Why Bell's Theorem Is Inapplicable to a Quantum System

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A review of the famous Bell's theorem leads us to conclude that it addresses one and only one specific issue: whether a body possesses a certain property or does not. The theorem certainly is not a statement about hidden parameters, non-locality or causality.

Over the years, Bell's theorem has played an important role in the interpretation of Quantum Mechanics. First we will prove the theorem, then we will look at its implications.

In Bell's theorem [1], we make two assumptions in the proof. These are:

A. Logic is valid - its derivation is strictly speaking mathematical in nature.

B. Its mathematical object is about a body having a property A or not having a property A.

The property in question is about anything that a particle possesses and can be measured. Consider a set of three measurements, A, B and C which are independent properties of an object. Example: A is the property that the earth is rotating about its own axis, B is the property of the earth having an atmosphere, and C is the property that the earth has water.

<u>Derivation of Bell's inequality</u>

Definition: if an object has property A, we denote it by A+; if not, we denote it by A−:

$$N(A+, B-) = N(A+, B-, C+) + N(A+, B-, C-) \quad (1)$$

This is true since an object must have the property C or does not have it, which is what the RHS says. In the above example: the LHS states that $N(A+, B-)$ is the earth rotating about its own axis and having no atmosphere. This is either true or false. The RHS reiterates that for this case, it's the sum of having an atmosphere ($C+$) or not having one ($C-$).

We assert that since all three terms in equation (1) are positive, then

$$N(A+, B-) \geq N(A+, B-, C-) \quad (2)$$

That is, $N(A+, B-)$ cannot be smaller than zero.

Also,

$$N(B+, C-) = N(A+, B+, C-) + N(A-, B+, C-) \quad (3)$$

Similar reasoning as above: an object must have the property A or does not have it, which is what the RHS says.

Therefore,

$$N(B+, C-) \geq N(A+, B+, C-) \quad (4)$$

Similar reasoning as in equation (2), that is, $N\,(B+, C-)$ cannot be smaller than zero.

Adding inequalities (2) and (4),

$$N(A+, B-) + N(B+, C-) \geq N(A+, B-, C-) + N(A+, B+, C-) \quad (5)$$

But the RHS of (5) is:

$$N(A+, B-, C-) + N(A+, B+, C-) = N(A+, C-) \quad (6)$$

That is, an object must have the property B or does not have it. Substituting (6) into (5), we get,

$$N(A+, B-) \ + \ N(B+, C-) \ \geq \ N(A+, C-) \qquad (7)$$

And that completes the proof.

Now the major problem is in the setup of the equations, which as we will argue, doesn't apply to a Quantum system.

Here's the problem: Suppose now that property A stands for the x-component of an electron's spin, B for the y-component of the spin, while C for its z-component.

So $N(A+, B-)$ would read as follows: the electron's spin has an x-component but no y-component???

Nowhere in Quantum Mechanics is it allowed to make such a statement.

The Heisenberg Uncertainty Principle states that if one component of the electron's spin is measured ($= \pm$½ ħ along whichever axis one so wishes to make that measurement) then the other two components are *indeterminate*.

Notwithstanding if you have an Alain Aspect experiment [2], in which Bell's inequality theorem is tested by measuring the polarization of photons along three different axes then the inequality would be "violated" as the theorem simply doesn't apply to a Quantum system. Even Bell had misgivings about what he was saying in regard to his own theorem – Wiseman indicated as much in his paper[3] that Bell himself was unsure about his own interpretation of his own theorem, unfortunately spawning in the intervening years different schools of interpretations.

Note: Claiming that a Quantum system "violates" Bell's theorem is an exaggeration and completely misses the point. In reality the theorem simply doesn't even apply to a Quantum system. The theorem certainly is not a statement about hidden parameters, non-locality or causality. None of that is present in its derivation. It's all about an object having a certain property or does not have that property, which is true for a Classical system but non-applicable to a Quantum system. It is the hope of this paper that the varying misinterpretations of Bell's theorem will subside in the coming years.

References

[1] J.S. Bell (1964), "On the Einstein-Podolsky-Rosen Paradox", Physics 1: 195–200.

[2]Alain Aspect, Philippe Grangier, Gérard Roger (1982), “Experimental Realization of Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment*: A New Violation of Bell's Inequalities”, Phys. Rev. Lett. 49 (2): 91–4.

[3] Howard M. Wiseman (2014), **“**The Two Bell's Theorems of John Bell”**,**

https://arxiv.org/abs/1402.0351