Calculation of Electric Field Intensity at Small Distances Between a Charge and a Source of Electric Field Based on a Macroscopic Mathematical Model

Vadim Khoruzhenko

August 23, 2024

Abstract

My work represents a macroscopic generalization of the Kaluza-Klein theories for quantum systems, concerning the existence of an additional fifth dimension in our space. This allowed Kaluza to unify the descriptions of gravity and electromagnetism within a single mathematical structure. Later, Oskar Klein added a crucial element to Kaluza's theory by proposing that the fifth dimension is compactified into a very small circle with a radius on the order of the Planck length. Due to its minuteness, this dimension remains invisible at ordinary scales. Additionally, I drew upon ideas proposed by Feynman, who suggested using the concept of energy density in space to describe electromagnetic interactions at the quantum level within the framework of Quantum Electrodynamics (QED). This theory helps explain how, at short distances, charge screening and renormalization occur due to interactions with virtual particles in the vacuum.

In my work, I attempt to describe the observable effects at the macroscopic level, such as the electric interaction of charges and the gravitational interaction of massive bodies, through an intuitively understandable process of spatial disturbance. This disturbance is first expressed in the disruption of the uniform distribution of spatial density and then in the curvature of the spherically symmetric structure of the disturbance. In formulating my theory at a macroscopic level, which implies a statistical averaging of the processes described by Kaluza, Klein, and Feynman at the quantum level, I was primarily inspired by the ideas proposed by these scientists. I aimed to formalize their mathematical concepts into simple and intuitively understandable postulates, which lead to observable effects in macro-objects. One such effect is the attenuation of the electric field at short distances between charges, similar to the processes occurring at the quantum level. Based on innovative theoretical insights, this work provides yet another alternative confirmation of the Theory of Relativity regarding the equivalence of energy and mass of elementary particles.

I Calculation of the Electric Field Based on the Macroscopic Space Density Model for Distances Between Charges Comparable to the Size of the Charges: Renormalization and Screening of Charges

To demonstrate the effectiveness of the proposed method for calculating the electric field, I will show that, using the concepts presented in this theory regarding the perturbation of an additional dimension of our space, which can be interpreted as the space density, I will easily predict such well-known effects in the framework of QED as renormalization and the screening effect of electric charges at small distances.

As a reminder, Richard Feynman, in his research on the interaction of elementary charges at the quantum level, studied the phenomenon of deviation in the interaction of electrons at small distances, comparable to the size of the classical electron, from what is predicted by Coulomb's law:

1. In one of Feynman's papers, he describes how the renormalization of charge and mass eliminates divergences arising in QED at higher orders of radiative corrections in the S-matrix. These processes are especially important for making the theory predictive and stable. Feynman notes that despite the elimination of divergences, the interaction between electrons at very short distances is enhanced compared to what is predicted by classical theory. This difference is caused by vacuum polarization, which leads to screening of the real charge at large distances but to its enhancement at short distances [1].

2. In his work on quantum electrodynamics, Feynman discusses how renormalization eliminates the need for counterterms for charge and mass, which are typically used to remove divergences. In this context, it is discussed how at small distances, the screening effect leads to significant changes in the charge and field behavior, which is significantly different from the predictions of classical electrodynamics [2].

3. In one of the key papers on quantum electrodynamics, Feynman describes how the interaction between electrons at short distances becomes stronger due to charge renormalization, which is caused by vacuum polarization. The screening effect that occurs at large distances weakens here, and the interaction becomes more intense. Feynman emphasizes that this enhancement of interaction at short distances is confirmed by S-matrix calculations and is critically important for an accurate description of electromagnetic interaction on small scales [3].

In my previous article [4], I established and, in my opinion, proved that the electric field strength created by the second sphere on the space density distribution created by the first sphere in a coordinate system associated with the center of the first sphere is determined by the formula:

$$\Delta W_{r_1'}(D) = -2\frac{R_2'\rho_2 V(R_2')}{(R_1' - D)^2}$$

At the same time, according to our assumptions, the space density distribution created by the second sphere does not act directly on the first sphere but through the change in the space density distribution created by the first sphere, compared to the second state of the "hypothetical" universe, when the first sphere was alone and the space density distribution created by it was spherically symmetric relative to the center of this sphere. In other words, the amount of interaction on the first sphere is determined by the effect of the space density distribution created by the first sphere due to the curvature of the spherical symmetry of this distribution by the influence of the space density distribution created by the second sphere in the third state of the "hypothetical" universe.

As can be seen from the presented formula, when $D \approx R_1'$, the value of the field will already differ from the value predicted by Coulomb's law, since our model already takes into account that our spheres are not point charges but have a finite size equal to R_1', which is considered when calculating the change in space density distribution occurring outside the shell of the spheres.

For simplicity of presentation, considering that the interaction of our spheres fully replicates Coulomb's law, which describes the interaction of electric charges, let us refer to our spheres as charges: accordingly, the first charge is the first sphere, and the second charge is the second sphere.

Let us now calculate how the field created by the second charge will act directly on the first charge. We have already found that the amount of interaction equals the integral of the gradient. Let us find the gradient of the space density distribution created by the second charge on the surface of the first sphere and take the integral of the resulting expression over the surface of this first sphere in the coordinate system of the first sphere. Thus, we obtain a formula for the classical interaction when it is assumed that the field of the second charge directly affects the first charge, without considering the curvature of the space density distribution (field) of the first charge by the field of the second charge.

We have the density distribution, found in the previous sections of this article, given by the formula:

$$\Delta\rho_{\text{decrease}}(r) = \frac{Q_2 \cdot R_2'}{4\pi r^4}$$

where r is the distance from the source to an arbitrary point, Q_2 and R_2' are constants.

We need to find the difference in density $\Delta\rho$ on the surface of the sphere.

1.1 Density Difference Inside and on the Surface of the Sphere

The density difference on the surface of a sphere of radius R_1' and inside it, as we defined when setting the problem, is given by the formula:

$$\Delta\rho = \rho_{\text{outside}}(r) - \rho_{\text{inside}} = \Delta\rho_{\text{decrease}}(r) - \rho_1$$

where $\rho_{\text{outside}}(r) = \Delta\rho_{\text{decrease}}(r)$.

1.2 Formula for the Space Density Distribution of the Second Charge on the Surface of the First Charge

To calculate the gradient of the density on the surface of the first sphere (first charge), we first find the density's dependence on the angle θ, using the formula for the distance on a sphere located at a distance D from the field source. The formula for the distance from the point to the source, obtained using the law of cosines, is well-known in geometry and does not require additional proof:

$$r(\theta) = \sqrt{R_1'^2 + D^2 - 2R_1' D \cos\theta}$$

Now we substitute this expression for $r(\theta)$ into the formula for the density $\Delta\rho_{\text{decrease}}(r)$:

$$\rho(\theta) = \frac{Q_2 \cdot R_2'}{4\pi \left(R_1'^2 + D^2 - 2R_1' D \cos\theta\right)^2}$$

1.3 Calculating the Gradient of Density on the Surface of the First Charge Created by the Second Charge

The density gradient on the surface of the sphere is defined as the derivative with respect to the angle θ:

$$\frac{d\rho(\theta)}{d\theta}$$

To calculate the density gradient with respect to the angle θ, we start with the expression for the density:

$$\rho(\theta) = \frac{Q_2 \cdot R_2'}{4\pi \left(R_1'^2 + D^2 - 2R_1' D \cos\theta\right)^2}$$

The gradient of density with respect to the angle θ will be equal to the derivative of this function with respect to θ:

$$\nabla_\theta \rho(\theta) = \frac{d\rho(\theta)}{d\theta}$$

Derivative of the Density with Respect to the Angle:

Let's start by writing the derivative with respect to θ of the expression $\rho(\theta)$. We have a composite function of the form $f(g(\theta))$, where $g(\theta) = R_1'^2 + D^2 - 2R_1' D \cos\theta$.

Applying the chain rule, the derivative of the function $\rho(\theta)$ will be:

$$\frac{d\rho(\theta)}{d\theta} = \frac{d}{d\theta}\left(\frac{Q_2 \cdot R_2'}{4\pi \left(g(\theta)\right)^2}\right) = \frac{Q_2 \cdot R_2'}{4\pi} \cdot \frac{d}{d\theta}\left(\frac{1}{\left(g(\theta)\right)^2}\right)$$

The derivative of $\frac{1}{(g(\theta))^2}$ with respect to $g(\theta)$ will be:

$$\frac{d}{dg(\theta)}\left(\frac{1}{(g(\theta))^2}\right) = -\frac{2}{(g(\theta))^3}$$

Now the derivative of $g(\theta)$ with respect to θ:

$$\frac{dg(\theta)}{d\theta} = \frac{d}{d\theta}\left(R_1'^2 + D^2 - 2R_1' D \cos\theta\right) = 2R_1' D \sin\theta$$

Finally, substituting these into the gradient:

$$\frac{d\rho(\theta)}{d\theta} = \frac{Q_2 \cdot R_2'}{4\pi} \cdot \left(-\frac{2}{(g(\theta))^3}\right) \cdot (2R_1' D \sin\theta)$$

Simplifying, we get:

$$\frac{d\rho(\theta)}{d\theta} = -\frac{Q_2 \cdot R_2'}{2\pi} \cdot \frac{R_1' D \sin\theta}{\left(R_1'^2 + D^2 - 2R_1' D \cos\theta\right)^3}$$

Now, we take the integral of this gradient over the entire surface of the first sphere to obtain the electric field acting on the first charge:

$$\text{Integral of the gradient:} \quad \int_0^{2\pi} \int_0^{\pi} \frac{d\rho(\theta)}{d\theta} \sin\theta \, d\theta \, d\phi$$

This integral gives us the total effect of the electric field, considering the perturbation of the space density distribution caused by the interaction of two charges. The resulting formula will describe how the renormalization of charges and screening effects manifest when the distance between charges becomes comparable to their size, which is in agreement with QED predictions.

1.4 Consider Each Integral Separately

1.4.1 Solution of the First Integral

Consider the first integral:

$$I_1 = \int_{-1}^{1} \frac{du}{\left(R_1'^2 + D^2 - 2R_1' D u\right)^3}$$

To solve it, we use the substitution:

$$v = R_1'^2 + D^2 - 2R_1' D u, \quad dv = -2R_1' D \, du$$

The limits of integration will change as follows: - When $u = -1$, $v = (R_1' - D)^2$. - When $u = 1$, $v = (R_1' + D)^2$.

Thus, the integral I_1 can be rewritten as:

$$I_1 = \frac{1}{2R_1' D} \int_{(R_1'+D)^2}^{(R_1'-D)^2} \frac{dv}{v^3}$$

This integral can be evaluated:

$$I_1 = \frac{1}{2R_1' D} \left[-\frac{1}{2v^2} \right]_{(R_1'+D)^2}^{(R_1'-D)^2}$$

Substitute the limits of integration:

$$I_1 = \frac{1}{4R_1' D} \left[\frac{1}{(R_1' - D)^4} - \frac{1}{(R_1' + D)^4} \right]$$

1.4.2 Solution of the Second Integral

Consider the second integral:

$$I_2 = \int_{-1}^{1} \frac{u^2\, du}{\left(R_1'^2 + D^2 - 2R_1' D u\right)^3}$$

To simplify the integral, we first make a substitution.
Use the substitution:

$$v = R_1'^2 + D^2 - 2R_1' D u$$

Then:

$$du = -\frac{dv}{2R_1' D}$$

Now express u in terms of v:

$$u = \frac{R_1'^2 + D^2 - v}{2R_1' D}$$

Changing the Limits of Integration
When $u = -1$:

$$v = R_1'^2 + D^2 + 2R_1' D = (R_1' + D)^2$$

When $u = 1$:

$$v = R_1'^2 + D^2 - 2R_1' D = (R_1' - D)^2$$

Thus, the limits of integration change from $v = (R_1' + D)^2$ to $v = (R_1' - D)^2$.
Express the integral in terms of the new variable and substitute everything into the integral:

$$I_2 = \int_{(R_1'+D)^2}^{(R_1'-D)^2} \frac{\left(\frac{R_1'^2 + D^2 - v}{2R_1' D}\right)^2 \cdot \left(-\frac{dv}{2R_1' D}\right)}{v^3}$$

Factor out the constant multipliers:

$$I_2 = -\frac{1}{8R_1'^3 D^3} \int_{(R_1'+D)^2}^{(R_1'-D)^2} \frac{(R_1'^2 + D^2 - v)^2}{v^3}\, dv$$

Now expand the square in the numerator:

$$I_2 = -\frac{1}{8R_1'^3 D^3} \int_{(R_1'+D)^2}^{(R_1'-D)^2} \frac{R_1'^4 + 2R_1'^2 D^2 + D^4 - 2(R_1'^2 + D^2)v + v^2}{v^3}\, dv$$

1.4.3 Decomposition into Three Integrals

Divide this integral into three separate integrals:

$$I_2 = -\frac{1}{8R_1'^3 D^3}\left[\int_{(R_1'+D)^2}^{(R_1'-D)^2} \frac{R_1'^4 + 2R_1'^2 D^2 + D^4}{v^3}\,dv - \right.$$
$$\left. - 2\int_{(R_1'+D)^2}^{(R_1'-D)^2} \frac{R_1'^2 + D^2}{v^2}\,dv + \int_{(R_1'+D)^2}^{(R_1'-D)^2} \frac{dv}{v}\right]$$

Now solve each of these integrals separately.

A. First Integral:

$$I_{21} = \int_{(R_1'+D)^2}^{(R_1'-D)^2} \frac{R_1'^4 + 2R_1'^2 D^2 + D^4}{v^3}\,dv$$

Factor out the constants from the integral:

$$I_{21} = (R_1'^4 + 2R_1'^2 D^2 + D^4)\int_{(R_1'+D)^2}^{(R_1'-D)^2} v^{-3}\,dv$$

Evaluate the integral:

$$I_{21} = (R_1'^4 + 2R_1'^2 D^2 + D^4)\left[-\frac{1}{2v^2}\right]_{(R_1'+D)^2}^{(R_1'-D)^2}$$

Substitute the limits:

$$I_{21} = -\frac{R_1'^4 + 2R_1'^2 D^2 + D^4}{2}\left[\frac{1}{(R_1' - D)^4} - \frac{1}{(R_1' + D)^4}\right]$$

B. Second Integral:

$$I_{22} = \int_{(R_1'+D)^2}^{(R_1'-D)^2} \frac{R_1'^2 + D^2}{v^2}\,dv$$

Factor out the constants from the integral:

$$I_{22} = (R_1'^2 + D^2)\int_{(R_1'+D)^2}^{(R_1'-D)^2} v^{-2}\,dv$$

Evaluate the integral:

$$I_{22} = (R_1'^2 + D^2)\left[-\frac{1}{v}\right]_{(R_1'+D)^2}^{(R_1'-D)^2}$$

Substitute the limits:

$$I_{22} = -(R_1'^2 + D^2)\left[\frac{1}{(R_1' - D)^2} - \frac{1}{(R_1' + D)^2}\right]$$

C. Third Integral:

$$I_{23} = \int_{(R_1'+D)^2}^{(R_1'-D)^2} \frac{dv}{v}$$

Evaluate the integral:

$$I_{23} = \ln\left[\frac{(R_1' - D)^2}{(R_1' + D)^2}\right] = 2\ln\left[\frac{R_1' - D}{R_1' + D}\right]$$

Substitute all three integrals into the expression and obtain the final expression for I_2:

$$I_2 = -\frac{1}{8R_1'^3 D^3}\Bigg\{ -\frac{R_1'^4 + 2R_1'^2 D^2 + D^4}{2}\left[\frac{1}{(R_1' - D)^4} - \frac{1}{(R_1' + D)^4}\right] - \\ - 2(R_1'^2 + D^2)\left[\frac{1}{(R_1' - D)^2} - \frac{1}{(R_1' + D)^2}\right] + 2\ln\left[\frac{R_1' - D}{R_1' + D}\right]\Bigg\}$$

Now let's simplify our expression for the integral I_2, which is:

$$I_2 = \frac{1}{2{R_1}^2 D^2} \cdot \frac{({R_1}^2 + D^2)^2}{(R^2 - D^2)^4} - \frac{1}{{R_1}^2 D^2} \cdot \frac{({R_1}^2 + D^2)}{({R_1}^2 - D^2)^3} + \frac{1}{4{R_1}^3 D^3}\ln\left(\frac{R_1 + D}{R_1 - D}\right)$$

After simplification, we obtain the final expression for the integral of the gradient of the space charge density created by the second charge over the surface of the sphere of the first charge:

$$\int_{S(R_1')} \nabla_\theta \rho(\theta)\, dS = -4Q_2 \cdot R_2' \cdot R_1'^3 D \int_0^\pi \frac{\sin^2\theta\, d\theta}{(R_1'^2 + D^2 - 2R_1' D\cos\theta)^3} = I_1 - I_2,$$

The solution for the first integral is:

$$I_1 = \frac{1}{4R_1' D}\left[\frac{1}{(R_1' - D)^4} - \frac{1}{(R_1' + D)^4}\right]$$

The solution for the second integral is:

$$I_2 = \frac{1}{2R_1'^2 D^2} \cdot \frac{(R_1'^2 + D^2)^2}{(R_1'^2 - D^2)^4} - \frac{1}{R_1'^2 D^2} \cdot \frac{(R_1'^2 + D^2)}{(R_1'^2 - D^2)^3} + \frac{1}{4R_1'^3 D^3}\ln\left(\frac{R_1' + D}{R_1' - D}\right)$$

If we don't factor out the exponent from the natural logarithm, we get the following expression:

$$\frac{1}{8R_1'^3D^3}\left[\frac{4R_1'D\left((R_1'^2+D^2)^2-2(R_1'^2+D^2)\cdot(R_1'^2-D^2)^2\right)}{(R_1'^2-D^2)^4}+\ln\left(\frac{(R_1'+D)^2}{(R_1'-D)^2}\right)\right]$$

1.5 Numerical Investigation of the Integral Expression

We will investigate the obtained expression for the integral numerically. To ensure that the values of the integral do not enter the imaginary part, we will keep the expression for both the numerator and the denominator of the logarithmic term squared. The fact that for $D < R_1'$ the values of the logarithm enter the imaginary part suggests that interactions in this region might be occurring in an additional hidden dimension.

Here is an interesting result we obtained, which, in my opinion, fully corresponds to what was obtained at the quantum level for two charges at small distances within the framework of quantum electrodynamics:

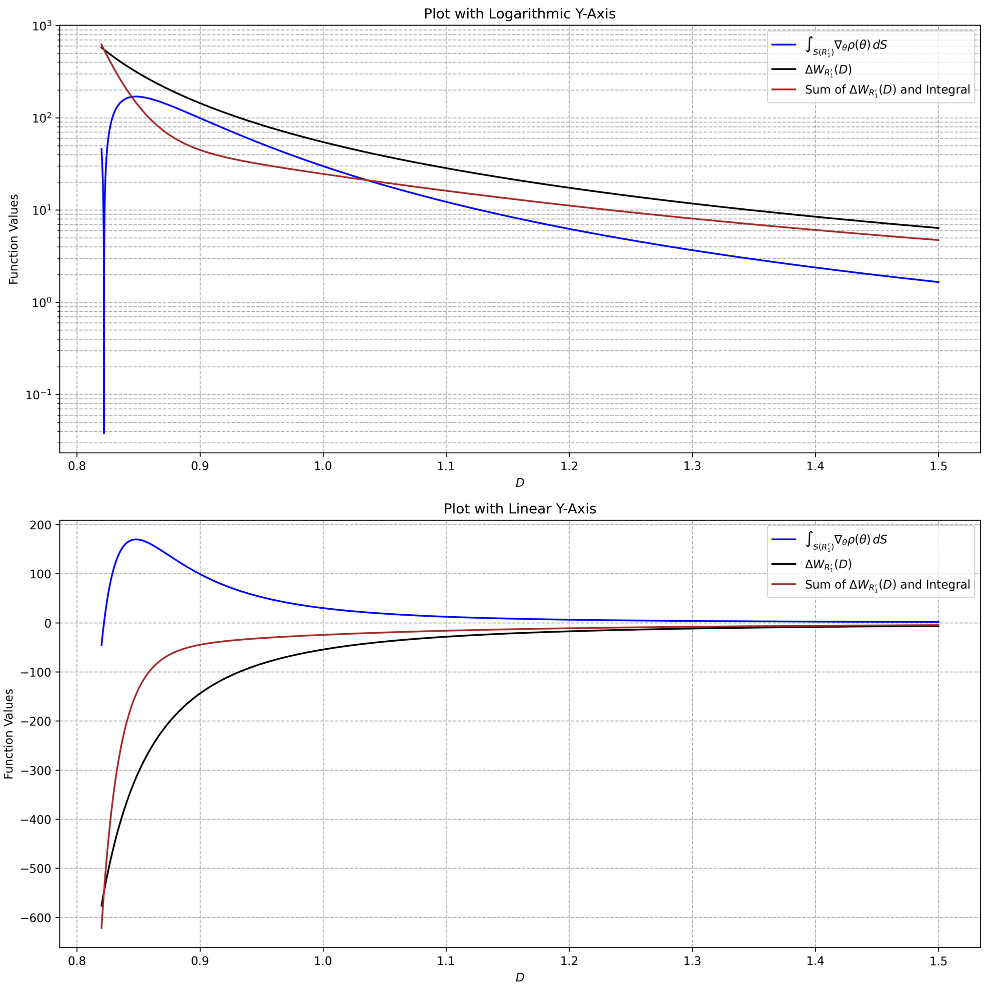

Figure 1: Graphs of the interaction quantity through the distortion of spherical symmetry of the field and through the direct interaction of the charge with the field of the second charge on the surface of the first charge's sphere

For comparison, here is how the electric field intensity graphs would look if we use the classical Coulomb's law without correction for point charges, not considering their size and geometric shape:

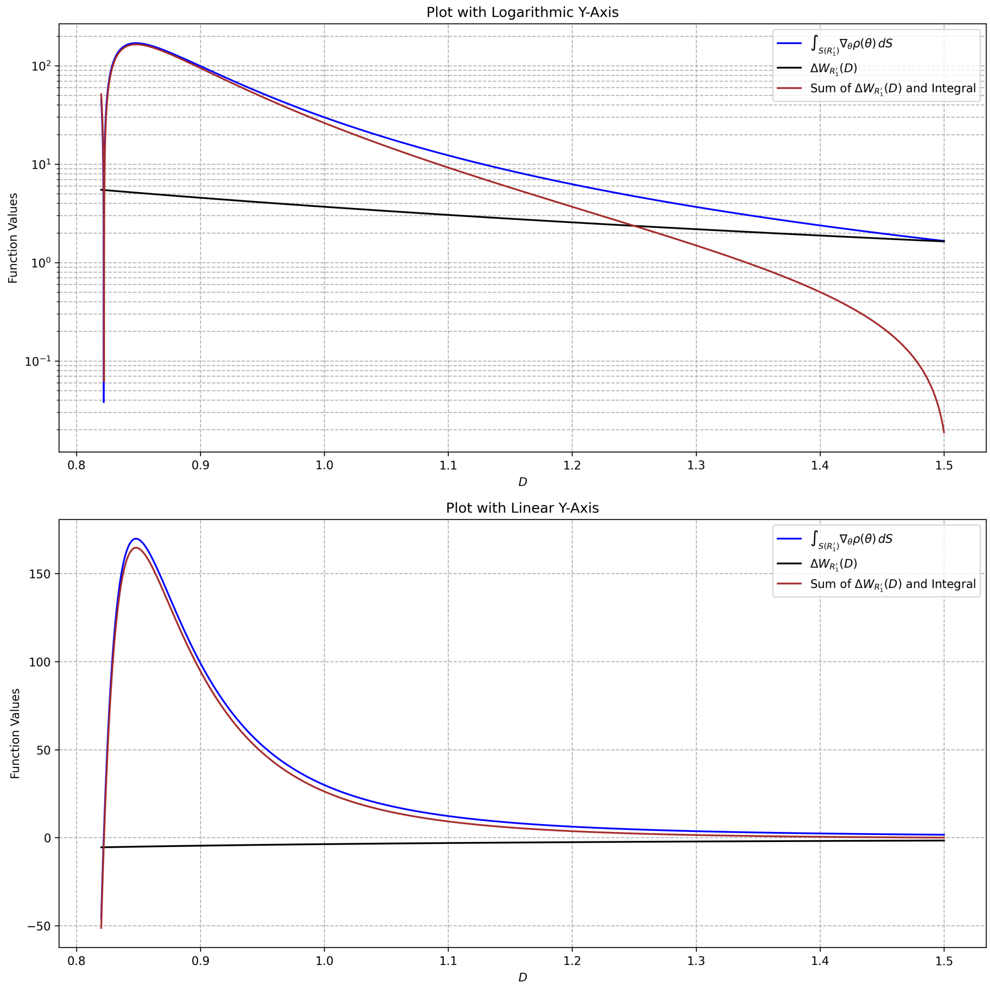

Figure 2: Graphs of the interaction quantity through Coulomb's law, with consideration for the interaction of the surface area of the first charge's sphere with the field of the second charge

We see that as the first charge approaches the field source, the field intensity starts to decrease, while the integral value of the gradient of the spatial density distribution over the surface of the first charge's sphere begins to increase at distances on the order of 5 times the size of the first sphere. As the field source approaches the surface of the first sphere, the expression we obtained for the integral of the gradient over the surface of the first charge starts to decrease sharply, and this value quickly moves into the negative range. At relatively small distances of the field source from the surface of the sphere, the value of the integral of the gradient over the surface of the sphere becomes greater in magnitude than the value for the amount of disturbance created by the second charge on the field of the first charge, meaning that at small distances the field starts to increase sharply, as predicted by quantum electrodynamics (QED) and confirmed by conducted experiments.

Thus, I have clearly shown that the proposed theory has a very wide range of applicability: it can correctly describe the behavior of the electromagnetic field at distances comparable to the size of the classical electron. The theory also correctly describes the interaction of charges as described by Coulomb's law at large distances.

Using the formula we obtained for the amount of energy required to compress a sphere — creating an electric charge, which we interpreted as the mass of the charge, one can derive equations for the distribution of the curvature coefficient of the space metric and thus obtain the gravitational equations.

In my opinion, the theory I proposed and the approach used in it are the missing link that will connect processes, both gravitational and electromagnetic, occurring at the quantum level with processes occurring at the macroscopic level. It can be said that the proposed theory is a statistical averaging of quantum mechanics at the macroscopic level, as it correctly accounts for the field effects associated with the sizes of the charges creating the field, the distribution of this field, and the interaction of charges both through direct interaction with the field and through the distortion of the field distribution created by each charge.

References

[1] R. P. Feynman, "The S Matrix in Quantum Electrodynamics," *Physical Review*, vol. 75, no. 11, pp. 1736–1753, 1949. DOI: 10.1103/PHYSREV.75.1736.

[2] R. P. Feynman, *The Feynman Lectures on Physics*, Addison-Wesley, 1965.

[3] R. P. Feynman, "The S Matrix in Quantum Electrodynamics," *Physical Review*, vol. 75, no. 11, pp. 1736–1753, 1949. DOI: 10.1103/PHYSREV.75.1736.

[4] V. A. Khoruzhenko, "Theoretical Justification of the Relationship Between Charge and Mass. Derivation of the Charge Internal Energy Equation." 2024 https://vixra.org/abs/2408.0053