A Classical Wave Model of Quasi-Static General Relativity

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Abstract

General relativity can be difficult for undergraduate students to comprehend, partly because the math is difficult and partly because it is not based on a simple physical model. However, in many situations general relativity can be interpreted as ordinary wave refraction in a non-uniform medium, with the refractive index (or wave speed) derived from only two independent components of a spatially isotropic diagonal spacetime metric. This work utilizes an elastic solid model of the vacuum to explain how the presence of wave energy would modify a medium to produce the metric variations of general relativity in a quasi-static environment. This analysis provides model-based explanations for many predictions of general relativity, including curved space, black holes, gravitational waves, and the different accelerations of light and massive objects.

1. INTRODUCTION

The nature of the vacuum has intrigued scientists for centuries. In the 17th century, Christian Huygens explained reflection and refraction by modeling light as waves, which were presumed to propagate like sound through an air-like aether.[1] Ole Rømer's discovery that light from Jupiter's moons travels with constant speed (calculated by Huygens) also supported the theory that light consists of waves.[2] However, Huygens' discovery that light has two different refractions in "Iceland crystal" led Isaac Newton to discount the wave theory of light and instead model light as asymmetric particles moving through empty space.[3] Thomas Young convincingly demonstrated the wave nature of light by forming an interference pattern from light beams emerging from two closely spaced slits.[4] Young suggested that light waves are similar to shear waves in a solid, which propagate with displacement in a plane perpendicular to the wave propagation direction. Maxwell used a more complicated model of the vacuum as rotating elastic cells to derive the equations of electromagnetism.[5]

Modeling of the vacuum as a solid material was problematic because it was believed that solid matter could not freely propagate through such a material. Stokes noted that fluids and solids are parts of a continuum,[6] so the vacuum came to be modeled as a sort of viscous fluid that has little drag for objects moving slowly, but responds like a solid to rapid vibrations. This model was disproven by the Michelson-Morley experiment.[7]

But Maxwell himself suggested that molecules might be "not substances themselves, but mere affections of some other substance."[8] In the early 20th century, it became clear that matter propagates through space as waves rather than as a collection of solid particles. However, this concept is so counterintuitive that many scientists still erroneously claim that the Michelson-Morley experiment disproved the existence of any type of aether.

Special relativity may be interpreted as an emergent consequence of the wave nature of matter, which satisfies Lorentz-covariant equations, rather than as an intrinsic property of spacetime. This approach is commonly referred to as "Lorentz ether theory." As Nobel laureate Robert Laughlin explained, "The modern concept of the vacuum of space, confirmed by everyday experiment, is a relativistic ether. But we do not call it this because it is taboo."[9]

Elastic waves in a solid have two types of momentum: intrinsic momentum due to motion of the solid medium, and wave momentum due to energy transport by the wave. Elastic waves also have two types of angular momentum: intrinsic angular momentum, or spin, due to rotational (compression-free) motion of the medium, and wave (or orbital) angular momentum associated with wave propagation and torque. Spin density ($\mathbf{s}$) is the vector field whose curl is equal to twice the intrinsic momentum density ($\mathbf{p} = \rho\mathbf{u}$ where ρ is inertial density and $\mathbf{u}$ is velocity):

$$\mathbf{p} = \frac{1}{2}\nabla \times \mathbf{s}\,. \tag{1}$$

The integrated spin angular momentum is equal to the integrated moment of momentum (assuming no contributions from infinity):

$$\int \mathbf{r} \times \mathbf{p}\, d^3r = \int \mathbf{s}\, d^3r\,. \tag{2}$$

Spin density is simply the coordinate-independent expression for ordinary classical angular momentum density.

The relationship between intrinsic momentum and spin angular momentum in Eq. 1 is implied by general relativity's requirement of a symmetric stress-energy tensor.[11,12] Rosenfeld stated that the separation of angular momentum into spin and orbital terms "has a direct physical meaning only for physical agencies that are endowed with inertia so that one could attach a system of reference that is at rest with respect to it."[12] In other words, without an inertial medium there would be no spin angular momentum. The Dirac operators for momentum and angular momentum density in an ideal elastic solid are identical (for the same normalization) to the corresponding operators in relativistic quantum mechanics.[10] Therefore we will describe elastic waves in terms of a polarization vector whose time derivative is equal to spin density.

Current practice is to make calculations using fields without reference to any particular physical model. However, models can still be useful pedagogical tools for teaching physically meaningful equations. We will use the model of the vacuum as an ideal elastic solid aether to explain the behavior of matter and light in quasi-static gravitational fields associated with stationary or slowly moving masses. The elastic solid model provides students with a familiar physical system that can be easily visualized, and the quasi-static environment simplifies the metric tensor to two independent components: one temporal and one spatial.

The idea that gravity results from spatial variation of the physical properties of the vacuum has been recognized at least since 1894, when George Fitzgerald wrote that "Gravity is probably due to a change in the structure of the aether, produced by the presence of matter."[13] Several physicists, including Einstein, subsequently proposed theories in which the speed of light is related to the gravitational potential.[14] In 1913, Einstein and Grossmann proposed that gravity not only causes acceleration, but also changes distance and time measurements through changes in the spacetime metric represented by a 4×4 matrix $g_{\alpha\beta}$.[15]

Using the spacetime coordinates $x^\alpha = (x^0, x^1, x^2, x^3) = (ct, x, y, z)$, the differential separation (ds) of special relativity is (summing over repeated indices):

$$(ds)^2 = g_{\alpha\beta} dx^\alpha dx^\beta \,. \tag{3}$$

with the spacetime metric $g_{\alpha\beta} = 1/g^{\alpha\beta} = \mathrm{diag}(1, -1, -1, -1)$. Different definitions of x^α and $g_{\alpha\beta}$ are common, so care should be taken when comparing references. Einstein and Grossmann proposed that gravitational fields alter the metric coefficients.

Trajectories are geodesics defined by:

$$\delta \int ds = 0 \,. \tag{4}$$

In general relativity, the metric coefficients are variable and Einstein's field equations relate the metric coefficients to the stress-energy tensor of matter.[16] Einstein's formulation of gravity can be summarized as modifying Fitzgerald's description to "Gravity is due to a change in the curvature of space-time, produced by the presence of matter."[17] This raises the question of whether changing the "structure of the aether" can produce the same effects as changing the "curvature of space-time." Furthermore, equations for matter waves and light waves share the characteristic wave speed "c" even though matter waves have group velocities with magnitudes less than "c". Hence it is plausible that light and matter can be modeled as waves propagating in a solid medium, and that gravitational modifications to the metric coefficients might also be derived from modifications to the medium carrying the waves.

Many scientists have described gravity as a variable index of refraction in space consistent with predictions of general relativity.[18–21] Engineers have even constructed models of black holes based on a radially inward increasing refractive index.[22]

In this paper, we consider only quasi-static gravitational fields with temporal variations and gradients small compared to the wave frequency and wave number, respectively. We also assume that the metric is diagonal and spatially isotropic so that the separation has only two independent components:[21]

$$(ds)^2 = g_{00}(\mathbf{r}) c_0^2 (dt)^2 + g_{11}(\mathbf{r}) |d\mathbf{r}|^2 \,, \tag{5}$$

with $g_{00} > 0$ and $g_{11} = g_{22} = g_{33} < 0$. We will derive expressions for g_{00} and g_{11} in terms of elastic constants.

2. METRIC COEFFICIENTS AND WAVE PARAMETERS

Let ∂_t indicate the partial derivative with respect to time (indices i, j, and k will be used for spatial dimensions). We start from a proposed equation for the evolution of the vector field $\mathbf{Q}(\mathbf{r}, \mathbf{t})$ whose time derivative is spin density $\mathbf{s} = \partial_t \mathbf{Q}$: [10]

$$\partial_t^2 \mathbf{Q} + \mathbf{u} \cdot \nabla \partial_t \mathbf{Q} - \mathbf{w} \times \partial_t \mathbf{Q} - c_1^2 \nabla^2 \mathbf{Q} = 0\,, \tag{6}$$

where $\mathbf{u} = (1/2\rho_1)\nabla \times \mathbf{s}$ is the local velocity of the solid material, and $\mathbf{w} = (1/2)\nabla \times \mathbf{u}$ is the angular velocity. For infinitesimal motion, the displacement is given by $\boldsymbol{\xi} = (1/2\rho_1)\nabla \times \mathbf{Q}$. Letting ρ_1 represent inertial density and μ_1 represent the shear modulus, the speed of elastic shear waves is $c_1 = \sqrt{\mu_1/\rho_1}$. We identify this with the speed of light $c_1 = 3.0 \times 10^8$ m/s. Eq. 6 states that spin density ($\partial_t \mathbf{Q}$) can change due to convection, rotation, and torque density ($c_1^2 \nabla^2 \mathbf{Q}$).

In regions of small amplitude ($|\mathbf{u}| << c_1$ and $|\mathbf{w}| << \omega_1$) we can neglect the nonlinear terms to obtain the simple wave equation:

$$\partial_t^2 \mathbf{Q} - c_1^2 \nabla^2 \mathbf{Q} = 0\,. \tag{7}$$

For simplicity, we will neglect changes of polarization and treat the wave as a scalar wave (e.g. choose a component Q_j of $\mathbf{Q}$ that is always perpendicular to the direction of wave propagation). Assume that a stationary particle-like solution has the separable form:

$$Q_j(\mathbf{r}, t) = R(\kappa_1 \mathbf{r}) \cos(\omega_1 t + \varphi_0) \tag{8}$$

with $c_1^2 \kappa_1^2 = \omega_1^2$. The wave equation can now be regarded as two separate equations:

$$(\partial_t^2 - c_1^2 \nabla^2) \cos(\omega_1 t + \varphi_0) = -\,\omega_1^2 \cos(\omega_1 t + \varphi_0)\,; \tag{9a}$$

$$(\partial_t^2 - c_1^2 \nabla^2) R = c_1^2 \kappa_1^2 R\,. \tag{9b}$$

Each of these equations is Lorentz-covariant.

A particle-like wave propagating in the x-direction is obtained via a Lorentz boost with velocity v_1 and $\gamma_1 = (1 - v_1^2/c_1^2)^{-1/2}$:

$$Q_j(\mathbf{r}, t) = R(\kappa_1 \gamma_1 (x - v_1 t), \kappa_1 y, \kappa_1 z) \cos\left(\gamma_1 \omega_1 t - \frac{\gamma_1 \omega_1}{c_1^2} v_1 x + \varphi_0)\right)\,. \tag{10}$$

The wave number is $k = (\gamma_1\omega_1/c_1^2)v_1$, consistent with relativity and quantum mechanics (wave momentum $p_w = \gamma m v = \hbar k$ with $m = \hbar\omega/c^2$).

Now suppose a time-independent, non-uniform index of refraction $n(\mathbf{r}) = c_1/c(\mathbf{r})$ varies slowly in space compared to the scale length ($|\nabla n| << \kappa$). As the wave travels through different positions, the values of c, κ, γ, and v can vary so that:

$$Q_j(\mathbf{r}, t) = R(\kappa\gamma(x - vt), \kappa y, \kappa z)\cos\left(\gamma_1\omega_1 t - \frac{n^2\gamma_1\omega_1}{c_1^2}vx + \varphi_0\right). \tag{11}$$

The frequency $\gamma_1\omega_1$ remains constant since the refractive index is independent of time. The width of the envelope function $R(\mathbf{r}, t)$ has been scaled by κ_1/κ. The wave number is $k = (\gamma_1\omega_1/c_1^2)n^2 v$. The propagation direction may gradually change, so the coordinates (x, y, z) must be interpreted as local coordinates relative to the direction of propagation $\hat{\mathbf{x}}$. Derivatives of $\hat{\mathbf{x}}$ are assumed to be negligibly small ($|\partial_i\hat{\mathbf{x}}| << \kappa$). The Lorentz factor γ is defined in terms of the local speed of light so that Eq. 9 becomes:

$$(\partial_t^2 - c_1^2\nabla^2)\cos\left(\gamma_1\omega_1 t - \frac{n^2\gamma_1\omega_1}{c_1^2}vx + \varphi_0\right) = -\,\omega_1^2\cos\left(\gamma_1\omega_1 t - \frac{n^2\gamma_1\omega_1}{c_1^2}vx + \varphi_0\right); \tag{12a}$$

$$(\partial_t^2 - c^2\nabla^2)R(\kappa\gamma(x - vt), \kappa y, \kappa z) = c^2\kappa^2 R(\kappa\gamma(x - vt), \kappa y, \kappa z)\,. \tag{12b}$$

Allowing variations of ρ and μ, the refractive index is $n = c_1/c = c_1\sqrt{\rho/\mu}$. In terms of metric coefficients (with i and j representing indices of spatial components) the wave equation becomes:

$$g^{00}\partial_t^2\mathbf{Q} + c_1^2 g^{ij}\partial_i\partial_j\mathbf{Q} = 0\,. \tag{13}$$

The metric is diagonal and spatially isotropic with $g^{11}/g^{00} = -1/n^2$.

Equation 12 becomes:

$$-g^{00}\gamma_1^2\omega_1^2 + g^{11}\frac{n^4\gamma_1^2\omega_1^2 v^2}{c_1^2} = -\,\omega_1^2 = -c_1^2\kappa_1^2\,; \tag{14a}$$

$$g^{11}c_1^2\kappa^2 = -\,c_1^2\kappa_1^2\,. \tag{14b}$$

Using $n^2 = g^{00}/g^{11}$, these can be reduced to:

$$g^{00} = \frac{\gamma^2}{\gamma_1^2} = n^2\frac{\kappa_1^2}{\kappa^2}\,; \tag{15a}$$

$$g^{11} = -\,\frac{\kappa_1^2}{\kappa^2}\,. \tag{15b}$$

3. METRIC COEFFICIENTS AND PHYSICAL PARAMETERS

3.1. Spatial Metric Coefficient

Now consider the behavior of inertial density and shear modulus in the presence of wave energy. The inertial density ρ is inversely proportional to volume, so as density increases, distances between corresponding wave features decreases. Therefore $\kappa/\kappa_1 = (\rho/\rho_1)^{1/3}$ and $g^{11} = -(\rho/\rho_1)^{-2/3}$ since the metric is associated with two factors of distance. The decrease in volume is attributed to the presence of transverse wave energy, so a good analogy is the decreased length of a twisted rubber band under constant tension. As the rubber band is twisted, the same length of rubber is packed into a shorter distance. A transverse wave on a string of fixed length would likewise slightly stretch the string. Under fixed tension, the time-averaged distance between the endpoints would decrease slightly. We assume that in three dimensions the compression is isotropic, or at least becomes isotropic as one moves away from the wave disturbance.

For small isotropic changes of length, the fractional changes in volume are simply proportional to the fractional change of length (e.g. for $\epsilon << 1$, isotropic contraction changes the volume of a cube of length D from D^3 to $(D(1-\epsilon))^3 \approx D^3(1-3\epsilon)$). Therefore we can assume that the effect of small displacements $\boldsymbol{\xi}(\mathbf{r})$ on g^{11} have the form:

$$\delta g^{11} \approx \frac{2}{3}\frac{\delta\rho}{\rho} \approx -2\frac{\delta D}{D} \approx -\frac{2}{3}\nabla\cdot\boldsymbol{\xi}\,. \tag{16}$$

More generally, the linear change in spatial metric coefficients has been identified as twice the strain tensor:[23]

$$\delta g_{ij} \approx -\delta g^{ij} \approx 2e_{ij} = \partial_i\xi_j + \partial_j\xi_i\,. \tag{17}$$

In a distorted crystal, spatial distances $ds = \sqrt{g_{ij}dx^i dx^j}$ would be measured by counting steps between atoms.

For shear wave motion, changes of length are proportional to the square of displacement (compare transverse waves on a string) and thus proportional to energy density $\sigma(\mathbf{r})$ for waves of a given frequency. Outside the region of shear wave disturbance (i.e. massive objects), changes in density ($\delta\rho$) would propagate as longitudinal waves:

$$\partial_t^2\delta\rho = c_L^2\nabla^2\delta\rho\,, \tag{18}$$

where c_L is the longitudinal wave speed. For quasi-static situations (averaging over the high-frequency transverse wave motion), we can ignore the time derivatives so that the density satisfies Laplace's equation $\nabla^2 \delta\rho = 0$. Assuming a spherically symmetric response to a disturbance then yields $\delta\rho \propto 1/r$. Combining contributions from wave energy in different locations yields:

$$\delta g^{11} \propto \int \frac{\sigma(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3r' \,. \tag{19}$$

Choosing the appropriate constant for general relativity then yields:

$$\delta g^{11} = \frac{2G}{c_1^2} \int \frac{\sigma(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3r' = -\frac{2}{c_1^2} \Phi_g \,, \tag{20}$$

where $G = 6.7 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$ is Newton's gravitational constant and Φ_g is the gravitational potential.

3.2. Temporal Metric Coefficient

The Steinberg-Cochran-Guinan (SCG) model predicts that the elastic modulus depends on density (ρ), pressure (P), and temperature (T) as:[24]

$$\mu = \mu_I + \frac{\partial \mu}{\partial P} \frac{P}{(\rho/\rho_I)^{1/3}} + \frac{\partial \mu}{\partial T}(T - T_I) \,, \tag{21}$$

where the subscript "I" indicates some initial condition. The relation with density is contained in the term involving pressure, so assume that the other terms are zero, that $\rho_1 = \rho_I$, and that:

$$\mu_1 = \frac{\partial \mu}{\partial P} P \tag{22}$$

with constant pressure P. The dependence of elastic modulus on density is then:

$$\frac{\mu}{\mu_1} = (\rho/\rho_1)^{-1/3} \,. \tag{23}$$

Hence the elastic modulus decreases as the density increases. To understand this, compare two springs wound from identical wires. The spring with the smaller pitch angle will have higher mass per unit of axial length and lower elasticity or spring constant. Eq. 23 is the three dimensional analogue of this example.

Combining Eq. 23 with the refractive index definition yields:

$$n = \sqrt{\frac{\mu_1}{\mu} \frac{\rho}{\rho_1}} = \left(\frac{\rho}{\rho_1}\right)^{2/3} . \tag{24}$$

Combining with Eq. 15 and using $\kappa/\kappa_1 = (\rho/\rho_1)^{1/3}$ then yields:

$$g^{00} = \frac{\gamma^2}{\gamma_1^2} = n\,; \tag{25a}$$

$$g^{11} = -\left(\frac{\rho_1}{\rho}\right)^{2/3} = -1/n\,. \tag{25b}$$

For $n \approx 1$, the magnitude of small changes are equal for g^{00} and g^{11} as in general relativity.[25] Therefore, the change in g^{00} is also proportional to nearby energy density:

$$\delta g^{00} = n - 1 \approx \frac{2G}{c_1^2}\int \frac{\sigma(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d^3r' = -\frac{2}{c_1^2}\Phi_g\,. \tag{26}$$

In terms of physical parameters, we have already seen that $g^{11} = -(\rho_1/\rho)^{2/3}$. This implies that g^{00} should contain a factor of $(\rho/\rho_1)^{1/3}$ to account for the factor of ρ in the original wave equation. Accounting for the factor of μ in the wave equation then requires $g^{00} = (\mu_1/\mu)(\rho/\rho_1)^{1/3}$, so the complete metric is:

$$g^{\mu\nu} = 1/g_{\mu\nu} = \left\{(\mu_1/\mu)(\rho/\rho_1)^{1/3}, -(\rho_1/\rho)^{2/3}, -(\rho_1/\rho)^{2/3}, -(\rho_1/\rho)^{2/3}\right\}\,. \tag{27}$$

This is consistent with simple dimensional analysis using $\rho \sim \mathit{mass}/\mathit{length}^3$ and $\mu \sim \mathit{mass}/(\mathit{length}\cdot \mathit{time}^2)$ since the mass of a solid region is unchanged by distortion (here "mass" refers to inertia of the solid medium, not to particle-like waves propagating through it). Substitution into the wave equation results in:

$$(\mu_1/\mu)(\rho/\rho_1)^{1/3}\partial_t^2 Q - (\rho_1/\rho)^{2/3}c_1^2\nabla^2 Q = 0\,. \tag{28}$$

This is equivalent to the usual wave equation with $c^2 = \mu/\rho$.

Using $\gamma^2 = (1 - n^2v^2/c_1^2)^{-1}$ and solving Eq. 25 for v^2 yields:

$$v^2 = \frac{c_1^2}{n^2}\left(1 - \frac{1}{n\gamma_1^2}\right)\,. \tag{29}$$

This is equivalent to the prediction of general relativity with metric $g^{\mu\nu} = 1/g_{\mu\nu} = \{n, -1/n, -1/n, -1/n\}$. In particular, Eq. 29 is equivalent to Eq. 14 in Ref. 21 for the same metric.

3.3. Magnitudes

The above analysis relates the vacuum refractive index to changes of inertial density, elasticity, and the gravitational potential. To appreciate the scale of the distortion of vacuum, we can use Eq. 26 to calculate the change of refractive index at the surface of the sun. The gravitational potential is $\Phi_g = -GM_S/R_S$, where $M_S = 2.0 \times 10^{30}\,$kg is the mass of the sun, and $R_S = 7.0 \times 10^8\,$m is the radius of the sun. The change of refractive index just outside the sun's surface is:

$$\frac{\delta n}{n} = \frac{2GM_S}{R_S c^2} = 4.2 \times 10^{-6}. \tag{30}$$

The distortion at the edge of the sun is quite small. This is the accumulated effect of distortions that can mostly be attributed to protons within the sun. Since the sun is spherical, we can attribute the distortion to a sphere of protons packed tightly around the center of the sun. The number of protons in the sun is approximately $N_p \approx M_S/m_p$, where $m_p = 1.67 \times 10^{-27}\,$kg is the proton mass. The radius of a proton is generally accepted to be $r_p = 8.4 \times 10^{-16}\,$m, so the radius of an equivalent sphere of protons (treated as cubes for tight packing) would be approximately:

$$R_p \approx \left(\frac{M_S}{m_p}\right)^{1/3} r_p \approx 9000\,\mathrm{m}. \tag{31}$$

Since the gravitational potential falls off as $1/r$ for a given mass, the change of refractive index at the edge of this sphere of protons would be:

$$\frac{\delta n}{n} \approx \frac{R_S}{R_p}(4.2 \times 10^{-6}) = 0.3\,. \tag{32}$$

This is still less than one, but second-order terms would no longer be negligible for such an object.

The change of refractive index at the edge of a single isolated proton is:

$$\frac{\delta n}{n} \approx \frac{2Gm_p}{c^2 r_p} = 3 \times 10^{-39}\,. \tag{33}$$

The smallness of this number implies that vibrations of a proton are very nearly volume-preserving in this model.

4. WAVE PROPAGATION

Fermat's principle requires that neighboring paths interfere constructively along a ray, as expressed by the variation:

$$\delta \int k\, d\ell = 0\,, \tag{34}$$

where $d\ell$ is the differential path length. Following Ref. 26, we parameterize the length using the "stepping parameter" a such that $d\ell = (d\ell/da)da$. Factoring out the constant factor $(\gamma_1\omega_1/c_1^2)$ from the wave number (k) yields:

$$\delta \int n^2 v \frac{d\ell}{da}\, da = 0\,. \tag{35}$$

Letting the variation correspond to a change of path $\delta\boldsymbol{\ell}$:

$$\int \left((\nabla(n^2 v) \cdot \delta\boldsymbol{\ell}) \frac{d\ell}{da} + n^2 v \delta \frac{d\ell}{da} \right) da = 0\,. \tag{36}$$

To first order, the change in $d\ell/da$ is only due to the component of $\delta\boldsymbol{\ell}$ parallel to $d\boldsymbol{\ell}/da$.

$$\int \left((\nabla(n^2 v) \cdot \delta\boldsymbol{\ell}) \frac{d\ell}{da} + n^2 v \frac{d\boldsymbol{\ell}/da}{d\ell/da} \cdot \frac{d\delta\boldsymbol{\ell}}{da} \right) da = 0\,. \tag{37}$$

Integrating the second term by parts yields:

$$\int \left((\nabla(n^2 v) \frac{d\ell}{da} - \frac{d}{da} \left(n^2 v \frac{d\boldsymbol{\ell}/da}{d\ell/da} \right) \right) \cdot \delta\boldsymbol{\ell}\, da = 0\,. \tag{38}$$

This has a very simple form if the stepping parameter a is chosen to be $da = dt/n^2$ so that $d\ell/da = n^2 d\ell/dt = n^2 v$.[26] For arbitrary variation da, the integrand must be zero and therefore:

$$\frac{1}{2}\nabla(n^4 v^2) = \frac{d^2\boldsymbol{\ell}}{da^2} = n^2 \frac{d}{dt}\left(n^2 \frac{d\boldsymbol{\ell}}{dt} \right)\,. \tag{39}$$

Substituting the value of v^2 from Eq. 29 yields:

$$\frac{c_1^2}{2}\nabla \left(n^2 \left(1 - \frac{1}{n\gamma_1^2} \right) \right) = n^2 \frac{d}{dt}\left(n^2 \frac{d\boldsymbol{\ell}}{dt} \right)\,. \tag{40}$$

Although we normally regard gravitational acceleration as independent of velocity, a velocity-dependence is contained explicitly in the γ_1^2 factor and implicitly in the derivative dn^2/dt (since $d/dt = \partial_t + \mathbf{v} \cdot \nabla$). For non-relativistic speeds ($v << c$) the velocity dependence is negligible.

Consider some special cases with propagation in the $x - z$ plane with $n \approx 1$ and $\nabla n = \partial_z n\, \hat{\mathbf{z}}$. First, consider non-relativistic massive particles with initial $\gamma_1 \approx 1$ and $dn^2/dt \approx 0$. The acceleration is:

$$\frac{d^2\boldsymbol{\ell}}{dt^2} = \frac{c_1^2}{2}\nabla n\,. \tag{41}$$

This relates the local gravitational acceleration g to the refractive index: $g = -c_1^2 \partial_z n/2$. In terms of the gravitational potential Φ_g of Eq. 26, this is:

$$\frac{d^2\boldsymbol{\ell}}{dt^2} = -\nabla\Phi_g\,. \tag{42}$$

For waves propagating at or near the speed of light perpendicular to the gradient, we have $dn^2/dt \approx 0$ and $\gamma_1 \to \infty$ so that:

$$\frac{d^2\boldsymbol{\ell}}{dt^2} = c_1^2 \nabla n = -2\nabla\Phi_g \,. \tag{43}$$

Hence light (or a relativistic particle) propagating perpendicular to the gradient of n has twice the acceleration of non-relativistic massive particles.

At the edge of a black hole, light traveling tangentially would be refracted into orbit with centripetal acceleration of:

$$\frac{d^2\boldsymbol{\ell}}{dt^2} = -\frac{c_1^2}{r}\hat{\mathbf{r}} = 2\partial_r \frac{GM}{r}\hat{\mathbf{r}} \,. \tag{44}$$

Solving this equation yields the Schwarzschild radius $r = 2GM/c_1^2$. For the mass of the sun, the Schwarzschild radius is about 3000 m.

The above examples have $dn^2/dt \approx 0$, but for light (or a relativistic particle) propagating vertically (up or down) we have: $dn^2/dt \approx \pm 2nc_1 dn/dz$. In this case $d\boldsymbol{\ell}/dt = \pm c\hat{\mathbf{z}}$ and:

$$\frac{d^2\boldsymbol{\ell}}{dt^2} = c_1^2\nabla n - 2c_1^2\nabla n = -c_1^2\nabla n = 2\nabla\Phi_g \,. \tag{45}$$

This result simply states that the speed of light increases as n decreases and vice versa, as also given by:

$$\frac{dc}{dt} = \frac{d}{dt}\left(\frac{c_1}{n}\right) = -c_1^2\frac{\nabla n}{n^2} \approx -c_1^2\nabla n \,. \tag{46}$$

Interestingly, the direction of acceleration is “upward”.

5. DISCUSSION

The predicted refraction of light passing near the sun was first verified in 1919 when a group led by Arthur Eddington measured the shifted positions of stars whose light passed close to the sun during a solar eclipse.[27] More direct measurements have since been made using radio waves, which do not require an eclipse.[28] Since light speed slows near the sun, there is also a delay in the signal as compared with propagation in free space. This delay has been measured and is in agreement with theoretical predictions.[29]

An interesting aspect of a spatially varying absolute (or “coordinate”) speed of light is that measurements of spatial dimensions yield non-Euclidean geometry. Suppose that distances are measured by timing light propagation to an object and back. A satellite in orbit around a planet would measure an altitude higher than reality since light reflected from the surface would slow down near the surface, lengthening the propagation time. Light propagating at a fixed altitude would have no such slowing, so the measured circumference of a circle around the planet would be less than 2π times the measured radius. Using rulers instead of light propagation to measure distances would yield the same non-Euclidean effects since a ruler consists of standing waves that would contract in regions of slower light speed. This phenomenon is commonly called "curved spacetime", but could just as well be called "compressed space". It is a simple consequence of spatial variation of the speed of light.

Gravitational waves, first discovered in 2015,[30] have a simple interpretation as deformations of a solid aether. These are incompressible transverse waves that can be modeled over short time intervals by displacements in the radial direction with azimuthal dependence $\cos(2\phi - \omega t)\hat{\mathbf{r}}$ (see e.g. Ref. 31). Since these are transverse waves, aether theory (like general relativity) predicts that they should travel with the same speed as light waves. Recent measurements have established the difference to be within 3% with 90% confidence.[32]

6. CONCLUSIONS

We have derived many aspects of general relativity by analyzing waves in a nonuniform elastic solid. The results presented here apply to quasi-static gravitational fields with a diagonal and spatially isotropic spacetime metric. Further work is needed to extend the analogy between vacuum and elastic solid to more general situations. However, the present work should be sufficient to give students an intuitive understanding of how the presence of energy produces gravitational effects on light and matter.

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AUTHOR DECLARATIONS

The author has a financial interest in a book that would be referenced in the Introduction following review.

DATA AVAILABILITY STATEMENT

No new data were generated or analyzed in this study.

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