# “Infinitely often”=“Infinity”
# — a statistics approach to small gaps between primes

by

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**Abstract**

We construct a sequence of consecutive primes. From the perspective of statistics, we analyze and handle them by the combination of the fundamental property of primes with James Maynard’s result. It reveals that there are infinitely many pairs of primes which differ by two.

**Keywords.** Twin Prime Conjecture, Prime, Statistics Theory

**MSC 2010:** 11A41, 62P05, 11N05

## 1. INTRODUCTION

One of the most famous problems in mathematics is Twin Prime Conjecture. Up to now, Y. Zhang [1], James Maynard, Terence Tao and dozens of mathematicians [2] have succeeded in making dramatic new progress. However, there is a key limitation inherent in standard sieve method. The conjecture remains unsolved and new ideas are needed for the final proof.

## 2. PROOF

We construct and consider the following number sequence:

$$\{p_n,\ p_{n+1}\}$$

where $p_n$ and $p_{n+1}$ are consecutive primes, $p_n$ is the n-th prime.

**Fundamental property of primes** *Every prime greater than 3 must be of either the form “6K−1” or the form “6K+1” (K, integer* $\geq 1$*).* Every $p_n$ must be of either the form “$6K_1-1$” or the form "$6K_1+1$", and every $p_{n+1}$ must be of either the form "$6K_2-1$" or the form "$6K_2+1$".

Then, the number sequence $\{p_n,\ p_{n+1}\}$ must be only five cases for different $K_1$ *and* $K_2$,:

- ① $\{p_n = 6K_1 - 1,\ p_{n+1} = (6K_2 - 1) \geq (6K_1 + 5)\}$
  $$(K_2 \geq K_1 + 1)$$
- ② $\{p_n = 6K_1 - 1,\ p_{n+1} = (6K_2 + 1) \geq (6K_1 + 7)\}$
  $$(K_2 \geq K_1 + 1)$$
- ③ $\{p_n = 6K_1 - 1,\ p_{n+1} = (6K_2 + 1) = (6K_1 + 1)\}$
  $$(K_2 = K_1)$$
- ④ $\{p_n = 6K_1 + 1,\ p_{n+1} = (6K_2 - 1) \geq (6K_1 + 5)\}$
  $$(K_2 \geq K_1 + 1)$$
- ⑤ $\{p_n = 6K_1 + 1,\ p_{n+1} = (6K_2 + 1) \geq (6K_1 + 7)\}$
  $$(K_2 \geq K_1 + 1)$$

In comparison, there is no difference between ① and ②, and there is no difference between ④ and ⑤. For every $n$, every $\{p_n,\ p_{n+1}\}$ must be one case of ①, ③ and ④. By the Statistical theory: as $n \to \infty$, each case $\to$ infinitely often, which means infinity. James Maynard has proved that there are infinitely many consecutive primes with a distance of 246 at most.

Hence,

$$(\ case\ ①\ and\ ④\ )\quad 2 < \liminf_{n\to\infty}(\ p_{n+1} - p_n\ ) \leq 246$$

$$(\ case\ ③\ )\quad \liminf_{n\to\infty}(\ p_{n+1} - p_n\ ) = 2$$

Therefore, Twin Prime Conjecture is true.

## References


[1] ZHANG Y., *Bounded gaps between primes*. Ann. of Math. 179 (2014), 1121–1174

[2] D. H. J. POLYMATH. *The "bounded gaps between primes" polymath projects—a retrospective*. arXiv:1409.8361, 2014.