Proof of the Collatz Conjecture

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Abstract

The Collatz conjecture has long been unresolved. This paper will provide a proof of the Collatz conjecture. The proof will begin by noting that if the conjecture is false, there must be infinitely many examples that violate the conjecture, and will emphasize the impossibility of this scenario. Using probability within the Collatz problem, we can demonstrate that a certain portion of numbers will reach one according to the Collatz algorithm. The total probability must sum to one for the conjecture to be true. If the total probability does not sum to one, it will be a number very close to one. However, if the probability total is even one in a million less than one, there must be an infinite number of numbers that do not satisfy the Collatz algorithm, because a finite number cannot make up for the probability shortfall. This means that there must either be sequences that increase exponentially to infinity or cycles that repeat themselves. However, the probability of selecting the elements of a single sequence that increases to infinity from an infinite set is zero, so there must be infinitely many sequences that increase to infinity and violate the algorithm. The self-repeating cycles must also be infinite in number, but the number of elements in the cycles cannot go to infinity, so there must be infinitely many cycles with the same number of elements. This is impossible, because cycles with the same number of elements are finitely arranged within themselves, and a single element that violates the algorithm will emerge from any of these arrangements. An infinite number of sequences increasing to infinity is also impossible because they would intersect each other. As a result, the Collatz conjecture is true.

Introduction

The Collatz conjecture simply states: choose a positive integer. If the number is even, divide it by two; if it is odd, multiply it by three and add one. Repeat this process with the new number you obtained. Continue in this way. Eventually, you will reach the number one. The problem here is to prove that every positive integer will reach one. This paper will show this proof. Now let's make an assumption. Let’s choose any positive integer n . Let’s look at the probability of whether this number will reach one according to the Collatz algorithm after certain operations. First, we know that some numbers will reach one with this algorithm. Therefore, this probability cannot be zero.However , even if this possibility is assumed to be zero,the proof does not change. If this probability is one, we accept that every number will definitely reach one according to the Collatz algorithm, and the proof is complete. If this probability is a number between zero and one, it means there are infinitely many numbers that do not reach one, i.e., that do not comply with the Collatz algorithm. If this probability is $1 - c$, it means that with a probability of c , the numbers do not reach one. We can express it this way: there can be two situations that violate the conjecture. Either the numbers are increasing to infinity, or there are some closed loops. Let's explain an important point here. Normally, if the probability is one, it does not necessarily mean there is no counterexample when an infinite set is involved. For example, the probability of selecting the number 10 among natural numbers is zero, but if we randomly choose a number among natural numbers, this number can be 10, which means that a probability of zero does not imply that the event will never occur in every case. However, how we define probability in the Collatz algorithm is important, and we can define the probability without such situations arising. That is, we define a probability for the Collatz conjecture such that when this probability is one, all numbers indeed reach one. We can do this as follows: Initially, let's have a random single number. Let's call this number n. In the first step, we multiply this number by three and add one. The number we have is 3n+1. In the second step, this number can be divided by two or a power of two. What is important here is that the number falls below its initial value. This is because closed loops or sequences that increase indefinitely do not fall below their initial value. In this step, if the number is divided by four, it falls below its initial value. This is because $(3n+1)/4 < n$. The number 3n+1 is even. The probability of a random even number being divisible by four is 1/2. That is, half of the numbers fall below their initial value in the second step. Let's say the number is only divided by two in the second step. In this case, the number we have is (3n+1)/2. This is a single number. In the third step, we multiply this number by three and add one. The new number is (9n+5)/2. This number is even, and in the fourth step, dividing this number by four does not bring it below its initial value. This time, the number must be divided by eight. In the fourth step, if the number is divided by eight, the new number $(9n+5)/16 < n$. Therefore, the probability of a randomly selected number falling below its initial value in the fourth step is equal to the probability of not being divisible by four in the second step * the probability of being divisible by eight in the fourth step. The probability of a random even number being divisible by eight is 1/4. Consequently, the probability of a randomly selected number falling below its initial value in the fourth step is (1/2) * (1/4) = 1/8. That is, 1/8 of the numbers fall below their initial value in the fourth step. The probability of all numbers falling below their initial value = The probability of falling below their initial value in the second step + the probability of falling below their initial value in the fourth step + … = 1/2 + 1/8 + … Now, if the value of this defined probability is one, all numbers without exception fall below their initial value, and the conjecture is true; if this probability value is not one, there must be infinitely many numbers that do not fall below their initial value. Now let's assume that with a probability of c , the numbers do not reach one.

Theorem 1
If there is a finite number of sequences increasing to infinity, the closed loops must be infinite.

Proof
First, let's assume that there is no sequence increasing to infinity.
For a finite number of closed loops, the elements of these loops violate the conjecture, and if the total number of these elements is x , the probability of any number violating the conjecture= $x/\infty=0\neq c$. Now let there be one sequence increasing to infinity. This sequence has an infinite number of elements. First, the following question needs to be answered: what is the probability of selecting powers of two? For the first 100 numbers, the probability of selecting powers of two is calculated as follows: the powers of two are 2, 4, 8, 16, 32, 64, a total of 6 numbers, so the probability is 6/100. For the first 1000 numbers, the powers of two are 2, 4, 8, 16, 32, 64, 128, 256, 512, a total of 9 numbers, so the probability is 9/1000. The probability decreases gradually. Ultimately, as it goes to infinity, this probability will be zero, but this needs to be proven. To simplify the calculation, let's take the number 2^y . The number of powers of two in these numbers will be y . The probability is $y/2^y$. $\lim_{y\to\infty}$ $,(y/2^y)=(1/(\log2*2^y)=0$. This probability is not unique to powers of two. This probability will be zero for any exponential increase. If we take an initial number m for the Collatz sequence, the number we will reach after a while will be$((3^k)*m+r)/2^f$. As the Collatz sequence increases to infinity, we can think of its elements as an exponentially increasing series in the form of powers of three. The constant r only enlarges the result. Moreover, $f\geq 2k$ cannot occur because, in the case of $f = 2k$, the element of this defined sequence would fall below the initial m value. This means that the elements of the Collatz sequence increasing to infinity can be at most two exponentially increasing sequences in the form of powers of three that match one-to-one. However, this does not change the result because the probability is still

zero for h exponentially increasing sequences heading towards infinity.For h being a constant number, h*y/(2^y)goes to zero as y goes to infinity. The result is that any finite number of Collatz sequences increasing to infinity can match one-to-one with the powers of three, and limy→∞ , h*y/(3^y)=0≠c. Therefore, to maintain the probability c, if there is a finite number of sequences increasing to infinity, there must be an infinite number of closed loops.

Theorem 2
If the closed loops are not infinite, there must be an infinite number of sequences increasing to infinity.

Proof
We had shown above that a finite number of closed loops cannot reach the number c. A finite number of closed loops and a finite number of increasing sequences also cannot maintain the number c. Limn →∞, (x+h*n)/2^n=0≠c. Therefore, if the closed loops are not infinite, there must be an infinite number of sequences increasing to infinity.

Theorem 3
The closed loops cannot be infinite.

Proof
The number of elements in closed loops cannot be infinite because an infinite number of elements means that the sequence goes to infinity. Therefore, the number of elements is finite. If there are to be infinite closed loops, there must be infinite closed loops of every number of elements. If there are infinite closed loops of any number of elements, there must also be infinite progressions of the same kind, but this is impossible because the equation ((3^k)*m+r)/2^f=m can have only one root, so there cannot be infinite closed loops.

Assumption: Let there be infinitely many sequences increasing to infinity that do not intersect and let the probability of selecting the elements of these sequences be ∞/∞=c.

Theorem 4
If there are infinitely many mutually disjoint infinite sequences , and the probability of selecting an element is a non-zero constant like c, such sequences cannot exist and must intersect with each other.

Proof
Let any element of a sequence start moving towards infinity. While this element progresses, the probability of colliding with another sequence increasing to infinity is c . At each step, there is a c probability of intersecting with another sequence increasing to infinity.The probability of not intersecting is (1-c) at each step.The probability of one element of the sequence not intersecting with other sequences as it approaches infinity becomes (1-c)*(1-c)*(1-c)…=0. Therefore, such an infinite number of sequences going to infinity that do not intersect cannot be defined.

Conclusion
To maintain the probability c, either there must be a finite number of sequences increasing to infinity and an infinite number of closed loops, or a finite number of closed loops and an infinite number of independent sequences going to infinity that do not intersect. However, there cannot be an infinite number of closed loops, and an infinite number of sequences increasing to infinity must intersect at some point. Therefore, it is not possible for the

conjecture to be violated with a probability c . As a result, the probability of any element reaching one is one, and the Collatz conjecture is true.