para complexes numbers

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Abstract:

In this article, I study the numbers that I have titled: para-complex numbers.

A para- complex number consists of a real part and para- imaginary part.

Algebraic form of a para- complex number: $z = a + jb$ with $j^2 = 1$ and $j \neq \pm 1$

"J : is a pure para-imaginary."
a, is the real part; b, is the imaginary part.

In an orthonormal coordinate system, the x-axis represents the real numbers, while the y-axis represents the para-imaginary number.

This work allows me to find a large number of mathematical formulas. This is just the beginning; I hope that researchers will improve it and derive more interesting mathematical formulas that will serve science.

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Para- complexes numbers

Introduction : suppose that there exists a number which is neither real, and not complex, such, that its square is equal to 1.

Notation1 : this number is noted: j , such as: $j^2 = 1$ and $j \neq \pm 1$

j , is called para -imaginary number.

Représentation in an orthonormal reference plan :

a para -complex plan , is plan there the abscissa axis , represents the reel number , the ordinate axis the para imaginary number $\ll j \gg$

Notation2: **Algebraic form of a complex number:** $z = a + jb$, with $j^2 = 1$ and $j \neq \pm 1$, $a;b$, reals.

a, real part; b para- imaginary part.

Noticed1 : almost all the properties that relate to complex numbers are valid for para complex numbers

Notation3: **Trigonometric Form of para complex number** :

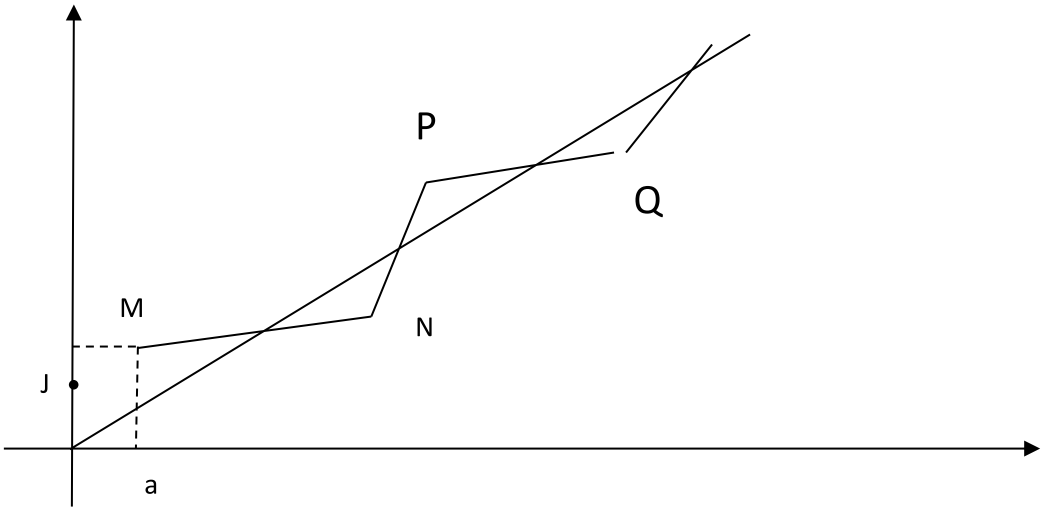
$z = r(\cos\alpha + j\sin\alpha)$; r is the module, α the argument of z

Let the first bisector of the equation $y = x$; either the point $M(a,b)$

1st case : $b \succ a$ the point M is above the first bisector.

$(a + jb)^2 = a^2 + b^2 + 2jab$; we observe that : $a^2 + b^2 \succ 2ab$

The point M image, passes below the first bisector. So on until it is on the first bisector when n increases.



N is image for M , P is image for N, Q is image for P.....etc.

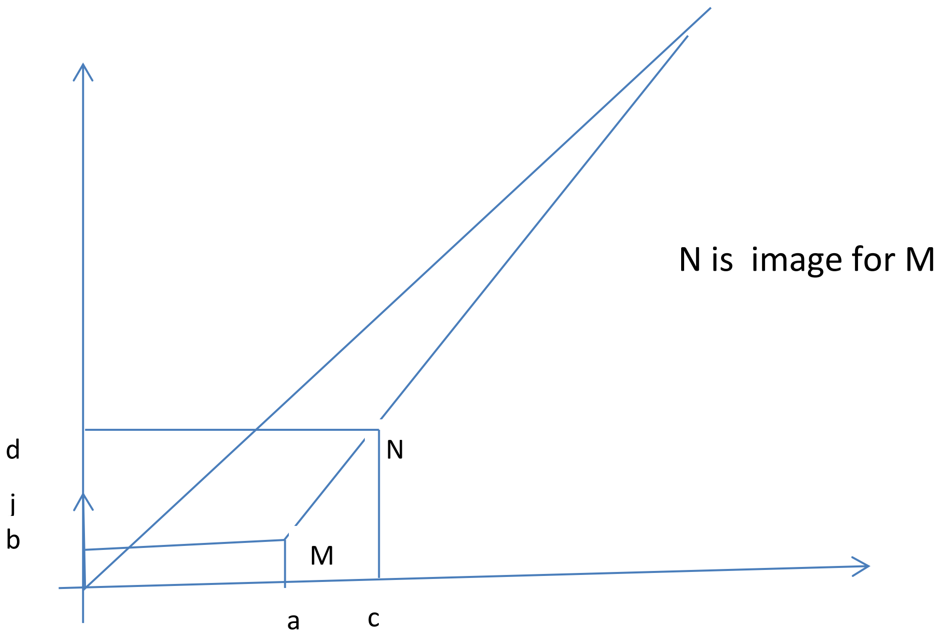
By raising to successive powers, the points images of the para complex number, take alternate position with respect to the first bisector of equation: $y = x$, until the image point is on the right: $y = x$.

$(a + jb)^{2n-1}$; $n \succ 1$, para imaginary part, greater than real part

$(a + jb)^{2n}$; real part greater than imaginary part.

<u>2nd case</u> when $b \prec a$ in this case the point M image of successive powers

Stay below the right: $y = x$



<u>**Noticed 2**</u> : $(a+jb)(1+j)=(a+b)(1+j)$

$(1+j)^n=2^{n-1}(1+j)$

<u>**Général formula**</u> **1**: $a,b\in\Re$, $j^2=1$; $j\neq\pm1$; $n\in\mathbb{N}$

$*(a+jb)^n=(b+aj)^n=\frac{1}{2}\left[(a+b)^n+(a-b)^n\right]+\frac{1}{2}J\left[(a+b)^n-(a-b)^n\right]$, n even

Application 1: $(2+3j)^4=(3+2j)^4=313+312j$

By applying the formula 1:

$$(2+3j)^4=\frac{1}{2}\left[\left(2+3\right)^4+(2-3)^4\right]+\frac{1}{2}\left[\left(2+3\right)^4-(2-3)^4\right]=$$
$$\frac{1}{2}(625+1)+\frac{1}{2}(625-1)=313+312j$$

$*(a+jb)^n=j(b+aj)^n=\frac{1}{2}\left[(a+b)^n+(a-b)^n\right]+\frac{1}{2}J\left[(a+b)^n-(a-b)^n\right]$,
n odd number

Application 2:

$$(3-2j)^3=j(-2+3j)^3=$$
$$=\frac{1}{2}\left[(3-2)^3+(3+2)^3\right]+\frac{1}{2}J\left[(3-2)^3-(3+2)^3\right]=-62+63j$$

Square root of a Para complex number.

<u>**Examples :**</u>

$\sqrt{5+4j}=a+jb$

$a^2+b^2+2jab=5+4j$ by identification : $a+b=\pm3$; $a-b=\pm1$

$$\sqrt{5+4j}=2+j$$
$$\sqrt{5+4j}=-2-j$$
$$\sqrt{5+4j}=1+2j$$
$$\sqrt{5+4j}=-1-2j$$

$$\sqrt[3]{28+36j}=\frac{1}{2}\left[\sqrt[3]{64}+\sqrt[3]{-8}\right]+\frac{1}{2}j\left[\sqrt[3]{64}-\sqrt[3]{-8}\right]=1+3j$$

Application for the trigonometric form : i **:** a pure imaginary number

$$\left[(\cos x+i\sin x)+j(\sin x-i\cos x)\right]^n=\left[(\cos x+i\sin x)(1+\frac{j}{i})\right]^n=$$

$$=(\cos x+i\sin x)^n(1+\frac{j}{i})^n$$, According to the general formula. (1)

$$\left[(\cos x+i\sin x)+j(\sin x-i\cos x)\right]^n=$$

$$\frac{1}{2}(\cos x+\sin x)^n(1-i)^n+\frac{1}{2}j(\cos x-\sin x)^n(1+i)^n$$, because

$$(\cos x+\sin x)+i(\sin x-\cos x)=(\cos x+i\sin x)(1-i)$$
$$(\cos x+\sin x)-i(\sin x-\cos x)=(\cos x-i\sin x)(1+i)$$

We set
$$\cos x+\sin x=\sqrt{2}\cos x$$
$$\cos x-\sin x=\sqrt{2}\sin(\frac{\pi}{4}-x)$$

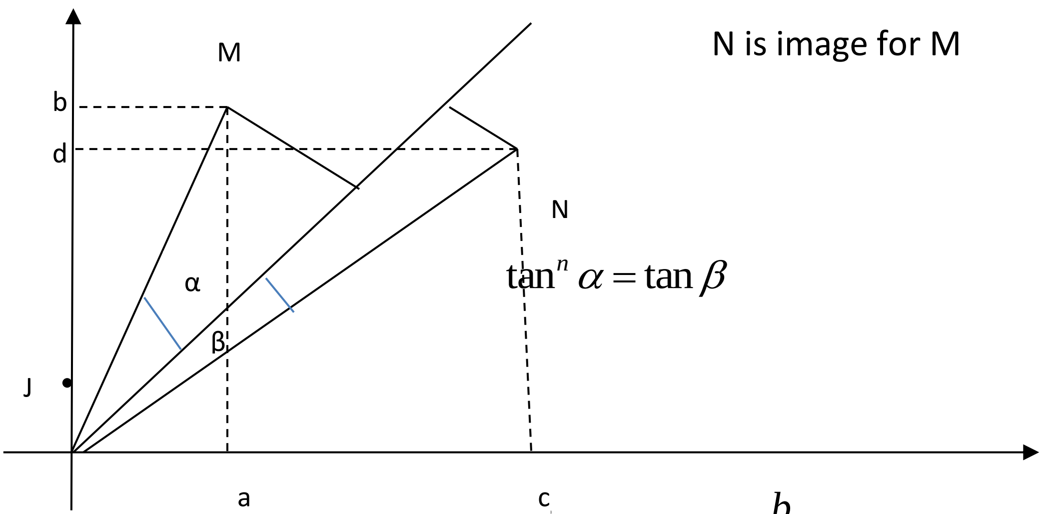
$$\left[(\cos x+i\sin x)+j(\sin x-i\cos x)\right]^n=$$
$$\frac{(\sqrt{2})^n}{2}(\cos x)^n(1-i)^n+\frac{(\sqrt{2})^n}{2}j(\sin(\frac{\pi}{4}-x))^n(1+i)^n=(\cos x+i\sin x)^n(1+\frac{j}{i})^n.$$

With:
$$(i+j)^{2n}=(2ij)^n$$
$$(i+j)^{2n+1}=(2ij)^n(i+j),\quad n\geq 0$$

<u>trigonométri Formula</u> : if $0 \prec \theta \prec \frac{\pi}{2}$, A, C reals or complexes

$$\frac{\cos n\theta}{\sqrt{2}}\sqrt{A^2+C^2+2AC\sin 2n\theta+(A^2-C^2)\cos 2n\theta}-$$
$$\frac{\sin n\theta}{\sqrt{2}}\sqrt{A^2+C^2-2AC\sin 2n\theta-(A^2-C^2)\cos 2n\theta}=A$$

$$\frac{\sin n\theta}{\sqrt{2}}\sqrt{A^2+C^2+2AC\sin 2n\theta+(A^2-C^2)\cos 2n\theta}-$$
$$\frac{\cos n\theta}{\sqrt{2}}\sqrt{A^2+C^2-2AC\sin 2n\theta-(A^2-C^2)\cos 2n\theta}=C$$



Let's us now, take the case where: $j^2 = \frac{a^2}{b^2}$ as such $y = \frac{b}{a}x$ that is to say the image of point M, alternate on either side of the equation line : $y = \frac{b}{a}x$

$$(\cos\theta + j\sin\theta)^2 = \cos^2\theta + \frac{a^2}{b^2}\sin^2\theta + 2j\sin\theta\cos\theta$$
$$\tan\alpha = \frac{b\cos\theta - a\sin\theta}{b\sin\theta + a\cos\theta}$$
$$\tan\beta = \frac{b^3\cos^2\theta + a^2 b\sin^2\theta - ab\sin 2\theta}{b^3\sin 2\theta + a(b^2\cos^2\theta + a^2\sin^2\theta)}$$

In this case: $\tan^n \alpha = g \tan \beta$, g real.

Noticed : θ is the argument of para- complex number. $\arg(z) = \theta$

Generalisation: $j^2 = \dfrac{a^2}{b^2}$

j , is called pseudo imaginary of first kind

$J_1^2 = j$, is called pseudo imaginary of second kind

$J_2^2 = j_1$, is called pseudo imaginary of third kind

$J_3^2 = j_2$, is called pseudo imaginary of fourth kind .

A**pplication** :

$$(1 + jj_1)^3 = (1 + 3j)(1 + j_1)$$
$$(j + j_1)^3 = (3 + j)(1 + j_1)$$
$$(j - j_1)^3 = (3 + j)(1 - j_1)$$

Other case : Z para complex à three dimensionel. $z = a + bj + ck$

$j^2 = uj + vjk$, $(u,v) \in IR$, $n \in IN$, $(A,B) \in IR$

k para imaginary such as $k^2 = 1$; $k \neq 1$

$$(Aj + Bjk)^n = \frac{1}{2} j\left[(u+v)^{n-1}(A+B)^n + (u-v)^{n-1}(A-B)^n\right] + \frac{1}{2}kj\left[(u+v)^{n-1}(A+B)^n - (u-v)^{n-1}(A-B)^n\right]$$

Applications :

Example 1 : $A = 1, B = 1, u = 1, v = 1, n = 2$, $j^2 = j + jk$

$$(j+jk)^2=\frac{1}{2}j\left[(2)^1(2)^2\right]+\frac{1}{2}kj\left[(2)^1(2)^2\right]$$
$$j^2(1+k)^2=4j+4kj$$
$$(j+jk)(1+k)^2=4j+4jk$$
$$(j+jk)(1+k^2+2k)=4j+4jk$$
$$(j+jk)(2+2k)=4j+4jk$$
$$2j(1+k^2+2k)=4j+4jk$$
$$4j(1+k)=4j+4jk$$

Example 2 : $A=1,B=2,u=1,v=-1,n=2$, then : $j^2=j-jk$

$$(j+2jk)^2=\frac{1}{2}j2(-1)^2-\frac{1}{2}kj2(-1)^2$$
$$(j-jk)(1+2k)^2=j-jk$$
$$j(1-k)(5+4k)=j-jk$$
$$j(5+4k-5k-4k^2)=j-jk$$
$$j(1-k)=j-jk$$

Example 3 : $A=1,B=2,u=2,v=-1,n=3$, $j^2=2j-jk$

$$(j+2jk)^3=\frac{1}{2}j\left[(1)^2(3)^3\right]+\frac{1}{2}kj\left[(2)^2(-1)^3\right]$$
$$j^3(1+2k)^3=9j+18kj$$
$$(2j-jk)(2-k)(1+2k)^3=9j+18jk$$
$$j(5-4k)(13+14k)=9j+18jk$$
$$j(65-56+18k)=9j+18jk$$
$$9j+18jk=9j+18jk$$

Example 4 : $A=4;B=-2;u=2;v=-1;n=4$

$$j^2=uj+vjk\Rightarrow j^2=2j-jk$$

$$(j-2jk)^2 = \frac{1}{2}j\left[(1)^3(-1)^4+(3)^3(3)^4\right]+\frac{1}{2}jk\left[(1)^3(-1)^4-(3)^3(3)^4\right]$$

$$(j-2jk)^4 = j^2(1-2k)^4 = (2j-jk)^2(1-2k)^4 = j^2(2-k)^2(1-2k)^4 =$$
$$(2j-jk)(2-k)^2(1-2k)^4 = j(2-k)^3(1-2k)^4 = j(1094-1093k)$$

$$= \frac{1}{2}j\left[1+3^7\right]+\frac{1}{2}jk\left[1-37\right] = j(1094-1093k)$$