SPHERICAL ELECTRONS AND
THE WHEELER-FEYNMAN TIME-SYMMETRIC THEORY

JAMES CONOR O'BRIEN
ORCID iD 0009-0000-8895-2345
(27/07/24)

Keywords: Wheeler-Feynman time-symmetric theory (W.F.T.S.T.), spherical electrons, electron electric dipole moment (EDM), longitudinal electromagnetic scalar potential waves, Wheeler-Feynman electromagnetic four-vector potential time symmetric theory (W.F.E.M.F.V.P.T.S.T.).

Abstract:

This paper is a follow up to previous work *A Dynamical Theory of the Electromagnetic Potential* [1][2][3][4] where I modelled fermions as longitudinal electromagnetic scalar potential waves travelling in $\mathbb{R}^{1,3}$ accompanied by perfectly spherical charged and fermions in $\mathbb{R}^3$ labelling this as the W.F.E.M.F.V.P.T.S.T. model. In this present paper I'm going to suggest these perfectly spherical fermions provide a definitive test for the W.F.E.M.F.V.P.T.S.T. model because aspherical electrons would violate a fundamental condition concerning the conservation of phase and energy for the W.F.E.M.F.V.P.T.S.T. model thus breaking that model.

§1 W.F.E.M.F.V.P.T.S.T.

First a recap of the W.F.E.M.F.V.P.T.S.T. model and it's derivation from the Wheeler-Feynman time-symmetric theory, if you've read the previous papers (O'Brien 2018) then skip this section.

The electromagnetic four-vector potential is split between the time axis and the spatial axis and leads directly to the Four-momentum p^μ,

$$\begin{aligned} \mathbb{R}^{1,3} &= (t,\ \mathbf{x},\ \ \mathbf{y},\ \ \mathbf{z}) \\ A_\mu &= (\frac{\phi}{c}, \mathbf{A}_x, \mathbf{A}_y, \mathbf{A}_z) \\ p^\mu = qA_\mu &= (\frac{E}{c}, \mathbf{p}_x, \mathbf{p}_y, \mathbf{p}_z) \end{aligned} \tag{1}$$

From eqn(1) it can be seen the scalar potential φ lies *entirely along the axis of time*, and to quote David Griffiths [5], *"There is a very peculiar thing about the scalar potential in Coulomb gauge: it is determined by the distribution of charge right now. If I move an electron in my laboratory, the potential V on the moon immediately records this change. That sounds particularly odd in the light of special relativity, which allows no message to travel faster than c. The point is that V by itself is not a physically measurable quantity – all the man in the Moon can measure is **E**, and that involves **A** as well. Somehow it is built into the vector potential(in the Coulomb gauge) that whereas V instantaneously reflects all changes in* ρ*, the combination* $-\nabla V - (\partial \mathbf{A}/\partial t)$ *does* not*; **E** will change only after sufficient time has elapsed for the "news" to arrive."*

This is the pivotal step in the W.F.E.M.F.V.P.T.S.T. model as the *scalar potential is longitudinal to the axis of time* while Maxwell's electromagnetic *fields are transverse to the axis of time* [6].

In contrast to Wheeler and Feynman [7][8] use of fields $\mathbf{E}_{adv}, \mathbf{E}_{ret}$, it was shown[1] the Wheeler-Feynman summation (WFS) can also be written for the four-vector potential A_μ by summing over the advanced A_μ^{adv} and retarded A_μ^{ret} potentials.

$$A_\mu^{int} = \sum \frac{A_\mu^{ret} - A_\mu^{adv}}{2} + \sum \frac{A_\mu^{adv} + A_\mu^{ret}}{2} \tag{2}$$

Note the $(A_\mu^{adv}, A_\mu^{ret})$ are here called the extrinsic or external potentials A_μ^{ext}, but the resultant sum is called the internal or intrinsic potential A_μ^{int}, however, the external magnetic vector potential $\mathbf{A}_\mu^{ext}$ does not lie along along the axis of time only the advanced and retarded scalar potentials φ^{ext} are summed—so we can drop $\mathbf{A}_\mu$,

$$\varphi^{int} = \varphi^{ext} = \varphi^{ret} = \sum \frac{\varphi^{\text{ret}} - \varphi^{\text{adv}}}{2} + \sum \frac{\varphi^{\text{adv}} + \varphi^{\text{ret}}}{2} \tag{3}$$

In the frame of reference of the resultant particle φ is in the Coulomb gauge, and to balance Maxwell's equations we must reintroduce the intrinsic magnetic vector potential $\mathbf{A}_\mu^{int}$, as we as the fields ($\mathbf{E}^{int}$,$\mathbf{B}^{int}$) and the electron current flow $\mathbf{J}^{int}$ and charge by

$$\begin{aligned}
\mathbf{A}^{int} \cdot \hat{\mathbf{n}} &= -\frac{1}{c}\frac{\partial \varphi^{ext}}{\partial t} \\
\mathbf{E}^{int} &= -\nabla \varphi^{ext} - \frac{\partial \mathbf{A}^{int}}{\partial t} \\
\mathbf{B}^{int} &= \nabla \times \mathbf{A}^{int} \\
\nabla \times \mathbf{B}^{int} &= \mu_0 \mathbf{J}^{int} + \mu_0 \eta_0 \frac{\partial \mathbf{E}^{int}}{\partial t}
\end{aligned} \tag{4}$$

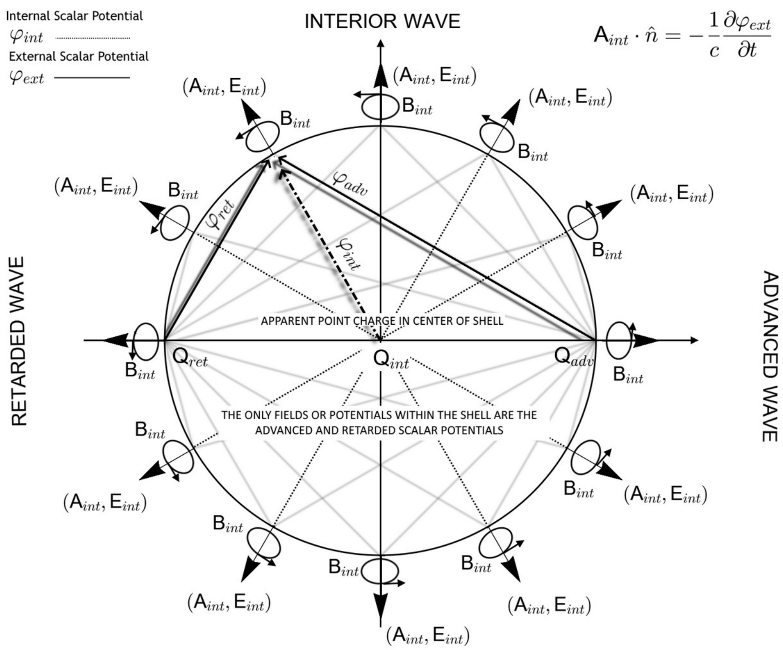
A_μ now reappears as $A_\mu^{int} = (\varphi^{int}/c, \mathbf{A}^{int})$, where $|\varphi^{int}| = |\varphi^{ext}|$; $|\mathbf{A}^{int}| = |\mathbf{A}^{ext}|$; and the Lorenz gauge then gives us the electromagnetic wave equations for electrons in **J** and φ,

$$\begin{aligned} \Box^2 \mathbf{A}^{int} &= \left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right]\mathbf{A}^{int} = \mu_0 \mathbf{J}^{int} \\ \Box^2 \varphi^{int} &= \left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right]\varphi^{int} = \frac{1}{\varepsilon_0}\rho^{int} \end{aligned} \tag{5}$$

All of these fields and potentials are derived from the intrinsic potentials on the surface of a spherically charged particle. The speed v_{int}, is merely Wheeler-Feynman average of the advanced and retarded velocities after equating momentums with $|m\, v_{adv}| = |m\, v_{ret}|$,

$$\begin{aligned} mv_{int} &= \sum \frac{mv_{ret} + mv_{adv}}{2}_{int} + \frac{mv_{ret} - mv_{adv}}{2}_{ext} \\ v_{int} &= \sum \frac{v_{ret} + v_{adv}}{2}_{int} + \frac{v_{ret} - v_{adv}}{2}_{ext} = v_{ret} \end{aligned} \tag{6}$$

Giving us all the quantities necessary for an electron with mass, charge, spin, velocity, and is pictured as a locally charged, hollow, perfect sphere covered with fields and potentials.



IN THIS MODEL AN APPARENT POINT charge Q_{INT} APPEARS IN THE CENTER OF THE SHELL

In the Coulomb gauge everything is a particle while in the Lorenz gauge everything is a wave, and this suggests a charged particle in terms of φ in $\mathbb{R}^3$ and a wavefunction in terms of A_μ in $\mathbb{R}^{1,3}$. Now the use of the four-vector potential formulation becomes clear as it also allows us to immediately write a wavefunction for the particle,

$$\psi^{ext} = \exp(-\tfrac{iq}{2\hbar}\int A_\mu^{ext} dx^\mu) \tag{7}$$

The ½ in eqn (7) appears as a result of splitting the A_μ into two parts along the axis of Time the anterograde direction and the retrograde direction in agreement with Wheeler and Feynman argument, however, we now include the epoch angles Advanced (ε_{adv}) and Retarded (ε_{ret}) as,

$$\begin{aligned}\psi_{\text{ret}} &= \exp(-\frac{iq}{2\hbar}\int A_\mu^{\text{ret}}\text{dx}^\mu + \varepsilon_{\text{ret}})\\ \psi_{\text{adv}} &= \exp(-\frac{iq}{2\hbar}\int A_\mu^{\text{adv}}\text{dx}^\mu + \varepsilon_{\text{adv}})\end{aligned} \tag{8}$$

Giving a total wavefunction of the WFS as,

$$\psi_{int} = \exp(-\tfrac{iq}{2\hbar}\int A_\mu^{\text{ret}}\text{dx}^\mu + \varepsilon_{\text{ret}}) * \exp(-\tfrac{iq}{2\hbar}\int A_\mu^{\text{adv}}\text{dx}^\mu + \varepsilon_{\text{adv}}) \tag{9}$$

Without loss of generality set the epoch angles equal to the Retarded and Advanced potentials of the Wheeler-Feynman free terms.

$$\begin{aligned}\varepsilon_{\text{ret}} &= \exp(-\frac{iq}{2\hbar}\int A_\mu^{\text{adv}}\text{dx}^\mu)\\ \varepsilon_{\text{adv}} &= \exp(-\frac{iq}{2\hbar}\int A_\mu^{\text{ret}}\text{dx}^\mu)\end{aligned} \tag{10}$$

Conservation of phase requires the epoch angles $\varepsilon_{adv} + \varepsilon_{ret} = 0$, thereby giving the wavefunctions a zero phase difference, these were labelled the interference terms, allowing eqn (2) to be expressed as a wavefunction,

$$\psi_{int} = \exp(-\tfrac{iq}{2\hbar}\int A_\mu^{\text{ret}} - A_\mu^{\text{adv}} + A_\mu^{\text{adv}} + A_\mu^{\text{ret}}\ \text{dx}^\mu) \tag{11}$$

Adding the half-wave functions together drops the ½ and results in a unitary wavefunction Ψ_{tot}, thus allowing the WFS to be written in a simpler wavefunctions form,

$$\Psi_{\text{total}} = \psi_{\text{total}} \cdot \psi_{\text{interference}} \tag{12}$$

Now we chose some wavefunction identities to conserve energy and phase,

$$\begin{gathered} |\psi_{\text{ret}}| = |\psi_{\text{adv}}| \\ \psi_{\text{ret}} = \psi^*_{\text{adv}} \\ \psi_{\text{adv}} = \psi^*_{\text{ret}} \\ \psi_{\text{ret}} \cdot \psi^*_{\text{ret}} = I \\ \psi_{\text{total}} = \psi_{\text{ret}} \cdot \psi_{\text{adv}} \\ \psi_{\text{interference}} = \psi_{\text{ret}} \cdot \psi^*_{\text{adv}} = I \end{gathered} \tag{13}$$

These identities reduce the WFS to its most elegant form,

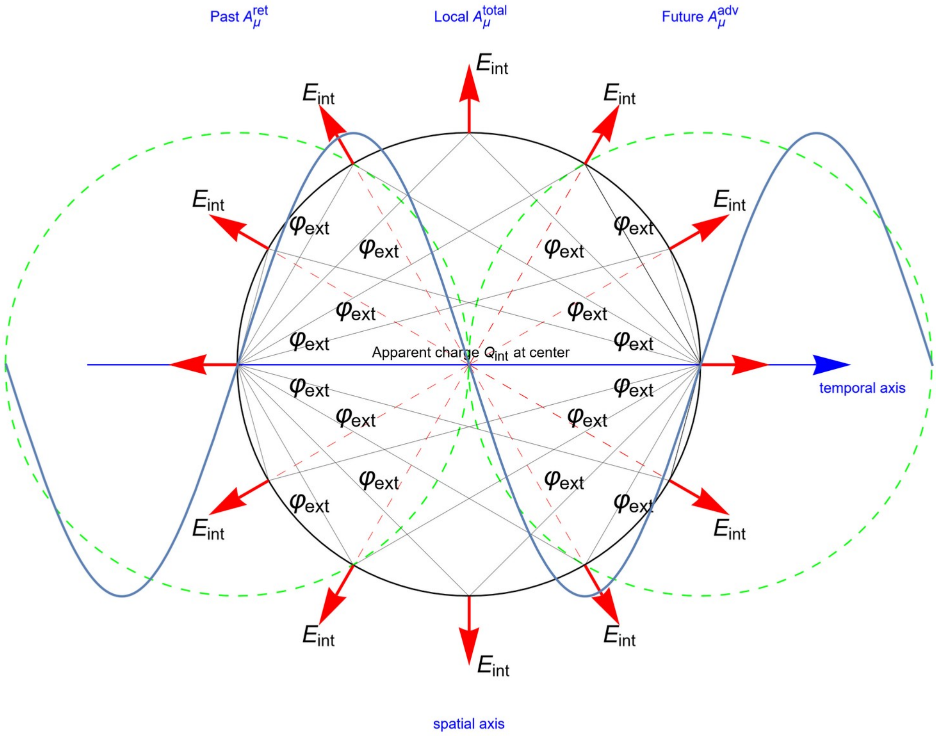
$$\begin{aligned} \Psi_{\text{total}} &= \psi_{\text{total}} \cdot \psi_{\text{interference}} \\ &= \psi_{\text{ret}} \cdot \psi_{\text{adv}} \cdot \psi_{\text{ret}} \cdot \psi^*_{\text{adv}} \\ &= \psi_{\text{ret}} \cdot \psi^*_{\text{ret}} \cdot \psi_{\text{ret}} \cdot \psi^{**}_{\text{ret}} \\ &= \psi_{\text{ret}} \cdot \psi^*_{\text{ret}} \cdot \psi_{\text{ret}} \cdot \psi_{\text{ret}} \\ &= \psi_{\text{ret}} \cdot I \cdot \psi_{\text{ret}} \\ &= \Psi_{\text{ret}} \end{aligned} \tag{14}$$

Where,

$$\Psi_{\text{total}} = \Psi_{\text{int}} = \Psi_{\text{ext}} \tag{15}$$

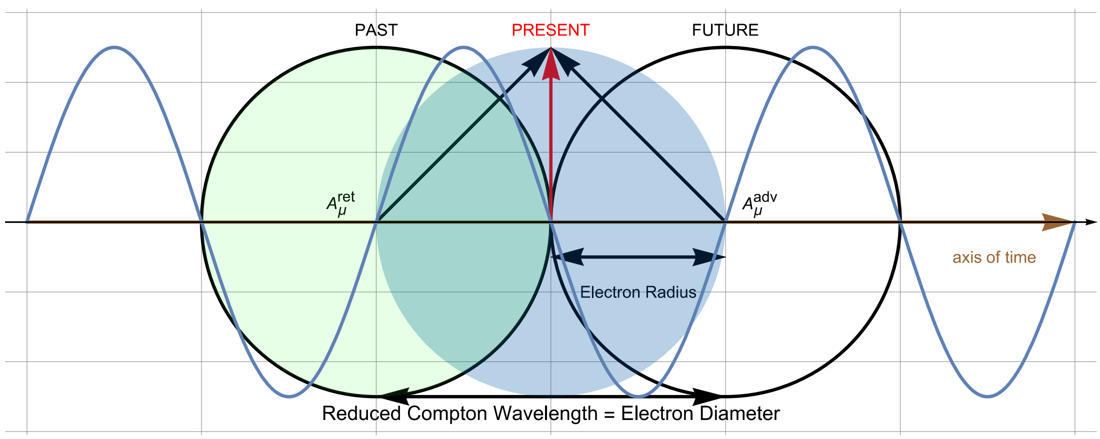
This appears as a series of charged empty spheres progressing along the axis of time, and despite the apparent complexity of the following diagram the spherical electron model behaves in $\mathbb{R}^{1,3}$ as an longitudinal electromagnetic wave just as Maxwell's electromagnetic wave evolves as a transverse electromagnetic wave in $\mathbb{R}^3$ (Maxwell 1864)[6] and was shown to result in an exact value of the Reduced Compton wavelength for the electron cf. [1][2][3][4].

This was labelled as the W.F.E.M.F.V.P.T.S.T., or the Wheeler-Feynman Electromagnetic Four-Vector Potential Time Symmetric Theory, and diagrammatically (cf next page) results in a perfectly spherical charged fermion in $\mathbb{R}^3$ travelling as an electromagnetic wavefunction in $\mathbb{R}^{1,3}$.



*Legend "electromagnetic wavefunction = blue lines"; "intrinsic scalar potentials = thin black lines"; "resultant intrinsic **E** fields = red arrows "; and "shells of adv and ret extrinsic potentials = dashed green lines".*

Or more simply,



Now we can address the problem of the spherical electron.

§2 Aspherical Electrons and the W.F.E.M.F.V.P.T.S.T.

Consider the zero phase difference of the free terms

$$\varepsilon_{adv} + \varepsilon_{ret} = 0 \tag{16}$$

If $\varepsilon_{adv} \neq -\varepsilon_{ret}$ then the A_μ^{adv} and A_μ^{ret} potentials are intrinsically out of phase and the Wheeler-Feynman summation is off the mass-shell, but because we are looking for real solutions for on mass-shell particles we must exclude the $\varepsilon_{adv} \neq -\varepsilon_{ret}$. To do this we note that as interference terms are functions of A_μ we only need to show the A_μ^{adv} and A_μ^{ret} potentials are asymmetric because if the extrinsic potentials were asymmetric then their WFS would violate the conservation of energy and once more would be off the mass-shell. Since the sphericity of the electron is determined by the A_μ which is proportional to the scalar potential,

$$\varphi = \frac{1}{4\pi\varepsilon_0}\frac{e}{r_e} \tag{17}$$

We only need note to the electron radius is given by,

$$r_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{m_e c^2} \tag{18}$$

and–after rearranging–we find the scalar potential has a constant value,

$$\varphi = \frac{m_e c^2}{e} \tag{19}$$

This is the same for all of its semi-axes a,b,c where $r_e = a, b, c$ and therefore it follows the eccentricity of the electron must be zero,

$$\epsilon = \sqrt{1 - \frac{b^2}{a^2}} = 0 \tag{20}$$

Since the magnitude of the semi-axes of the A_μ^{adv} and A_μ^{ret} potentials are intrinsically equal it follows the eccentricity of the advanced and retarded electrons are zero and the resultant Wheeler-Feynman summation must yield a perfectly spherical particle, otherwise the interference would be non-zero. Therefore electrons must be perfectly spherical. In other words, the zero interference equates to perfectly spherical particles, and this gives a definitive test for the W.F.E.M.F.V.P.T.S.T. model, for if electrons are shown to be aspherical the model would fail.

§3 Tests for Spherical Electrons and the Electron Electric Dipole Moment

To demonstrate this we need to consider the experimental results for the Electron Electric Dipole Moment EDM (d_e).

The d_e is proportional to the asymmetry of the electron, as aspherical electrons would exhibit a measurable electron electric dipole moment across their diameter, for if $d_e \geq 0$ then the interaction energy U would be greater than zero,

$$U = -\vec{d_e} \cdot \vec{E} \geq 0 \tag{21}$$

In recent experiments[9][10[11] the spherical shape of the electron has been measured to extraordinary precision by probing the d_e:

"*Treating the statistical and systematic errors on equal terms, we can extract an upper bound on the size of the EDM,* $|d_e| < 10.5 \times 10^{-28}$ *e·cm with 90% confidence."* [9]

"*Our result,* $d_e = (-1.3 \pm 2.0_{stat} \pm 0.6_{syst}) \times 10^{-30}$ e cm, *is consistent with zero and gives an upper bound of* $|d_e| < 4.1 \times 10^{30}$ e cm *at 90% confidence."*[10]

"*The result of this second-generation EDM measurement using ThO is* $\omega^{\mathcal{NE}}$ = -510 ±373_{stat} ± 310_{syst} µrad s^{-1}. *Using* $d_e = -\hbar\omega^{\mathcal{NE}}/\mathcal{E}_{eff}$ *and*[16,17] $E_{eff} \approx$ 78 $GVcm^{-1}$ *results in* d_e = 4.3 ± 3.1_{stat} ± 2.6_{syst} × 10^{-30}*e cm*" [11].

It can be seen the this puts the experimental upper limit of the $|d_e| < (10.5 \times 10^{-28}, 4.1 \times 10^{-30}, 4.3 \times 10^{-30})$ e cm, and since the EDM is a measure of the eccentricity of the electrons and given its extraordinarily low value which we can take to be near zero this implies electrons are perfectly spherical.

These results seem to strongly confirm the validity of the W.F.E.M.F.V.P.T.S.T. and should future experiments give a significant eccentricity then the model would be broken, and therefore it is predicted electrons are perfectly spherical.

REFERENCES

[1] O'Brien, J.C., "A Dynamical Theory of the Electromagnetic Potential". viXra:1912.0173 submitted 2018.

[2] O'Brien, J.C., "The Revolution of Matter" (2nd ed.). Amazon Press, (September 1, 2022).

[3] O'Brien, J.C., "Zitterbewegung and the Wheeler-Feynman Time-Symmetric Theory" viXra:2407.0007 submitted 2024.

[4] O'Brien, J.C., "The Arrow of Time and the Wheeler-Feynman Time-Symmetric Theory". (viXra:2312.0029 submitted 2023).

[5] Griffiths, D.J., "Introduction to Electrodynamics" (4th ed.). Cambridge University Press. (2017).

[6] Maxwell, James Clerk., "A Dynamical Theory of the Electromagnetic Field". Philosophical Transactions of the Royal Society of London. 155: 459–512. (Paper read at a meeting of the Royal Society on 8 December 1864).

[7] Wheeler, J.A.; Feynman, R.P., "Interaction with the Absorber as the Mechanism of Radiation", Reviews of Modern Physics. 17 (2-3): 157-181, (April 1945).

[8] Wheeler, J. A.; Feynman, R. P., "Classical Electrodynamics in Terms of Direct Interparticle Action", Reviews of Modern Physics. 21 (3): 425-433. (July 1949).

[9] Hudson J.J., Kara D. M., Smallman I.J., Sauer B.E., Tarbutt M.R., Hinds E.A., "Improved measurement of the shape of the electron". Nature (2011); 473 (7348): 493 DOI: 10.1038/nature10104.

[10] Roussy T.S., Caldwell L., Wright T., Cairncross W.T., "A new bound on the electron's electric dipole moment". Science 381, 46 (2023).

[11] ACME Collaboration. "Improved limit on the electric dipole moment of the electron". Nature 562, 355–360 (2018). https://doi.org/10.1038/s41586-018-0599-8