

Brain Scale Free Criticality and Renormalization Extended for Unification and Consciousness Theories.

Tariq Khan
Omaha, Nebraska, USA

Recent articles summarizing research noting the proclivity of renormalization techniques to solve brain self-organized criticality and deep learning could intimate that the challenges with the unification of Quantum Mechanics and Relativity, and with finding a formal solution for quantum decoherence or even consciousness, could be chimeras if the human observed nature of reality itself are renormalizations of an infinite fractal space-time that is intractable with current techniques.

Two articles in 2014 by Jennifer Ouellette and Natalie Wolchover [1] [2] that reference renormalization and fractals in both Per Bak's famous paper on self-organized criticality [3] and Pankaj Mehta and David Schwab's paper on A.I. deep learning [4], could be pointing to an even larger application of renormalization techniques in fundamental physics. The mathematical technique called *renormalization*, in essence, raises analysis "up a level" to ignore infinities that prevent a quantitative solution.

First note this key passage from Oulette's article on the work of Per Bak and others uniting the concepts of self-organized criticality in the brain and renormalization:

"Through deep learning, there is also the hope of a better theoretical understanding of human cognition. Vijay Balasubramanian, a physicist and neuroscientist at the University of Pennsylvania, said he and other experts who span his two fields have long noticed the conceptual similarity between renormalization and human perception. "The development in Pankaj and David's paper might give us the tools to make that analogy precise," Balasubramanian said. For example, the finding appears to support the emerging hypothesis that parts of the brain operate at a "critical point," where every neuron influences the network as a whole. In physics, renormalization is performed mathematically at the critical point of a physical system, explained Sejnowski, a professor at the Salk Institute for Biological Studies in La Jolla, Calif. "So, the only way it could be relevant to the brain is if it is at the critical point." There may be an even deeper message in the new work. Tishby sees it as a hint that renormalization, deep learning and biological learning fall under the umbrella of a single idea in information theory. All the techniques aim to reduce redundancy in data. Step by step, they compress information to its essence, a final representation in which no bit is correlated with any other. Cats convey their presence in many ways, for example, but deep neural networks pool the different correlations and compress them into the form of a single neuron." [1]

Also, consider this statement below from Oulette's article in the context of the collapse of the Schrödinger wave function upon a quantum system's observation or interaction with the environment:

"The brain is an incredibly complex machine. Each of its tens of billions of neurons is connected to thousands of others, and their interactions give rise to the emergent process we call "thinking." According to Bak, the electrical activity of brain cells shift back and forth between calm periods and avalanches - just like the grains of sand in his sand pile - so that the brain is always balanced precariously right at that the critical point." [1]

Then, consider this sentence below in Oulette's article compared to the nature of dual-slit interferometer experiments, the hallmark of the essence of quantum mechanics, where the interference pattern is generated on the screen by the delivered photon, unless *which-path* knowledge is known that is perhaps a situation analogous to a self-organized criticality "critical point," where any randomness or ambiguity of the photon path (as a wave) is eliminated.

"A brain that is not critical is a brain that does exactly the same thing every minute, or, in the other extreme, is so chaotic that it does a completely random thing, no matter what the circumstances," Chialvo said." [1]

Also, note these key passages on deep learning referencing fractals and renormalization from Wolchover's article on Mehta and Schwab's work:

"The new work... demonstrates that a statistical technique called "renormalization," which allows physicists to accurately describe systems without knowing the exact state of all their component parts, also enables the artificial neural networks to categorize data as, say, "a cat" regardless of its color, size or posture in a given video. Renormalization is a systematic way of going from a microscopic to a macroscopic picture of a physical system, latching onto the elements that affect its large-scale behavior and averaging over the rest. Fortunately for physicists, most microscopic details don't matter; describing a table doesn't require knowing the interactions between all its subatomic quarks. But a suite of sophisticated approximation schemes is required to slide up the distance scales, dilating the relevant details and blurring out irrelevant ones along the way. Mehta and Schwab's breakthrough came ... when they decided to focus on a procedure called variational or "block-spin" renormalization... method involves grouping components of a system into larger and larger blocks, each an average of the components within it. The approach works well for describing fractal-like objects, which look similar at all scales, at different levels of resolution; Kadanoff's canonical example was the two-dimensional Ising model - a lattice of "spins," or tiny magnets that point up or down. He showed that one could easily zoom out on the lattice by transforming from a description in terms of spins to one in terms of blocks of spins." [2]

Philosophical Implications

Notice how renormalization, as an approximation and methodology, is similar to Isaac Newton's use of the *limit* in Calculus as well as techniques created by Benoit Mandelbrot with fractal math [5]. Benoit Mandelbrot's famous paper "How long is the coast of Britain?" shows how fractal mathematical techniques are similar to the renormalization concept of "rising up a level" (e.g., an observer viewing a fractal) thus averaging out the coarse items to find higher order relationships for optimization of speed (reduction of calculation time) or to get a holistic "view," and ultimately a solution, for what seemed to be a problem hidden in infinities - as seen in the example for the approximate answer to the length of the coast line of Britain.

Thus, perhaps the similarities between the techniques used to solve fractals and renormalization intimate a deeper connection in Nature? Could it be that the nature of the smallest units of space-time (let us say a theoretical Planck-length space-time "pixel", as opposed to a fundamental particle like an electron) is actually not a "square" or "cube," in terms of its shape or dimension, but is, rather, a fractal shape. Would a space-time pixel that is fractal in nature possibly provide a sink-like surface area where gravity dilutes and is, thus, so comparatively "weak" to the other three fundamental forces in the Standard Model, as it diffuses into the "cracks" of the fractal nature of space-time? Could this model possibly also answer the major questions of why we cannot solve the major challenges of the nature of Quantum Decoherence and the dreamed of a grand unification of quantum mechanics and general relativity as, in this model, they simply cannot be united, vis a vis how the infinite fractal coastline cannot be measured directly with its infinite fractal nature? Is a similar renormalization process thus needed to explain the "creation" of our daily "reality" by a conscious mind that, in essence, "elevates" our calculation attempts out of the Quantum world into our daily macroscopic reality scale and we thus "push" the infinities of the Schrödinger equation (i.e., the required continuous number line for the wave function) under the proverbial rug? Could this be the long sought paradigm shift needed that has left the best minds in the world unable to solve or find the mathematical unification equation or solution as maybe no equation or precise mathematical answer exists (again vis a vis the measurement of the fractal coastline length) but, while involving apparent or even actual infinities, the reality of the world does exist, just like a fractal exists?

If we re-examine Roger Penrose's famous "three worlds" model of reality, as noted in his book *The Road to Reality* [6], of a metaphysical Platonic World of fundamental mathematical laws or equations, perfect circles, π , and infinity that make the foundational rules required of a miraculously consistent Physical World filled with an abundance of identical particles and quantized forces with this physical reality of atoms and energy as the material required to make a human body needed to make a human brain needed as a vessel for a human mind or consciousness that exists in a Mental World that might be the only thing or place or mechanism possible to create or discover or hold the first Platonic World of ideas and mathematical laws and infinite concepts? Might this new model of a fractal space-time perhaps solve or reduce Penrose's three

world model to a Universe or reality of only fractal (infinite) objects and conscious minds that can (or must) renormalize them to literally “think outside the infinite box” and experience or create a reality? By existing in a “higher” level of a “vertical” renormalized fractal reality what might then logically lie “above” us along this same existential axis?

From a different or opposite perspective, perhaps reality itself undergoes a form of phase transition, akin to the changes in water from ice to liquid to gas, as Nature scales from the quantum mechanics modeled *small*, to the general relativity governed *large*. Perhaps reality itself is quantized in the sense of measurements at a scale of a certain number of total bits or memory follow quantum mechanics, while larger measurements or observations cross a threshold and, thus, require relativity to model them? Imagine Nature where the math itself transitions from discrete to continuous beyond a critical inflection point. While we know of the macroscopic effects of quantum tunneling and fusion reactions in stars, when we measure them in the microscopic realm, perhaps Nature “becomes” discrete. Discrete space-time may actually “emerge” from a continuous reality upon observation only. Maybe challenges with confirming the Dark Energy expansion rate of the Universe intimate an additional higher level or scale of transition into yet another form of mathematics? Cosmologists have reported that the Universe itself will undergo a phase transition, so perhaps this approach is not so outlandish. [7]

Scale Free Consciousness

Let us consider further the “scale free” or scale invariance aspects of *self-organized criticality* and the critical brain hypothesis. [8] At the critical point in the brain, the pattern of signals exists at every scale like a fractal. Self-organized criticality is defined as a property of dynamical systems where “[t]heir macroscopic behavior thus displays the spatial or temporal scale-invariance characteristic of the critical point of a phase transition, but without the need to tune control parameters to a precise value, because the system, effectively, tunes itself as it evolves towards criticality.” [9] Woodrow L. Shew et al., noted in their 2009 research that “findings suggest that in the cortex, (1) balanced excitation and inhibition establishes criticality, which maximizes the range of inputs that can be processed, and (2) spontaneous activity and input processing are unified in the context of critical phenomena.” [10] John M. Beggs, in his 2009 book *The Cortex and the Critical Point*, notes:

“One of the main consequences of being near a critical point is scale-free property. These are hypothesized to lead to optimal information transmission and also thought to optimize dynamic range, sensitivity to inputs, information storage, and computational power. Just being near the critical point will produce optimality over the scale of the brain (pg. 22-23).” [11]

Now, let us make an analogy to modern Information Technology “cloud computing” where processing is done at a remote “compute” location and results transported back to a user. Theoretical physicist Dr. Leonard Susskind at Stanford University in his lecture on Quantum Gravity describes the Holographic Paradigm:

“A hologram is a two-dimensional mathematical representation of a three-dimensional portion of the world. ... How do you take a three-dimensional thing and map it into two dimensions? Well, it’s possible but it is always at the cost of the two-dimensional image looking like a random hash. ... The information of what is in the hologram is in the bulk. The Holographic Principle states that the information encoding everything taking place in this three-dimensional world is encoded on the boundary of that region as a kind of quantum hologram.” [12]

Some of the principles that drove the creation of cloud computing include its resiliency and redundancy. These are traits also common to Nature or gene pools focused on long term survival. Thus, we might extend the analogy to brain criticality. Brains are optimized at the critical point and consciousness fades as the brain drifts away from it. Adam J. Eisen et al. note in their 2024 research in the journal *Neuron*:

“Thus, stability, and hence consciousness, needs to be understood in terms of a dynamic brain. Here, we approach the analysis of anesthetic unconsciousness through the lens of dynamic stability (henceforth stability), a fundamental concept in dynamical systems theory and control. Essentially, dynamic stability

is a measure of the robustness of a dynamical system. The system needs to be able to recover from disturbances (e.g., distractions, random fluctuations in activity) to its normal state... We validated the model's estimates of changes in dynamic stability in systems for which the ground truth stability is known. We found that propofol-induced unconsciousness is associated with destabilized neural dynamics. ... Propofol disrupts the balance between cortical excitation and inhibition. This balance is known to be critical for maintaining the stability of cortical dynamics. Combined with our findings, this paints a picture in which propofol tampers with this balance, causing widespread cortical instabilities and thereby disrupting the brain's capacity for information processing. Overall, our analysis suggests a mechanism for anesthesia that involves destabilizing brain activity to the point where the brain loses the ability to maintain conscious awareness." [13]

At the critical point, signaling is *scale invariant* (fractal pattern going down to the smallest scale). Thus, might it be the case that Nature has found a location for optimal processing? Might it be the case that human consciousness does not actually lie inside our physical skulls but, rather, lies in the aforementioned quantum bulk boundary of the Holographic Paradigm? As Michael Brooks notes in a 2024 physics article in *New Scientist*, "the goings-on in space can be fully described by data held on the outer surface, or boundary, of that space, a phenomenon known as holographic duality." [14]

Is the real reason for the existence of brain criticality that it reaches a state of scale invariance at which point actual thought and intelligence, occurring at the bulk, is thus replicated or communicated "up the fractal ladder" to the scale of neurons in our physical world? Consider the comments from Dr Henry Markram of Blue Brain Project at the Swiss Federal Institute of Technology:

"If the changes that occur in the brain only make sense if you map them to a higher dimensional structure, then that's what you are going to have to do," he says. "Memory may be hiding in high-dimensional structures." As the Blue Brain team continues its effort to create a larger and more accurate digital brain, Markram thinks that one day the topological approach could even help crack that hardest problem of all - consciousness. "When we see a phenomenon that looks mysterious and difficult and intractable, there is a scientific possibility that what we are seeing and experiencing is a shadow projection from higher-dimensional representations," he says. "We need mathematics to climb up into those higher dimensions. Then we'll understand how those shadows emerge. Consciousness may be a shadow." [15]

The brain at its critical point, where all scales are identical fractals (i.e., synchronized), becomes synonymous with a "vertical" telecommunications channel:

"When a continuous stream of fixed-length frames are sent, a synchronized receiver can in principle identify frame boundaries forever. In practice, receivers can usually maintain synchronization despite transmission errors; bit slips are much rarer than bit errors. Thus, it is acceptable to use a much smaller frame boundary marker, at the expense of a lengthier process to establish synchronization in the first place." [16]

If all particles and forces that create our reality can be encoded in a boundary, then why not consciousness as well? Colloquially this might imply that every consciousness is part of a single consciousness (the single boundary) and perhaps even add some metaphysical credence to the idea of an "after-life" of one's conscious self as it may already, and inherently, exist external to the physical world anyway.

Metaphysics

We may consider Nature less as an "inventor" and more of an entity that commandeers, almost always, out of necessity. An example of important recent research that supports this claim is work showing how intelligence has likely evolved at least twice on planet earth amongst vertebrate animals e.g., birds versus mammals. Yasemin Saplakoglu notes in *Quanta Magazine* how a "series of studies published in February 2025 provides the best evidence yet that birds and mammals did not inherit the neural pathways

that generate intelligence from a common ancestor, but rather evolved them independently. This suggests that vertebrate intelligence arose not once, but multiple times." [17] [18]

We can envision Nature with an ecosystem on planet earth that becomes increasingly large, diverse, complex, and intelligent, using whatever "tools" it can find for any competitive advantage e.g., neurons, memory, categorization and labelling, planning, learning, language, etc. Then we can imagine Nature seeking improvements on each of these including long term memory, modeling, written language, and possibly even holographic and/or fractal memory as a fast, resilient, and perhaps abundant memory source.

But human intelligence is different from that of birds. Human brains are capable of conscious experiences. However, as noted, consciousness can occur only when a human brain is at a "critical point." Why is this the case? Why is there an apparent association between the human brain's ability to become conscious and it reaching a fractal critical point? Why fractal?

We can speculate that it is the fractal structure that allows Nature to utilize yet another higher level tool or substrate. We can imagine Nature using fractal structures for not only optimal memory storage but perhaps also to access an ultimate memory or ultimate "library of libraries" akin to Carl Jung's *collective unconscious* [19]. There is an obvious analogy here between the self-similar and infinite regression of a fractal, and the recursive nature of consciousness where a mind can ask or ponder: why it asks why it asks why. As Carl Jung stated in a lecture in 1936:

"My thesis then, is as follows: in addition to our immediate consciousness, which is of a thoroughly personal nature and which we believe to be the only empirical psyche (even if we tack on the personal unconscious as an appendix), there exists a second psychic system of a collective, universal, and impersonal nature which is identical in all individuals. This collective unconscious does not develop individually but is inherited. It consists of pre-existent forms, the archetypes, which can only become conscious secondarily and which give definite form to certain psychic contents." [20]

Jung also has claimed that the "unconscious has no time. There is no trouble about time in the unconscious. Part of our psyche is not in time and not in space. They are only an illusion, time and space, and so in a certain part of our psyche time does not exist at all." [21]

Note, to consciously *experience* and to *understand* appears to require more than just mechanical tools vis a vis neurons and a body. While an animal may cry, as far as we can tell, it does not ask why it does. We can further speculate that just as A.I. gains its power from its ability to quickly and efficiently reference a massive database of historical information, perhaps the human subconscious mind gains the same from access to the ultimate "repository of experience" that may be stored in spacetime itself. Again, note that consciousness occurs in the brain, after anesthesia, only after the brain reaches the fractal critical point. We can imagine then a fractal *link* or *pipe* being created from our macroscopic world "all the way down" to the quantum Planck scale microscopic world. But is there any evidence that the Universe is fractal at a fundamental microscopic scale? Modern theoretical research does support this claim. [22] [23][24][25]

Oliver Lauscher et al. note in their 2005 paper:

"It is also shown that asymptotic safety implies that spacetime is a fractal in general, with a fractal dimension of 2 on sub-Planckian length scales. ... Spacetime seems to be more strongly curved at small distances than at larger ones. The scale-free relation suggests that at distances below the Planck length the QEG spacetime is a special kind of fractal with a self-similar structure. ... But since the quantum spacetime is characterized by the infinity of metrics, it can acquire very nonclassical and in particular fractal features. In fact, every proper distance calculated from $g_{\mu\nu} k$ is unavoidably scale dependent. This phenomenon is familiar from fractal geometry, a famous example being the coastline of England whose length depends on the size of the yardstick used to measure it (pgs. 15-16)." [22]

Jan Ambjorn et al., note in their 2005 research how a "closer look at the quantum geometry reveals a number of highly nonclassical aspects, including a dynamical reduction of spacetime to two dimensions on short scales and a fractal structure of slices of constant time." [25]

Modern theoretical physics has also claimed that the universe is fundamentally holographic in nature with information perhaps stored on a two dimensional boundary. [26] Is this vision compatible with a fractal spacetime? Again, research appears to support a universe that is both fractal and holographic. Both holograms and fractals share the properties of self-similarity and of being able to encode information at

different scales. Fractals exhibit self-similarity (their parts resemble the whole), while holograms encode information about an object's entire structure across different sizes or scales. [27] This similarity has led to the exploration of "fractal holographic principles," suggesting a possible holographic universe where information is encoded in fractal patterns. [27]

Mary Alexandria Kelly et al., in their 2013 research proposed the idea of human brains using holographic reduced representations (HRRs) or associative memory. They note:

"...a holographic associative memory is a computational memory based on the mathematics of holography. Holographic associative memory has been of interest to cognitive psychologists because: 1) associative memories are content-addressable, allowing items to be retrieved without search, in a manner similar to the fast, parallel retrieval of memories in the human mind. 2) just as human memory can store complicated and recursive relations between ideas, holographic associative memories can compactly store associations between associations. 3) holographic associative memories have what is called lossy storage, which is useful for modelling human forgetting. ...The mathematics of holography has long been suggested as the principle underlying the neural basis of memory... HRRs have also been used to model how humans understand analogies and the meaning of words, how humans encode strings of characters, and to model how humans perform simple memory and problem-solving tasks." [28]

A key point of this essay is that Nature was not predestined to create conscious human minds. But as the entire planet, ecosystem, gene pool, and human species evolved to the maturity and scale to permit it, and as competitive pressures drove the need for increased memory (and the need for increased intelligence), Nature, checking every corner, not only went far and wide, but also deep. We can speculate that in *the deep*, a spark was found and then never lost. The fitness or performance advantage from an ability to comprehend and remember experiences (if not subconsciously all collective experience, a la Jung) thus led to the eventual success of humans over nearly all of Nature on this planet.

If minds are fractal, or fractal and holographic, many open questions still remain. Has a microscopic consciousness or collective subconscious always existed? What is the process or protocol by which memories are stored as fractals? Can we observe, prove, or even change fractal or holographic memory storage in a brain and/or in spacetime? Can a machine be built to do the same? Does this model imply that a consciousness could exist after the death of a physical body or brain? Is there a limit to the amount of memory in spacetime? Could an increase in the amount of memory stored in a fractal spacetime be what is driving a Dark Energy expansion of the Universe?

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