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Best estimate of density sum error of the prime number theorem for a sequence of $x_n$well chosen

Abstract

$\pi(x)$The TNP prime number counting function

$P(x)=\frac{x}{Lnx}$Gaussian approximation for prime numbers

We will establish the following density framework for the sequence of $x_n= e^{n^s}$or n a natural integer greater than 7 and s > 1.

$$\alpha \leq \sum_7^{+\infty} \left(\frac{\pi(x_n)}{x_n} - \frac{P(x_n)}{x_n}\right) \leq \beta$$

$$Alors \quad 0 \leq \beta - \alpha \leq 0.04$$

Introduction

* We will use Conclusion 1 (vixra number theory article khazri bouzidi fethi)

For s > 1 ξ(s) = $\sum_1^{+\infty} \frac{P(e^{n^s})}{e^{n^s}}$where P is the Gauss approximation $P(x)=\frac{x}{Lnx}$and ξ the classical Riemann function.

* The framework of Dusart 1999: $\frac{x}{lnx}(1+ \frac{1}{Lnx}) \leq \pi(x) \leq \frac{x}{Lnx}(1+\frac{1.2762}{Lnx})$

Calculation

The Dusarat 1999 inequality gives $\frac{x}{lnx}(1+ \frac{1}{Lnx}) \leq \pi(x) \leq \frac{x}{Lnx}(1+\frac{1.2762}{Lnx})$, the reduction is true for x $\geq$ 599and the increase for x > 1. We a x = 599$\simeq e^{6.39}$.In the following we will always take x $\geq e^7$ and P(x) = $\frac{x}{Lnx}$

$\frac{x}{lnx}(1+ \frac{1}{Lnx}) \leq \pi(x) \leq \frac{x}{Lnx}(1+\frac{1.2762}{Lnx})$ which gives

$\frac{x}{Lnx\, Lnx} \leq \pi(x) - P(x) \leq \frac{1.2762\, x}{Lnx\, Lnx}$ divide by the real x with x $\geq e^7$

$\frac{1}{Lnx\,Lnx} \leq \frac{\pi(x)}{x} - \frac{P(x)}{x} \leq \frac{1.2762}{Lnx\,Lnx}$ replace x by $x_n = e^{n^s}$.with n ≥ 7and s > 1.

$\frac{1}{n^{2s}} \leq \frac{\pi(x)}{x} - \frac{P(x)}{x} \leq \frac{1.2762}{n^{2s}}$ with n ≥ 7and s > 1 let's go to the sum between 7 and +∞

$\sum_7^{+\infty} \frac{1}{n^{2s}} \leq \sum_7^{+\infty} (\frac{\pi(x)}{x} - \frac{P(x)}{x}) \leq \sum_7^{+\infty} \frac{1.2762}{n^{2s}}$

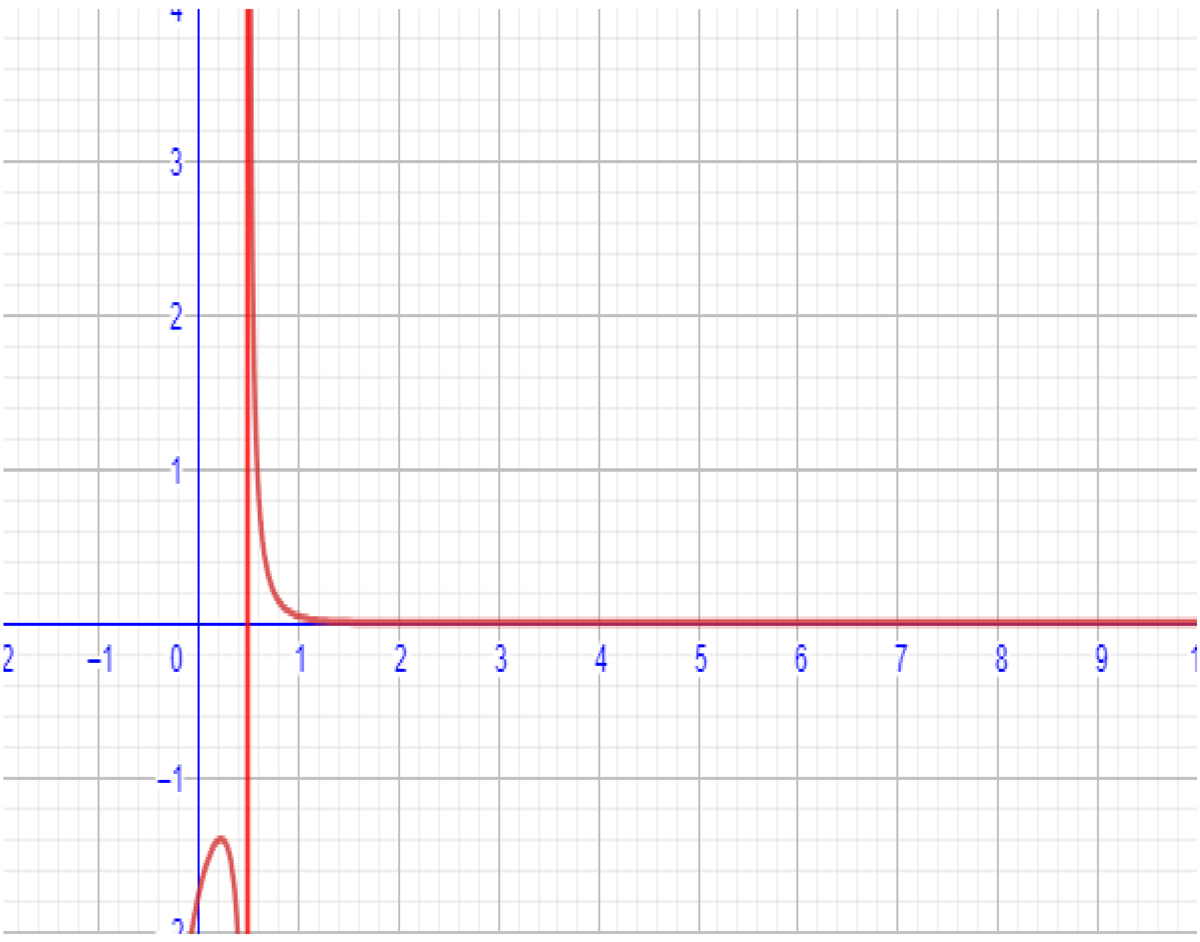
$\xi(2s)$- $(1+ \frac{1}{2^{2s}}+ \frac{1}{3^{2s}}+ \frac{1}{4^{2s}}+ \frac{1}{5^{2s}}+\frac{1}{6^{2s}}) \leq \sum_7^{+\infty} (\frac{\pi(x)}{x} - \frac{P(x)}{x}) \leq 1.2762(\xi\ (2s) - \left(1 + \frac{1}{2^{2s}} + \frac{1}{3^{2s}} + \frac{1}{4^{2s}} + \frac{1}{5^{2s}} + \frac{1}{6^{2s}}\right))$

The deference between the two framing terms gives the errors

$$\mathbf{R(s) = 0.2762(\xi\ (2s) - \left(1 + \frac{1}{2^{2s}} + \frac{1}{3^{2s}} + \frac{1}{4^{2s}} + \frac{1}{5^{2s}} + \frac{1}{6^{2s}}\right))}$$

Examples for s=1 a calculation with Géogebra gives R (1) = 0.04

For s large enough R(s) ≃ 0



Graphical representation of R(s) with Géogebra

Conclusion

$\pi(x)$ The TNP prime number counting function

$P(x)= \frac{x}{Lnx}$ Gaussian approximation for prime numbers

For the sequence of $x_n= e^{n^s}$or n a natural integer greater than 7 and s > 1.

(That has to say x ≥ 600 des Nombres, T. A. (2022). *Habilitation à Diriger des Recherches* (Doctoral dissertation, Université de Limoges). des Nombres, T. A. (2022). *Habilitation à Diriger des Recherches* (Doctoral dissertation, Université de Limoges).)

$$\alpha \leq \sum_7^{+\infty} (\frac{\pi(x_n)}{x_n} - \frac{P(x_n)}{x_n}) \leq \beta$$

The error $\beta - \alpha = 0.2762(\xi\ (2s) - \left(1 + \frac{1}{2^{2s}} + \frac{1}{3^{2s}} + \frac{1}{4^{2s}} + \frac{1}{5^{2s}} + \frac{1}{6^{2s}}\right))$

References

des Nombres, T. A. (2022). *Habilitation à Diriger des Recherches* (Doctoral dissertation, Université de Limoges).