A simple macroscopic model of ice sheet dissolution

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Abstract

This paper explores the impact of rising global temperature on the melting of ice floes and ice sheets in the Arctic Ocean, Greenland and Antarctic. This paper notes that the current understanding of the impact of climate change on Arctic, Greenland and Antarctic ice floes and ice sheets may be significantly underestimated. First, this paper analyzes the relationship between global temperature change and Global Mean Sea Level (GMSL) rise after the end of the Last Glacial Maximum (LGM). The current rate of global temperature rise is now 10 times faster than after the end of the LGM. This also means that the current rate of GMSL rise will also be likely to be 10 times faster than the rate of GMSL rise at that time. In order to better and accurately analyze the relationship between global temperature rise and GMSL rise, a simple macro model of ice sheet dissolution is established. In this model, we believe that the main cause of the dissolution of the ice sheet is the convective heat transfer from the air. Due to the presence of huge glacial lakes in Greenland and Antarctica, most of the meltwater from the ice sheet is temporarily stored in these glacial lakes. If global temperatures continue to rise, these glacial lakes could cause dam failures and cause catastrophe. We used this model to estimate the rate of dissolution of ice floe in the Arctic Ocean, and the estimates were in good agreement with the actual observations. We then used this model to estimate the rate of dissolution of the Greenland and Antarctic ice sheets. Estimates suggest that the risk of significant GMSL rise and dam failure of glacial lakes is very high in the coming decades.

Content

1 Introduction

Many models of climate or ice sheets can be found online and in academic databases. The scope is very wide, and the results are varied.

For example, the model established by the IPCC (www.ipcc.ch) is based on the relationship between climate change and ecosystems, biodiversity, and human society (Zhou. 2021). Through this model, it is possible to determine the direct interaction between climate change and the way of life of human society, sustainable development methods, etc. After considering the interactions between the atmosphere, ice sheets, and oceans, Park et al. developed a more complex atmosphere-ocean-ice-sheet coupling model (Park, Schloesser & Timmermann, et al. 2023) to make more accurate predictions of Global Mean Sea Level (GMSL) rise.

The Community Earth System Model (CESM) (www.cesm.ucar.edu) was first released in 1983 by the NSF-backed National Center for Atmospheric Research (NCAR). This is a climate model that considers a lot of factors, in which a variety of factors such as the atmospheric system, rivers, land, ocean, ice sheet, ice floes, etc., are comprehensively considered, and the models established by these factors are coupled together to form a complex model of the earth systems. These kind of models can be used to predict changes in global temperature and atmospheric carbon dioxide concentrations. For example, the predictions of similar Community Climate System Model (CCSM) (Smith, Jones & Yeager. 2010) in 2001 show that such models are relatively accurate. (Blackmon, Boville & Washington. 2001).

In addition to the more typical climate models mentioned above, there are of course other models (Flato, Marotzke & Collins, et al. 2014), as well as some very characteristic models, such as NeuralGCM, which uses neural networks to predict climate change. (Kochkov, Yuval & Hoyer. 2024). Overall, however, one of the obvious shortcomings of these models is the lack of a very macro model of the impact of climate on the dissolution of ice sheets. After all, observing the impact of climate change on the ice sheet on Earth will be very different from watching the impact of climate change on the ice sheet as a whole in universal space. Therefore, the macro model in this paper can be a good supplement to the various climate models that have been established, such as CESM.

2 The current GMSL rise can be compared to the GMSL rise at the end of the LGM

The Last Glacial Maximum (LGM) occurred about 19,000 years ago, when the GMSL was about 80-120m lower than it is today. The end of the last glacial period was about 11,700 years ago, that is, it lasted for more than 7,000 years. In those 7,000 years, global temperatures have risen by about eight degrees Celsius. This allows us to estimate that GMSL will rise by more than 1*m* over a period of 100 years. However, the situation is very different now, and since the Industrial Revolution, the global temperature rise has reached nearly 1.5°C.

According to the findings of Osman, Denton et al. (Osman. 2021) (Denton, Anderson & Putnam. 2010), the global temperature and GMSL changes have been relatively flat for more than 10,000 years, but in the past 100 years, that is, since the Industrial Revolution, the global temperature has suddenly risen sharply by nearly 1.5°C, and such a large temperature increase actually corresponds to the temperature increase more than 1000 years after the end of the LGM. In other words, temperatures are rising at least 10 times faster than they have since the end of the LGM.

If we take into account the rise in GMSL caused by the rise in atmospheric temperature over the past 100 years, which can be approximated linearly for the entire physical law, it means that the current GMSL rise rate will be 10 times higher than the GMSL rise after the end of the LGM. This means that if the GMSL rises by more than 1 meter every 100 years in the end of LGM, then at the rate of global temperature rise since the Industrial Revolution, sea levels should now rise by more than 1 meter every 10 years and by more than 10 meters every 100 years. Of course, the very complex interaction between the Greenland and Antarctic ice sheets and global temperatures means that the rate of increase will be slower in some years over the course of the century. The slower rate of rise does not mean that these ice sheets, which have less melt, will not be compensated for in the future. Conversely, uneven rates of sea-level rise can lead to more dramatic changes in climate and geology.

From the above analysis, we can also see that during the GMSL rise after the end of the LGM, GMSL rise of 1 meter per 100 years means that the GMSL rise is 1 centimeter per year. The current rate of GMSL rise, measured by relatively accurate satellite measurement techniques, is only a few millimeters per year. In other words, if the GMSL rise is measured to exceed $1cm$ in a given year, it means that the rate of GMSL rise may soon exceed the rate of GMSL rise after the end of the LGM.

Of course, GMSL rise is uneven, with some regions seeing even greater GMSL rise. For example, CNN reported that the GMSL of the Pacific Ocean off the coast of China rose by $0.94cm$ from 2021 to 2022 (Gan. 2023). This is very close to the level of GMSL rise at the end of LGM.

3 The macro model of ice floes and ice sheet dissolution

3.1 Factors influencing the dissolution of ice sheets and the establishment of the model

3.1.1 Factors influencing the dissolution of ice sheets

The first thing that can be determined is that the amount of heat generated by human activities is much smaller than the amount of heat radiated to the Earth by sunlight. Therefore, if the atmosphere, ice sheets or ice floes, seawater or rocks are considered as a system, the temperature of the entire Earth is essentially constant while the radiation energy received from the sun is unchanged. Of course, the carbon dioxide produced by human activities prevents infrared heat from radiating out. However, carbon dioxide also prevents the infrared rays contained in the sun's rays from radiating into the Earth. Considering that the energy of solar radiation is greater than that of the earth, this is the basic reason why the earth has been cooling for billions of years. Therefore, the infrared part of the energy contained in the solar radiation energy also has the same law. So we believe that the two are essentially equivalent.

Of course, in the case of Venus, the concentration of carbon dioxide in the air is very large, so it is basically difficult for the radiant energy of the sun's rays to reach the ground, and relying entirely on the heat inside Venus for heating, the temperature of the surface of Venus can reach 400 degrees Celsius. From the heat conduction equation, it can be estimated that the time required to heat the surface of the planet by heat conduction alone will be calculated in millions of years. At present, more than 100 years have passed since the industrial revolution of mankind. Therefore, it is certain that the current cause of atmospheric warming is not only the greenhouse effect caused by carbon dioxide produced by human combustion of fossil fuels, but also the heat generated by human activities is also the direct cause of global warming.

So we can understand this earth system in this way, because the magnitude of the thermal conductivity is in order

$$Rocks > ice > seawater > atmosphere$$

It can be seen that when the global temperature rises, the heat released by human activities heats the atmosphere, causing the temperature of the atmosphere to rise. Heat from the atmosphere is transferred to seawater, land rocks, ice sheets and ice floes in the Arctic and Antarctic by means of convective heat transfer. This heat is further transferred to the Earth's interior until it reaches equilibrium with the heat released from the Earth's core. In this way, it is relatively clear that it is mainly the convective heat transfer from the atmosphere and the oceans that causes the dissolution of the Arctic and Antarctic ice sheets or ice floes. The process by which atmospheric convective heat transfer heats ice sheets or ice floes is relatively certain. However, if there is less seawater and the flow velocity is relatively low, the convective heat transfer effect of seawater will be weaker. Especially for the Arctic ice floes, the sunlight shines through the ice floes into the seawater, first of all, the ice floes absorb a lot of heat, and the heat used to heat the sea water is relatively small. Therefore, it is expected that the direction of convective heat transfer in seawater and the direction of convective heat transfer in the atmosphere will be opposite. That is, the ice floes provide heat to the seawater, which slows down the dissolution rate of the ice floes.

From the perspective of thermodynamics, the transfer of heat mainly involves two modes:

convective heat transfer and heat conduction. Convective heat transfer is caused by the flow of fluids. It is related to the material and flow rate of the fluid. The heat of convective heat transfer can be analyzed using Newton's laws of cooling. namely

$$\frac{dQ}{dt} = hA\Delta T$$

Wherein: Q is the heat transfer by convection, h is the convective heat transfer coefficient, and T is the temperature difference.

Heat conduction, on the other hand, can be calculated using Fourier's law. namely

$$\frac{dQ}{dt} = \kappa A\frac{dT}{dx}$$

where κ is the thermal conductivity, A is the area of thermal conduction, and x is the distance of thermal conduction. From the comparison of these two formulas, since the thermal conductivity per unit length is much smaller than the convective heat transfer coefficient, ie

$$h \gg \frac{\kappa}{\Delta x}$$

The amount of heat transferred as a result of heat conduction will be much smaller than the heat transferred by convective heat transfer. For air, if the velocity of its flow reaches 10 m/s, its convective heat transfer coefficient reaches $h = 37Wm^{-2}K^{-1}$. The thermal conductivity of air $\kappa \approx 0.024\sim0.028\ Wm^{-1}K^{-1}$. That is, the heat due to convective heat transfer is much greater than the heat from the slow heat conduction of air. The same is true for seawater. For example, the thermal conductivity of seawater is about $\kappa \approx 0.6\ Wm^{-1}K^{-1}$, and the thermal conductivity of seawater per unit length is much smaller than the convective heat transfer coefficient of air. Therefore, in the presence of convective heat transfer in fluids, the heat transfer is mainly based on convective heat transfer.

In this way, we can analyze in detail the factors affecting the Arctic and Antarctic ice sheets or ice floes, which mainly involve the following aspects:

1. Convective heat transfer and heat conduction of air

First of all, we should make it clear that the thickness of the atmosphere is 100 kilometers relative to the ice sheet of Greenland or the Antarctic continent, so the thickness of the atmosphere is relatively non-negligible. The greater the thickness of the atmosphere, which means that warmer air from farther distances is easier to flow over the South and North Poles. Of course, since the specific heat capacity of air is much smaller than that of seawater, this also means that the heat carried by the air is transferred to the ice sheet and cools down quickly. However, because the atmosphere flows at a relatively fast speed, hot air from the equator will quickly replenish the heat. Especially when there is turbulence throughout the atmosphere, such as the occurrence of El Niño, the efficiency of convective heat transfer will be higher. Therefore, the heat transfer affecting the Arctic and Antarctic ice sheets or ice floes is mainly based on the convective heat transfer of air. The heat conduction of air is essentially negligible.

2. Convective heat transfer and heat conduction of seawater

At room temperature, the convective heat transfer coefficient of seawater is about $h = 50 \sim 500 Wm^{-2}K^{-1}$, which is larger than that of air. However, the depth of the sea beneath the Arctic ice floes is only about a thousand meters. This depth is not large compared to millions of square kilometers of ice floe. In addition, considering that the relative movement of sea ice and seawater is relatively slow, the exchange between the seawater under the Antarctic ice floe and the seawater outside the ice floe is not very frequent, so the temperature of the seawater below the ice floe will basically remain relatively stable. Of course, these seawater also generate some ocean currents, so the heat exchange of convective heat transfer is much greater than the heat conduction. However, this convective heat transfer is mainly to take away some of the heat from the inside of the ice floe and slow down the dissolution rate of the ice floe.

3. Heat conduction of rocks

Rocks do not flow, so they transfer heat mainly through heat conduction. When the temperature of the entire earth system is relatively stable, the temperature inside is higher the further you go towards the inner core of the earth. Therefore, the temperature on the surface of the rock contact with the sea or ice sheet should be relatively low. However, as global temperatures rise, the ice sheets are heated, and these heated ice sheets transfer heat to the rocks below. But if the heat is transferred only by heat conduction, it takes a very long time. Using the heat conduction equation, it can be estimated that it will take about several hundred years for this heat to be gradually transferred to the interior of the rock. However, for the ice sheet in Greenland or the Antarctic continent, when the ice sheet is dissolved by air convection heat transfer, a large amount of thick meltwater from the ice sheet will fall to the bottom of the ice sheet, and a very large glacial lake will form at the bottom of the ice sheet. The temperature of these glacial lakes will be relatively high, which can release a certain amount of heat to reduce the temperature gradient inside the rocks, which in turn will alleviate the dissolution of the ice sheet to a certain extent.

3.1.2 The macro model

From the analysis in the previous section, it can be seen that the factors influencing the dissolution of the Greenland and Antarctic ice sheets are mainly the convective heat transfer of air, while seawater and rocks can alleviate the dissolution of the ice sheet to some extent. Based on such a consideration, we can build a very macro model. In this model, we can ignore the differences in temperature in different parts of the Earth's atmosphere as a whole. In addition, considering that the area of the North and South Pole ice sheets is relatively large, we can use a relatively simple one-dimensional model, and the coordinate axis is the conduction of heat along the radial direction of the earth. This macro model allows us to ignore the influence of many details.

Of course, we can also use a very complex model. Considering that the thermodynamic mechanisms by which the ice floes and ice sheets of the entire Earth interact with other Earth's materials, etc., are so complex, very complex differential equation models may be required to describe them. The advantage of this sophisticated model is that it is possible to detail the impact of every tiny factor on the ice sheet. For example, the change of salt concentration of seawater caused by the freezing and dissolution of ice floe on the sea surface, and the change of salt concentration affects the change of temperature gradient in seawater. However, this kind of detailed description can easily lead us to pay too much attention to detail and ignore the changes at the macro level. Unlike today's geophysics, which is mainly about the fine structure of the earth, global warming is a much more macro problem. At a very macro level, if there are too many details involved, like when we use Newton's laws, we also have to consider what shape the wave function of the electron is, which can make the whole problem very complicated.

Therefore, the model used in this article will be kept as simple as possible. The advantage of a macro model is that it allows us to address climate change at a much macro level.

The simpler macro model also means that our model is more scalable, which means that if we find that there are various fine structure problems involved in the earth's climate change in the research process, then we can use the macro effects generated by these fine structures to make appropriate modifications to the model, so that the conclusions drawn by the model can better describe the real problems. It's like the tunneling effect of quantum mechanics, and the macroscopic effects it produces allow us to make macroscopic instruments like the Josephson interferometer.

Of course, a simple macro model has the advantage of being easy to modify. In other words, when we use such a macro model to solve a problem, we can easily find the problem when we encounter trouble. For example, in this model, we currently ignore the heating effect of seawater and rocks on ice floes or ice sheets. In the future, if there is sufficient evidence to prove that the heating effect of seawater or rocks on ice floes or ice sheets is very significant, then it is sufficient to add two factors: seawater and rocks to this model.

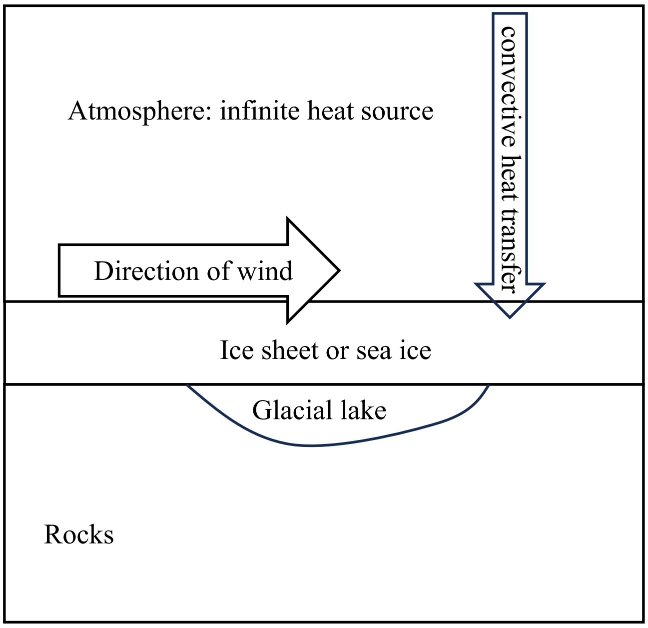

Fig. 1. Effect of atmospheric convective heat transfer on ice sheets or sea ice

The structure of the whole model is shown in Fig. 1, where we can see that the atmosphere acts as an infinite heat source to provide persistent heat to the Arctic ice floes or ice sheets. The effect of this infinite heat source means that the heating of the ice floes and ice sheets in the Arctic and

Antarctic will not cause the temperature of the atmosphere to drop significantly. Of course, this is a more idealistic state. Because when the global temperature rise is not very large, the ice floes or ice sheets of the Arctic and Antarctic can adjust the global climate appropriately. In other words, when global temperatures rise, the cooling effect of the Arctic and Antarctic ice floes and sheets can keep the global temperature at the right temperature. This may also be the reason why the global temperature did not rise significantly in the early days after the start of the Industrial Revolution. However, if we stretch the time span very long, to decades or even hundreds of years, then the temperature of the atmosphere will always maintain an upward trend due to the continuous heating of the atmosphere and the greenhouse effect of carbon dioxide, and it will be difficult to reverse it in a short time. The dissolution of ice floes and sheets is relatively disposable, which is why we can think of the entire global atmosphere as an infinite source of heat.

On the other hand, through calculations, we can find that the thermal conductivity of seawater or rock is very large, about 10-50 times that of air. In other words, the temperature is easily maintained in the atmosphere. However, if this temperature is absorbed by the ice floes or ice sheets, the heat is quickly transferred to the seawater or rocks. These seawater or rocks are connected to the sphere of the whole earth. The size of the Earth is very large, so even a 3°C increase in global atmosphere temperature will not cause a significant increase in the temperature of the entire Earth's sphere. So we can think of the seawater and rocks on Earth as a thermostat. In other words, its temperature is largely unaffected by the rise in global climate temperatures. Of course, there is already evidence that the average global sea temperature is also rising rapidly. Without a continuous supply of heat and the effects of atmospheric greenhouse gases, this rise is likely to be short-lived. If there is not enough heat supply, then the heat of the seawater will spread out through the rocks on the seabed, eventually lowering the temperature of the seawater. Of course, the thermal conductivity of seawater is somewhat smaller than that of rock. The thermal conductivity of rocks is about three to four times that of seawater, which means that the temperature in seawater can be maintained for a relatively long time.

Therefore, the rise in the temperature of the seawater, especially the Arctic seawater covered by ice floe, absorbs less energy from the sun, and we think it is a passive warming process. It must heat up more slowly than the ice floes. Under conditions where there is not enough sunlight in the Arctic and Antarctic, the effect of sunlight heating the seawater will be much less. At this time, the heat in the atmosphere is mainly used to heat the Arctic ice floes by means of heat transfer, and then transfer this heat to the sea water below the ice floes.

From the results of the following calculations, we can see that such an estimate is basically reasonable. Our current model is like ice cubes placed on a shallow plate in a room, and then there is a forced convection in the air at a wind speed of about 10 meters per second. There is also a stove in the room to heat the whole room. In this way, we can see that the water dissolved by the ice cubes will remain on the plate at the beginning, but once the water of the ice cubes has dissolved to a large amount, the water will overflow and flow to the floor of the whole room, which is equivalent to the GMSL starting to rise.

With such a simplified model, it is easier to calculate the impact of global atmospheric warming on the melting of ice floes and ice sheets in the Arctic, Greenland and Antarctic. The fluidity of the atmosphere under consideration is very strong, so the way the atmosphere transfers heat to the ice floe and ice sheet is mainly convection. Here we can use Newton's laws of cooling to do the calculations. The parameters are also relatively easy to determine. For example, from some studies, we can find that the wind speed in the Arctic is generally about 10m/s, so the convective heat transfer coefficient can be determined to be $h = 37Wm^{-2}K^{-1}$. The thermal conductivity, specific heat capacity, and density of substances such as ice and air are all known, so the whole calculation process is very simple and straightforward. Let's start with an analysis of the dissolution of Arctic ice floe. The results of the calculations are basically consistent with the actual observations. This also verifies the correctness of this model to a certain extent. Then we applied this model to the dissolution of the ice sheets in Greenland and Antarctica. Estimate the rate of dissolution of the Greenland and Antarctic ice sheets.

3.2 The rate at which Arctic ice floes dissolve

Judging from the dissolution of Arctic ice floe, in 1988 the area of Arctic ice floes over four years old reached A=3.12 million square kilometers. By 2019, the area of ice floes with a life span of more than four years was only 89,000 square kilometers (NASA Scientific Visualization Studio). In other words, more than 3 million square kilometers of Arctic ice floes that are more than four years old have dissolved in 31 years.

According to NASA data (SMD Content Editors), between 1988 and 2019, global temperatures rose by approximately 0.93-0.31=0.62 (°C). We can approximate the rise in temperature during this period as a linear rise process, so that

$$\Delta T = kt = \frac{0.62}{31 \times 365 \times 24 \times 3600} t = 6.342 \times 10^{-10} t$$

Of these, 31 represents the 31-year period from 1988 to 2019.

In addition, when heat is fed into the ice floe, the ice floe dissolves. Considering that the ice floes are very thin, the area of the ice floes will be reduced. In this way, the relationship between the heat absorbed and the area of the ice floe is:

$$dQ = -H_{of}dM = -4H_{of}\rho dA$$

The thickness of the ice floe is 4 meters. The minus sign reflects the decrease in heat in the air that causes the ice floes to dissolve. Where *A* is the area of the Arctic ice floe, *M* is the mass of the Arctic ice floe, *Q* is the heat absorbed by the ice floe. H_{of} is the pyrolysis of ice, ρ is the density of ice. Then we consider Newton's law of cooling

$$\frac{dQ}{dt} = hA\Delta T$$

Since the Arctic wind speed can generally reach 10m/s, a relatively large air convection heat transfer coefficient $h = 37Wm^{-2}K^{-1}$ can be used.

We have

$$-4H_{of}\rho dA = (0.62hA \times 6.342 \times 10^{-10}t)dt$$

Then we can get

$$lnA - lnA_0 = -5.44 \times 10^{-18}t^2$$

Here we set 1988 to $t = 0$. A_0 is the area of ice floes in the Arctic that were more than four years old in 1988. So

$$A = A_0 \exp(-5.44 \times 10^{-18}t^2) = 1.65 \times 10^{10}(m^2) = 16500(km^2)$$

In 2019, only about 89,000 square kilometers of Arctic ice floes remained (NASA Scientific Visualization Studio). Considering that there may be other factors involved, such as the role of the relatively new ice floe in the surrounding area to absorb heat, the convective heat transfer of seawater also absorbs a large part of the heat in the ice floe, which leads to a decrease in the amount of ice floe dissolves, this result is largely consistent by orders of magnitude.

This also proves that the main cause of the dissolution of ice floe or ice sheet is the convective heat transfer of air. The temperature rise or melting effect of ice floes or ice sheets caused by heat conduction in seawater and rocks is very weak.

3.3 Estimation of the rate of dissolution of the Greenland ice sheet

We can borrow the dissolution model of the Arctic Ocean ice floes to estimate the rate of dissolution of the Greenland ice sheet. This is because the Arctic ice floes are underneath seawater, while the Greenland ice sheet is rocky underneath. Except that seawater can flow and is therefore able to transfer heat by convection to some extent, the other properties are basically the same. In addition, considering that the Greenland ice sheet is more than 1,000 meters thick, it can be seen from the above estimates that the heat in the atmosphere is basically only consumed in the ice sheet, and the amount of heat transferred to the rock is negligible.

The amount of heat from atmospheric convective heat transfer can be estimated using Newton's law of cooling.

$$\frac{dQ}{dt} = hA\Delta T$$

Considering that the loss of ice sheets in Greenland and Antarctica during a period of limited temperature rise is relatively small and does not result in a significant reduction in the area of the ice sheet, so we can assume that area A is essentially constant.

Since the surface of the Greenland ice sheet is not completely flat, the actual surface area is A_s larger than that covered by the Greenland ice sheet. Take $A_s = 1.3A$. From the empirical formula of air convective heat transfer coefficient, we can obtain air convective heat transfer coefficient $h = 37Wm^{-2}K^{-1}$.

Among them, the Greenland ice sheet covers an area of $1.834 \times 10^{12} m^2$

Ice volume in Greenland is $2.75 \times 10^{15} m^3$

In this way, according to the global atmospheric warming of 1.5°C, the heat brought by the air can be reached

$$\frac{dQ_{air}}{dt} \approx 37 \times 1.3 \times 1.8 \times 10^{12} \times 1.5 \approx 1.3 \times 10^{14} (W)$$

Therefore

$$\Delta Q_{air} \approx 1.3 \times 10^{14} \Delta t$$

Considering that the total mass of the Greenland Ice Sheet is $M = 2.75 \times 10^{18}$, the time required to dissolve the entire Greenland Ice Sheet is

$$\Delta t = \frac{MH_{of}}{\Delta Q_{air}} = \frac{2.75 \times 10^{18} \times 3.341 \times 10^5}{1.3 \times 10^{14}} \approx 7.07 \times 10^9 (s) \approx 224 (years)$$

This seems to be a long time. However, considering that only 1/10 of Greenland's ice sheet can cause GMSL to rise by 0.7 meters, that is, Greenland's ice melt will cause GMSL to rise by about 0.7 meters in about 22 years. This also means that by around 2046, GMSL could rise by nearly one meter since the Industrial Revolution as a result of Greenland's melting ice. This is a considerable increase. And we are not taking into account the possible exponential rise in global temperatures here. If global temperature rise accelerates in the future, it means that the Greenland ice sheet will dissolve faster. However, these estimates are based on a 1.5-degree rise in global temperatures, with the temperature difference between the atmosphere and the ice sheet remaining unchanged. But in fact, when the ice sheet dissolves to a certain extent, the temperature difference between the atmosphere and the ice sheet will gradually shrink and reach an equilibrium, at which time the ice sheet of the Arctic and Antarctic will no longer melt and grow, and the GMSL will stabilize.

Let's take a look at the water storage capacity of Greenland's glacial lakes. If the global temperature rises by 1.5°C and is maintained, the amount of ice melt in Greenland per year due to air convection heat transfer will be

$$m = \frac{\Delta Q_{air}}{H_{of}} \approx \frac{1.3 \times 10^{14} \times 31536000}{3.34 \times 10^{5}} \approx 1.23 \times 10^{16} (kg)$$

According to NASA, the current mass of Greenland's ice sheet flowing out of the ocean per year is $2.67 \times 10^{14} kg$

It can be seen that after the Greenland ice sheet dissolved, most of the meltwater did not flow out into the ocean. This part of the unflowing ice sheet meltwater is usually stored in Greenland in the form of glacial lakes. If the ice sheet dissolves further, once the volume of water in these glacial lakes is large enough, it can cause the glacial lakes to burst their banks, creating very large floods. Of course, if global temperatures drop, these glacial lakes will re-condense into ice caps.

This is the same effect as the construction of many dams in the sixties and seventies of the last century, when major countries in the world built. According to the results of existing studies, the rate of rise in GMSL as a whole did slow down significantly in the sixties and seventies (Frederikse, Landerer & Wu. 2020). This has to do with the dam's role in storing water. In addition, after the end of LGM, a large number of glacial lakes formed on the Qinghai-Tibet Plateau in China also have a relatively strong water storage effect, thereby slowing down the rise of sea level. At present, there are many geological relics of such glacial lakes on the Qinghai-Tibet Plateau. For example, the Tarim Basin covers an area of 400,000 square kilometers. If much of the Qinghai-Tibet Plateau was covered by glaciers during the last glacial period, the meltwater from the glaciers would form an inland lake in the Tarim Basin that is larger than the Caspian Sea. And if the dam fails, it will cause very large floods in the middle and lower reaches of the Yangtze River in China.

We can refer to the area of the Tarim Basin to estimate the maximum water storage of the glacial lakes in Greenland. Based on 400,000 square kilometers and a water storage depth of 1,000 meters, the weight of water that can be stored is

$$m_t = 40 \times 10^{10} \times 1000 \times 1000 = 4 \times 10^{17}(kg)$$

That's about 1/6 of the entire Greenland ice sheet. If all of this water were to flow into the ocean, it should raise GMSL by about 1 meter.

With global temperatures rising by 1.5°C and maintained, the amount of ice melt remaining in Greenland is about $1.23 \times 10^{16}kg$ per year, which means that these glacial lakes can be filled with meltwater from the ice sheet in about 32.6 years. Under the condition that the glacial lakes of Greenland can no longer hold more glacial meltwater, the amount of ice melt from the ice sheet will cause the sea to rise by 3.4 centimeters per year.

3.4 Dissolution trends of the Antarctic ice sheet

The area of the Antarctic ice sheet $A = 1.24 \times 10^{13}m^2$, the total mass of the ice sheet is $2.45 \times 10^{19}kg$, and the global temperature warms by 1.5°C, taking $A_s = 1.3A$. Take the air convective heat transfer coefficient $h = 37Wm^{-2}K^{-1}$, according to the above calculation formula, the annual heat conduction of the atmosphere to the ice sheet is

$$\frac{dQ_{air}}{dt} \approx 37 \times 1.3 \times 1.24 \times 10^{13} \times 1.5 \approx 8.95 \times 10^{14}(J)$$

The time it takes for the Antarctic ice sheet to dissolve completely

$$\Delta t = \frac{MH_{of}}{\Delta Q_{air}} = \frac{2.75 \times 10^{19} \times 3.341 \times 10^5}{8.95 \times 10^{14}} \approx 8.4 \times 10^{10}(s) \approx 266(years)$$

At 1.5°C of global warming, the annual dissolution mass of the ice sheet in Antarctica due to air convection is

$$m = \frac{\Delta Q_{air}}{H_{of}} \approx \frac{8.95 \times 10^{14} \times 31536000}{3.34 \times 10^5} \approx 8.44 \times 10^{16}(kg)$$

Since the total dissolution of the Antarctic ice sheet can raise sea levels by about 60 meters, over a period of about 27 years, sea levels may rise by about 6 meters due to the dissolution of the Antarctic ice sheets.

Combined with the dissolution of Greenland's ice sheet and the dissolution of other continental glaciers, it is still very likely that GMSL will rise by about 7 meters by about 2050. Of course, this is without taking into account the effect of glacial lakes on the Antarctic continent.

Based on the rise in GMSL after the end of the LGM, a global temperature rise of 1.5 degrees means that the limit of GMSL rise is about 15 meters. This means that when the GMSL reaches about 15m, the dissolution of the ice sheets in Greenland and Antarctica will reach a dynamic equilibrium, and excess meltwater will no longer be discharged into the ocean.

Of course, the above estimates do not take into account the effects of glacial lakes in Antarctica. Therefore, let's estimate the water storage capacity of Antarctic glacial lakes.

Considering that about half of Antarctica is plain, this is somewhat similar to the topography of Chinese mainland. Therefore, it can be estimated in terms of the area of the Qinghai-Tibet Plateau in China. Assuming that the area of Antarctica that can form glacial lakes is about 2 million square kilometers, and it can store 2,000 m of water, then the amount of water can be reached

$$m_t = 200 \times 10^{10} \times 1000 \times 1000 = 2 \times 10^{18}(kg)$$

It takes about 23.7 years to fill these glacial lakes with water. After that, all the meltwater from Antarctica began to be gradually discharged into the ocean. With an annual displacement of $8.44 \times 10^{16} kg$, the ocean can rise by 23.5 cm per year. Unless global temperatures stop rising after that, the risk of dam failure of glacial lakes is always there.

Therefore, between now and 2057, the annual inflow of small amounts of glacial meltwater into the oceans will be mainly through glaciers river. Because Greenland has a longer coastline than the Antarctic mainland, Greenland will export about twice as much glacial meltwater as Antarctica. After 2057, the GMSL rise will gradually increase to 23.5 centimeters. In about nine years, the glacial lakes of Greenland will also fill in, after which GMSL rise will reach 27 centimeters. In other words, in about 20~30 years, we will probably see the GMSL rise by at least $1m$ or more than the current level, and the rate of rise will be accelerated rapidly. GMSL rise will rise rapidly to around 6 meters by about 2080.

4 Conclusions

Available data confirm that global temperatures are now rising ten times faster than they have since the end of the LGM. This also means that the rate of GMSL rise could now also increase by a factor of 10. After the end of the LGM, the GMSL rise of $1m$ every 100 years corresponds to the current GMSL rise of 1 meter per 10 years. The rate of GMSL rise is very fast. Surely why aren't we seeing such a large GMSL rise now? In fact, available data suggest that the current rise in GMSL since the Industrial Revolution is only nearly 30 centimeters. This should have something to do with the thermodynamic mechanism of the Antarctic and Greenland ice sheets. Because The Greenland and Antarctic are very large and the lands are uneven, and it is easy to form very large glacial lakes that can temporarily store some of the glacial meltwater, thus limiting the direct discharge of glacial meltwater into the ocean. On the other hand, the rise in global temperature since the Industrial Revolution has been relatively stable, not suddenly rising to nearly 1.5 degrees. This means that in the process of gradual accumulation of temperature, it will not be able to have a more serious impact on the Greenland and Antarctic ice sheets immediately. On the contrary, the Antarctic ice floes and ice sheets can also regulate and stabilize the temperature of the global atmosphere to a certain extent. That's why GMSL aren't rising as fast at the moment. However, for some nonlinear physical phenomena, if certain critical points are breached, a very large and rapid GMSL rise process may be imminent. Such a rapid rise in sea levels would be catastrophic. From our analysis, this inflection point could occur around 2057, when the glacial lakes of the Antarctic continent have been filled with meltwater.

In addition, the rate of GMSL rise should be of particular concern if it exceeds 1 centimeter per year. This is because this rate has exceeded the average annual rate of GMSL rise after the end of the LGM.

From the estimates in this paper, it can be seen that at a global warming of nearly 1.5°C, it could melt away 20% of the Greenland and Antarctic continental ice sheets in about a few decades. This rate of melting is at least ten times faster than the rate at which ice sheets dissolve after the end of the LGM. Given the rapid rise in temperatures since the Industrial Revolution, this conclusion is reasonable.

Of course, there is a premise for this conclusion to be true, that is, if the global temperature in the next few decades can maintain the current 1.5°C increase compared to the pre-industrial revolution, according to the dissolution of the ice sheets during the last glacial period, the GMSL corresponding to the limit dissolved ice will rise to about 15 meters and stop rising. However, in view of the current acceleration of the global temperature rise, it is believed that the global temperature will rise by more than 3 degrees Celsius in the future, which is also a very high probability. In this case, it is also possible that all the ice sheets in the Greenland and Antarctic will melt due to uncontrolled warming. Naturally, the challenges to human society will be greater.

Compared with other studies, the advantage of our model is that it is a very macro model. Therefore, the influence of various details can be minimized as much as possible, allowing us to pay more attention to the essence of the problem. Of course, this article only makes it clear that tackling the accelerating rise in sea levels is an urgent task facing humanity, and that we need to start acting now. This may involve very large migrations of people, or it may involve the construction of very large coastal dams. However, we believe that no matter which plan it is, it should reach the stage where it can be implemented immediately.

The shortcomings of this study are also obvious. The first shortcomings of this study are whether it is appropriate to compare the current rate of GMSL rise with the amount of GMSL rise after the end of the LGM. If there is a nonlinear relationship between the rise in global temperature and the rise in sea level, it may involve more complex physical laws. Secondly, in comparison with the rise in GMSL after the end of the LGM, a global temperature rise of 1.5°C could consume about 20% of the ice sheet in Antarctica and Greenland, resulting in a GMSL rise of about 15 meters. However, from the above analysis, it can also be seen that the glacial lakes above Antarctica and Greenland also have a very large water storage capacity, reaching $2.4 \times 10^{18} kg$. This is equivalent to a GMSL rise of 6.67 meters. Therefore, if the water storage effect of glacial lakes is subtracted, the eventual rise in GMSL caused by a temperature rise of 1.5°C could be only about 9 meters. We also do not take into account the weight of glacial lake meltwater accumulated between the Industrial Revolution and around 1988, which could lead our calculations to underestimate the impact of glacial lake dam failures on GMSL rise. Of course, since this is a macroscopic model, we do not fully consider the fine structures involved in some geophysics. Therefore, there may be a relatively large error in the results. This means that there should be plenty of leeway to implement measures to combat GMSL rise based on our research.

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