A Navigator's Solution to the Michelson-Morley Problem

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Abstract

The Michelson-Morley experiment and its resolution by the special theory of relativity form a foundational truth in modern physics. In this paper we examine and generalise the geometry of the sequence of events within a standard MM interferometer to arrive at a geometry that merges the perspectives of the rest and moving frames within a common stationary circle in space. Further we show that this theoretical approach leads us into spherical trigonometry that supplies a simple solution of the Michelson-Morley problem that is compatible with Einstein's paradigm.

1 Introduction

The aim of this paper is to conduct an in-depth theoretical re-visitation of the paradigm shifting Michelson-Morley (MM) experiment, its famous null result [1] and the resulting paradox of space and time whose solution [2] forms the foundational basis of modern physics. We will examine arguments that show that the event sequence within an MM interferometer may be theorised by the rest frame in an unconventional fashion. This approach will demonstrate that under inertial conditions and independent of its orientation or its relative velocity with respect to the rest frame, the locus of all points in space where a reflection event can occur within an MM interferometer is a stationary circle in space. Restricting the discussion to inertial conditions, we attempt to reconcile the MM paradox in a fashion that retains this circular geometry while remaining compatible with the theoretical consequences of special relativity.

2 Euclidean Geometry

On a flat surface [2], we draw any angle θ at origin Q bounded by two equal length line segments $QB = QB' = h$. We join points B and B' to points A and C such that the line segment AC is perpendicular to QB and centred at Q. We will restrict our arguments to the domain $x < h$. Fig. 1 illustrates.

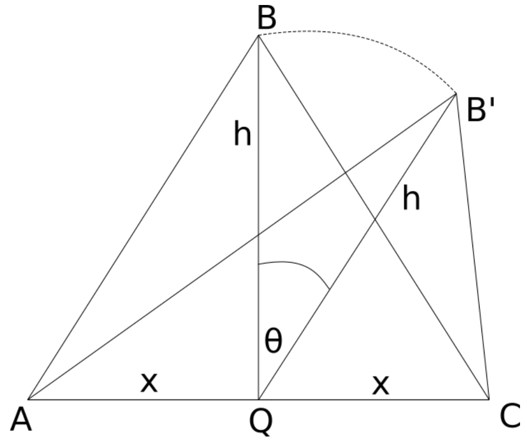

Figure 1: Triangles ABC and $AB'C$ rendered on a flat surface.

From fig. 1, we posit the following:

1. If $x > 0$, physical measurements will verify the theoretical statement $AB + BC \neq AB' + B'C$ remains true for all $\theta \neq 0, \pi, 2\pi$...
2. Since h is constant, curve BB' will take the form of a circle as $0 \leq \theta \leq 2\pi$ independent of x.
3. If $x > 0$, physical measurements will verify the theoretical statement $\angle AB'Q \neq \angle QB'C$ remains true over all $\theta \neq 0, \pi/2, \pi$...

3 A Template of the MM Experiment

Now we turn to theoretical aspects of relativistic optical interferometry to demonstrate that the geometry and sequence of events within an MM interferometer always templates to that of fig. 1.

3.1 Frames of Reference

Consider two imaginary euclidean reference frames that are in relative motion with respect to each other. Let us arbitrarily assume one of these frames is at rest and the other moves with some velocity v with respect to the rest frame. Accordingly we refer to fig. 1 and declare,

1. A rest frame I_0 centered at point Q.
2. A moving frame I_1 that translates from point A to point C with some velocity v relative to rest frame I_0.

3.2 Geometry and Sequence of Events

Now let us consider the structure of an MM interferometer [1](see fig. 2). By fixing $\angle B_1'QB_2' = \pi/2$, line segments QB_1' and QB_2' form the arms of the interferometer. Mirrors B_1 and B_2 are aligned perpendicular to their respective arms. The apparatus may be rotated about its source and consequently each arm subtends its own angle θ_i measured from a perpendicular to line segment AC. Let us affix moving frame I_1 to the source of the interferometer. Now let us imagine this interferometer moving through space under inertial rules such that,

1. v remains constant ($AQ = QC$).

2. The interferometer orientation (θ_i) with respect to line segment AC remains constant.

Reference frame I_1 (affixed to the source) translates with constant velocity v from point A to point C. From the perspective of the rest frame I_0, a discrete event cycle begins with the source at point A marking the simultaneous emission of a pair of photons (wavelength=λ). As the entire apparatus moves with some constant ($AQ = QC$) velocity v relative to origin Q along line segment AC, the photons are emitted at point A, reflect from mirrors B_1 and B_2 to finally arrive simultaneously (in phase with each other) at point C. This geometry and sequence of events remains true over all possible orientations θ of an MM interferometer [3] and over all $0 \leq v < c$ where c represents the velocity of light in free space [4].

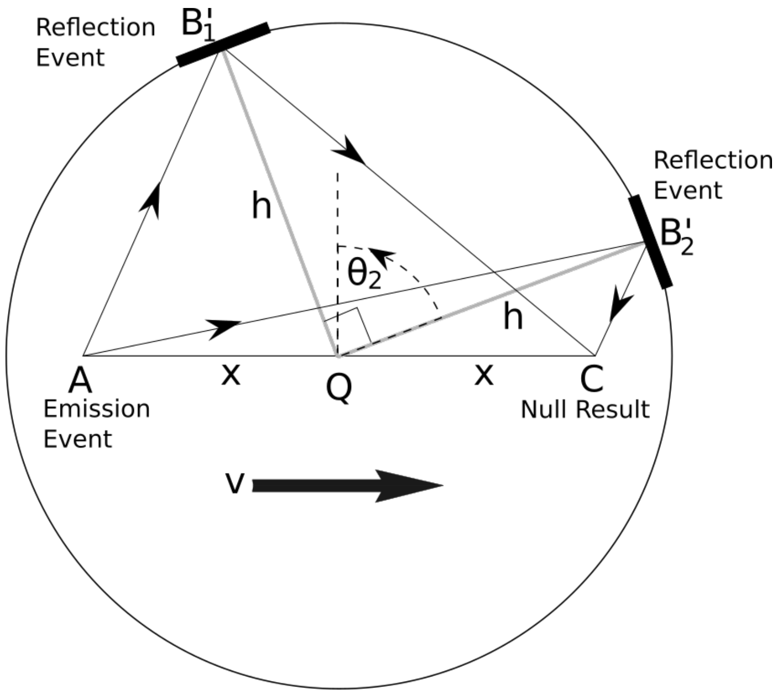

Figure 2: Geometry of the Michelson-Morley experiment depicting the general case $v \neq 0$ and $\theta_i \neq 0, \pi/2, \pi....$ Point Q is chosen as the origin. Only the events within the interferometer that are relevant to relativistic discussion are shown. Independent of the orientation of the interferometer, rest frame I_0 will find triangle $AB_i'C$ is a generalisation of triangle $AB'C$ in fig. 1. Identical to fig.1, physical measurements of the geometry of events will confirm that $AB_i' + B_i'C \neq AB_j' + B_j'C$ for all $\sin\theta_i \neq \sin\theta_j$ (inequality in path lengths) and $\angle AB'Q \neq \angle QB'C$ (inequality in angles of incidence and reflection) for all $\theta_i \neq 0, \pi/2, \pi...$ By setting $v = 0$ ($x = 0$), the figure represents the observational perspective of moving frame I_1. By setting $v > 0$ ($x > 0$), the figure represents the observational perspective of rest frame I_0. It is evident from fig. 1 that curve BB' will take the form of a stationary circle of radius h about point Q independent of θ_i (i.e. orientation) and v (i.e. frame of reference).

4 Preliminary Analysis

At this stage of investigation, rest frame I_0 recognises the inequalities in path lengths depicted in fig. 1 coupled with the experimental null result of the MM experiment to arrive at a well understood paradox of space and time that is traditionally reconciled by selecting point A as the origin followed by the application of special relativity [5]. But we may also posit that by selecting instead point Q as the origin, rest frame I_0 and *any* moving frame I_i moving with velocity v_i along the AC or CA directions are all assured that over all $0 \leq \theta_i \leq 2\pi$ and $0 \leq v_i < c$, the locus of all points in space where a reflection event can occur is a common stationary circle of radius h about point Q. Invoking the symmetry of the circle, a rest frame I_0 may also rotate fig. 2 in entirety about point Q by any angle $0 \leq \phi \leq 2\pi$ and may incorporate any number of moving frames $I_1, I_2, I_3, ...I_i$, each moving in any possible direction ϕ_i and each set an any orientation $0 \leq \theta_i \leq 2\pi$ for all $0 \leq v_i < c$ within a single common stationary circle i.e. curve BB'. Further, invoking the superposition property of waves [6], we may posit [7] that this single circle BB' is capable of hosting an infinite number of MM null result cycles moving at all possible velocities, in all possible directions, simultaneously [8]. To this end, let us theorise a model of space upon which rest frame I_0 is able reconcile the paradox of unequal path lengths presented above in a manner that retains the geometry of curve BB'.

5 Spherical Trigonometry

Let us first recall that fig. 1 is drawn on a flat surface [2] and that the value of x in this figure is physically determined by measuring rod and assigned the unit metre(m). Noting also that light obeys the properties of travelling waves [9], we recognise the governing function of sinusoidal travelling waves, i.e. $\sin(x)$, takes an argument x that must be in an angular unit of measure i.e. radians. With these in mind, let rest frame I_0 project fig. 1 onto the surface of an imaginary sphere of arbitrary radius R such that the shortest distance path between any two points are described by great circles on the sphere [10]. Thus the magnitude of distances $x, h, AB', B'C$ are measured analytically in radians subtended at the centre of this sphere. Curve BB' takes the form of a small circle on the surface of this sphere having radius h radians and centred at point Q.

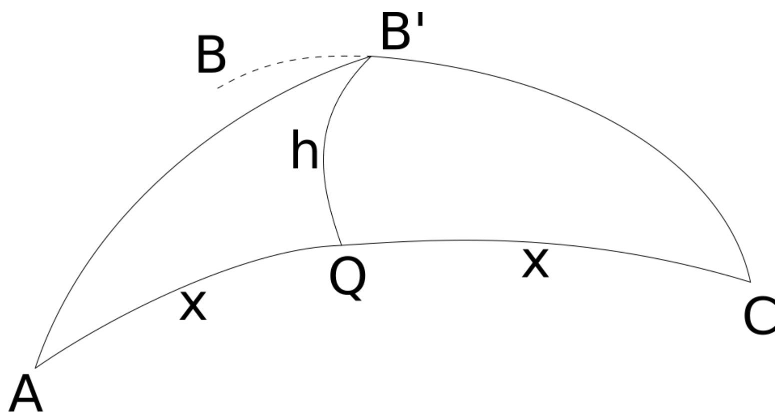

Figure 3: Spherical Trigonometry.

5.1 Analysis of Spherical Model

From fig. 3 and the rule of sines for spherical triangles [11], rest frame I_0 finds in $\triangle AB'Q$:

$$\frac{\sin AB'}{\sin(\pi/2 + \theta)} = \frac{\sin h}{\sin A} = \frac{\sin x}{\sin i} \tag{1}$$

where $i = \angle AB'Q$. Similarly for $\triangle CB'Q$:

$$\frac{\sin CB'}{\sin(\pi/2 - \theta)} = \frac{\sin h}{\sin C} = \frac{\sin x}{\sin r} \tag{2}$$

where $r = \angle CB'Q$.

From equations, 1 and 2 rest frame I_0 finds in all spherical triangles of the form $AB'C$:

$$\frac{\sin(AB')}{\sin(CB')} = 1 \tag{3}$$

Referring now to fig. 2, eq. 3 guarantees that by interpreting the MM null result geometry with this approach, rest frame I_0 is assured the theoretical statement $AB'_i + B'_iC = AB'_j + B'_jC$ remains true independent of v, h, θ. Thus the paradox of unequal path lengths presented by physical measurements of fig. 1 vanishes independent of frame of reference v_i or orientation of the interferometer θ_i. Further by selecting point Q as a common origin, every frame of reference $I_0, I_1, I_2...I_\infty$, whether at rest or moving are all assured that the commonality and the circularity of curve BB' (refer sec. 4) remain unaffected if projected onto this theoretical model of space.

6 Conclusion

Equation 3, demonstrates the total analytical light path $AB' + B'C$ is always equal to the maximal value i.e. π radians. Further eq. 3 remains true independent of h, θ and remains valid over all $0 \leq v/c < \infty$. This model of analytical space is presented for scrutiny in support of Einstein's assertion that "the velocity of light in our theory plays the part, physically, of an infinitely great velocity" [2][8].

7 Statements and Declarations

The author has no competing interests to declare that are relevant to the content of this article. There are no data associated with this article.

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