Have they got it wrong about black holes?

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Abstract

By recognising that Newtonian gravity is a manifestation of the time curvature in a curved four-dimensional Lorentzian manifold such as in Einstein's general theory of relativity(GR), it can be shown that space and time are completely regular in the neighbourhood of a static point mass, and therefore a black hole and event horizon are just mathematical artefacts. In addition, this also leads to the conclusion that superluminal velocities do not occur in reality and that gravity does not diverge to infinity as masses approach each other, which also removes the non-physical singularity at the coordinate origin.

1 Introduction

Black holes play a central part in the current paradigm in cosmology and astrophysics, and are believed to be ubiquitous in the universe. The solution using Einstein's theory of general relativity for the curvature of spacetime near a point mass can be used to justify their prediction, and in the meantime, observational evidence for their existence has been claimed via two experimental techniques: gravitational wave signals [1], and the so-called EHT telescope [2]. However, looking back over the history of the subject, I am convinced that a crucial error was made that resulted in the concept of a black hole becoming falsely promoted. It is this theoretical aspect that I am going to concern myself with in this paper.

2 Background theory

The first person to obtain a solution for the gravitational field outside a point mass using general relativity (GR) was Karl Schwarzschild in 1916 [3], less than a year after Albert Einstein published his *GR* theory [4]. A spacetime increment may be written in spherical polar coordinates (t, r, θ, ϕ) as:

$$d\tilde{s}^2 = c^2 dt'^2 = A\, c^2 dt^2 - B\, dr^2 - C(d\theta^2 + \sin^2\theta\, d\phi^2) \tag{1}$$

where A, B and C are radially dependent functions describing the curvature of the time, radial and angular metric coefficients, respectively; c is the speed of light, dt' is an increment of proper time, dt an increment of coordinate time, and dr is a radial increment. This is, I believe, the most general description for a spherically symmetric space that must prevail in the neighbourhood of an isolated static point mass. However, as I shall explain later, a different form for the metric is usually adopted, in which C is replaced by r^2, and r should then be construed as a different radial coordinate.

Next, from the metric, the calculus of variations is used to obtain geodesic equations for the four coordinate variables (t, r, θ, ϕ) using the proper time t' as the Lagrangian parameter that extremises the path of a particle in the gravitational field. One obtains:

$$\begin{aligned}
\ddot{t} + \frac{A'}{A}\,\dot{r}\dot{t} &= 0 \\
\ddot{r} + \frac{A'}{2B}\,c^2\dot{t}^2 + \frac{B'}{2B}\,\dot{r}^2 - \frac{C'}{2B}\left(\dot{\theta}^2 + \sin^2\theta\,\dot{\phi}^2\right) &= 0 \\
\ddot{\theta} + \frac{C'}{C}\,\dot{\theta}\dot{r} - \sin\theta\cos\theta\,\dot{\phi}^2 &= 0 \\
\ddot{\phi} + \frac{C'}{C}\,\dot{\phi}\dot{r} + 2\cot\theta\,\dot{\theta}\dot{\phi} &= 0
\end{aligned} \tag{2}$$

The equation of interest for describing the radial free-fall of a test particle directly towards the mass at the origin is the second of these expressions, which leads to the following equation of motion for free-fall:

$$\ddot{r} + \frac{A'}{2B}\,c^2\dot{t}^2 + \frac{B'}{2B}\,\dot{r}^2 = 0 \tag{3}$$

where $\ddot{r}\,(= d^2r/dt'^2)$ is the proper acceleration, $\dot{r}\,(= dr/dt')$ is the proper velocity, $A' = dA/dr$ and $B' = dB/dr$. Using the radial part of the metric in Equation 1 to eliminate $\dot{t}$ (with $d\theta = d\phi = 0$), we can then reformulate Equation 3 as:

$$\ddot{r} + \frac{A'}{2AB}\,c^2 + \left(\frac{A'}{2A} + \frac{B'}{2B}\right)\dot{r}^2 = 0 \tag{4}$$

Einstein's field equations of GR enable us to find the way A, B and C relate to each other. Christoffel curvature coefficients are found from the geodesic equations and then used to obtain the Ricci tensor components R_{ab}. The components that are not trivially zero are found to be, as follows:

$$R_{tt} = c^2 \frac{A''}{2B} - c^2 \frac{A'B'}{4B^2} - c^2 \frac{A'^2}{4AB} + c^2 \frac{A'C'}{2BC} \ ; \ g_{tt} = -c^2 A$$
$$R_{rr} = -\frac{A''}{2A} + \frac{A'B'}{4AB} + \frac{A'^2}{4A^2} + \frac{B'C'}{2BC} - \frac{C''}{C} + \frac{C'^2}{2C^2} \ ; \ g_{rr} = B$$
$$R_{\theta\theta} = 1 - \frac{C''}{2B} + \frac{B'C'}{4B^2} - \frac{A'C'}{4AB} \ ; \ g_{\theta\theta} = C \tag{5}$$

with an equivalent equation to the third for $R_{\phi\phi}$. For the vacuum outside the point mass, R_{ab} are set to zero to satisfy Einstein's GR field equations. Accordingly, we then obtain from Equations 5 the following pair of simultaneous equations relating A, B and C:

$$\frac{A'}{A} + \frac{B'}{B} = \frac{2C''}{C'} - \frac{C'}{C} \ ; \quad \frac{A'}{A} - \frac{B'}{B} = -\frac{2C''}{C'} + \frac{4B}{C'} \tag{6}$$

Since there are only two independent equations for the three functions, A, B, and C, it means they cannot be solved explicitly, without some additional condition or assumption.

To circumvent this problem, as mentioned above, C is replaced by r^2, which in effect removes one of the unknowns and allows an exact solution to be obtained with two unknowns and two simultaneous equations. The angular part of the metric then appears as it would for a flat spatial metric, and A and B are constrained in some way to be related to each other. This procedure essentially amounts to introducing a new radial coordinate, and the coordinates are then called Schwarzschild coordinates (in honour of Schwarzschild's name). This new radial coordinate is not the same as r in Equation 1, but is measured as the circumference divided by 2π of a sphere centred around the massive body (e.g. [5],[6]).

Replacing C with r^2, Equations 6 become

$$\frac{A'}{A} + \frac{B'}{B} = 0 \ ; \quad \frac{A'}{A} - \frac{B'}{B} = -\frac{2(B-1)}{r} \tag{7}$$

which can now be solved to give:

$$A = \frac{1}{B} = 1 - \frac{\alpha}{r} \tag{8}$$

where α is a constant of integration.

To relate the geometry of GR expressed via the constant of integration α to physical quantities such as Newton's gravitational constant G and the mass of the object causing gravity M, it is customary to make use of Newton's law of gravitation, in what is called a weak-field approximation. Inserting $B = 1/A$ from Equation 8 into the free-fall equation of motion (Equation 4), one then obtains:

$$\ddot{r} + \frac{A'c^2}{2} = 0$$

or

$$\ddot{r} = -\frac{\frac{1}{2}\,\alpha c^2}{r^2} \tag{9}$$

where $A' = \alpha/r^2$ from Equation 8. This appears to agree with the inverse-square behaviour of Newton's law of gravitation, in which free-fall acceleration is $-GM/r^2$, giving $\alpha = 2GM/c^2$, which is a positive quantity known variously as the gravitational radius or Schwarzschild radius. The quantity α turns out to represent only a small distance, about $2.9\,km$ for a star the mass of the Sun and $8.7\,mm$ for Earth. However, for a supermassive object, such as the centre of the Milky Way galaxy it could be approximately 12 million kilometres.

The solution in Equation 8 thus predicts that A and B change sign at this radial coordinate $r = \alpha$. For a point (or highly compacted) mass with physical radius less than α, it therefore seems that a discontinuity may occur in spacetime, and the distance α has become known as the event horizon of a black hole.

3 Discussion and further theory

Historically, Schwarzschild recognised the presence of a mathematical discontinuity in his solution, but by defining a suitable auxiliary radial coordinate he forced the discontinuity to be at the origin, since he believed it to be non-physical. Shortly afterwards Droste [7] and Weyl [8] provided a solution, but restricted the range of r to $r > \alpha$. Subsequently, Hilbert [9] extended Droste and Weyl's solution to the region $r < \alpha$ on the grounds that a coordinate transformation does not alter the physics of the situation, and GR is supposed to be a generally covariant theory. It is essentially Hilbert's solution allowing for a change in sign of A and B, that is accepted today

Throughout his life, Einstein himself considered the black-hole solution to be unphysical both because of the strange behaviour near the event horizon and because there is a singularity at the centre of the black hole. He was on a quest to find a unified theory that would eliminate the singularity in his theory of gravity. Other authors, such as Moffat [10], claim to have discovered exact, non-singular solutions of Einstein's field equations. For example, in Moffat's modified gravity theory an exotic field energy is added that supposedly permeates all of spacetime, and causes a negative force to be exerted on a collapsing star and prevents the formation of a black hole.

I am convinced, however, that there is a simpler explanation that falsifies the irregular behaviour of spacetime. Firstly, the geometry of Newton's inverse-square law of gravitation is undeniably strictly Euclidean (or flat). In other words, spatial curvature plays no part in Newtonian or classical gravity. On the other hand, Einstein's *GR* is a geometrical theory explaining gravity through the curvature of both space and time. Logically, then, if we relate Newton's law with *GR* to obtain correspondence, Newtonian gravity is that contribution to gravity resulting exclusively from the curvature of time. William Unruh puts it this way: "gravity is the uneven running of clocks at different places" [11]. When comparing *GR* with Newtonian gravity, it is therefore incorrect to use the reciprocity of space and time curvature dictated by *GR* from Equation 8, not even approximately. To compare *GR* with classical gravity you have to write $B = 1$, as for a Euclidean space, and then the equation of free-fall from Equation 4 becomes:

$$\ddot{r} + \frac{A'}{2A}\left(c^2 + \dot{r}^2\right) = 0 \qquad [B = 1] \tag{10}$$

Solving this differential equation and inserting Newtonian expressions for free-fall acceleration and velocity then gives the following expression for A:

$$A = \left(1 + \frac{\alpha}{r}\right)^{-1} \qquad ; \qquad \alpha = 2GM/c^2 \tag{11}$$

where the constant of integration α is again equal to $2GM/c^2$. Thus, we have quantified the insight that the gravitational force or acceleration in Newton's law relates strictly to the curvature of the time coordinate in *GR*. Clearly, as shown in Figure 1, the function A in Equation 11 (regular solution in the figure) shows no discontinuity for any value of the radial coordinate r.

Now writing the Schwarzschild radial coordinate in the *GR* solution of Equation 8 as r^*, so as not to confuse the two coordinates, we may write

$$A = 1 - \frac{\alpha}{r^*} = \left(1 + \frac{\alpha}{r}\right)^{-1} \tag{12}$$

from which it follows that

$$r^* = r + \alpha \tag{13}$$

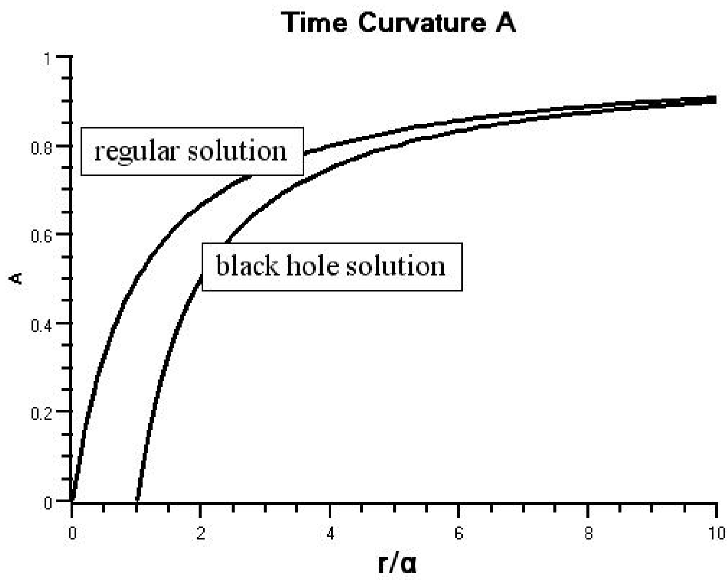

Figure 1: Time curvature A: In the regular solution the time clock stops ($A = 0$) at the point mass; in the black-hole solution the clock stops at a distance $r = \alpha$ from the point mass, and then becomes negative or space-like

The difference between r^* and r is extremely small for $r >> \alpha$, and distinguishing between the two then becomes irrelevant. But when r is of the order of α the situation is crucially different. While the range of r goes from zero to ∞, the range of r^* is from α to ∞. The spacetime manifold therefore does not exist for $r^* < \alpha$, Hilbert's extension to $r^* < \alpha$ is therefore meaningless, and there is no event horizon.

The solution $A = (1 + \alpha/r)^{-1}; B = 1$ does not accurately satisfy Einstein's vacuum field equations of GR, but we do not expect it to, and there is no requirement that it should, since Newtonian physics does not treat a curved space *per se*. As stated, Newton's inverse-square law of gravity describes that aspect of gravity caused exclusively by the curvature of the time coordinate, and this is manifestly dominant for most cases we consider, such as planetary motion. However, space curvature becomes significant when speeds approach the speed of light, and distances to the central mass become small, and this will modify gravity from being purely Newtonian. The use of GR is then to describe phenomena that Newton's law does not describe, such as the perihelion rotation of the planet Mercury, and the bending of starlight passing near the Sun. To satisfy GR, we then require $B = 1/A$ with $C = r^{*2}$, which gives the following complete solution for the curvature of spacetime due to a static point mass:

$$A = \frac{1}{B} = \left(1 + \frac{\alpha}{r}\right)^{-1} \qquad ; \qquad C = (r+\alpha)^2 \tag{14}$$

This could also have been obtained directly by substituting A from Equation 11 into Equations 6.

We now have a new way of visualising the situation. Regarding the point mass as fixed in space at $r = 0$, since $r^* = r + \alpha$, the origin of Schwarzschild coordinates $r^* = 0$ lies somewhere on a sphere at radius $r = \alpha$. This has also been addressed previously by Leonard Abrams [12] and Stephen Crothers [13] in various papers. Thus, taking any point P in space, the Schwarzschild coordinate $r^* = 0$ lies a distance α behind $r = 0$ on the line joining P to $r = 0$. A free-falling object along this line therefore reaches the point mass before it would reach the singularity of the Schwarzschild coordinate origin, which in any case does not exist as part of the spacetime manifold.

4 A limiting velocity and gravitational force

Some further predictions are now apparent. By integrating Equation 4 and substituting $B = 1/A$, it is straightforward to show that the proper velocity of free-fall is given as

$$\frac{\dot{r}^2}{c^2} = 1 - A$$

where the asymptotic condition $A \to 1$, $r \to \infty$ has been used. Then, substituting my solution $A = (1 + \alpha/r)^{-1}$, we obtain

$$\dot{r} = c\sqrt{\frac{\alpha}{r+\alpha}} \tag{15}$$

This means we have $\dot{r} \to c$ for $r \to 0$. It contrasts fundamentally with the black hole solution, substituting $A = 1 - \alpha/r$, where we obtain

$$\frac{\dot{r}^2}{c^2} = 1 - A = \frac{\alpha}{r}$$

or

$$\dot{r} = c\sqrt{\frac{\alpha}{r}} \tag{16}$$

which gives $\dot{r} \to c$ for $r \to \alpha$, and $\dot{r} \to \infty$ for $r \to 0$. The solution presented here thus predicts a limiting free-fall velocity of c, in contrast to the black-hole solution (Hilbert's extension) and Newton's law that both predict infinite speed as $r \to 0$.

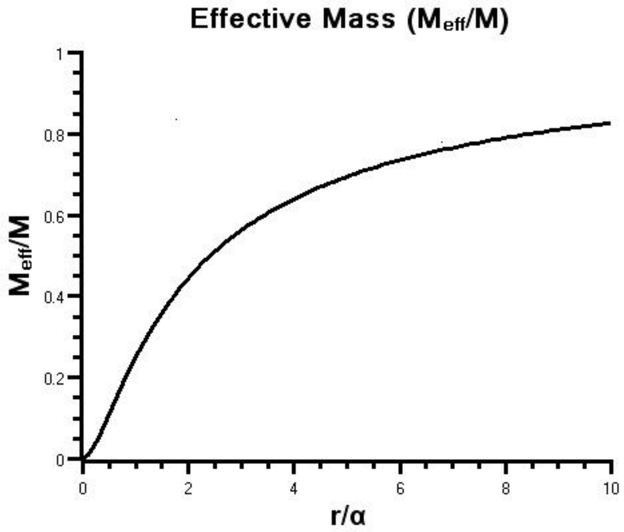

Figure 2: Effective gravitational mass

Furthermore, in my model the radial free-fall acceleration is given by

$$\ddot{r} = -\frac{1}{2}c^2\frac{\alpha}{(r+\alpha)^2} = -\frac{GM}{(r+2GM/c^2)^2} \tag{17}$$

which shows classical Newtonian behaviour $-GM/r^2$ for $r \gg \alpha$ but deviates (decreases) from inverse-square law behaviour for r of the order of α.

Finally, I shall define an effective gravitational mass ratio M_{eff}/M as the acceleration in my model divided by the acceleration in Newton's law, from which we obtain:

$$\frac{M_{eff}}{M} = \frac{r^2}{(r+\alpha)^2} = \left(1+\frac{\alpha}{r}\right)^{-2} \tag{18}$$

We thus have for large r, Newton's law being obeyed, i.e. the effective gravitational mass is constant ($M_{eff} = M$), but as r decreases to zero, the effective mass goes to zero, and the singularity you expect to find at $r = 0$, where the laws of physics would break down, disappears (see Figure 2).

5 Conclusion

The deductions outlined here deviate from the current paradigm. The idea of a black hole and event horizon is a mathematical possibility, but has been shown to be non-physical by correct correspondence of Newton's law in conjunction with Einstein's GR theory. The present model is much more intuitive and rules out superluminal velocities. Gravity does not diverge to infinity as masses approach each other, which removes the physically inexplicable issue of a singularity in spacetime.

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