Measurement of Physical Quantities under Differently Calibrated Rulers and Clocks

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Abstract:

The theory of special relativity is developed with two stipulations that any propagating electromagnetic wave travels at the same constant speed c, with respect to all inertial reference frames irrespective of their relative velocities and any IRF shall synchronize its spatially separated clocks by the assumption or convention that the one way speed of light within that IRF is constant and equal to c in all directions. The mathematical development of these concepts lead us to the principle of the relativity of simultaneity, the space-time continuum and the block universe that implies the existence of past, present, and future in a four-dimensional space-time continuum. The principle of relativity of simultaneity essentially means that the tenses, past, present, and future, are an illusion. Time order of events are subjective and thus all events in the universe exist together on the continuum. We show that the characteristics of light propagation are the same whether we use the Lorentz Transformation (LT) or the Galilean Transformation (GT) in the sense that the amplitude of the propagating wave at any space-time location remains the same in both the transformations. We argue that the space and time coordinates assigned to any space-time point are different in LT and GT but the identity of a space-time point is not compromised. The different numbers of space and time coordinates assigned to a space-time point by LT and GT arise out of calibration differences and do not indicate any altered reality.

Keywords: Physical Measurements, Coordinate Systems, Calibration, Synchronisation, Special Relativity Theory

1. Introduction

1.1 Basic Concept of the manuscript

Consider a inertial reference frame (IRF) K, with event coordinates (x,t). Consider another IRF K' moving at a uniform speed v along the x axis of K with its x' axis aligned with x axis of K. All objects in K' are observed to be moving at v along the x axis by observers in K. The most general event coordinate transformation between K and K' is

$x' = a (x-vt)$ ----- (1.1)

remarks: whenever $x'=0$, $x = vt$ as the origin of K' is observed to be moving at velocity/speed v by observers in K.

$t' = p x + q t$ …. (1.2)

Equation 1.1 aligns with the statement that all objects in K' are observed to be moving at v along the x axis by observers in K. Equation 1.2 is most general in its specifications. The three parameters of the transformation are a,p and q, as v is defined by the velocity between IRF K' and IRF K.

If K' uses a differently calibrated system of measurements, the same can be described as

$x'' = A (x-vt)$ ----- (1.3)

$t'' = Px + Qt$ ------- (1.4)

The IRF K' and K" are identical without any movement between them.
From above four equations, the transformation between K' and K" is easily obtained as

$x'' = (A/a) x'$ ------ (1.5)

$t'' = [(Pq-Qp)/(aq+apv)]\, x' + [(Pv+Q)/(pv+q)]\, t'$ ---- (1.6)

Equation (1.5) states that there is no relative velocity between IRF K' and IRF K" and hence both K' and K" are identical as IRFs. Equation (1.6) states that there is a complex relationship in the time measurement of the two systems due to clock calibration differences and synchronisation procedures adopted by K' and K" although both these IRFs are physically identical.

It is to be noted that Galilean Transformation (GT) with $a = q = 1$, $p = 0$, Tangherlini Transformation (TT) with[1] $a = \gamma$, $q = (1/\gamma)$, $p = 0$, and Lorentz Transformtion with $a = q = \gamma$, $p = -v\gamma/c^2$ are in conformity with the form of above equations 1.1 to 1.4 and any transformation between LT to GT, GT to LT, LT to TT, TT to LT, GT to TT or TT to GT will follow the pattern prescribed in equations 1.5 and 1.6 (please see appendix for details). It is needless to mention that K' and K" will observe the events in the universe identically although they may assign spatial and temporal coordinated differently as these "two" IRFs are essentially identical and same IRF with only calibration differences. Thus whether we use GT, LT or TT as a transformation between K and K', the observation of events in the universe by observers in K' will not alter essentially.

1.2 Physical Concepts

Propagation of light waves at a constant speed c with respect to all reference frames that are in relative uniform movement with respect to each other is the accepted principle of physics in the modern era. This leads to the principle of the relativity of simultaneity, the space-time

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[1] $(1/\gamma) = [1-(v^2/c^2)]^{(1/2)}$

continuum and the block universe that implies the existence of past, present, and future in a four-dimensional space-time continuum. The principle of relativity of simultaneity essentially means that the tenses, past, present, and future, are an illusion. Time order of events are subjective and thus all events in the universe exist together on the continuum, also known as the 'Block Universe' [1].

The definition of simultaneity as a result of a convention has been discussed in detail in [2]. This definition was proposed by Einstein [3]. He, as well as researchers of his era recognized the difficulties in synchronizing non-co-located (spatially separated) clocks within an inertial frame of reference in the context that clocks may tick slower when moved. In the context of isotropy of light propagation within an inertial frame of reference, Einstein in his first paper [3] had stated thus "*But an examination of this supposition would only be possible if we already had at our disposal the means of measuring time. It would thus appear as though we were moving here in a logical circle.*"

Recognizing the fact that in order to measure the one-way speed of light, one required spatially separated synchronized clocks already in position at various locations within an inertial reference frame, Einstein [3] goes on to define each and every inertial frame to be isotropic in the context of light propagation. He states in [3] "*That light requires the same time to traverse the path A -> M as for the path B-> M (M is the midpoint of the line AB) is in reality neither a supposition nor a hypothesis about the physical nature of light, but a stipulation which I can make of my own free will in order to arrive at a definition of simultaneity.*"

As a consequence of this definition, an elegant derivation of the Lorentz Transformation was derived by him in the appendix in [3]. Thus, by a definition, the three concepts, namely, relativity of simultaneity, apparent length contraction, and apparent slow running of 'moving' clocks got enshrined into the fundamentals of Physics. More importantly, this definition assumes isotropy of light propagation in all observing inertial reference frames irrespective of the state of motion of the inertial frame wherein the light source was stationary.

The awareness of the flow of time is related to human consciousness and the relationship between physical processes both internal and external to the human body. However, temporal order or time order does not measure duration but only the before, after, or simultaneous nature of two events, and does not require the measurement of the magnitude of the time interval. It has been suggested that between two events A and B, if we are unable to recall or specify the amount of elapsed time, whether A happened before B or otherwise cannot be altered [4].

Special Relativity theory proposes that moving rods appear to be contracted and moving clocks appear to run slow. H.A. Lorentz constructively derived contraction of moving rods along the line of motion by considering electromagnetic phenomena. [5, 6]. These principles were incorporated by Einstein [3] into the theory of special relativity; however in his theory, the contraction was apparent and reciprocal between two reference frames. Tangherlini [7] proposed a transformation that incorporates length contraction and time dilation without the relativity of simultaneity. His theory maintains the round trip speed of light as constant.
A detailed discussion on the experimental proofs of relativistic length contraction and their possible alternative interpretations is given in [8].

The reality or apparentness of the contraction of a moving rod is discussed scientifically and historically, in detail in [9].

Assignment of a number to a spatial location from a chosen origin is a subjective matter in that it could be for example in feet or meters. Suppose we use a meter ruler and if the meter ruler contracts (say for example in the winter) and we use the ruler without being aware that the ruler has contracted, we may assign slightly different numbers to locations in space. However, this will not alter the basic structure of the universe. Events such as meeting of two persons or collision of two objects remain essentially unaffected in their characteristics by the assigned numbers to the spatial and temporal perspective by different systems of measurement.

We show in the following sections, while different systems may alter the perception of wavelength, frequency and speed of propagation (in a given reference frame), the amplitudes of the wave will remain unaltered. Here by different measurement systems, we mean two different systems adopted to measure lengths and time within a reference frame. That implies there is no movement involved between the two systems. They both measure lengths and time in the same reference frame.

Section 2 shows there is same phase of the wave remains the same under the Lorentz and Galilean transformations. Section 3 demonstrates the same principle under the Tangherlini transformation. Section 4 shows that the same conclusion for an arbitrary point on the propagation path remains unaltered under these three transformations. Section 5 describes a general linear transformation of event coordinates within an inertial frame. Section 6 describes the transformation between Galilean to Lorentz as a single matrix within a reference frame. Section 7 details interpretation of the results. In Section 8 we offer our conclusions.

2. Phase of the wave under Lorentz and Galilean transformations

Consider an instant of a wave front with maximum amplitude which leaves the origin at t = 0, in the reference frame co-moving with the source (K). Event 1, occurs at (x = 0, t = 0).

This wave front reaches the location (*ct*, *t*) in reference frame K (we call this Event 2), propagating at the speed *c*, the speed of light in vacuum.

The phase difference between Events 1 and 2 is shown to be zero below.

The propagation equation is Amplitude = $A_{max}\, Sin\, 2\pi\ [\ (x/\lambda) - \omega t]$. …….. (2.1)

With $\omega = c/\lambda$ and $x = ct$, the expression $(x/\lambda) - \omega t = 0$ and as expected the same instant of the wave propagating at *c*, retains its phase between Event 1 (0, 0) and Event 2 (*ct*, *t*).

2.1 Phase of the wave under Lorentz transformation

Consider Events 1, 2 transformed from the frame K to another reference frameK', moving at velocity *v* along the *x* axis. When we assume that the Lorentz transformation is the correct one, we get for a receding wave from any standard text, New wave length in $K' = \lambda \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}} = \lambda_L$ ……….. (2.2)

New frequency in $K' = \omega \frac{\sqrt{1-v/c}}{\sqrt{1+v/c}} = \omega_L$ ………… (2.3)

The product of the above two remains *c*, the propagation speed as stipulated by the synchronization convention maintaining the speed of light as c in K' as well. The subscript L indicates that these are in the moving frame K'using the Lorentz transformation between K to K'.

The two events, Event 1 (0, 0) and Event 2 (*ct*, *t*) in frame K are transformed in frame K'

by the Lorentz transformation to

(0, 0) and $(ct - vt)\gamma, \left(t - \frac{vct}{c^2}\right)\gamma$, where $\gamma = 1/\sqrt{1 - v^2/c^2}$ ………… (2.4)

The phase difference $(x'/\lambda_L) - \omega_L t' = ((ct - vt)\gamma/\lambda_L) - \omega_L\,(t - \frac{vct}{c^2})\gamma$ …….. (2.5)

Noting that $\omega_L = c/\lambda_L$ this becomes

Phase difference = $\left(\frac{\gamma}{\lambda_L}\right)[(ct - vt) - c\,\left(t - \frac{vct}{c^2}\right)] = \left(\frac{\gamma}{\lambda_L}\right)[(ct - vt) - (ct - vt)] = 0$ .. (2.6)

Thus the phase difference between Events 1 and 2 is zero in frame K' as it is in frame K.

2.2 Phase of the wave under Galilean transformation

In the Galilean transformation the Event 2 coordinates in frame K' transform to $[(ct - vt), t]$. The wavelength remains the same as λ as the instant remains unchanged between K and K' in the Galilean transformation. There is no contraction of rulers (apparent or real), and so the distance between two peaks of the wave remain the same in K and K'.

Thus $\lambda_G = \lambda$.

The speed of propagation is $(c - v)$ in the Galilean case and thus the frequency becomes $\omega_G = (c - v)/\lambda$.

The phase difference = $(x'/\lambda) - [(c - v)/\lambda)]t'$ ………. (2.7)

Noting that $t' = t$ in the Galilean transformation and $x' = ct - vt$, we obtain the phase difference under the Galilean transformation to also be zero.

2.3 Summary

Thus we see while the Galilean transformation altered the frequency and speed of propagation but did not alter the wavelength, the Lorenz transformation altered the wavelength and frequency but not the speed of propagation. However, the phase of the wave front was retained at the location (x, t) when transformed either by the Lorentz or Galilean transformations.

Anisotropy in a moving frame with respect to the propagation of EM (electro magnetic) waves, will reflect irrespective of the transformation or calibration choices. While the Lorentz Transformation (LT) preserves isotropy of speed of propagation, it allows anisotropy in observed wavelength and frequency. The Galilean Transformation (GT) (as well as the Tangherlini Transformation (TT)) preserve isotropy of the wavelength but allow anisotropy in speed of propagation and frequency. Whichever choice one makes, the observations about events of happening in the universe will not be altered.

For measurement of any physical quantity at a point x,t in inertial reference frame (IRF) K, that can be represented as f (x,t)

The same need to be F (x',t') in IRF K'.

Since this physical measurement is external to both K and K' as in the **amplitude** of a propagating Electromagnetic wave,

f (x,t) must be equal to F (x',t')

For EM propagation

$(x-ct)/\lambda = (x' - c^{*}t')/\lambda^{*}$ ------- (2.3.1)

Taking a general linear transformation of the variables x,t to x',t' with a only constraint that the velocity of objects in K' as observed by observers in K is v

We have $x' = a(x-vt)$ and $t' = px + qt$ ----- (2.3.2)

Substituting (2.3.2) into (2.3.1) we have

$(x-ct)/\lambda = [a(x-vt) - c^{*} (px+qt)] / \lambda^{*}$ ------- (2.3.3)

Comparing and equating coefficients of x and t, we get

$\lambda^{*} = \lambda (a - pc^{*})$ ---- (2.3.4)

$c^{*} = (ac-av)/(q+pc)$ ---- (2.3.5)

substituting (2.3.5) into (2.3.4)

$\lambda^{*} = \lambda (aq+avp)/(q+pc)$ ---- (2.3.6)

equations (5) and (6) give the propagation speed and wavelength in IRF K'.

These are consistent with equation (2.3.1) in that the amplitude observed at any given space-time point is measured to be the same independent of the values of a,p and q.

Following table illustrates the dispensation for GT, TT and LT

Table – 1 Relationship between quantities when the choice is GT, TT or LT

sl no of parameter/ result		GT (Galilean)	TT (Tangherlini)	LT (Lorentz)
1	a	1	γ	γ
1	p	0	0	$-v\gamma/c^2$
3	q	1	$1/\gamma$	γ
4	c^* eq 2.3.5	c-v	$(c-v)\gamma^2$	c
5	λ^* eq 2.3.6	λ	$\lambda\gamma$	$\lambda\gamma[1+(v/c)]$
6	Length Contraction	NO	YES	YES (but only apparent or illusory)

Table – 1 Relationship between quantities when the choice is GT, TT or LT

3. Transformation from isotropic frame by Tangherlini transformation

The Tangherlini transformation is well known to maintain the round trip speed of light as constant and maintain the same synchronization as the isotropic frame. The resultant coordinates lead to an anisotropic frame with onward propagation speed as $c/(1+ v/c)$ and the return propagation speed as $c/(1- v/c)$ with the average round trip speed, which is the harmonic mean of the onward and return speeds as c.

The Tangherlini transformation from an isotropic frame to another (anisotropic) frame with the round trip speed of light as constant is given by

$x' = (x - vt)\gamma$ ………. (3.1)

$t' = t/\gamma$ …………… (3.2)

The speed of propagation in the onward path is $(ct - vt)\gamma/(t/\gamma) = c/(1+ v/c)$ …. (3.3)

In the return path the speed of propagation is $c/(1- v/c)$. ……. (3.4)

On the onward path, two peaks at (0, 0) and (λ, 0) in the isotropic frame K will be observed at (0, 0) and ($\gamma\lambda$, 0) by substituting x = 0 and x = λ and t = 0 in the above equations. Thus the observed wave length is $\lambda' = \gamma\lambda$.

Observed frequency = speed of propagation/wavelength $\omega' = c/[(1+ v/c)\,\gamma\lambda]$ ….. (3.5)

For any point (x, t) in the isotropic frame, the phase of the wave is

$(x/\lambda) - \omega t = (x - ct)/\lambda$ since ω, the frequency is c/λ.

The same is translated by the Tangherlini transformations to

$(x'/\lambda') - \omega' t' = (x - vt)\gamma/\gamma\lambda - c\ (t/\gamma)/[\,(1+ v/c\,)\,\gamma\lambda]$ …….. (3.6)

This equation, when simplified, by noting that $\gamma = 1/\sqrt{1- v^2/c^2}$ gives the same phase for the wave front as in the isotropic frame, that is $(x - ct)/\lambda$.

Thus we find that the phase and thus the amplitude observed at any space-time point remains the same under Lorentz, Galilean, and Tangherlini transformations.

4. Phase of the wave at an arbitrary point (x, t) on the propagation path

In this section we derive the phase of the wave at an arbitrary point on the propagation path.

4.1 Phase of the wave at (x, t)

Consider an arbitrary point (x, t) on a light wave's propagation path.
The equation of the light wave propagation is

Amplitude $A(x,t) = A_0 \, Sin\, 2\pi\, [(x/\lambda) - \omega t]$, where ω = propagation speed/wave length.

For the isotropic frame, the propagation speed is c, and therefore

Amplitude $A(x,t) = A_0 \, Sin\, 2\pi\, [(x/\lambda) - (c/\lambda)\, t] = A_0 \, Sin\, 2\pi \left(\frac{x-ct}{\lambda}\right)$ (4.1)

4.2 Galilean transformation

The point (x, t) transforms to $x' = (x - vt), t' = t$

Amplitude $A\,(x', t') = A_0 \, Sin\, 2\pi[\left(\frac{x'}{\lambda'}\right) - \omega' t]$ ……… (4.2)

In this case wavelength remains same ($\lambda' = \lambda$) and speed of propagation is $(c - v)$ for the receding wave.

Frequency = propagation speed/wavelength = $(c - v)/\lambda$. ….. (4.3)

The value of t remains unaltered in Galilean transformation.

Therefore, amplitude $A\,(x, t) = A_0 \, Sin\, 2\pi\, [\left(\frac{x-vt}{\lambda}\right) - \left(\frac{c-v}{\lambda}\right) t]$. …. (4.4)

After cancelling vt/λ we get the phase of the wave to be the same as equation (4.1).

4.3 Lorentz transformation

Here due to Doppler effect λ becomes $\lambda' = \lambda \sqrt{\frac{1+\left(\frac{v}{c}\right)}{1-\frac{v}{c}}} = \lambda \sqrt{\frac{1+\beta}{1-\beta}}$ where $\beta = v/c$ …… (4.5)

The propagation speed remains c.

Frequency $\omega' = \frac{c}{\lambda'} = \omega\sqrt{\frac{1-\beta}{1+\beta}}$ whereby $\lambda\omega = \lambda'\omega' = c$ …… (4.6)

After transforming *(x, t)* to *(x - vt) γ*, $(t - vx/c^2)\,\gamma$, we find

Amplitude = $A_0\ Sin\ 2\pi$ [{ $(x - vt)\gamma/\lambda'\}$ − $(c/\lambda')\{\ (t - vx/c^2)\gamma\}]$

Substituting for $\lambda'\ as\ \lambda\sqrt{\frac{1+\beta}{1-\beta}}$ and grouping the *x* and *t* terms we get

Amplitude= $A_0 Sin\ 2\pi\left\{\left[\ x(1+\ v/c)\,\gamma/(\lambda\sqrt{\frac{1+\beta}{1-\beta}}\,)\right] - (ct/\lambda)\ (\ 1+\ v/c)\ \gamma/\sqrt{\frac{1+\beta}{1-\beta}}\right\}$ …(4.7)

Recognizing that $\gamma = \frac{1}{\sqrt{(1-\beta)(1+\beta)}}$ and β = *v*/c, and after algebraic simplifications we get

Amplitude = $A_0\ Sin\ 2\pi\left(\frac{x-ct}{\lambda}\right)$ which is the same as equation (4.1)

4.4 Tangherlini transformation

For Tangherlini transformation, the onward speed of propagation is $c^2/(c + v)$ and the return speed is $c^2/(c - v)$, making the round trip speed of propagation of light to be *c*. The harmonic mean of $c^2/(c + v)$ and $c^2/(c - v)$ that is

$$\frac{1}{\frac{1}{2}\left(\frac{c+v}{c^2}+\frac{c-v}{c^2}\right)} = c$$ ……… (4.8)

The wavelength is increased by a factor γ due to contraction of rulers by the factor $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

Therefore, wavelength = λγ. ……. (4.9)

In Tangherlini transformation, moving clocks run slow by a factor of γ and there is no asynchronization between the frames. Thus $t' = t/\gamma$.

Frequency = propagation speed/ wavelength = $(c^2/(c+v))/(\lambda\gamma) = (c/\lambda)\sqrt{\frac{1-v/c}{1+v/c}}$

Phase = $((x - vt)\ \gamma/(\lambda\gamma)) - ((c/\lambda)\sqrt{\frac{1-v/c}{1+v/c}}\ t')$ ………. (4.10)

Substituting $t' = t/\gamma$ and noting that (1/γ) = $\sqrt{\left(1-\frac{v}{c}\right)\left(1+\frac{v}{c}\right)}$,

and after simplification, the above phase is equal to *(x - ct)/λ*, the same as equation (4.1).

Therefore the phase of the wave remains unaltered in all the three cases for the point *(x, t)*, original, transformed by Galilean, Lorentz, or Tangherlini.

For the details of the transformations within the moving frame, see appendix as well as section 1.1

5. General transformation of event coordinates within an inertial frame

A general linear transformation of event coordinates is

$X = ax - bt$

$T = px + qt$

X may be rewritten as $a\big(x - (bt/a)\big)$.

So whenever X = 0, x = *bt/a*, making the origin of (X, T) coordinate system move at a speed of *(b/a)* as observed by the (x, t) coordinate system.

Thus if we require a general transformation that has zero relative speeds, that is within a reference frame, then it becomes

$X = ax$ (5.1)

$T = px + qt$ (5.2)

We can call the (x, t) coordinates as A and the (X, T) coordinates as B, (instead of using K and K') emphasizing that there is no movement between the system of coordinates A and B. But there is relativity of simultaneity between A and B when p is not equal to zero.

Let us assume that in the coordinate system A, light waves are propagating at speed *c* with wave length λ and frequency ω, such that λ ω = *c*.

Let us consider a peak that leaves from (0, 0) and another peak that is at (λ, 0) in coordinate system A. The peak at (λ, 0) is propagating with coordinates

$(\lambda + \Delta x, \Delta x/c)$

When transformed to coordinate system B, this propagation is transformed by equations (5.1) and (5.2) above to

$\Lambda = a(\lambda + \Delta x)$ and $0 = p(\lambda + \Delta x) + (q/c)\,\Delta x$ as we require the distance between two peaks at the same instant in coordinate system B, for determining Λ, the wave length in coordinate system B.

From $p\,(\lambda + \Delta x) + (q/c)\,\Delta x = 0$, we get $\Delta x = -p\lambda/(p + (q/c))$

And therefore $\Lambda = a\lambda\left(1 - \frac{p}{p+\frac{q}{c}}\right) = \frac{a\lambda q}{pc+q}$

When $p = 0$, the synchronization shift is absent and Λ = *a* λ.

In general $\Lambda = \frac{a\lambda q}{pc+q}$ (5.3)

Speed of propagation in coordinate system B will be *(ct, t)* in A goes to *(act, (pct + qt))* in B from equations (5.1) and (5.2)

Thus the speed of propagation in B = *ac/(pc+q)*

Frequency in B = speed/ wavelength = *ac/ (pc+q) /* Λ

= *ac/ (pc+q)* / ($\frac{a\lambda q}{pc+q}$); Cancelling *(pc+q)* and *a* we get

= *(c/λq)*

= ω/q

Therefore the frequency observed by coordinate system B is the frequency observed by coordinate system A divided by *q*.

That is Ω = ω/q (5.4)

5.1 Phase and Amplitude of the propagating wave as observed by the systems A and B

With the standard wave propagation equation the phase of the wave at *(x, t)* in the coordinate system A is *(x/λ) – ωt.* When *c* is the propagation speed as determined by A, this becomes

Phase difference (between location (0, 0) and (x, t)) = *(x - ct)/λ*

When transformed to coordinate system B, it becomes

Phase of wave in system B = (X/Λ) – ΩT ………… (5.5)

Substituting from equations (5.1) and (5.2) for X and T and noting that Ω = ω/q from equation (5.4),

Phase of wave in system B = $(ax/\Lambda) - (\omega/q)(px + qt)$

Substituting for Λ as $\frac{a\lambda q}{pc+q}$ from equation (5.3),

Phase of the wave = $\left(ax/(\frac{a\lambda q}{pc+q})\right) - ((\omega/q)\ (px + qt))$

Noting that ω = c/λ,

Phase of the wave = $\frac{x(pc+q)}{\lambda q} - \frac{c(px+qt)}{\lambda q}$ ………… (5.6)

After simplifying, we obtain

Phase of the wave = *(x - ct)/λ* as observed by coordinate system B as well as in coordinate system A.

Thus the phase of the wave observed by both the coordinate systems A and B are identical and equal to *(x - ct)/λ*.

In other words, calibration differences in rulers and clocks and distant synchronization differences (relativity of simultaneity) leave the observed phase and amplitude of the wave unaltered when an inertial reference frame is using two coordinate systems A and B with no relative motion between A and B. Table 2 summarizes all the conclusions reached so far.

Table – 2 Wavelength, Speed of Propagation and Frequency

Sl Number of Parameter		Coordinate system A (x, t)	Coordinate system B (X,T)
1	Wave length	λ	$\frac{a\lambda q}{pc+q} = \Lambda$
2	Frequency	ω	$\Omega = \omega/q$
3	Speed of propagation	$c = \lambda\omega$	$(ac)/(pc + q) = c^*$
4	Phase at x, t	$(x - ct)/ \lambda$	$(X - c^*T) / \Lambda = (x - ct)/ \lambda$

Table 2 Wavelength, Speed of Propagation and Frequency

Remarks $X = ax$ and $T = px + qt$

It may also be noted that the alternate pathways for the decomposition of the Lorentz transformation have been discussed in [10] using matrix notation. We shall follow a similar matrix approach in the next section to reiterate the conclusions derived so far.

In a simple example, when we consider a uni-dimensional space that is x axis, If the propagation speed of a particle or wave is s_1 along the forward direction and s_2 along the return direction, s_1 not being equal to s_2, the propagation speed can be made equal to a hypothetical value s in the following method.

The average round trip speed of the movement is the harmonic mean of s_1 and s_2 that is

Round trip speed = $2s_1s_2/(s_1+s_2)$

The ratio of the above speed with the hypothetical target speed is $s(s_1+s_2) / 2s_1s_2$

By recalibrating both rulers and clocks by a factor

$f = [s(s_1+s_2) / 2s_1s_2]^{1/2}$ …………. (5.7)

the round trip speed can be increased to s. Without going into details one can see that by altering the synchronisation of spatially separated clocks, the onward and return speeds can be made equal, thus realising (apparent) isotropy. Such changes in the calibration of rulers, the clocks and the synchronisation scheme does not alter the characteristics of the events happening in the universe, within the inertial reference frame or elsewhere.

When $s_1 = c+v$ and $s_2 = c-v$, this whole scheme reduces to the special theory of relativity, whence the factor f in equation 5.7 becomes $\gamma = 1/[1-(v^2/c^2)]^{1/2}$. The modifications to the coordinate system made by the factor f (or γ in the special relativity theory) in equation 5.7 are calibration related and thus external to the events and do not modify the characteristics observed about the events.

5.2 Extension to light wave traveling along a line in a plane

Consider a wave propagating at angle θ to x axis in the (x, y, t) coordinate system.
The same wave propagates at angle θ' to X axis in the (X, Y, T) coordinate system

Let the transformation between the two systems be as earlier with the addition Y = y

$X = ax$ ………. (5.8)
$Y = y$ .... (5.9)
$T = px + qt$ ……..(5.10)

with no movement between the two systems.

Table 3 The various connections between the two systems.

Sl No	Property	x, y, t system	X, Y, T system	Remarks
1	Coordinates	x, y, t	X = ax Y = y T = px + qt	
2	Cylindrical Coordinates	r cos θ, r sin θ , t	R cos θ', R sin θ', T	
3	Relation between r, R	r	$R=r\sqrt{(a^2 cos^2\theta + sin^2\theta)}$	
4	Angle of propagation (aberration)	θ	Cos θ' = $\frac{a\cos\theta}{\sqrt{(a^2 cos^2\theta + sin^2\theta)}}$	With x, X axis
5	Sin θ and Sin θ'		Sin θ' = $\frac{Sin\,\theta}{\sqrt{(a^2 cos^2\theta + sin^2\theta)}}$	
6	Speed of Propagation	c	$\frac{c\sqrt{(a^2 cos^2\theta + sin^2\theta)}}{(pc\cos\theta + q)}$	= C
7	Wave Length	λ	$\frac{\lambda q\sqrt{(a^2 cos^2\theta + sin^2\theta)}}{(pc\cos\theta + q)}$	= Λ By considering instant in X, Y, T
8	Phase at any location	(r-ct)/λ	(R – CT) / Λ	Both are same as can be shown by algebraic simplification. **See derivation below this table.**

Table 3 The various connections between the two systems.

<u>Phase calculations in the X,Y,T system</u> = (R – CT) / Λ

= $\{r\sqrt{(a^2 cos^2\theta + sin^2\theta)}$ - $\frac{c\sqrt{(a^2 cos^2\theta+ sin^2\theta)}}{(pc\cos\theta+q)}$ (px+qt) } / $\{\frac{\lambda q\sqrt{(a^2 cos^2\theta+ sin^2\theta)}}{(pc\cos\theta+q)}\}$ …… (5.11)

$= r\sqrt{(a^2 cos^2\theta + \ sin^2 \theta)}$ (pc cos θ +q) / $\lambda q\sqrt{(a^2 cos^2 \theta + \ sin^2 \theta)}$ ……….. (5.12)

- $c\sqrt{(a^2 cos^2 \theta + \ sin^2 \theta)}$ (px+qt) / $\lambda q\sqrt{(a^2 cos^2 \theta + \ sin^2 \theta)}$ …………….......(5.13)

$= r\sqrt{(a^2 cos^2\theta + \ sin^2 \theta)}$ (pc cos θ +q) / $\lambda q\sqrt{(a^2 cos^2 \theta + \ sin^2 \theta)}$ ………...(5.14)

- $c\sqrt{(a^2 cos^2 \theta + \ sin^2 \theta)}$ (pr cos θ +qt) / $\lambda q\sqrt{(a^2 cos^2 \theta + \ sin^2 \theta)}$ …………(5.15)

$= \frac{r(pc\cos\theta + q\ )}{\lambda q} - c\,\frac{pr\cos\theta + qt}{\lambda q}$ ……………. (5.16)

= (r – ct)/ λ …………………. (5.17)

Thus (R – CT) / Λ = (r – ct)/ λ , meaning the phase of the wave is unaffected by the general transformation X = ax ; T = px +qt

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Wavelength

Consider a wave propagating in an arbitrary direction in the x, y plane when transformed to X, Y plane.

Notes: here *c* is referring to speed of light in the x, y, t system and it need not necessarily be the *c* that is the universal constant.

Speed of Propagation in (X, Y, T) system:

A peak starting at (0, 0, 0) and traveling at angle θ with respect to x axis goes to (ct cos θ, ct sin θ, t) after time t in the x, y, t reference frame.

The same events are recorded at (a ct cos θ, ct sin θ, pct cos θ + qt) in X, Y, T reference frame.

Thus the speed of propagation is

$\sqrt{(a^2c^2\ cos^2\theta + \ c^2 sin^2\theta)}/(pc\ cos\theta + q)$

Speed of propagation = $c\sqrt{(a^2 cos^2\theta + \ sin^2\theta)}/(pc\ cos\theta + q)$ ………. (5.18)

Wave length in (X, Y, T) system:

Consider two peaks in the (x, y, t) system at (0, 0, 0) and (λ cos θ, λ sin θ, 0) at instant t = 0.

The second peak propagates by the equation [(λ + Δr) cos θ, (λ + Δr) sin θ, Δr/c)]

The same is transformed in the X, Y, T system as [a (λ + Δr) cos θ, (λ + Δr) sin θ, p (λ + Δr) cos θ + q Δr/c)]

In order to get the location of the second peak at the instant T = 0, we set p (λ + Δr) cos θ + q Δr/c) = 0

Δr = - (p λ cos θ) / (p cos θ +(q/c))

Thus the second peak at instant T = 0 in the (X, Y, T) system becomes

$$X = a(\lambda - \frac{p\lambda \cos\theta}{p\cos\theta + \left(\frac{q}{c}\right)}) \cos\theta$$

$$Y = (\lambda - \frac{p\lambda \cos\theta}{p\cos\theta + \left(\frac{q}{c}\right)}) \sin\theta$$

T = 0

Evaluating $X^2 + Y^2$ and taking the square root, we get

$$\Lambda = (\lambda - \frac{p\lambda \cos\theta}{p\cos\theta + \left(\frac{q}{c}\right)}) \sqrt{a^2\, cos^2\,\theta + sin^2\,\theta}$$

$$= \frac{\lambda q}{c} \frac{1}{p\cos\theta + (\frac{q}{c})} \sqrt{a^2\, cos^2\,\theta + sin^2\,\theta}$$

$$\Lambda = \frac{\lambda q}{pc\cos\theta + q} \sqrt{a^2\, cos^2\,\theta + sin^2\,\theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.19)$$

(as given in sl no 7 in Table 2 in section 5)

6. Transformation between Galilean to Lorentz as a single matrix within K'

Suppose we have a choice in reference frame K' to transform event coordinates of frame K

by Galilean or Lorentz, these two choices can be transformed into one another by a single matrix as detailed below. The leftmost transformation in the equation below transforms a Lorentz transformation (LT) to a Galilean transformation.

$$\begin{pmatrix} 1/\gamma & 0 \\ v\gamma/c^2 & \gamma \end{pmatrix} \begin{pmatrix} \gamma & -v\gamma \\ -v\gamma/c^2 & \gamma \end{pmatrix} = \begin{pmatrix} 1 & -v \\ 0 & 1 \end{pmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6.1)$$

In the same way the leftmost transformation in the equation below transforms a Galilean transformation to a Lorentz transformation.

$$\begin{pmatrix} \gamma & 0 \\ -v\gamma/c^2 & 1/\gamma \end{pmatrix} \begin{pmatrix} 1 & -v \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \gamma & -v\gamma \\ -v\gamma/c^2 & \gamma \end{pmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6.2)$$

and

$$\begin{pmatrix} 1/\gamma & 0 \\ v\gamma/c^2 & \gamma \end{pmatrix} \begin{pmatrix} \gamma & 0 \\ -v\gamma/c^2 & 1/\gamma \end{pmatrix} = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.3)$$

The two matrices listed below, (the first matrix in 6.1 and 6.2) do not result in any movement:

$$\begin{pmatrix} 1/\gamma & 0 \\ v\gamma/c^2 & \gamma \end{pmatrix} \text{ and } \begin{pmatrix} \gamma & 0 \\ -v\gamma/c^2 & 1/\gamma \end{pmatrix} \qquad (6.3)$$

They are only assigning different numbers to spatial and time measurements in the *same inertial reference frame*. They will not change the here and now magnitudes of the amplitudes of propagating waves. These two matrices in (6.1) conform to the analysis given in Section 5 in that there is zero relative speed in both these matrices. With regard to the asynchronization element *p*, as mentioned in Section 5, this may be viewed in the context of the importance of correct synchronization in observing event coordinates as emphasized in [11,12].

7. Interpretation of the Results

As analysed in the previous sections, the Phase of any propagating wave remains same at a particular space-time event under LT (Lorentz Transformation), GT (Galilean Transformation) and TT (Tangherlini Transformation). Two waves w1 and w2 will retain their phases at all space-time points under all three transformations GT, LT and TT; Although the recorded coordinates may vary under the three transformations. As discussed in section 5 (equation 5.7), adjustments in calibration can produce apparent isotropy and alter the magnitude of speed as required. The recorded coordinates are directly related to calibration issues and they do not affect the characteristics or physical measurements (such as the amplitude of the wave) at any event.

Many Physicists [2,4,7,8,9] consider TT to be a 'reasonable' alternative to LT. The difference between TT and GT is only calibration. And that also essentially doesn't change the observation about events. So moving from LT to TT (synchronization change), and further to GT (change in calibration of rulers and clock ticks), is within the same IRF with different calibration and synchronisation convention and these aspects do not alter the observations about events in the universe.

8. Conclusion

The transformations Galilean, Lorentz, and Tangherlini differ in the scale of spatial and temporal measurements and distant clock synchronization convention. The amplitude of the waves are external to these assignments of spatial and temporal coordinates. Although wavelength, frequency, and speed of propagation may differ under these three transformations, the amplitude at a given space-time point remains unaltered.

A general coordinate transformation that describes a calibration and synchronisation change is described below.

$$\begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \quad \ldots\ldots\ldots\ldots\ldots.. \quad (8.1)$$

The three zeros appearing in the last column specify that there is no movement between the two measuring systems. There are only calibration differences and temporal asynchronization in the spatial directions as indicated by the elements a_{41}, a_{42} and a_{43} in the three spatial directions. It is easy to deduce that such a transformation will not alter the phase of a propagating wave when it reaches a spatial location, even though the two systems may assign different spatial and temporal coordinates to that spatial location. Evidently, the inverse of above 4x4 transformation will be of the form

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} & 0 \\ b_{21} & b_{22} & b_{23} & 0 \\ b_{31} & b_{32} & b_{33} & 0 \\ b_{41} & b_{42} & b_{43} & a_{44} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix} \quad \ldots\ldots\ldots\ldots\ldots \quad (8.2)$$

Once again indicating that there is no movement or velocity between the two measurement systems. When the three spatial coordinate axis of measurement systems A and B are aligned (collinear), the non-diagonal elements of the first three rows will be zero. The non-diagonal elements of the fourth row a_{41}, a_{42}, a_{43}, b_{41}, b_{42}, and b_{43} refer to temporal asynchronization along the three spatial axis. The diagonal elements refer to the calibration scale differences along the three spatial and one temporal axis. Although the two sets of observers may differ in the spatial coordinates and temporal recordings of a given event, the characteristics of the event will be identical as observed by the two sets of observers. One following the x, y, z, t system and the other following the X, Y, Z, T system with no movement as indicated by the three zeros in the last column of the above two matrices. For example let x, y, z, t be the measurement system (A) in K' transformed by a Galilean transformation from a reference frame K, that is in uniform motion with respect to reference frame K', and let X, Y, Z, T be the measurement system (B) in K', transformed by a Lorentz transformation from K. Thus we have two measurement systems A and B in K'. The coordinates of any event may differ in systems A and B, but the characteristics of the observation about the event will not differ.

For any event observed by both reference frames K and K' where K and $K'$are in relative motion, the observations will retain the same characteristics irrespective of whether we use Galilean, Lorentz or Tangherlini transformations between K and K' to designate the spatial and time coordinates in K'. The event and its characteristics are independent of the transformation we use.

If one makes (deliberately or as a genuine correction) the one meter ruler to read as 1.1 meter that will not change the event characterestics.
Correct or incorrect calibration of rulers and clocks will only affect the measurement of wavelength, propagation speed and frequency. Speed of propagation will be frequency multiplied by wavelength always. All this correctness or incorrectness in measurements will not affect other physical quantities measured at a space-time point, that is independent of any mistakes or genuine corrections one may do or not do in length and time measurements, because the amplitude and such other measurements (unrelated to length measurements) of the event happening at a space-time point.
Students will appreciate that calibration (of rulers and clocks) and synchronization (of distant clocks) issues can alter perception of speed and linear measurements but cannot change the observation about events. The use of GT is more appropriate as it does not produce real or illusory length contraction [12].

Data Availability Statement:

All data generated or analysed during this study are included in this published article that is the manuscript as above.

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Appendix A: Conversion to different calibration systems with the same observed velocity

Lorentz Transformation (LT) $\begin{pmatrix} \gamma & -v\gamma \\ -v\gamma/c^2 & \gamma \end{pmatrix}$

Galilean Transformation (GT) $\begin{pmatrix} 1 & -v \\ 0 & 1 \end{pmatrix}$

Tangherlini Transformation (TT) $\begin{pmatrix} \gamma & -v\gamma \\ 0 & 1/\gamma \end{pmatrix}$

Table – 4 Conversion between differently calibrated systems but in the same IRF

To From	Lorentz	Tangherlini	Galilean
Lorentz	I	$\begin{pmatrix} 1 & 0 \\ v/c^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1/\gamma & 0 \\ v\gamma/c^2 & \gamma \end{pmatrix}$
Tangherlini	$\begin{pmatrix} 1 & 0 \\ -v/c^2 & 1 \end{pmatrix}$	I	$\begin{pmatrix} 1/\gamma & 0 \\ 0 & \gamma \end{pmatrix}$
Galilean	$\begin{pmatrix} \gamma & 0 \\ -v\gamma/c^2 & 1/\gamma \end{pmatrix}$	$\begin{pmatrix} \gamma & 0 \\ 0 & 1/\gamma \end{pmatrix}$	I

Table – 4 Conversion between differently calibrated systems but in the same IRF

Example: To convert Lorentz to Tangherlini, take the matrix in row "Lorentz" and column "Tangherlini" and we get as below

$$\begin{pmatrix} 1 & 0 \\ v/c^2 & 1 \end{pmatrix} \begin{pmatrix} \gamma & -v\gamma \\ -v\gamma/c^2 & \gamma \end{pmatrix} = \begin{pmatrix} \gamma & -v\gamma \\ 0 & 1/\gamma \end{pmatrix}$$

Lorentz Tangherlini

All entries (matrices) in the above table are having zero velocity (element 12 of every matrix is zero), that means they operate within a inertial frame, affecting wavelength, frequency and propagation speed. The amplitude of the propagating wave is unaffected at any spatial location - instance.

Note: Instance is different from instant. Instance is a event occurring at a spatial location may be at different instants (in different measurement systems in the same reference frame). The event and its characteristics are unaffected by the coordinates assigned by different measurement systems within the same inertial reference frame. No velocity is involved between the measurement systems viz Lorentz, Galilean and tangherlini as indicated by the null element 12 in all the above matrices.