 times slower than with $2\ a$ distance (a is defined as the electron Compton length $\frac{ħ}{m_e c}$). The extremely fast electron wavepacket dynamics in the vacuum fabric may be observed in the future with the new attosecond electron microscopy[28].

We further assume that the electron charge polarizes the pion tetraquark tetrahedrons sphere since the chargeless pion tetraquark tetrahedrons have electric dipoles. The polarization of the vacuum pion tetraquark tetrahedrons cloud adds a long-range harmonic potential term ~ $\frac{ħ\omega x^2}{L^2}$, where L is the length scale of the pion tetraquarks cloud. The electron and pion tetraquark tetrahedron quark cloud potential in one dimension with 10 sites and with increasing long-range harmonic potential term, 0.25, 1, 2 and 4 $\frac{ħ\omega x^2}{L^2}$ are shown below.



Figures 4a-d illustrates the electron and pion tetraquark tetrahedron quark cloud potential in one dimension with 10 fabric sites and increasing long- range harmonic potential term ((a) 0.25, (b) 1, (c) 2 and (d) 4 $\frac{\hbar\omega x^2}{L^2}$).

The two lowest symmetric, ψ_1 , and antisymmetric, ψ_2 , eigenfunctions are shown below for the four long-range harmonic potential values. Note that the eigenfunctions peaks are localized in the fabric wells and that in the two lower states, the electron is not localized in a single well, it has a finite probability to be found in few adjacent wells in the fabric.

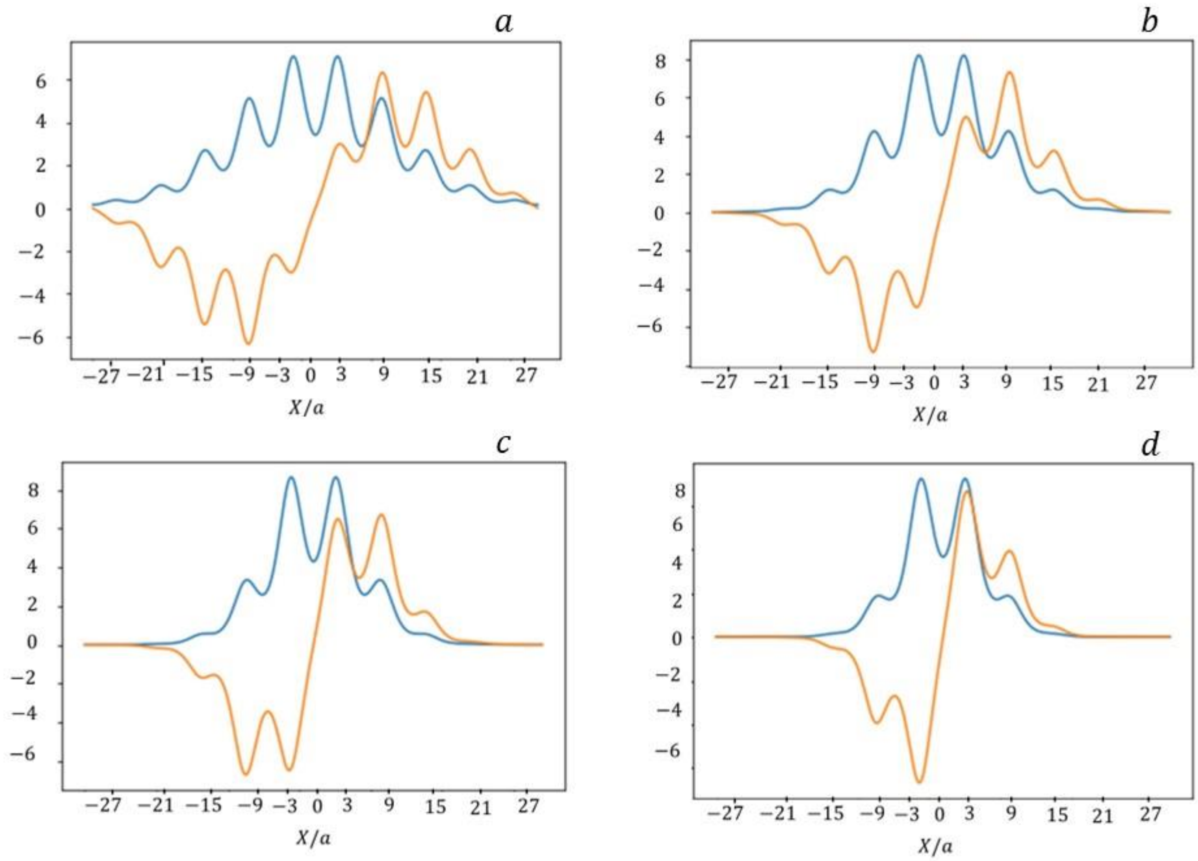


Figure 5a-d illustrates the first and second symmetric ψ_1 and antisymmetric ψ_2 eigenfunctions for the four long -range harmonic potential values ((a) 0.25, (b) 1, (c) 2 and (d) 4 $\frac{\hbar\omega x^2}{L^2}$).

An initial wavepacket may be formed by a superposition of the two lower eigenstates, $\psi_0 = \frac{1}{\sqrt{2}}(\psi_1 + j\psi_2)$. The electron wavepacket has high probability to be found in the first few wells on the left initially. After half a period, the wavepacket tunnels to the right-hand side wells (in blue). Note that with higher value of the long-range harmonic potential the low eigenstates are localized in fewer fabric sites as shown in figures 6c and 6d.

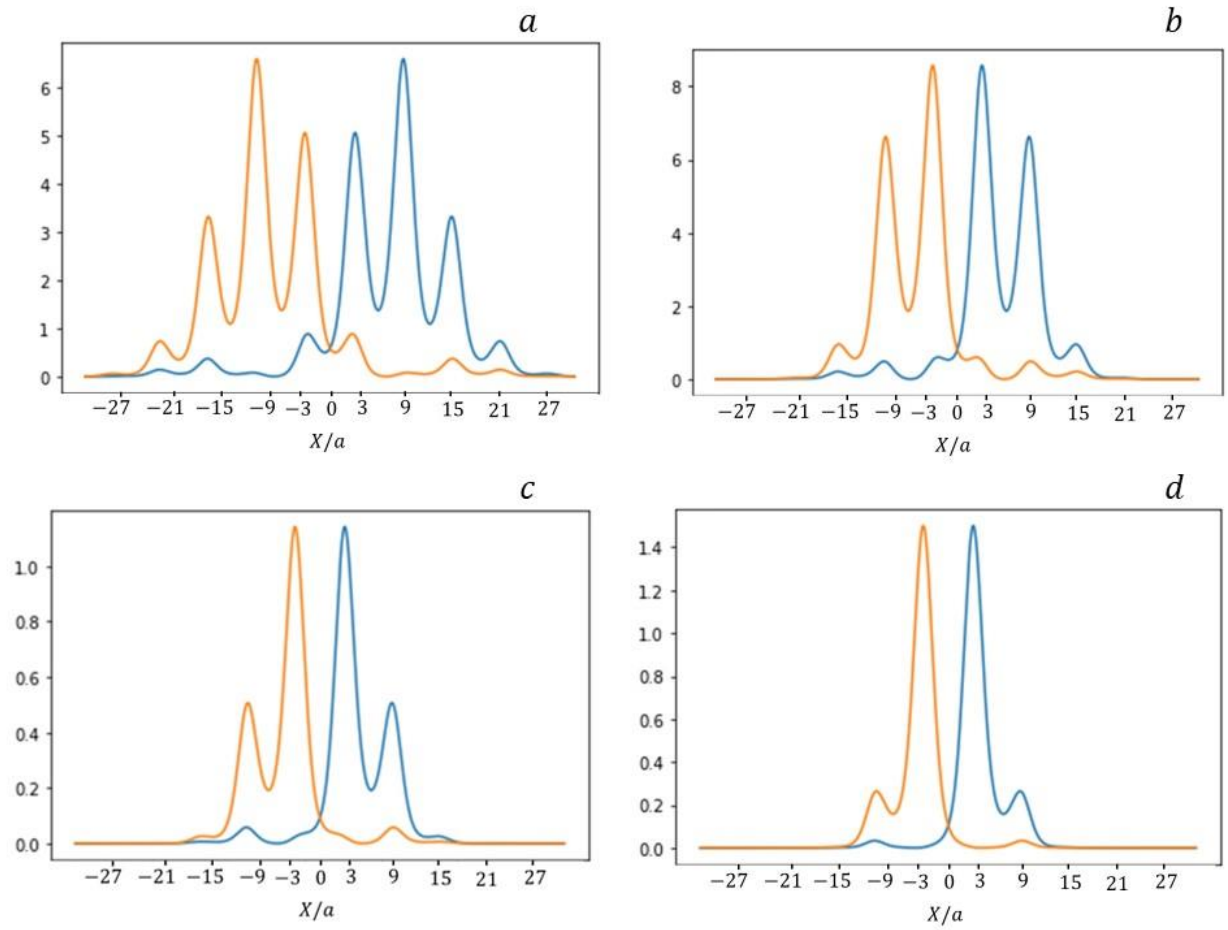


Figure 6a-d illustrates the electron wavepacket at t=0 (in orange) and after a half time period (in blue) for the four long -range harmonic potential values ((a) 0.25, (b) 1, (c) 2 and (d) 4 $\frac{\hbar\omega x^2}{L^2}$).

The position expectation value of the electron wavepacket for 5 time periods for the four long-range harmonic potential values calculated according to equation 8 are shown in figures 7a-d.

$$X(t) = \langle \psi_t | \hat{X} | \psi_t \rangle \qquad (8)$$

The position expectation value oscillates between the left-hand side wells to the right-hand side wells and with higher long-range harmonic potential values the oscillation amplitude decreases since the wavepackets are more localized in the first two wells.

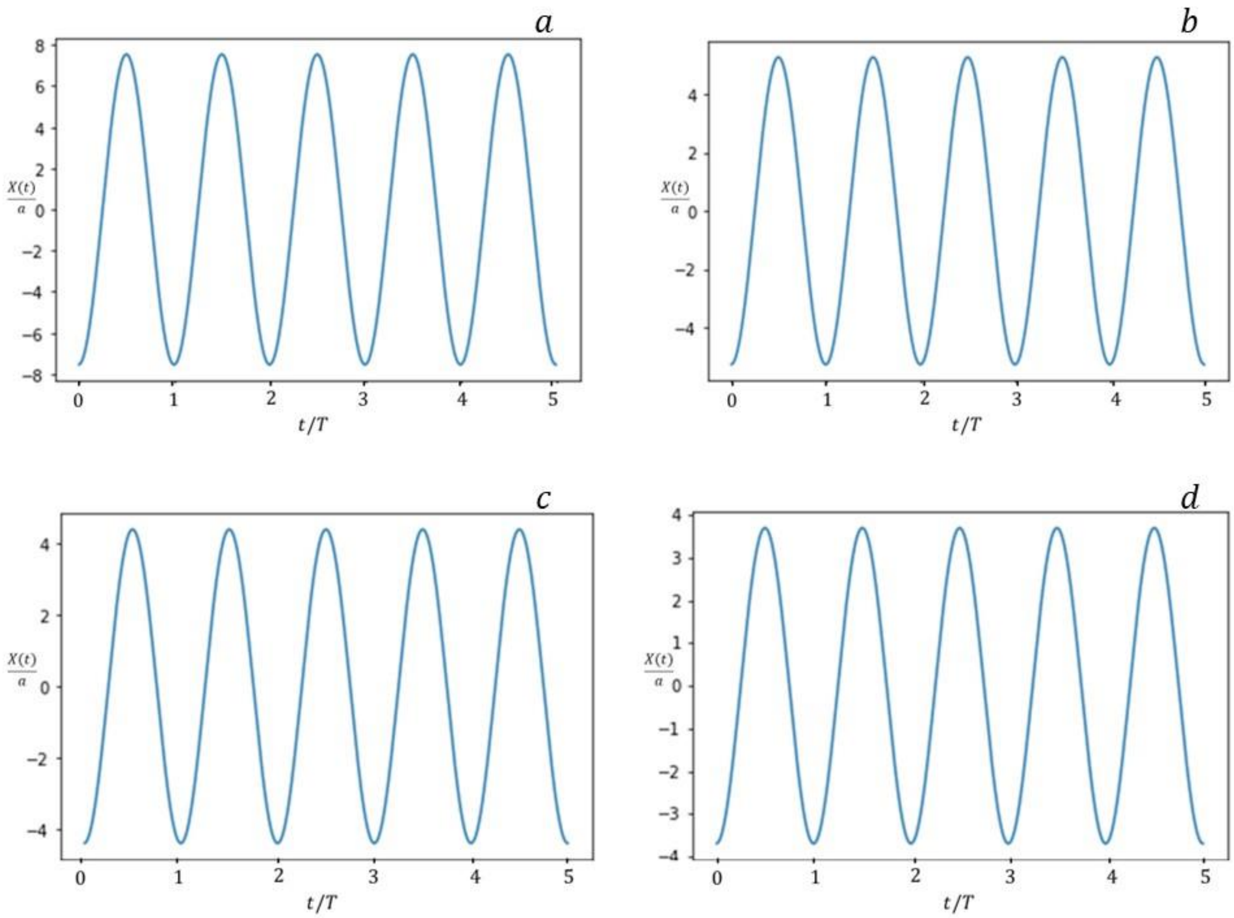


Figure 7a-d illustrates the position expectation value of the electron wavepacket for 5 time periods for the four long -range harmonic potential values ((a) 0.25, (b) 1, (c) 2 and (d) 4 $\frac{\hbar\omega x^2}{L^2}$).

The higher symmetric ψ_7 and antisymmetric ψ_8 eigenstates are localized in the outer wells. The superposition, $\psi_0 = \frac{1}{\sqrt{2}}\,(\,\psi_7 + \mathrm{j}\psi_8)$, is shown below where the tunneling occurs between the outer wells.

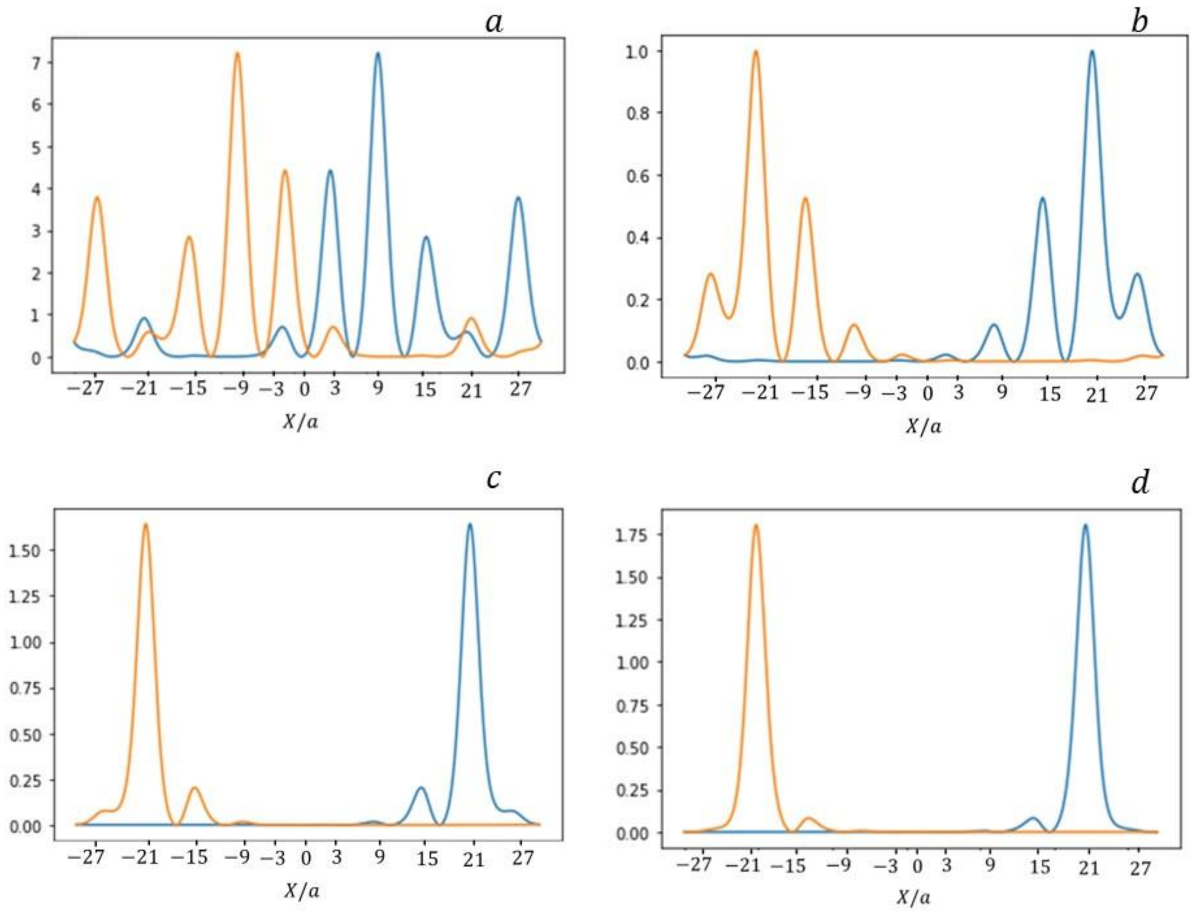


Figure 8a-d illustrates the electron wavepacket at t=0 (in orange) and after half time period (in blue) for the four long -range harmonic potential values ((a) 0.25, (b) 1, (c) 2 and (d) 4 $\frac{\hbar\omega x^2}{L^2}$).

We assume that the electrodynamic polarization effect of the charged electron on the vacuum fabric is to rearrange the pion tetraquark tetrahedron sphere with the speed of light, where the long-range harmonic potential shown in figures 4a-d are induced by the electron charge almost instantly in its new position after the electron tunnels. We can adjust numerically the length a ($\sim 0.46 \frac{\hbar}{m_e c}$ for example) such that the calculated electron velocity according to equation 7, $v_e = \frac{2a\,(E_a - E_s)}{\pi\hbar}$, will be close to the speed of light. In this case, the rearrangement of the pion tetrahedrons cloud and the tunneling of the electron wavepacket occurs at maximal speed and the electron speed is limited to c since the pion tetrahedrons fabric cannot rearrange faster.

The long-range harmonic potential value determines the uncertainty in the electron position, where with higher long-range harmonic potential the electron wavepacket is localized more. With smaller harmonic potential the electron wavepacket spreads over more fabric sites. However, the minimal width of the electron wavepacket is about the distance between fabric sites, the electron tunnels from site to site extremely fast with the zitterbewegung frequency and its position cannot be determined with higher accuracy than that. The long-range harmonic potential value may depend on the classical electron velocity, where with higher electron velocity the polarization of the pion sphere that surrounds the electron is stronger and the electron wavepacket width is smaller.

Schrödinger suggested that particles would be described with wavepackets and found that a Gaussian wavepacket formed by a linear combination of plane waves gets wider linearly with time in free space[1]. The electron wavepacket simulations above do not prove the proposed quark model for the electron and pion tetraquark tetrahedron vacuum fabric but show that a confined electron wavepacket may be obtained with the duplicated double well potential model that represents the electron and the vacuum pion tetraquark tetrahedron cloud. However, lattice QCD may prove the quark model and allow calculating the mass of the electron tetraquark and the pion tetraquark. With the proposed quark model of the electron and the vacuum fabric, the electron is never bare like Dirac assumed and we suggest that the perturbative free particle bare propagator may be the cause for the divergence of the vacuum polarization and self-energy Feynman integrals[29-31].

6. The Positron Tetraquark Tetrahedron

The positrons tetraquark tetrahedrons have a positive charge of a $u\ and\ \tilde{d}$ quarks replacing the negative charge of a $\tilde{u}\ and\ d$ quarks of the electron tetraquark tetrahedrons as shown below in figures 9 (a-b) for the electrons on the left and for the positrons on the right in figures 9 (c-d). Two positron enantiomers, e_R^+ and e_L^+, may exist like the two electron tetraquark tetrahedron enantiomers e_R^- and e_L^-. In the four cases, an exchange of two quark flavors, transform the electrons, or the positrons, to a pion tetraquark tetrahedron π^{Td} conserving charge and tetraquark state.

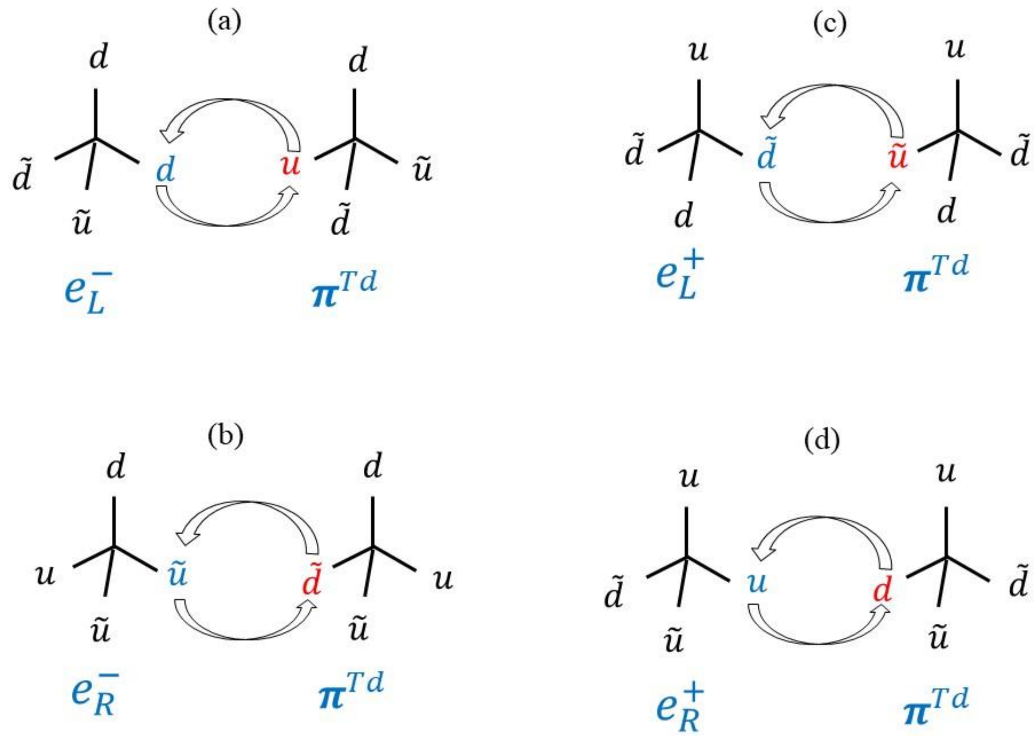


Figure 9 illustrates electron tetraquark tetrahedron enantiomers (a) and (b) and positron tetraquark tetrahedron enantiomers (c) and (d) exchanging quarks with pion tetraquark tetrahedrons with symmetric reactions such that the electrons and positrons transform to pion tetraquark tetrahedrons and vice versa conserving charge and chiral state.

7. Electron-Positron Annihilation on the Vacuum Fabric

Electron-positron tetraquarks tetrahedrons annihilation on the pion tetraquark tetrahedron fabric may occur by scattering of an electron tetraquark tetrahedron and a positron tetraquark tetrahedron on the quantum fabric forming two pion tetraquark tetrahedrons that become part of the quantum vacuum pion tetrahedron fabric as shown in equation 9.

$$\tilde{u}d\tilde{d}d(e_L^-) + u\tilde{d}\tilde{u}u(e_R^+) \rightarrow \tilde{u}du\tilde{d}(\pi^{Td}) + \tilde{d}d\tilde{u}u\,(\pi^{Td}) \qquad (9)$$

Hence if an electron tetrahedron in site i on the fabric collides with a positron on adjacent site j, the outcome is that in both i and j sites after the collision there will be pion tetraquark tetrahedrons, where the electron and positron charge and spins are annihilated. The extra energy of the electron and positron may be transferred to the vacuum pion tetraquark tetrahedron fabric as electromagnetic wave energy. Note that in equation 9 the number and flavor of the quarks are conserved. The quarks are not destroyed or created in the proposed quark exchange reactions[9-13].

8. Lattice QCD and the Quark Model of the Electron

Lattice QCD is a non-perturbative computation scheme for the strong force in low energies[32-33]. Perlovsek et al[34] write that the only hadron states found so far are two quark mesons and three quarks baryon states and that no exotic tetraquark, pentaquark, hybrid meson-gluon or molecular meson states have been confirmed beyond doubt yet. However, there are several candidates in the light tetraquarks and hidden charm sectors, and for example, Perlovsek et al studied with lattice QCD the light tetraquarks that may be the experimentally observed σ and κ light mesons.

The quark exchange reaction between two adjacent pion tetraquark tetrahedrons in lattice sites i and j may be seen as hadron scattering reactions. The d and u quarks are exchanged between the two pion tetraquarks and the scattering reaction is symmetric since the products are the same as the reactants. Note that the quark exchange reaction may flip the vacuum pion tetraquark tetrahedrons chirality and break the QCD vacuum chiral symmetry.

$$\tilde{u}d\tilde{d}u\,(\pi^{Td})_i + \tilde{u}d\tilde{d}u\,(\pi^{Td})_j \rightarrow \tilde{u}u\tilde{d}d\,(\pi^{Td})_i + \tilde{u}d\tilde{d}u\,(\pi^{Td})_j \qquad (10)$$

Equation 1 above (on page 6) describes a two tetraquarks scattering reaction where the electron tetraquark is transformed to a pion tetraquark and vice versa. The two tetraquark scattering reactions (equations 1 and 10) may be studied with lattice QCD and the mass of the pion tetraquark and the electron tetraquark may be calculated[34-35].

9. Summary

We assume that the answers to the three questions raised in section 2 above are positive and consider them as axioms of the new quark model of the electron and the vacuum fabric. Accordingly, the electron is not an elementary, point like and not a single particle. The electron tetraquark tetrahedron forms with the vacuum pion tetraquark tetrahedrons fabric a polarized cloud. The quantum vacuum has a structure formed by pion tetraquark tetrahedrons fabric with varying density. The massive pion tetraquark tetrahedrons are made of 50% matter and 50% antimatter particles, and hence the vacuum fabric is not made of regular matter particles. The electron motion occurs via u and d quark flavor exchange tunneling between electron tetraquark tetrahedrons and pion tetraquark tetrahedrons through a double well potential in a first electron tetraquark state, and the exchanges of $\tilde{u}$ and $\tilde{d}$ antiquark flavors in the second electron tetraquark state. Three conclusions derived from the proposed quark model for the electron and the vacuum fabric are: (a) Quark numbers and flavors are conserved quantities in the quark exchange reactions like the conservation of atoms and electrons in molecular reactions[9-13]; (b) Gluon dynamics may break the QCD ground state symmetry by flipping the chirality of the pion tetraquark tetrahedron enantiomers; and (c) Lattice QCD may prove the quark model for the electron and the vacuum fabric and may allow calculating the mass of the proposed pion and electron exotic tetraquarks.

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