# 3d Quantum Gravity, Localization and Particles Beyond Standard Model

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**Abstract**

We review a 3d quantum gravity model, which incorporates massive spinning fields into the Euclidean path integral in a Chern-Simons formulation. Fundamental matter as defined in our previous preon model is recapped. Both quantum gravity and the particle model are shown to be derivable from the supersymmetric 3d Chern-Simons action. Forces–Matter unification is achieved.

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# Contents



# 1 Introduction

The purpose of this article is to show how a 3d Chern-Simons (CS) quantum gravity model of other authors leads by localization procedure to our CS model of elementary particles beyond the standard model. The vector supermultiplets in both cases are counterparts.

The key element in our model is unbroken supersymmetry which gets "hidden" below a scale $\Lambda_{cr} \sim 10^{10} - 10^{16}$ GeV. This construction differs significantly from the community accepted models like the minimal supersymmetric standard model (MSSM), see the commendable review [1] and references therein. In our scenario, there are no squarks or sleptons. Instead, all superpartners are constituents of quarks and leptons themselves (see tables 1 and 2). Fayet's Forces $\longleftrightarrow$ Matter unification is obtained.

This note is organized as follows. In section 2 the 3d quantum gravity model is briefly reviewed in terms of a path integral, the partition function containing Wilson loops of Chern-Simons theory. Exact all-order and perturbative calculations can be done by introducing supersymmetric localization techniques in the path integral. Our Chern-Simons theory based composite particle model, proposed some time ago, is recapped in section 3. Finally, in section 4 the results are summarized. Two appendices are provided for background information. The nature of the note is phenomenological.

# 2 Chern-Simons Gravity

## 2.1 Wilson Spool

Low-dimensional gravity provides interesting tests of the gravitational path integral. In two and three space-time dimensions, there is no propagating graviton and all of the effective degrees of freedom are long-range. This is what happens in pure Einstein gravity with a cosmological constant (of either sign) as a Chern-Simons gauge theory [2]. A full leveraging of this fact allows the exact evaluation of the gravitational path integral either about a saddle point [3] or as a non-perturbative sum over saddles [4]. While Chern-Simons gravity is not a UV-complete[1] model of quantum gravity [4], its all-loop exactness provides strong tests for potential microscopic models in the spirit of e.g. [5].

This question of gravity-matter coupling is made precise in [6], where massive scalar fields minimally coupled to gravity with a positive cosmological constant are considered. The key result is the expression of the one-loop determinant (or partition function, see appendix A) of a massive scalar field coupled to a background metric, $g_{\mu\nu}$, as a gauge invariant object of the Chern-Simons connections, $A_{L/R}$:

$$Z_{\text{scalar}}[g_{\mu\nu}] = \exp \frac{1}{4}\mathbb{W}[A_L, A_R] \ . \tag{1}$$

The object $\mathbb{W}[A_L, A_R]$, coined the Wilson spool, is a collection of Wilson loop operators wrapped many times around cycles of the base geometry. The equality in (1) is expected to apply to three-dimensional gravity of either sign of cosmological constant.

The importance of (1) is also practical. It was shown in [6] that certain exact methods in Chern-Simons theory (such as Abelianisation [7] (not discussed here) and supersymmetric localization [8]) extend to three-dimensional de Sitter ($dS_3$) Chern-Simons gravity with the Wilson spool inserted into the path integral. This allows a precise and efficient calculation of the quantum gravitational corrections to $Z_{\text{scalar}}$ at any order of perturbation theory of Newton's constant, $G_N$.

The generalization of (1) for massive spinning fields is the following. Consider the local path integral,[2] $Z_{\Delta,\mathsf{s}}$, of a spin-$\mathsf{s}$ field $\Phi_{\mu_1\mu_2\cdots\mu_s}$ with mass

$$m^2/\Lambda = (\Delta + \mathsf{s} - 2)(\mathsf{s} - \Delta) \ , \tag{2}$$

minimally coupled to a metric geometry, $(M_3, g_{M_3})$, where $\Delta$ is the conformal dimension (the eigenvalue of the dilatation operator $D$) and $M_3$ is topologically either Euclidean BTZ [9] or Euclidean $\mathrm{dS}_3$. In [6] it is proposed that

$$\log Z_{\Delta,\mathsf{s}}[g_{M_3}] = \frac{1}{4}\mathbb{W}_{j_L,j_R}[A_L, A_R] \ , \tag{3}$$

[1] A pragmatic attitude is taken here to UV-completeness.

[2] Including any additional Stückelberg fields to fix its invariances and associated ghosts.

where

$$\mathbb{W}_{j_L,j_R}[A_L,A_R] = \frac{i}{2}\int_{\mathcal{C}} \frac{\mathrm{d}\alpha}{\alpha}\frac{\cos\left(\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}\left(1+\mathsf{s}^2\sin^2\left(\frac{\alpha}{2}\right)\right)\times \\ \sum_{\mathsf{R}_L\otimes\mathsf{R}_R}\mathrm{Tr}_{\mathsf{R}_L}\left(\mathcal{P}e^{\frac{\alpha}{2\pi}\oint_\gamma A_L}\right)\mathrm{Tr}_{\mathsf{R}_R}\left(\mathcal{P}e^{-\frac{\alpha}{2\pi}\oint_\gamma A_R}\right) \ . \tag{4}$$

In (4), the Chern-Simons connections $A_{L/R}$ are related to the metric $g_{M_3}$ in (3) through the usual Chern-Simons gravity dictionary [10] and they are integrated over a non-trivial cycle, $\gamma$, of the base geometry. The representations, $\mathsf{R}_{L/R}$, appearing in the Wilson loops are summed over a set determined by the mass and spin, $(\Delta, \mathsf{s})$, of (3) and labeled by weights $(j_L, j_R)$. Lastly, the parameter $\alpha$ is integrated along a contour $\mathcal{C}$ determined by a regularization scheme appropriate for the sign of cosmological constant. The ultimate effect of the $\alpha$ integral is to implement a winding of the Wilson loop operators around $\gamma$; this occurs through the summing the residues of the poles of its measure (as well as any of representation traces themselves). The above object, (4), is coined the spinning Wilson spool.

## 2.2 $N = 2$ Supersymmetric Localization

We now describe an alternative route to the exact calculation of the Chern-Simons partition function through localization techniques. We will focus particularly on $\mathcal{N} = 2$ supersymmetric localization [11]. One benefit of this approach is that much of the basic machinery has been established with a non-trivial background connection, $a$, in mind allowing a fairly straightforward incorporation of $a \neq 0$. However: the situations with non-trivial background connections have historically arisen on manifolds with interesting topology and many of the explicit results for $S^3$ have been established with $a = 0$. Below we collect and synthesize these results in a way that is useful for $dS_3$ gravity.

Supersymmetry in the context of de Sitter is a contentious subject, with much of the difficulty arising from realizing unitary representations of the supersymmetry algebra in Lorentzian signature. We will take a somewhat agnostic stance on this topic by working directly in Euclidean signature, we are ultimately discussing $SU(2)_k$ Chern-Simons theory on $S^3$ whose $\mathcal{N} = 2$ supersymmetric extension is well-established. We use the existence of this symmetry to our advantage to localize the path integral all while verifying that the extension to $\mathcal{N} = 2$ does not alter essential features of the original partition function.

Much of what follows mirrors the helpful review [8]. The vector multiplet of three-dimensional $\mathcal{N} = 2$ gauge theory is given by fields

$$\{A_\mu,\ \boldsymbol{\sigma},\ \mathfrak{D},\ \lambda,\ \bar{\lambda}\}\ , \tag{5}$$

where $A$ is a $\mathfrak{g} = \mathfrak{su}(2)$ connection, $\boldsymbol{\sigma}$, $\mathfrak{D}$ are scalars, and $\lambda$, $\bar{\lambda}$ are Dirac spinors. All fields are $\mathfrak{g}$-valued and by convention we will take them all to be anti-Hermitian,[3] with supersymmetry variations parameterized by two Grassmann variables $\bar{\epsilon}$ and $\epsilon$ as specified in [8]. The supersymmetric Chern-Simons action is

$$S_{\mathrm{SCS}} = \frac{1}{4\pi}\int \mathrm{Tr}\left(A\wedge \mathrm{d}A + \frac{2}{3}A^3\right) - \frac{1}{4\pi}\int d^3x\sqrt{g}\,\mathrm{Tr}\left(\bar{\lambda}\lambda - 2\mathfrak{D}\boldsymbol{\sigma}\right) \ , \qquad (6)$$

and enters the path-integral multiplied by the level $k$

$$Z_k^{\mathrm{SCS}}[S^3] = \int \frac{\mathcal{D}A}{\mathcal{V}_G}\mathcal{D}\bar{\lambda}\mathcal{D}\lambda\mathcal{D}\mathfrak{D}\mathcal{D}\boldsymbol{\sigma}\, e^{ikS_{\mathrm{SCS}}} \ . \qquad (7)$$

To make subsequent notation more fluent, we will drop the "$[S^3]$" above with it understood that we are always working on the three-sphere. Note that on a formal level, as far as the function dependence on $k$ is concerned, the addition of the auxiliary fields in the multiplet does not alter $Z_k^{\mathrm{SCS}}$ with respect to the non-supersymmetric path-integral $Z_k$ [8].

The deformation that allows us to localize the path-integral $Z_k^{\mathrm{SCS}}$ is the super-Yang-Mills action

$$\begin{aligned} S_{\mathrm{SYM}} = &-\int \mathrm{Tr}\left(\frac{1}{2}F\wedge\star F + D\boldsymbol{\sigma}\wedge\star D\boldsymbol{\sigma}\right) \\ &- \int d^3x\sqrt{g}\,\mathrm{Tr}\left(\frac{1}{2}\left(\mathfrak{D}+\boldsymbol{\sigma}\right)^2 + \frac{i}{2}\bar{\lambda}\gamma^\mu D_\mu\lambda - \frac{1}{2}\bar{\lambda}[\boldsymbol{\sigma},\lambda] - \frac{1}{4}\bar{\lambda}\lambda\right) \ , \end{aligned} \qquad (8)$$

where $D_\mu$ is the gauge-covariant derivative and $\gamma_\mu$ can be taken to be the Pauli-matrices acting on spinor indices. $S_{\mathrm{SYM}}$ is itself a super-derivative and therefore $Q$-exact. Adding this to the path-integral with coefficient $\mathtt{t}$, i.e.,

$$Z_k^{\mathrm{SCS+SYM}}(\mathtt{t}) = \int \frac{\mathcal{D}A}{\mathcal{V}_G}\mathcal{D}\bar{\lambda}\mathcal{D}\lambda\mathcal{D}D\mathcal{D}\boldsymbol{\sigma}\, e^{ikS_{\mathrm{SCS}}-\mathtt{t}S_{\mathrm{SYM}}} \ , \qquad (9)$$

is then innocuous: $Z_k^{\mathrm{SCS+SYM}}(\mathtt{t}) = Z_k^{\mathrm{SCS}}$ for any $\mathtt{t}$, including in the limit $\mathtt{t}\to\infty$ where the path-integral localizes on the saddle of $S_{\mathrm{SYM}}$.

## 2.3 Localization Locus

In the $\mathtt{t}\to\infty$ limit, the path-integral localizes on the following equations of motion

$$F = 0 \ , \qquad D\boldsymbol{\sigma} = \mathrm{d}\boldsymbol{\sigma} + [A,\boldsymbol{\sigma}] = 0 \ , \qquad \mathfrak{D}+\boldsymbol{\sigma} = 0 \ . \qquad (10)$$

We expand the solutions around a flat connection $a = g^{-1}\mathrm{d}g$, for some group element $g$. Again, $g$ may not be single-valued and $a$ may possess a holonomy

$$\mathcal{P}\exp\left(\oint_\gamma a\right) = \exp(2\pi\mathfrak{m}) \ , \qquad (11)$$

[3] In comparison to the notation of [8], a field here is related to a field there by $\Phi_{\mathrm{here}} = i\Phi_{\mathrm{there}}$.

for some curve $\gamma$. The other fields that have saddle solutions to (10) are given by

$$\boldsymbol{\sigma}_0^{(g)} = g^{-1}\boldsymbol{\sigma}_0 g \ , \qquad \mathfrak{D}_0 = -\boldsymbol{\sigma}_0^{(g)} \ , \qquad \lambda_0 = 0 \ , \qquad \bar{\lambda}_0 = 0 \ , \tag{12}$$

for $\boldsymbol{\sigma}_0$ a constant element of $\mathfrak{g}$. We require $\boldsymbol{\sigma}_0^{(g)}$ to be single-valued and so the constant element defining the saddle must obey

$$[\mathfrak{m}, \boldsymbol{\sigma}_0] = 0 \ . \tag{13}$$

With this we can take $\boldsymbol{\sigma}_0$ to be in a Cartan subalgebra containing $\mathfrak{m}$. We will scale fluctuations as

$$\begin{aligned} A =& a + \frac{1}{\sqrt{\mathtt{t}}} B \ , \qquad \boldsymbol{\sigma} = \boldsymbol{\sigma}_0^{(g)} + \frac{1}{\sqrt{\mathtt{t}}}\hat{\sigma} \ , \qquad \mathfrak{D} = -\boldsymbol{\sigma}_0^{(g)} + \frac{1}{\sqrt{\mathtt{t}}}\hat{\mathfrak{D}} \ , \\ & \lambda = \frac{1}{\sqrt{\mathtt{t}}}\hat{\lambda} \ , \qquad\qquad \bar{\lambda} = \frac{1}{\sqrt{\mathtt{t}}}\hat{\bar{\lambda}} \ , \end{aligned} \tag{14}$$

and perturb the action (6) around the saddle (12) as $\mathtt{t} \to \infty$. The leading contribution to $S_{\mathrm{SCS}}$ is

$$\lim_{\mathtt{t}\to\infty} S_{\mathrm{SCS}} = S_{\mathrm{CS}}[a] - \frac{\mathrm{Vol}(S^3)}{2\pi}\mathrm{Tr}\boldsymbol{\sigma}_0^2 \ . \tag{15}$$

Meanwhile the leading contribution to $\mathtt{t}\, S_{\mathrm{SYM}}$ is

$$\begin{aligned} \mathtt{t}\, S_{\mathrm{SYM}} = & - \int \mathrm{Tr}\left(\frac{1}{2}\mathrm{d}_a B \wedge \star \mathrm{d}_a B + (\mathrm{d}_a\hat{\sigma} + [B, \boldsymbol{\sigma}_0^{(g)}]) \wedge \star(\mathrm{d}_a\hat{\sigma} + [B, \boldsymbol{\sigma}_0^{(g)}])\right) \\ & - \int d^3x\sqrt{g}\mathrm{Tr}\left(\frac{1}{2}\left(\hat{\mathfrak{D}} + \hat{\sigma}\right)^2 + \frac{i}{2}\hat{\bar{\lambda}}\gamma^\mu D_\mu^{(a)}\hat{\lambda} - \frac{1}{2}\hat{\bar{\lambda}}[\boldsymbol{\sigma}_0^{(g)}, \hat{\lambda}] - \frac{1}{4}\hat{\bar{\lambda}}\hat{\lambda}\right) + \dots \end{aligned} \tag{16}$$

where $\mathrm{d}_a$ is the background exterior derivative, and $D_\mu^{(a)}$ is the spinor covariant derivative with fixed connection, $a$. This action can be made Gaussian under a suitable gauge-fixing and then path-integrated in standard fashion. Details can be found [8] and references therein.

## 2.4 Gauge Choice

We will choose the gauge[4]

$$\mathcal{G}_a[B] = \mathrm{d}_a^\dagger B \equiv - \star \mathrm{d}_a \star B = 0 \ , \tag{17}$$

whose Fadeev-Popov determinant, $\Delta_a[B]$, can be enacted through adding ghosts $\bar{c}, c$:

[4]This gauge-fixing is only consistent when $a$ is a flat-connection, implying that $\mathrm{d}_a^2 = 0$ defines an equivariant cohomology.

$$Z_k^{\rm SCS+SYM} = e^{ikS_{\rm CS}[a]} \int \mathrm{d}\boldsymbol{\sigma}_0\, e^{-i\frac{k}{2\pi}\mathrm{vol}(M_3)\mathrm{Tr}\boldsymbol{\sigma}_0^2} \times \int \frac{\mathcal{D}B}{\mathcal{V}} \mathcal{D}\hat{\bar{\lambda}}\mathcal{D}\hat{\lambda}\mathcal{D}\hat{\mathfrak{D}}\mathcal{D}\hat{\sigma}\mathcal{D}\bar{c}\mathcal{D}c\, \delta[\mathrm{d}_a^\dagger B]\, e^{-\mathsf{t}S_{\rm SYM}-S_{\rm ghost}} \;, \quad (18)$$

with action

$$S_{\rm ghost} = \int \mathrm{Tr}\left(\bar{c}\wedge\star\mathrm{d}_a^\dagger\mathrm{d}_{a+\mathsf{t}^{-1/2}B}\, c\right) = \int d^3x\sqrt{g}\,\mathrm{Tr}\left(\bar{c}\wedge\star\Delta_a^0\, c\right) + O(\mathsf{t}^{-1/2}) \;, \quad (19)$$

where $\Delta_a^0 = \mathrm{d}_a^\dagger\mathrm{d}_a$ is the $a$-deformed Laplacian acting on $\mathfrak{g}$-valued zero-forms.[5] The ghost determinants simply cancel the determinants from $\hat{\mathfrak{D}}$ and $\hat{\sigma}$ (as well as a Jacobian from $\delta[\mathrm{d}_a^\dagger B]$) and so we arrive at the promised Gaussian path-integral:

$$Z_k^{\rm SCS+SYM} = e^{ikS_{\rm CS}[a]} \int \mathrm{d}\boldsymbol{\sigma}_0\, e^{-i\frac{k}{2\pi}\mathrm{vol}M_3\mathrm{Tr}\boldsymbol{\sigma}_0^2}\, Z_{\rm Gauss}[\boldsymbol{\sigma}_0] \;, \quad (20)$$

with

$$Z_{\rm Gauss}[\boldsymbol{\sigma}_0] := \int [\mathcal{D}B]_{\ker\mathrm{d}_a^\dagger}\mathcal{D}\hat{\bar{\lambda}}\mathcal{D}\hat{\lambda}\, e^{\frac{1}{2}\int\mathrm{Tr}(\mathrm{d}_aB)^2+\int\mathrm{Tr}[B,\boldsymbol{\sigma}_0^{(g)}]^2-\int\mathrm{Tr}\left(\frac{i}{2}\hat{\bar{\lambda}}\gamma_\mu D_\mu^{(a)}\hat{\lambda}-\frac{1}{2}\hat{\bar{\lambda}}[\boldsymbol{\sigma}_0^{(g)},\hat{\lambda}]-\frac{1}{4}\hat{\bar{\lambda}}\hat{\lambda}\right)} \;. \quad (21)$$

# 3 Composite Supersymmetric Particles

Now we come to the point of this note. The localization procedure of subsection 2.2 is not only a calculational tool but the vectormultiplet (5) should be literally realized on the matter sector of the corresponding particle model. In fact, this supersymmetric mattter structure was anticipated on phenomenological basis some time ago in [12–14]. The setup for this particle scenario is as follows:

(i) Unbroken supersymmetry is adopted for fundamental particles. The divisive point between the Minimal Supersymmetric SM and our model (for visible and dark matter) is the following: supersymmetry is unbroken and superpartners are included in constructing the standard model particles. There are no squarks or sleptons to be dicovered.[6] This can be achieved only if standard model fermions are split into three preons. A binding mechanism for preons has been constructed using spontaneously broken 3d Chern-Simons theory.

Preons, or here chernons, are free particles above the energy scale $\Lambda_{cr}$, numerically abou t $\sim 10^{10} - 10^{16}$ GeV. It is close to reheating scale $T_R$ and the grand unified theory (GUT) scale. At $\Lambda_{cr}$ chernons make a phase transition by an attractive Chern-Simons model interaction into composite states of standard model quarks and leptons, including gauge interactions. Chernons have undergone "second quarkization".

[5]It is tacit in (18) that the zero modes of $\bar{c}, c$ under $\Delta_a^0$ are not to be integrated over.

[6] The MSSM leads rather to particle "double counting".

(ii) Gravity must be present on quantum level with proper global symmetries.

(iii) The scenario must match the cosmological standard model with preheating observational data and baryon asymmetry of matter [14].

The chernon scenario with the three interactions is described by the following Lagrangians. To include charged matter we define the charged chiral field Lagrangian for fermion $m^-$, complex scalar $s^-$ and the electromagnetic field tensor $F_{\mu\nu}$[7]

$$\mathcal{L}_{QED} = -\frac{1}{2}\bar{m}^-\gamma^\mu(\partial_\mu + ieA_\mu)m^- - \frac{1}{2}(\partial s^-)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \ . \tag{22}$$

We assign color to the neutral fermion $m \to m_i^0$ $(i = R, G, B)$. The color sector Lagrangian is then[8]

$$\mathcal{L}_{QCD} = -\frac{1}{2}\sum_{i=R,G,B}\left[\bar{m}_i^0\gamma^\mu(\partial_\mu + igG_\mu^a t_a)m_i^0\right] - \frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu} \ . \tag{23}$$

We now have the supermultiplets shown in table 1.

| Multiplet | Particle, Sparticle |
|---|---|
| chiral multiplets spins 0, 1/2 | $s^-, m^-$; $\sigma_i, m_i^0$; a, n |
| vector multiplets spins 1/2, 1 | $m^0$, $\gamma$; $m_i^0, g_i$ |

**Table 1:** The particle $s^-$ is a charged scalar particle. The particles $m^-, m^0$ are charged and neutral, respectively, Dirac spinors. The a is axion and n axino [15]. $m^0$ is color singlet particle and $\gamma$ is the photon. $m_i$ and $g_i$ (i = R, G, B) are zero charge color triplet fermions and bosons, respectively.

Note that in table 1 there is a zero charge quark triplet $m_i$ but no gluon octet. Instead, supersymmetry demands the gluons to appear only in triplets at this stage (before reheating) of cosmological evolution. The dark sector we get from axion sector {a, n} in table 1.

The matter-chernon correspondence for the first two flavors (r = 1, 2; i.e. the first generation) is indicated in table 2 for left handed particles.

[7] The next two equations are in standard 4D form. They are not used quantitatively below.

[8] More complete CS Lagrangians are discussed in [8]

| SM Matter 1st gen. | Chernon state |
|---|---|
| $\nu_e$ | $m_R^0 m_G^0 m_B^0$ |
| $u_R$ | $m^+ m^+ m_R^0$ |
| $u_G$ | $m^+ m^+ m_G^0$ |
| $u_B$ | $m^+ m^+ m_B^0$ |
| $e^-$ | $m^- m^- m^-$ |
| $d_R$ | $m^- m_G^0 m_B^0$ |
| $d_G$ | $m^- m_B^0 m_R^0$ |
| $d_B$ | $m^- m_R^0 m_G^0$ |
| W-Z Dark Matter | Particle |
| boson (or BC) | $s$, axion(s) |
| $e'$ | axino $n$ |
| meson, baryon $o$ | $n\bar{n}, 3n$ |
| nuclei (atoms with $\gamma'$) | multi $n$ |
| celestial bodies | any dark stuff |
| black holes | anything (neutral) |

**Table 2:** Visible and Dark Matter with corresponding particles and chernon composites. $m_i^0$ (i = R, G, B) is color triplet, $m^{\pm}$ are color singlets of charge $\pm 1/3$. $e'$ and $\gamma'$ refer to dark electron and dark photon, respectively. BC stands for Bose condensate. Chernons obey anyon statistics.

After quarks have been formed by the process described in [14] the SM octet of gluons will emerge because it is known that fractional charge states have not been observed in nature. To make observable color neutral, integer charge states (baryons and mesons) possible we proceed as follows. The local $SU(3)_{color}$ octet structure is formed by quark-antiquark composite pairs as follows (with only color charge indicated):

$$\text{Gluons}: \mathrm{R\bar{G}, R\bar{B}, G\bar{R}, G\bar{B}, B\bar{R}, B\bar{G}}, \frac{1}{\sqrt{2}}(\mathrm{R\bar{R} - G\bar{G}}), \frac{1}{\sqrt{6}}(\mathrm{R\bar{R} + G\bar{G} - 2B\bar{B}})\ . \tag{24}$$

With the gluon triplet the first hunch is that they form, with octet gluons now available, the $3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$ bosonic states with spins 1 and 3. These three gluon coupling states would need a separate investigation.

Finally, we introduce the weak interaction. After the SM quarks, gluons and leptons have been formed at scale $\Lambda_{cr}$ there is no more observable supersymmetry in nature [16]. To avoid a more complicated vector supermultiplet in table 1, we may append the standard model electroweak interaction in our model as a $SU()2)_Y$ Higgs extension with the weak bosons presented as composite pairs like gluons in (24). The standard model has now been heuristically derived.

Standard model and dark matter is formed by chernon composites in the very early universe at temperature about the reheating value $T_R$. Due to spontaneous symmetry breaking in three dimensional QED$_3$ by a heavy Higgs-like particle the

Chern-Simons action can provide a binding force stronger than Coulomb repulsion between equal charge chernons. Details of chernon binding and a mechanism for baryon asymmetry in the Universe are presented in [14, 17]. Here we mention the action used

$$S = \frac{k}{4\pi}\int_M \mathrm{tr}(\mathrm{A}\wedge \mathrm{dA} + \frac{2}{3}\mathrm{A}\wedge\mathrm{A}\wedge\mathrm{A})\ , \tag{25}$$

and the gauge invariant effective potential for chernon scattering obtained in [18, 19]

$$V_{\mathrm{CS}}(r) = \frac{e^2}{2\pi}\left[1-\frac{\theta}{m_{ch}}\right]K_0(\theta r) + \frac{1}{m_{ch}r^2}\left\{l - \frac{e^2}{2\pi\theta}[1-\theta r K_1(\theta r)]\right\}^2\ , \tag{26}$$

where $K_0(x)$ and $K_1(x)$ are the modified Bessel functions and $l$ is the angular momentum ($l = 0$ in this note). In (26) the first term [ ] corresponds to the electromagnetic potential, the second one $\{\ \}^2$ contains the centrifugal barrier ($l/mr^2$), the Aharonov-Bohm term and the two photon exchange term.

# 4 Summary

Lattice methods have been developed for CS theory canonical quantization [20, 21]. These make possible developing numerical methods for calculations. The generation question is likely to be solved by an additional symmetry group or excitation interaction.

The supersymmetric Chern-Simons action (6) with matter in (5) and the action (25) with matter in table 1 provide a framework for a new kind of unified theory of matter with the standard model gauge interactions and quantum gravity. This means opening a new layer of matter below SM where topological concepts become essential. The CS action in 3d space-time is the backbone of the theory. One has to choose what kind of universe one wants to build on top of it by defining a proper vector supermultiplet for matter. The supermultiplet considered here yields the present universe with the anticipated unification in [1] "$Forces \longleftrightarrow Matter$"[9] . In our scenario, this unification is achieved above $\Lambda_{cr}$. Below $\Lambda_{cr}$ supersymmetric chernons are organized into ordinary SM particles and no supersymmetry can be observed any more.

The present discussion is precursory. Details of this framework have to be studied consistently, and among other things, introduce the fourth dimension by reheating. A single CS action to build all particles and interactions indicates an element of a theory of "everything" - to the extent it can be defined.

[9] "The idea looks attractive, even so attractive that supersymmetry is frequently presented as uniting forces with matter. This is however misleading at least at the present stage [MSSM], and things do not work out that way." [1]

# A Partition Function Perturbatively

The Euclidean partition function $Z$ of three-dimensional quantum field theory is computed perturbatively below. It is defined as [22]

$$Z = \int D\phi e^{-g^{-2}S(\phi)} \ , \tag{27}$$

where $\phi$ is a free quantum field propagating in a fixed background $\mathcal{M}$ which is locally Anti-de Sitter ($AdS_3$) and hyperbolic space $\mathbb{H}_3$. The field $\phi$ may be either a scalar, an $U(1)$ gauge field or a linearized metric perturbation. An explicit factor of $g^{-2}$ (proportional to $1/\hbar$) in front of the action has been included for convenience. If we consider Eulcidean $AdS_3$ with the identification $t_{Euclidean} \sim t_{Euclidean} + \beta$, then (27) is the partition function of Thermal AdS.

The path integral (27) may be expanded around a classical solution $\phi_0$ to the equations of motion as

$$\log Z = -g^{-2}S^{(0)} + S^{(1)} + g^2 S^{(2)} + \dots \ .$$

Here $S^{(0)} = S(\phi_0)$ is the action of the classical solution, and $S^{(i)}$ denotes the correction to this saddle point action at $i^{th}$ order in perturbation theory. The goal is to compute the one loop action $S^{(1)}$ expanded around the classical vacuum solution $\phi = \phi_0$ for any locally hyperbolic space. For gauge fields and metric perturbations this calculation is quite technical, although the end result is relatively simple.

Perhaps the most interesting application of the results described above is the problem of three dimensional quantum gravity with a negative cosmological constant. The Euclidean action of the theory is

$$S = -\frac{1}{16\pi G}\int d^3x\sqrt{g}\left(R + \frac{2}{\ell^2}\right) \ , \tag{28}$$

where the length $\ell$ is related to the cosmological constant $\Lambda = -2/\ell^2$. Solutions to the equations of motion are metrics of constant negative curvature $R = -6/\ell^2$. The theory has a single dimensionless coupling constant, $k = \ell/16G$. We will use units where $\ell = 1$.

The partition function of quantum gravity is calculated with asymptotically AdS boundary conditions at a given temperature $\beta^{-1}$ and angular potential $\theta$. The canonical ensemble partition function at finite $\beta$ and $\theta$ can be thought of as the Euclidean functional integral

$$Z(\tau) = \int_{\partial\mathcal{M}=T^2} Dg e^{-kS(g)} \ , \tag{29}$$

where we integrate over metrics whose conformal boundary a torus $T^2$ with modular parameter $\tau = \frac{1}{2\pi}(\theta + i\beta)$. In writing (29) we have pulled out an overall factor of $k$ from the action. At leading order in $k$, this partition function is found by computing the classical action of a Euclidean solution to the equations of motion. The simplest such solution is just Euclidean AdS space, with periodically identified time coordinate. The contribution of this geometry to the partition function can be expanded in perturbation theory

$$Z_{saddle}(\tau) \sim e^{-kS^{(0)}+S^{(1)}+k^{-1}S^{(2)}+\dots} \ . \tag{30}$$

The classical action $S^{(0)}$ is [23]

$$e^{-kS^{(0)}} = |q|^{-2k} \ . \tag{31}$$

where $q = e^{2\pi i\tau}$.

The one loop correction $S^{(1)}$ was introduced following the logic of [24]. The authors argued that the symmetry group relevant to general relativity with asymptotically $AdS_3$ boundary conditions is two copies of the Virasoro algebra. This means that the partition function (30) must be some representation of the Virasoro algebra:

$$Z_{saddle}(\tau) = \mathrm{Tr}\, q^{L_0}\bar{q}^{\bar{L}_0} \ . \tag{32}$$

Since the operators $L_0 + \bar{L}_0$ and $L_0 - \bar{L}_0$ are identified with energy and angular momentum operators, respectively, this is just the usual expression for a canonical ensemble partition function at fixed temperature and angular potential. The classical action (31) is interpreted as the contribution to (32) of a ground state $|0\rangle$ of weight $L_0 = \bar{L}_0 = -k$. The trace in equation (32) is over the Hilbert space of perturbative excitations around this $AdS_3$ background, and the other states appearing in this trace will give the subleading corrections appearing in (30). These states are the Virasoro descendants of the ground state, found by acting on $|0\rangle$ with some combination of the Virasoro operators $L_{-n}$. Including these states in the trace gives

$$Z_{saddle} = |q|^{-2k} \prod_{n=2}^{\infty} \frac{1}{|1-q^n|^2} \ . \tag{33}$$

The additional terms appearing in (33) are identified as $e^{S^{(1)}}$.[10]

The one-loop partition function (33) is computed directly y the heat kernel method in [22]. In particular, the one-loop determinant $\det \Delta^{(2)}$ is calculated, where $\Delta^{(2)}$ is the kinetic operator for linearized graviton fluctuations around the background metric. In computing the partition function, one must also include the Fadeev-Popov determinants arising due to gauge fixing. These involve the

[10] In fact, this expression must be one loop exact, because there is a unique representation of the Virasoro algebra with lowest weight. So there is no possible modification of the formula (33) – aside from a renormalization of the coupling $k$ – which is consistent with the Virasoro symmetry.

determinants of a scalar Laplacian $\Delta^{(0)}$ and a vector field Laplacian $\Delta^{(1)}$. Although the intermediate stages of this computation are quite complicated, the final answer takes a simple form:

$$e^{-kS^{(0)}+S^{(1)}} = e^{-kS^{(0)}} \frac{\det \Delta^{(1)}}{\sqrt{\det \Delta^{(0)} \det \Delta^{(2)}}} = |q|^{-2k} \prod_{n=2}^{\infty} \frac{1}{|1-q^n|^2} \ . \tag{34}$$

This computation demonstrates directly that the structure of a conformal field theory emerges from quantum gravity in Anti-de Sitter space.

# B Euclidean $dS_3$

Three-dimensional de Sitter space-time is a maximally symmetric spacetime with two-sphere spatial slices expanding into future and past infinity [10]. Due to this expansion, inertial observers have access to a finite portion of space-time called the static patch. A coordinate patch covering one-half of this patch is given by $(t, \rho, \varphi)$ with $\rho \in [0, \frac{\pi}{2})$, $t \in (-\infty, \infty)$, and $\varphi \in [0, 2\pi)$ with $\varphi$ a periodic coordinate. The observer's origin and causal horizon lie at $\rho = 0$ and $\rho = \frac{\pi}{2}$, respectively. The corresponding metric of the static patch is

$$\frac{\mathrm{d}s^2}{\ell_{\mathrm{dS}}^2} = -\cos^2\rho\, \mathrm{d}t^2 + \mathrm{d}\rho^2 + \sin^2\rho\, \mathrm{d}\varphi^2 \ . \tag{35}$$

Above, the de Sitter radius, $\ell_{\mathrm{dS}}$, sets the length scale for this maximally symmetric space-time.

For the purpose of moving between the following Chern-Simons description and the metric description, as well for computing one-loop determinants, it will be useful to go to Euclidean signature through the Wick-rotation, $t = -i\tau$. Under this rotation the static-patch rotates to a three-sphere in torus coordinates

$$\frac{\mathrm{d}s^2}{\ell_{\mathrm{dS}}^2} = \cos^2\rho\, \mathrm{d}\tau^2 + \mathrm{d}\rho^2 + \sin^2\rho\, \mathrm{d}\varphi^2 \ . \tag{36}$$

Absence of a conical singularity at the horizon, $\rho = \frac{\pi}{2}$, sets $\tau \sim \tau + 2\pi$. The isometry group of this Euclidean space is $SU(2)_L \times SU(2)_R/\mathbb{Z}_2$ with the $L/R$ denoting left/right group action. We will denote the generators of these two actions as $\{L_a\}_{a=1,2,3}$ and $\{\bar{L}_a\}_{a=1,2,3}$, respectively.

In accordance with the split structure of this isometry group, we will describe Euclidean $dS_3$ gravity with a pair of $SU(2)$ Chern-Simons theories

$$S = k_L\, S_{\mathrm{CS}}[A_L] + k_R\, S_{\mathrm{CS}}[A_R] \ , \tag{37}$$

with

$$S_{\mathrm{CS}}[A] = \frac{1}{4\pi} \mathrm{Tr} \int \left( A \wedge \mathrm{d}A + \frac{2}{3} A \wedge A \wedge A \right) \ , \tag{38}$$

and the trace taken in the fundamental representation. The basic ingredients of the dictionary between this Chern-Simons description and the more familiar "metric description" is given by relating the gauge connections, $A_{L/R}$, to a dreibein, $e^a$, and (dual) spin-connection, $\omega^a = \frac{1}{2}\epsilon^{abc}\omega_{bc}$, via

$$A_L = i\left(\omega^a + \frac{1}{\ell_{\rm dS}}e^a\right)L_a\ , \qquad\qquad A_R = i\left(\omega^a - \frac{1}{\ell_{\rm dS}}e^a\right)\bar{L}_a\ . \tag{39}$$

The levels, $k_{L/R}$, are written as

$$k_L = \delta + is\ , \qquad k_R = \delta - is\ . \tag{40}$$

We can rewrite the action (37) as

$$iS = -I_{\rm EH} - i\delta I_{\rm GCS}\ , \tag{41}$$

with

$$I_{\rm EH} = -\frac{s}{4\pi\ell_{\rm dS}}\int \epsilon_{abc}e^a\wedge\left(R^{bc} - \frac{1}{3\ell_{\rm dS}^2}e^b\wedge e^c\right) \tag{42}$$

the Euclidean Einstein-Hilbert term in the first-order formalism[11] with positive cosmological constant, $\Lambda = \ell_{\rm dS}^{-2}$. Thus $s \equiv \frac{\ell_{\rm dS}}{4G_N}$ is the (inverse) gravitational coupling; in this paper we will find it convenient to keep it written as $s$ and keep in mind the classical limit is $s\to\infty$. $I_{\rm GCS}$ is the gravitational Chern-Simons term

$$I_{\rm GCS} = \frac{1}{2\pi}{\rm Tr}\int\left(\omega\wedge{\rm d}\omega + \frac{2}{3}\omega\wedge\omega\wedge\omega\right) + \frac{1}{2\pi\ell_{\rm dS}^2}{\rm Tr}\int e\wedge T\ , \tag{43}$$

with $T$ the torsion two-form. Under quantization the levels will undergo a finite renormalization $k_{L/R}\to r_{L/R}\equiv k_{L/R}+2$ [25] amounting to a renormalization of the $I_{\rm GCS}$ coupling, $\delta\to\hat{\delta}=\delta+2$. For the rest of this paper we will always work directly with the renormalized levels.

Appropriate classical flat background connections, $a_{L/R}$, describing the $S^3$ are given by

$$\begin{aligned} a_L =& iL_1\,{\rm d}\rho + i\left(\sin\rho L_2 - \cos\rho L_3\right)({\rm d}\varphi - {\rm d}\tau)\ ,\\ a_R =& -i\bar{L}_1\,{\rm d}\rho - i(\sin\rho\bar{L}_2 + \cos\rho\bar{L}_3)({\rm d}\varphi + {\rm d}\tau)\ . \end{aligned} \tag{44}$$

An important aspect of the above connections is that they each possess ring singularities at $\rho = 0$ and $\rho = \frac{\pi}{2}$ where the $\varphi$ and $\tau$ coordinates degenerate, respectively. These Wick-rotate to the worldline at the static patch origin and to the causal horizon bifurcation surface, respectively. There is potential for holonomy around these singularities which we will write generically as

$$\mathcal{P}\exp\oint_\gamma a_L = g_\rho^{-1}e^{i2\pi L_3\mathsf{h}_L}g_\rho\ , \qquad \mathcal{P}\exp\oint_\gamma a_R = \bar{g}_\rho\,e^{i2\pi\bar{L}_3\mathsf{h}_R}\bar{g}_\rho^{-1}\ , \tag{45}$$

[11] Here $R^{ab}$ is the Riemann two-form.

for periodic group elements, $g_\rho = e^{iL_1\rho}$ and $\bar{g}_\rho = e^{i\bar{L}_1\rho}$, and holonomies, $\mathsf{h}_{L/R}$. Given the solution (44), it is easy to deduce that for cycles, $\gamma_{\text{orig}}$, wrapping the static-patch origin at $\rho = 0$

$$\gamma_{\text{orig}}: \qquad \mathsf{h}_L = 1 \ , \qquad \mathsf{h}_R = 1 \ , \tag{46}$$

while for cycles, $\gamma_{\text{hor}}$, wrapping the causal horizon at $\rho = \frac{\pi}{2}$,

$$\gamma_{\text{hor}}: \qquad \mathsf{h}_L = 1 \ , \qquad \mathsf{h}_R = -1 \ . \tag{47}$$

Lastly we point out that these singularities give delta function sources of curvature[12] that is important for reproducing the on-shell action

$$ir_L S_{\text{CS}}[a_L] + ir_R S_{\text{CS}}[a_R] = \frac{\pi \ell_{\text{dS}}}{2G_N} \ , \tag{48}$$

which is the tree-level de Sitter entropy.

# References


[1] Pierre Fayet, The Supersymmetric Standard Model, Advanced Series on Directions in High Energy Physics, The Standard Theory of Particle Physics, pp. 397-454 (2016) Open Access Chapter 20: The Supersymmetric Standard Model.

[2] E. Witten, (2+1)-Dimensional Gravity as an Exactly Soluble System, Nucl. Phys. B 311 (1988) 46.

[3] Collier, Scott and Eberhardt, Lorenz and Zhang, Mengyang, Solving 3d gravity with Virasoro TQFT, SciPost Phys., 15, 4 (2023). DOI: 10.21468/SciPostPhys.15.4.151. arXiv:2304.13650

[4] Castro, Alejandra and Lashkari, Nima and Maloney, Alexander, A de Sitter Farey Tail, Phys. Rev. D 83, 124027 (2011). DOI: 10.1103/PhysRevD.83.124027. arXiv:1103.4620

[5] Anninos, Dionysios and Denef, Frederik and Law, Y. T. Albert and Sun, Zimo, Quantum de Sitter horizon entropy from quasicanonical bulk, edge, sphere and topological string partition functions, JHEP 01, 088 (2022). DOI: 10.1007/JHEP01(2022)088. arXiv:2009.12464

[6] A. Castro, I. Coman, J. R. Fliss, and C. Zukowski, Keeping matter in the loop in dS3 quantum gravity, JHEP 07 (2023) 120. arXiv:2302.12281

[7] Blau, Matthias and Thompson, George, Chern-Simons Theory on Seifert 3-Manifolds, JHEP 09, 033 (2013). DOI: 10.1007/JHEP09(2013)033. arXiv:1306.3381


[12] Importantly the metric geometry remains smooth everywhere.

[8] Marcos Mariño, Lectures on localization and matrix models in supersymmetric Chern–Simons–matter theories, J. Phys. A: Math. Theor. 44 463001 (2011). DOI: 10.1088/1751-8113/44/46/463001. arXiv:1104.0783

[9] Máximo Bañados, Claudio Teitelboim, Jorge Zanelli, The Black Hole in Three Dimensional Space Time, Phys.Rev.Lett. 69 (1992) 1849-1851. DOI: https://doi.org/10.1103/PhysRevLett.69.1849. arXiv:hep-th/9204099

[10] Robert Bourne, Alejandra Castro, and Jackson R. Fliss, Spinning up the spool: Massive spinning fields in 3d quantum gravity, J. Phys. A: Math. Theor. 58 (2025) 025402 (42pp). arXiv:2407.09608

[11] Kapustin, Anton and Willett, Brian and Yaakov, Itamar, Exact Results for Wilson Loops in Superconformal Chern-Simons Theories with Matter, JHEP 03, 089 (2010). DOI: 10.1007/JHEP03(2010)089. arXiv:0909.4559

[12] Risto Raitio, A Model of Lepton and Quark Structure. Physica Scripta, 22, 197 (1980). PS 22,197. viXra:1903.0224

[13] Risto Raitio, Supersymmetric preons and the standard model, Nuclear Physics B931, 283–290 (2018). DOI: 10.1016/j.nuclphysb.2018.04.021. arXiv:1805.03013

[14] Risto Raitio, A Chern-Simons model for baryon asymmetry, Nuclear Physics B Volume 990, May 2023, 116174. DOI: 10.1016/j.nuclphysb.2023.116174. arXiv:2301.10452

[15] Roberto D. Peccei and Helen R. Quinn, CP Conservation in the Presence of Pseudoparticles, Phys. Rev. Lett. 38 (25) 1440–1443 (1977).

[16] Risto Raitio, The fate of supersymmetry in quantum field theories, Journal of High Energy Physics, Gravitation and Cosmology, Vol.10, No.2 (2024). DOI: 10.4236/jhepgc.2024.102038. arXiv:2307.13017

[17] H. Belich, O. M. Del Cima, M. M. Ferreira Jr. and J. A. Helayel-Neto, Electron-Electron Bound States in Maxwell-Chern-Simons-Proca $QED_3$, Eur. Phys. J. B 32, 145–155 (2003). arXiv:hep-th/0212285

[18] Ya. I. Kogan, Bound States of Fermions and Superconducting Ground State in a $2+1$ Gauge Theory with a Topological Mass Term. JETP Lett. 49, 225 (1989).

[19] M. I. Dobroliubov, D. Eliezer, I. I. Kogan, G.W. Semenoff and R.J. Szabo, Mod. Phys. Lett. A, 8, 2177 (1993).

[20] Theodore Jacobson and Tin Sulejmanpasic, Canonical quantization of lattice Chern-Simons theory. arXiv:2401.09597

[21] Bingnan Zhang, Abelian Chern-Simons gauge theory on the lattice, Phys. Rev. D 105, 014507 (2022). arXiv:2109.13411

[22] Simone Giombi, Alexander Maloney, Xi Yin, One-loop Partition Functions of 3D Gravity, JHEP 0808:007,2008. DOI: https://doi.org/10.1088/1126-6708/2008/08/007. arXiv:0804.1773

[23] J. M. Maldacena and A. Strominger, AdS(3) black holes and a stringy exclusion principle, JHEP **9812**, 005 (1998). arXiv:hep-th/9804085

[24] J. D. Brown and Marc Henneaux, Central Charges in the Canonical Realization of Asymptotic Symmetries: An Example from Three Dimensional Gravity Commun. Math. Phys. 104, 207-226 (1986).

[25] Witten, Edward, Quantization of Chern-Simons Gauge Theory with Complex Gauge Group, Commun. Math. Phys., 137, 29-66 (1991). DOI: 10.1007/BF02099116.