

Potential and Entropy Defined By Caratheodory Axiomatization, Relative Rate of Change of Energy

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Abstract

At the beginning of this paper, by the generalized wave equation of electric field and the solution of equation. We apply Caratheodory's axiomatization to define potential function and entropy function of charged particles of the system. After introducing the relative rate of change of energy, their corresponding entropy function are defined by the potential energy and the quantum energy.

Key words: Caratheodory axiomatization of electric field, partition function and entropy, probability distribution.

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1 Potential and Entropy Defined By Caratheodory Axiom

First of all, we will apply Caratheodory axiomatization to define the potential function and entropy function of a charged system, let us only pay attention to the electric field wave equation of the electromagnetic field generalized wave equations of this pair [1, 2], and get its solution at the same time, whose are as follows, respectively

$$c^2 \frac{\partial^2 \mathbb{E}}{\partial t^2} + \frac{C}{c^2} \cdot \frac{\nabla \partial \mathbb{E}}{\partial t} - \nabla^2 \mathbb{E} = 0, \quad (1)$$

$$\mathbb{E} = \mathbb{E}_0 \exp[-\lambda k \cdot r(1+i)]. \quad (2)$$

$\exp[\lambda \omega t(1+i)]$ (λ is constant)

We get the average value of this electric field over a long period of time

$$\begin{aligned} \langle \mathbb{E}(r) \rangle_t &= \int_0^\infty \mathbb{E}(r, t) / t dt \\ &= \pi/2 \mathbb{E}_0 \exp[-\lambda k \cdot r(1+i)]. \end{aligned} \quad (3)$$

When this charged particle is coupled with a charged system, the electric field would also show wave numbers k changes. If the distance r between the particle and the system is a constant. From Eq.(3), the particle would show system-dependent potential as indicated below

$$\begin{aligned} \varphi &= \int_0^\infty \pi/2 \mathbb{E}_0 \exp[-\lambda k \cdot r(1+i)] \cdot dk \\ &= (-1+i)\pi/(4\lambda) |\mathbb{E}_0| r^{-1}, \end{aligned} \quad (4)$$

in which the first term denotes the field-generated static potential. Compared with the Coulomb's potential $\varphi_c = -q_0 r^{-1}$ [3, 4], we have

$$q_0 = \pi/(4\lambda) |\mathbb{E}_0|, \quad (5)$$

$$\varphi_c = -q_0 r^{-1}. \quad (6)$$

The Eq.(4) is an periodic potential. Hence we have Coulomb's potential

$$\varphi = -q_0 r^{-1} + i q_0 r^{-1} = (-1+i)\varphi_c, \quad (7)$$

If the system's electric charge $Q = Nq$ and potential $\Phi(r, k)$, the $\Phi(r, k)$

is a potential function of spatial position r and wave number k . We make electric field in instable state, $\mathbb{E}_k = \partial\Phi/\partial k$, force $\mathbb{F}_k = Q \mathbb{E}_k = Q \partial\Phi/\partial k$, then the energy of the particle becomes

$$\delta\mathcal{E} = Q \partial\Phi/\partial k \cdot dk. \quad (8)$$

From now on, we apply Caratheodory axiomatization to a system of charged particles [5], to showed that the potential function and entropy function.

By rewriting Eq.(8) into its component form, we get

$$\delta\mathcal{E} = \sum_j^n Q \mathbb{E}_{k_j} \cdot dk_j. \quad (9)$$

Equation (9) must be multiplied by a function $1/\gamma$ to make it an exact differential, from Eq.(9) [6] we obtain

$$\frac{1}{\gamma} \delta\mathcal{E} = \frac{1}{\gamma} \sum_j^n Q \mathbb{E}_{k_j} \cdot dk_j, \quad (10)$$

in Eq.(10) we get following solution

$$F = c \quad (c \text{ is constant}), \quad (11)$$

make $dF = 0$, from Eq.(10) then become

$$dF = \frac{1}{\gamma} \delta\mathcal{E} = \frac{1}{\gamma} \sum_{j=1}^n Q \mathbb{E}_{k_j} \cdot dk_j. \quad (12)$$

Take $n = 3$ for the spatial component, made $dF = 0$, Eq.(11) and Eq.(12) becomes

$$dF = \frac{1}{\gamma} (Q \mathbb{E}_{k_1} \cdot dk_1 + Q \mathbb{E}_{k_2} \cdot dk_2 + Q \mathbb{E}_{k_3} \cdot dk_3) = 0. \quad (13)$$

and

$$dF = \partial F/\partial k_1 \cdot dk_1 + \partial F/\partial k_2 \cdot dk_2 + \partial F/\partial k_3 \cdot dk_3 = 0, \quad (14)$$

the dF is a exact differential, let's compare Eq.(13) with Eq.(14) to get the following equations

$$\begin{aligned} (\partial F/\partial k_1)/Q \mathbb{E}_{k_1} &= (\partial F/\partial k_2)/Q \mathbb{E}_{k_2} = \\ (\partial F/\partial k_3)/Q \mathbb{E}_{k_3} &= 1/\gamma. \end{aligned} \quad (15)$$

For this case in which exist two independent systems, let's say system 1 with energy \mathcal{E}_1 and system 2 with energy \mathcal{E}_2 to make their exact differential, multiply by $1/\gamma_1$ and $1/\gamma_2$, respectively, which are as follows

$$dF_1 = \frac{1}{\gamma_1} \delta\mathcal{E}_1, \quad dF_2 = \frac{1}{\gamma_2} \delta\mathcal{E}_2. \quad (16)$$

Let's merge the two systems into one, their coupled potentials should be equal, when the two systems reach the equilibrium state of the potential. In the new system formed by the afore-listed two we have

$$dF = \frac{1}{\gamma} \delta \mathcal{E}. \quad (17)$$

Let τ be a variable of systems 1 and 2 in the equilibrium process, we get the following from Eq.(16) and Eq.(17) from energy conservation

$$dF = \frac{\delta \mathcal{E}}{\gamma} = \frac{\delta \mathcal{E}_1 + \delta \mathcal{E}_2}{\gamma} = \frac{\gamma_1}{\gamma} dF_1 + \frac{\gamma_2}{\gamma} dF_2, \quad (18)$$

and

$$dF = \partial F / \partial F_1 dF_1 + \partial F / \partial F_2 dF_2 + \partial F / \partial \tau d\tau, \quad (19)$$

comparing Eq.(18) with Eq.(19), we get

$$\begin{aligned} \partial F / \partial F_1 &= \gamma_1 / \gamma, \quad \partial F / \partial F_2 = \gamma_2 / \gamma \\ \text{and } \partial F / \partial \tau &= 0. \end{aligned} \quad (20)$$

It would not be difficult to see from Eq.(20) that F is irrelevant with a parameter τ during system 1 and system 2 merging into one system, by taking $\partial F / \partial \tau = 0$ or in other words these proportions $\partial(\gamma_l / \gamma) / \partial \tau = 0$

($l = 1, 2$), namely,

$$\partial(\gamma_1 / \gamma) / \partial \tau = \partial(\gamma_2 / \gamma) / \partial \tau = 0, \quad (21)$$

from Eq.(21) we have

$$\begin{aligned} \partial(\ln \gamma_1) / \partial \tau &= \partial(\ln \gamma_2) / \partial \tau = \\ \partial(\ln \gamma) / \partial \tau &= L(\tau), \end{aligned} \quad (22)$$

the $L(\tau)$ is their a common function of system, consider Eq.(22). We draw into a function $M(F_l)$ of the variable F_l from Eq.(20), by taking $\partial F / \partial \tau = 0$ and $\partial F_l / \partial \tau = 0$ ($l = 1, 2$). So we could write this as

$$\ln \gamma_l = \int L(\tau) d\tau + \ln M(F_l), \quad (23)$$

or

$$\gamma_l = M(F_l) \exp\left(\int L(\tau) d\tau\right) \quad (l = 1, 2). \quad (24)$$

Now we may define the electric charged system's potential function $\Phi(\tau)$

$$\Phi(\tau) = C \exp\left(\int L(\tau)d\tau\right) \quad (25)$$

(C is constant),

and entropy function S

$$S_l = \frac{1}{C} \int M(F_l)dF_l \quad (26)$$

$$dS_l = \frac{1}{C} M(F_l)dF_l \quad (l = 1, 2),$$

based on the law of energy conservation $\delta\mathcal{E} = \delta\mathcal{E}_1 + \delta\mathcal{E}_2 = \gamma_1 dF_1 + \gamma_2 dF_2$,

formulas Eq.(17), Eq.(24),

Eq.(25) and Eq.(26), we have

$$\delta\mathcal{E} = \sum_{l=1}^2 dS_l \Phi(\tau) = \sum_{l=1}^2 \frac{1}{C} M(F_l) dF_l \cdot \left[C \exp\left(\int L(\tau)d\tau\right) \right], \quad (27)$$

or the system's entropy

$$dS = \delta\mathcal{E}/\Phi. \quad (28)$$

By now, we may illustrate the order motion of the charged particles by using potential Φ , or by using the system's entropy S . In statistical mechanics, the thermodynamical motion

of particles should satisfy the Boltzmann equation, the disorder motions of particles could be represented by using entropy function $S = k_B \ln Z$ [7] If temperature T and potential Φ simultaneously exist in the same system, these particles would move respectively in both states of disorder thermodynamical motion and order potential motion.

2 Relative Change Rate of Energy

Now we assume that the system of a large number of particles move between two potential environments, in which the high-potential Φ_h with energy $Q\Phi_h$ and low-potential Φ_l with energy $Q\Phi_l$. System absorb energy $\mathcal{E}_h = Q\Phi_h$ from the high-potential environment state, and release energy $\mathcal{E}_l = Q\Phi_l$ to the low-potential environment state, where Q is the total charge of the system. Thus energy of

the system change as follows

$$\Delta\mathcal{E} = \mathcal{E}_h - \mathcal{E}_l = Q\Phi_h - Q\Phi_l. \quad (29)$$

Let's draw into relative change rate of energy of the system, we get

$$\eta = \frac{\mathcal{E}_h - \mathcal{E}_l}{\mathcal{E}_h} = 1 - \frac{\mathcal{E}_l}{\mathcal{E}_h}. \quad (30)$$

Take the second term of Eq.(30), meanwhile substitute into the energy $\mathcal{E} = Q\Phi$, thus

$$\frac{\mathcal{E}_l}{\mathcal{E}_h} = \frac{Q\Phi_l}{Q\Phi_h}, \quad (31)$$

after rearranging out above Eq.(31), becomes

$$\frac{\mathcal{E}_h}{\Phi_h} = \frac{\mathcal{E}_l}{\Phi_l}, \quad (32)$$

the exact differential is obtained from equation (32), we can define the entropy function of the system and calculus for the line integral along the closed curve, which is as follows

$$ds = \frac{\delta\mathcal{E}}{\Phi}, \oint ds = \frac{\mathcal{E}_h}{\Phi_h} - \frac{\mathcal{E}_l}{\Phi_l} = 0. \quad (33)$$

In addition to above entropy function of the system, if a particle carries energy $\mathcal{E} = \hbar\omega$, where the ω is angular frequency, the \hbar is Planck constant.

It still satisfies the energy change $\Delta\mathcal{E} = \mathcal{E}_h - \mathcal{E}_l$. So take the second term of Eq.(30), then we have

$$\frac{\mathcal{E}_h}{\mathcal{E}_l} = \frac{\hbar\omega_h}{\hbar\omega_l}, \quad (34)$$

when rearranged Eq.(34) becomes

$$\frac{\mathcal{E}_h}{\omega_h} = \frac{\mathcal{E}_l}{\omega_l}. \quad (35)$$

Similarly hereafter, we get the exact differential and the entropy function of the system, which is

$$ds = \frac{\delta\mathcal{E}}{\omega}, \oint ds = \frac{\mathcal{E}_h}{\omega_h} - \frac{\mathcal{E}_l}{\omega_l} = 0. \quad (36)$$

Thus we get two types of exact differential and entropy functions of system, respectively.

3 Partition, Entropy of normalized probability density ρ

In the phase space of the system, we assume that there is normalized probability density ρ , expected value \mathcal{E} of energy and entropy function S , which

are as follows, respectively

$$1 = \int_0^\infty dx^N \rho, \quad \mathcal{E} = \int_0^\infty dx^N \rho H$$

and $S = q \ln \rho = q \int_0^\infty dx^N \rho \ln \rho.$

(37) Substitute Eq.(43) into Eq.(39), so

By calculating the variation with Lagrangian multipliers α and μ_ρ , the

Eq.(37) become

$$\int_0^\infty [\alpha H + \mu_\rho - (q + q \ln \rho)] dx^N \delta \rho = 0, \quad (38)$$

therefore, we get

$$\mu_\rho - q + \alpha H - q \ln \rho = 0. \quad (39)$$

Adjust Eq.(39), then the above formula becomes

$$\rho = \exp[(\mu_\rho - q)/q] \exp(\alpha H/q). \quad (40)$$

The integral Eq.(40) is given

$$\exp[(q - \mu_\rho)/q] = \int_0^\infty \exp(\alpha H/q) dx^N. \quad (41)$$

By Eq.(41), we define the partition function of the system

$$Z = \exp[(q - \mu_\rho)/q]. \quad (42)$$

Let's take the logarithm of Eq.(42),

then Eq.(41) becomes

$$\mu_\rho - q = -q \ln Z. \quad (43)$$

$$-q \ln Z + \alpha H - q \ln \rho = 0. \quad (44)$$

Above equation is multiplied by potential function Φ , the energy relation of the system is obtained

$$-q\Phi \ln Z + \alpha\Phi H - q\Phi \ln \rho = 0. \quad (45)$$

In the system with a large number of charged particles there is free energy F , internal energy U and potential energy ΦS , they has energy relation as follows

$$F + U - \Phi S = 0. \quad (46)$$

Let's compare Eq.(45) with Eq.(46), we have

$$F = -qN\Phi \ln Z \quad \text{and} \quad \alpha = 1/\Phi. \quad (47)$$

In Eq.(33), when the energy is divided by the potential function, we defined

the entropy function of the particle, which is as following formula

$$S = \frac{F}{N\Phi} = -q \ln Z. \quad (48)$$

From now on, let us consider another type of energy $\mathcal{E} = \hbar\omega$, again starting with the normalized probability density of the system, the expected value of the energy and the entropy function, which are as respectively

$$\begin{aligned} 1 &= \int_0^\infty dx^N \rho, & \mathcal{E} &= \int_0^\infty dx^N \rho H \\ \text{and } S &= \hbar \ln \rho = \int_0^\infty dx^N \hbar \rho \ln \rho. \end{aligned} \quad (49)$$

Similarly, the following is obtained from Eq.(49), by calculating the variation and multiplying with Lagrangian multipliers α' and μ_ρ , there are

$$\int_0^\infty [\alpha' H + \mu_\rho - (\hbar + \hbar \ln \rho)] dx^N \delta \rho = 0. \quad (50)$$

Similarly to the calculation process of Eq.(38) and Eq.(39), we re-

arrange Eq.(50) to get its probability density

$$\rho = \exp[(\mu_\rho - \hbar)/\hbar] \exp(\alpha' H/\hbar), \quad (51)$$

then we integral Eq.(51), namely

$$\exp[(\hbar - \mu_\rho)/\hbar] = \int_0^\infty \exp(\alpha' H/\hbar) dx^N. \quad (52)$$

From this, we can define the partition function of the particle as

$$\begin{aligned} Z &= \exp[(\hbar - \mu_\rho)/\hbar] \text{ or } \mu_\rho - \hbar = \\ &= \hbar \ln Z, \end{aligned} \quad (53)$$

from Eq.(50) and Eq.(53), and rearrange, we get

$$-\hbar \ln Z + \alpha' H - \hbar \ln \rho = 0. \quad (54)$$

If we multiply the angular frequency ω by equation above, then the energy of the particle is equal to

$$-\hbar \omega \ln Z + \alpha' \omega H - \hbar \omega \ln \rho = 0. \quad (55)$$

Similarly, there is the following energy relation of the particle, where the U is internal energy, the F is free energy, the other ωS is the quantum energy, which is as follows

$$F + U - \omega S = 0. \quad (56)$$

By Eq.(36) we assume that the energy relation still be satisfied in the system of this particle. Compare Eq.(55) with Eq.(56), can yet define entropy function. In addition the Lagrangian multiplier of the energy is obtained, which is as follows

$$S = \frac{F}{\omega} = -\hbar \ln Z \quad \text{and} \quad \alpha' = 1/\omega. \quad (57)$$

So far we defined two kinds of partition functions and entropy functions of the particles, respectively, in terms of the probability.

4 Lagrangian Multiplier $1/q\Phi$ $1/\hbar\omega$ the Partition Function and Entropy Function

If a system is composed of a large number of countable particles, we classify the particles so that each part has N_i particles, and assume that in N_i per each of particles has an energy $p_i c$. Total energy of the system is \mathcal{E} , which are as follows

$$N = \sum_i N_i, \quad \mathcal{E} = \sum_i N_i p_i c, \quad (58)$$

we subdivide the phase space Ω of the system into ν units. In per unit volume, where the ν has N_i particles, and set its probability g_i . Then probability of the system is

$$W = \prod_i \frac{N!}{N_i!} g_i^{N_i}. \quad (59)$$

Applying the Stirling's approximate

formula $\ln N! \simeq (N + 1/2) \ln N - N + 1/2 \ln(2\pi)$ [8], when $N_i \gg 1$ by Eq.(59) of particles number is becomes

$$\exp(\mu) = N/Z. \quad (65)$$

$\delta \ln W = \sum_i (\ln g_i - \ln N_i) \delta N_i = 0.$ From this, we rewrite the number of particles as

$$(60)$$

By Lagrangian multipliers α_E and μ , from Eq.(58) and Eq.(60), with calculating its variation, which is as following

$$\sum_i (\alpha_E p_i c - \mu - \ln g_i / N_i) \delta N_i = 0, \quad (61)$$

$$N_i = g_i \exp(-\alpha_E p_i c) \exp(\mu), \quad (62)$$

hence, we have

$$N = \sum_i N_i = \exp(\mu) \sum_i g_i \exp(-\alpha_E p_i c). \quad (63)$$

By the Eq.(63) let us define the partition function Z [9], to get

$$Z = \sum_i g_i \exp(-\alpha_E p_i c). \quad (64)$$

As thus, the Lagrangian multiplier

$$N_i = N/Z g_i \exp(-\alpha_E p_i c). \quad (66)$$

Now the above equation of the differential form can be written from Eq.(66), as follows

$$N_i dp_i = N/Z g_i \exp(-\alpha_E p_i c) dp_i. \quad (67)$$

And we had known

$$N/Z g_i \exp(-\alpha_E p_i c) dp_i = d[p_i N/Z g_i \exp(-\alpha_E p_i c)] + N/Z \alpha_E p_i c g_i \exp(-\alpha_E p_i c) dp_i. \quad (68)$$

Let us compute the integral on the left-hand side of Eq.(68) to obtain the total number of the particles of the system as

$$N = \int N_i dp_i = \int N \frac{1}{Z} g_i \exp(-\alpha_E p_i c) dp_i. \quad (69)$$

The integral of the equation (69) can be written as the following is given

$$N = \int \frac{-1}{\alpha_E c} N \frac{\partial \ln Z}{\partial p_i} dp_i = \frac{-1}{\alpha_E c} N \ln Z + a. \quad (\text{let constant } a=0) \quad (70)$$

Now for let us compute the integral to the right-hand side of Eq.(68), the expected value of the energy pc is obtained , we get

$$N \alpha_E \langle pc \rangle = N \alpha_E \int_0^\infty p c \frac{1}{Z} g_i \exp(-\alpha_E pc) dp. \quad (71)$$

According to Eq.(69) and Eq.(71), we can get the relation between the total number of particles of the system, and the expected value of the energy pc , as show below

$$N = N \alpha_E \langle pc \rangle. \quad (72)$$

Owing to total electric charge of the system is $Q = Nq$, we have the potential energy $\mathcal{E} = Q\Phi = Nq\Phi$ [10], thus the average energy of individual particles becomes

$$\langle \epsilon \rangle = \mathcal{E}/N = q\Phi, \quad (73)$$

for the same particle, we think of that if particle energy ϵ meet the dual forms $\epsilon = pc = q\Phi$. then average energy $\langle \epsilon \rangle = \langle pc \rangle$, would be

$$\langle \epsilon \rangle = \langle pc \rangle = q\Phi. \quad (74)$$

Therefore formula Eq.(72) becomes

$$\alpha_E = \frac{1}{q\Phi}. \quad (75)$$

By now, the $\alpha_E = 1/(q\Phi)$ is Lagrangian multiplier of the energy, the Eq.(64) and Eq.(66) may be rewritten so that number of N_i particles and the partition function Z as become

$$Z = \sum_i g_i \exp\left(-\frac{p_i c}{q\Phi}\right), \quad (76)$$

$$N_i = \frac{N}{Z} g_i \exp\left(-\frac{p_i c}{q\Phi}\right). \quad (77)$$

This is the Lagrangian multiplier $\alpha_E = 1/(q\Phi)$ of the number of the charged particles in the statistical sense, which will play a vitally important role in the forthcoming discussion. By Eq.(70) we get

$$N = \frac{-1}{\alpha_E c} N \ln Z = N q\Phi \frac{-1}{c} \ln Z. \quad (78)$$

From Eq.(33) the energy \mathcal{E} divided by the potential function Φ , then the entropy function of the particle by Eq.(78) as

$$S = \frac{\mathcal{E}}{\Phi} = q \frac{-1}{c} \ln Z. \quad (79)$$

In Eq.(64), when the energy pc is replaced by energy $\hbar\omega$, so the partition function of the system becomes

$$Z = \sum_i g_i \exp\left(-\frac{\hbar\omega_i}{q\Phi}\right). \quad (80)$$

Similarly to Eq.(71), the expected value of energy $\hbar\omega$ can be obtained from the following equation

$$N = N \alpha_q \langle \hbar\omega \rangle = N \alpha_q \int_0^\infty \hbar\omega \frac{1}{Z} g_i \exp(-\alpha_E \hbar\omega) d\omega, \quad (81)$$

by Eq.(81) to get

$$\alpha_q = \frac{1}{\langle \hbar\omega \rangle}. \quad (82)$$

Thus the entropy function of the particle may be obtained from Eq.(70).

By Eq.(74) dual energy relation let $q\Phi = pc = \hbar\omega$, thus set $\alpha_E = \alpha_q$, which the energy is the following equation namely

$$\frac{-1}{\alpha_q c} N \ln Z = N \langle \hbar\omega \rangle \frac{-1}{c} \ln Z. \quad (83)$$

In the Eq.(36) we have defined the exact differential or quantum entropy, where divide the energy $\epsilon = \hbar\omega$ by the angular frequency ω . Then the entropy function of the particle from Eq.(83) is

$$S = \frac{\mathcal{E}}{N \langle \omega \rangle} = \hbar \frac{-1}{c} \ln Z. \quad (84)$$

So far, we derived Lagrangian multipliers of energy, as well as two classes of partition functions and their corresponding entropy functions.

5 Distributions, Partitions and Critical States of Countable Particles of a System

We assume that there is an i -th particle coupled with a large number of other particles, which potential energy $q\varphi_i$ satisfies the dual energy relation $q\varphi_i = p_i c$. If the N_i countable particles are distributed in a G_i cell of the phase space. Thus the first particle has G_i distribution pattern, the second particle has $G_i - a$ distribution pattern, and so on, with the final term is $G_i - (N_i - a)$, where a is constant. Let's suppose that the g_i is probability of these N_i particles, so the total number of the particles, whole energy and probability of the system are as follows, respectively

$$N = \sum_i N_i, \quad \mathcal{E} = \sum_i N_i p_i c, \quad (85)$$

by the L.Brillouin statistical methods [11], we get

$$\begin{aligned} W &= \prod_i \frac{[G_i(G_i - a) \dots (G_i - (N_i - 1)a)]}{N_i!} g_i^{N_i} \\ &= \prod_i \frac{G_i!}{N_i!(G_i - N_i a)!} g_i^{N_i}. \end{aligned} \quad (86)$$

From expanding the above probabilities formula, according to Stirling's approximation formula [12, 13], and calculate the variation of these equations, apply the Lagrangian multipliers α_E and μ , respectively, we have

$$\sum_i [(\ln \frac{G_i}{N_i} - a) + \ln g_i + \alpha_E p_i c - \mu] \delta N_i = 0. \quad (87)$$

Let us rearrange the Eq.(87), so that the number of particles is as follows

$$N_i = G_i g_i / [\exp(-\alpha_E p_i c + \mu) + a g_i] \quad Z = \sum_i G_i g_i \exp(-\alpha_E p_i c). \quad (92)$$

$$(a = \pm 1). \quad (88)$$

For ease of calculation, let's say $x = \exp(-\alpha_E p_i c + \mu)$ and expand the generating function to get

$$N_i = \frac{G_i}{a} - \frac{1}{a^2 g_i^2} G_i g_i x. \quad (89)$$

From this, the phase space Ω and the total number of particles of the system are obtained as follows

$$\Omega = \sum_i \frac{G_i}{a}$$

$$\text{and } N = \sum_i a^2 g_i^2 N_i = \sum_i G_i g_i x. \quad (90)$$

by Eq.(90) the second term, let's rewrite the total number of the particles in the system to get

$$N = \sum_i G_i g_i \exp(-\alpha_E p_i c) \exp(\mu). \quad (91)$$

Now we can define the partition function of the system as

By Eq.(91) and Eq.(92), thus we get the Lagrangian multiplier of number of particles

$$\exp(\mu) = \frac{N}{Z}. \quad (93)$$

From Eq.(70) and Eq.(75), the relation between partition function and the energy as

$$\frac{-1}{\alpha_E c} \ln Z = q \Phi \frac{-1}{c} \ln Z \quad (\alpha_E = \frac{1}{q \Phi}). \quad (94)$$

In a countable particles system, let the energy is divided by the potential function, we introduce into the entropy function from Eq.(33) and Eq.(94), to get

$$S = \frac{\mathcal{E}}{\Phi} = q \frac{-1}{c} \ln Z. \quad (95)$$

However, when the particle has dual relation of energy $p_i c = \hbar \omega_i$, by Eq.(92) the partition function of the system

can be written as

$$Z = \sum_i G_i g_i \exp(-\alpha_E \hbar \omega). \quad (96)$$

The i -th particle of the G_i cell of phase space carry the energy $p_i c$. This particle concurrent also has thermodynamic kinetic energy $p_i^2/(2m)$ in temperature T , we get

$$\begin{aligned} \mathcal{E}_1 &= \sum_i N_i p_i^2/(2m), \mathcal{E}_2 = \sum_i N_i p_i c \\ N &= \sum_i N_i. \end{aligned} \quad (97)$$

The following probabilities are the identical as in Eq.(86), if the N_i particles distribute in G_i cell of the phase space. Therefore, N_i particles would have the probability W as below

$$\begin{aligned} W &= \prod_i \frac{[G_i(G_i - a) \dots (G_i - (N_i - 1)a)]}{N_i!} g_i^{N_i} \\ &= \prod_i \frac{G_i!}{N_i!(G_i - N_i a)!} g_i^{N_i}. \end{aligned} \quad (98)$$

In the same way, according to Stirling's approximation formula, calculate the variation of these equations

and with the Lagrangian multipliers

β , α_E and μ , respectively, we have

$$\begin{aligned} \sum_i [(\ln \frac{G_i}{N_i} - a) + \ln g_i - \beta \frac{p_i^2}{(2m)} + \\ \alpha_E p_i c - \mu] \delta N_i = 0, \end{aligned} \quad (99)$$

$$\text{set } \beta = 1/(kT), \alpha_E = 1/(q\Phi).$$

We know that the thermal motion of these particles is disordered motion, thus take the Lagrangian multiplier to get a negative sign $\beta = -1/kT$.

Whereas for the motion of these particles in the potential field is an ordered motion, thus we should take plus sign the $\alpha_E = +1/q\Phi$. By Eq.(99) we make a rearrangement as

$$N_i = G_i g_i / [\exp(\frac{p_i^2}{2mkT} - \frac{p_i c}{q\Phi} + \mu) + a g_i], \quad (100)$$

take $a = 1, -1$, we have

$$N_i = G_i g_i / [\exp(\frac{p_i^2}{2mkT} - \frac{p_i c}{q\Phi} + \mu) \pm g_i] \quad (101)$$

There are both temperature T and potential functions Φ in the system.

From Eq.(100) we get Bose and Fermi statistical distributions of number of particles. For ease of calculation, let's say $y = \exp(-\beta p_i^2/2m + \alpha_E p_i c)$ and expand the generating function to get

$$\begin{aligned}\Omega &= \sum_i \frac{G_i}{a} \\ N &= \sum_i a^2 g_i^2 N_i = \sum_i G_i g_i y \exp(\mu).\end{aligned}\quad (102)$$

Now we can define the partition function of the system, we get

$$Z = \sum_i G_i g_i \exp(-\beta p_i^2/2m + \alpha_E p_i c).\quad (103)$$

Thus Lagrangian multiplier of number of these particles is

$$\exp(\mu) = \frac{N}{Z}.\quad (104)$$

When take $a = 0$, for the system concurrently exist temperature T and potential function Φ , by Eq.(100) the Boltzmann statistical distribution becomes

$$N_i = G_i g_i / \exp\left(\frac{p_i^2}{2mkT} - \frac{p_i c}{q\Phi} + \mu\right).\quad (105)$$

By the Eq.(103), we can define partition function of the system Z

$$Z = \sum_i G_i g_i \exp\left(-\beta \frac{p_i^2}{2m} + \alpha_E p_i c\right).\quad (106)$$

Then, by Eq(105) and Eq.(106) the Lagrangian multiplier of number of particles becomes

$$\exp(\mu) = \frac{N}{Z}.\quad (107)$$

If there is temperature of critical state $T = T_0$ in the system, then the equilibrium state of the system has $p_i^2/2mkT_0 = p_i c/q\Phi$, and thus making $\exp(p_i^2/2mkT_0 - p_i c/q\Phi) = 1$. thus, from Eq.(100) these particles will be distributed as follows

$$N_i = G_i g_i / [\exp(\mu) + a g_i],\quad (108)$$

By Eq.(108) total number of particles of the system, we have

$$N = \sum_i G_i g_i / [\exp(\mu) + a g_i].\quad (109)$$

Let us calculate integrate of Eq.(100)

$$\begin{aligned}N &= \int_0^\infty N_i dp = \int_0^\infty \frac{4\pi v p^2 g_i}{h^3} \\ &\frac{1}{\exp\left(\frac{p^2}{2mkT} - \frac{p c}{q\Phi} + \mu\right) + a g_i} dp.\end{aligned}\quad (110)$$

When these particles place in the potential Φ , their motion will be ordered state, if the system's temperature $T \rightarrow 0$, these particles disordered thermal motions would also approach ordered state. Namely, the particles would stay in the ground state. These particles of spin 1 would locate in the identical phase cell, would show the follows:

$$N = \frac{1}{\exp\left(\frac{p^2}{2mkT} - \frac{pc}{q\Phi}\right) - g_i} + \int_0^\infty \frac{4\pi v p^2 g_i}{h^3} \frac{1}{\exp\left(\frac{p^2}{2mkT} - \frac{pc}{q\Phi} + \mu\right) - g_i} dp. \quad (111)$$

For concurrent existence of temperature T and potential Φ in the system, and these particles at spin 1/2, hence we have the following particle number:

$$N = \int_0^\infty \frac{4\pi v p^2 g_i}{h^3} \frac{1}{\exp\left(\frac{p^2}{2mkT} - \frac{pc}{q\Phi} + \mu\right) + g_i} dp. \quad (112)$$

It is not difficult to see that in a system is composed of charged particles, in the environment potential Φ there is the probability distribution of the particles ordered motion. However, if in the system these particles are electrically neutral or at potential $\Phi = 0$ the potential energy $q\Phi = 0$, the motion of these particles will be in a state of the thermal motion of disorder. In other words that when exists only temperature T in the system, these particles only have thermodynamic kinetic energy, they are normally the Boltzmann, Bose and Fermi probability statistical distribution, the numbers of particle is as follows

$$N_i = \frac{4\pi v p^2 g_i}{h^3} \frac{1}{\exp\left(\frac{p^2}{2mkT} + \mu\right) + a g_i} \quad (a=0, -1, 1). \quad (113)$$

In this paper, we apply the Caratheodory axiomatization to de-

fined the potential function and entropy function of the system. We introduce the relative rate of change of energy and its corresponding entropy function. Based on the probability distribution density, by number of particles and expectations value of energy in the system, to calculate the variation. From multiplying of Lagrangian multipliers, we derived the parameters of energy $1/(q\Phi)$ and particle's number $\exp(\mu)$. Another, the partition function and the entropy function of the system are also defined. The whole entropy function of the system has been polymerization become $S = k_B \ln Z_t + q(-1/c) \ln Z_c + \hbar(-1/c) \ln Z_q$, where they are respectively, the $k_B \ln Z_t$ is thermodynamic entropy, the $q(-1/c) \ln Z_c$ is entropy of the system of charged particles and the $\hbar(-1/c) \ln Z_q$ is entropy of quantum of the system. When concurrent exist of temperature T and potential Φ in the system, with the L.Brillouin

statistical method of phase space to obtained probability distribution of number of particles, and the distribution of the particles of critical state in temperature $T = T_0$.

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