The MM Theory:
Explaining Quantum Mechanics Concepts through MM Theory
Part (3)

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(Submitted: July 3, 2024)

Summary: This paper presents a novel approach to understanding the intricate aspects of quantum mechanics via the applications of concepts from MM theory. What is presented as a conundrum within the field of quantum mechanics is logically explained and elucidated upon by MM theory via a deterministic view and explanation of the concepts, particularly centring on the classical conclusions and results derived from the double-slit experiment.

Abstract

This article examines and discusses concepts within the field of quantum mechanics. Specifically, this paper focuses on the idea of wave-particle duality within this field, by further examining the nature of light. The basic premise is rooted in MM theory, previously introduced and expanded upon in literature. Per MM theory, light is viewed as a wave of motion of momenta conveyed by an ensemble of M particles propagating harmonically in space. Through a deep analysis of the double-slit experiment, this paper aims to illustrate the manner by which MM theory provides a complete and thorough understanding of the nature of light and the basic fundamentals of quantum physics.

The discussion centres on highlighting how the MM theory offers a comprehensive interpretation of various observations such as interreference patterns and illuminates the underlying mechanisms at play. By viewing light as a wave of momenta, seemingly contradictory observations are logically addressed and reconciled. The theory further demonstrates how the detection process can disrupt the wave pattern and leads to the collapse of the interference patterns observed. Furthermore, this article challenges the fundamental aspects of quantum mechanics, such as indeterminism and the role of measurement. Via MM theory's examination of quantum measurements, the concepts of observer dependency and hidden variables are challenged. This ultimately suggests that quantum phenomena are inherently deterministic and can be predefined using known variables.

In conclusion, this paper challenges the fundamental framework of quantum mechanics, offering deeper insights into the nature of light and the wave-particle duality via the deterministic approach derived from the MM theory and paves the way for further experimentation and study in this field.

Keywords

Theory of Everything, MM Theory, M particle, Quantum Mechanics, Light, Wave, Particle, Double-slit Experiment

1. Introduction

Throughout history, the conceptualization of light has drastically transformed with scientific advancement. Beginning with ancient civilizations, it was initially entwined with philosophical and theological ideologies that associated it with vision and perception. Early Greek philosophers hypothesized that luminous objects emit light such that it propagates throughout space in straight trajectories. Empedocles viewed light as a divine element. These ideas formed the basis for early understanding of light [1] [2].

It was not until the 17th century that Sir Isaac Newton then posited the corpuscular theory whereby light is viewed as a set of discrete particles [3]. Concurrently, Dutch physicist Christiaan Huygens presented an alternative view of light as a manifestation of wave phenomena [4].

Subsequently, in the 19th century, Thomas Young conducted the pivotal double-slit experiment, which appeared to provide overwhelmingly convincing evidence for the wave-like behavior of light [5] [6]. Following this experiment, overall discourse on the nature of light transitioned towards acceptance of the wave-like behavior, and the previously dominant corpuscular model was challenged. The results of Young's experiment was then subsequently challenged in the 20th century following Albert Einstein's description of the photoelectric effect, which was thereafter cited as evidence for the particle nature of light [7].

In the contemporary understanding, the nature of light is viewed as a wave-particle like duality. Light is seen as a form of electromagnetic radiation, encompassing both electric and magnetic fields. These manifestations are the results of the framework of quantum electrodynamics and Maxwell's equations [8]. As such, light is seen as inherently dual, manifesting itself as either a wave or particle dependent on the type of experiment or observation being conducted.

Thus, from the early Greek philosopher's divine appreciation for light to the contemporary ideas of its dual wave-particle nature, the journey for the understanding of light has proved to be a constant quest of deeper knowledge as sought by humanity, marked by new discoveries and theories in this field. It is marked by ever changing reflections, adjustments, and new ideas about the nature of light and its behavior.

However, now MM Theory, a novel theory which has been previously presented by the author, provides for a new understanding of the nature and behavior of light [9]. MM theory asserts that "light is a wave of motion of momenta which are conveyed by an ensemble of M particles that acts harmonically and propagates throughout space. The direction of these motions is along the path of travel. Light as an emission is the propagation of longitudinal waves similar to sound waves."

In the realm of the field of quantum mechanics, the double-slit experiment stands as an enduring enigma that leaves many unanswered questions open to analysis and interpretation. There are two prominent contemporary schools of thought, namely the Copenhagen [10] and the Bohm interpretations [11] [12]. These each offer contrasting views of this seminal experiment. First, the Copenhagen interpretation, introduced by scientists Niels Bohr [13] [14] and Werner Heisenberg [15] rely on a probabilistic reasoning for their explanation and speak of what is termed as a wave function, said to collapse in the event of a measurement. In contrast, the Bohm school of thought, introduced by physicist David Bohm, underlies a deterministic path to explaining the experiment's results and where the particles are said to follow pre-determined trajectories as defined by a hidden wave function. As such, the double-slit experiment stands as a defining standard for challenging and testing the extent of these interpretations.

In recent years, further research appears to have been conducted on the Copenhagen interpretation. Namely, in 2011 an article was published by Sacha Kocsis et al., entitled "Observing the Average Trajectories of Single Photons in a Two-Slit Interferometer" [16]. The authors detailed a modified two-slit experiment using weak measurements in attempting to avoid disrupting the quantum states of photons, with their results supporting the probabilistic interpretation and the Copenhagen school of thought.

Despite significant work in the field of quantum mechanics, the interpretations of fundamental principles and the problem of the collapsing wave function remain enduring. As such, interpretations of Copenhagen and Bohmian continue to be viewed by many researchers as unsatisfying and inadequate. Specifically, the mechanism for the collapse of the wave function in the prominent Copenhagen interpretation continues to baffle scientists. Consequently, there is significant research gap for further theoretical and experimental work to be conducted to investigate and posit new ideas for deeper understanding of quantum theory.

Based on concepts taken from MM Theory, this paper begins by first explaining the nature and behavior of light as illuminated by this theory. Second, the discussion advances to focus on the so-called "duality of light" behavior as per the double-slit experiment and the intricacies of this experiment are carefully dissected through the lens of MM Theory, providing for a new perspective that can be taken on light. Thirdly, the behind-the-scene's reasoning for the collapse of wave function as per the classical interpretation of quantum mechanics, when re-evaluated and reviewed by the MM Theory is described. Combining all the above elements and taken together with the results, a revised interpretation of quantum mechanics is examined, newly addressed, and presented.

2. Theory

2.1. Nature and behavior of light

Light exhibits a fascinating duality, behaving both as a wave and as a particle. The experimental evidence and literature supporting the wave-like behavior of light is commonly cited as the double-slit experiment [17], diffraction experiments [18] [19], Newton's rings [20], Huygens' principle [4] [21], and polarization [22]. These experiments and hypotheses collectively provide strong evidence that light exhibits characteristic of waves.

Concurrently, light also is said to demonstrate particle-like properties, as notably shown in the photoelectric effect described by Albert Einstein. When a light of certain frequency shines on a metal surface, it ejects electrons where the energy of these electrons depends on the frequency of the incoming light, suggesting that light consists of discrete packets of energy called photons [7]. Further evidence for the particle nature of light is seen in Compton scattering, where X-rays scatter off electrons, resulting in a wavelength shift consistent with what would be expected in the behavior of light as particles [23] [24]. Additionally, Max Planck's study of blackbody radiation revealed that light emitted by a blackbody could only be explained if the energy was quantized into discrete packets and this ultimately led to the concept of photons [25].

Per the MM theory, we begin by noting that all of space is filled with M particles and the space between any two M particles varies when compared to any other two. A simple case as per [Figure 1] can be considered, where three M particles are shown with initial and equal magnitudes of linear momentums. These three particles transfer their momentums via collisions to other particles ahead. Therefore, any given particle's momentum is different from any other at virtually all times. In other words, all particles have varying magnitudes and directions in their momentums. This means that any single M particle's properties will be different in time and space and these properties will depend on how a particle interacts with the others during its collisions.

As can be seen, the particles' momentums do not travel in completely straight paths during the collisions of the M particles as may ideally be expected. The Figure illustrates that the linear momenta move in an expansive manner such that they gradually spread to other particles along the path of travel. In addition, as a result of the momentum transferring to more and more particles, the magnitude of the momentum in each particle also gradually decreases the further away the momenta travel to from the initial source. Along the path of travel, it is also noted that the sum of the linear momenta of all particles in the direction of travel at any point in time remains equal to that of the initial set of three particles with the initial momenta.

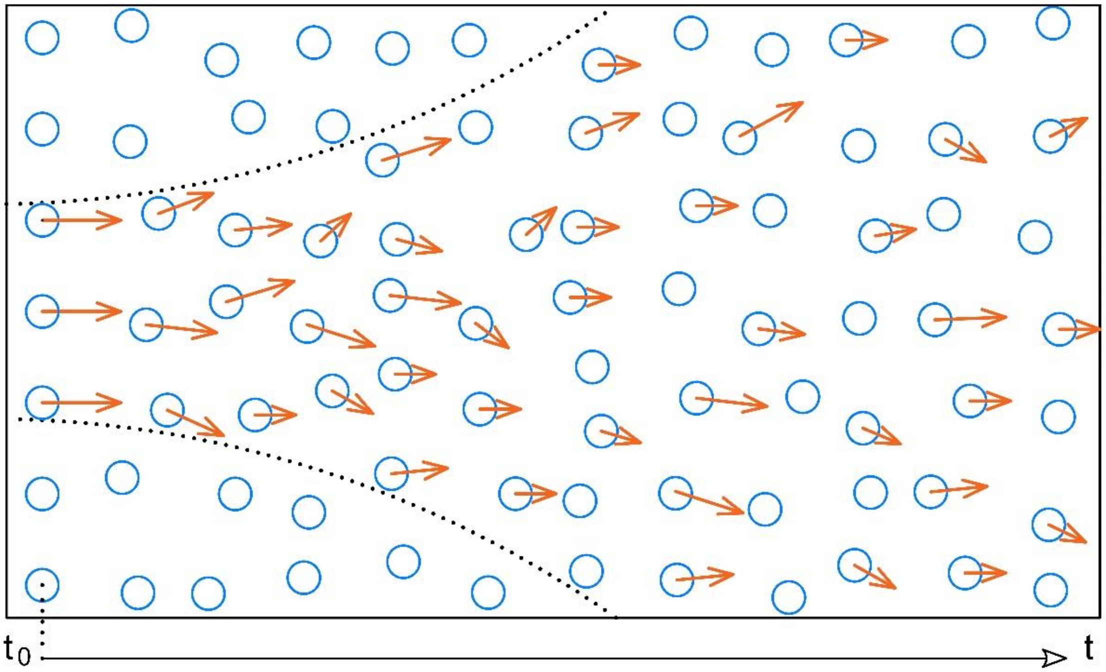

Fig. 1 A set of three M particles transferring their linear momenta to the particles ahead, and the propagation of these momenta in a manner that it is spreading out conically and the direction and the amount of momentum of each particle ahead is different from any other particle.

Now, per the MM theory, propagation of light from a single source is in a spherical manner and occurs via M particles [Figure 2]. When considering an excited ensemble of M particles from a light source transferring momentums to other M particles ahead, it is proposed that a certain number of particles gain momentum while others lose it to other particles via the collision interactions. This creates the basis for the understanding of light as a wave and wavefronts. Thus, the momentums of particles are not essentially always equal to that of other particles at any point in time, even those in contact with one another or even along the same wave front. As seen in [Figure 2], there are wave fronts of crests and troughs in the context of M particles when viewed along a plane. The crests have the maximum linear momenta of these particles and the troughs represent the minimums. Again, it can be seen here that the directions of these linear momentum are not always ideally in straight paths but is rather somewhat random and is generally directed along the path that the light source is taking.

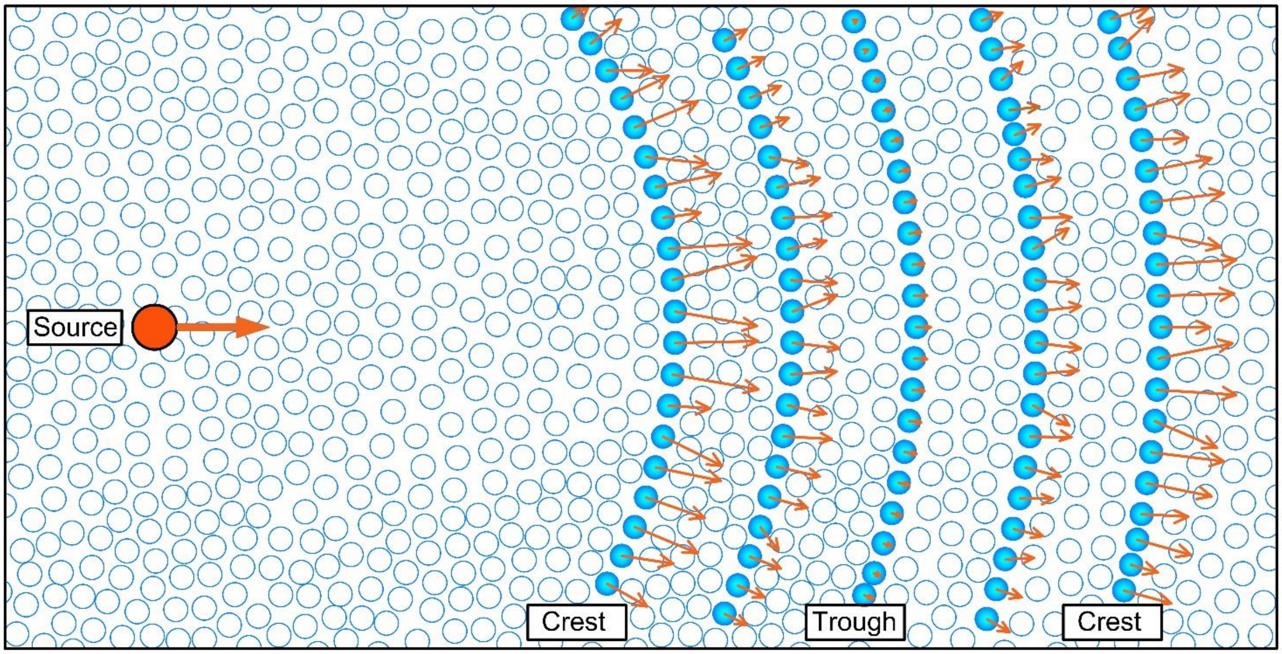

Fig. 2 Propagation of light via M particles from a light source, illustrating the crests and troughs of a longitudinally transferred light wave.

In regards to the detection of light as a particle, certain light detectors may be used. In the case of light as a particle and in the context of MM Theory, the M particles that have certain momentums or energies will be captured by the detectors (i.e., human/animal eye or machine). In physics, these particles are commonly termed as photons. In such a scenario, the collisions of M particles to the ones ahead causes differences in the momentums of such particles. Therefore, the positions of the particles that have their momentums detected as photons change at any point in time. For instance, any M particle that had a sufficient momentum to be detected as a photon, may at a later time have a different momentum while another M particle at another position may gain momentum and become a photon. As such, the position of a new photon may not be at the same location as that of a previous photon particle. As a result of this, detectors find the position of photons at different points in time and the positions are changed instantaneously. In other words, the position as well as the momentums of photons are continually varying because of the interactions of M particles with each other. Moreover, concurrent to this, the energy of photons does not always essentially equate to one another.

Now, the entities that can detect light fall into various categories, among them being human eyes, animal eyes, and man-made detectors. In all cases, the light that can be detected depends on the values of the particle’s momentums, the frequency of vibrations of M particles due to effects of frequency of light itself, the associated universal vibrations from other M particles, and the sensitivity of the detecting entities. For the human eye, it can be detected between a certain range of values. The same can be said for animal eyes and man-made detectors. For the animal eyes it varies on the species of animals. If the detector is not sensitive enough to detect momentum and vibration from any particle that is measured, then any effect from such a particle would be left undetected.

2.2. Novel Explanation for the Double-Slit Experiment

In the case of the double-slit experiment, when a ray of light is directed towards two narrow slits, what is seen on the screen is a classical interreference pattern that indicates light is acting in a wave-like fashion [Figure 3]. This experimental observation is a conundrum in the understanding of light, as when said photons are singularly emitted through the slits, they develop the same interreference pattern through numerous collisions at the screen.

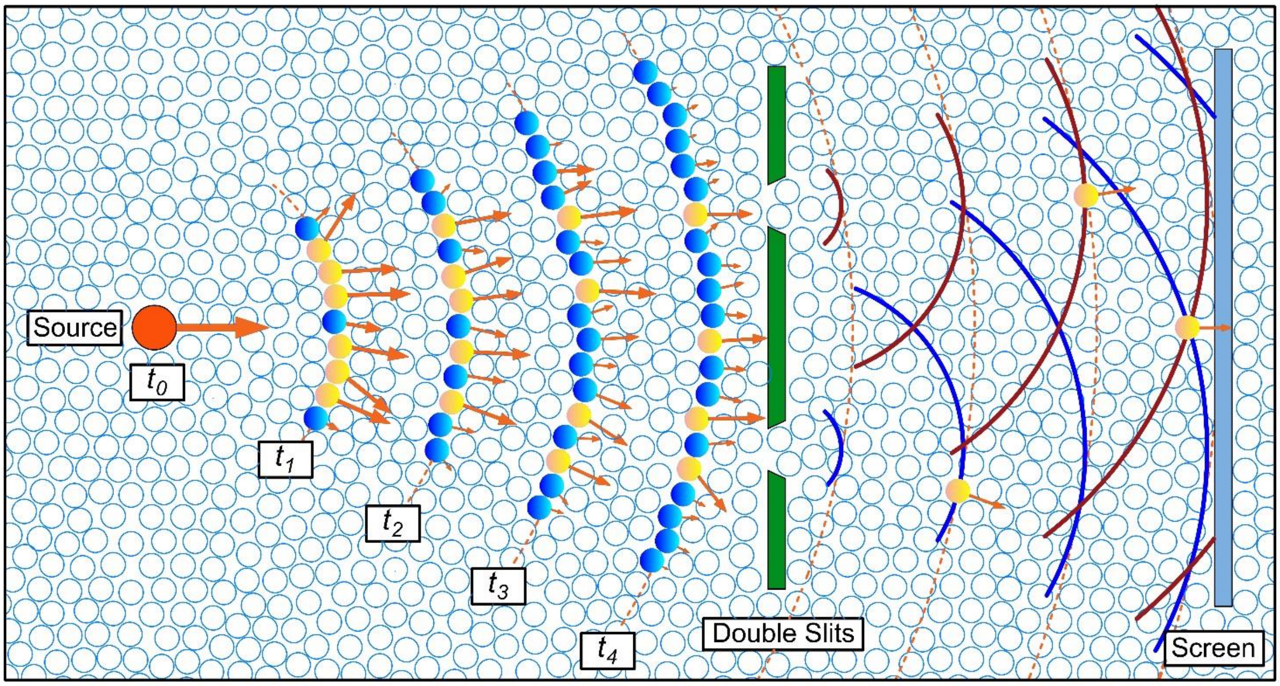

Fig. 3 Light from an initial emitting source t_0 travelling to wavefront crests at t_1, t_2, t_3, and t_4 before the double-slit barrier, subsequently forming an interreference pattern on the screen.

The MM theory when applied to the double-slit experiment explains that the motion and the transferring of the momenta in the case of light is always in a wave-like manner. However, if one focuses on studying a single "photon," what is detected is a particle rather than a wave. As described, the detectors can detect only the particles possessing certain momentums and vibrations within a distinct range of values. Photons are illustrated as yellow particles in [Figure 3]. In reality, there are other "photons" or light particles (i.e., M particles) coexisting but undetected alongside these particles. As such, they are not accounted for since they don't fall within the range of the detector.

With this new understanding, the observed wave-like behavior when photons are singularly emitted through the slits, forming an interreference pattern through numerous collisions on the screen, can now be explained from a new perspective. With the understanding that a photon is a particle from an ensemble of particles conveying momentums, it is theorized that they collectively behave as a wave.

This is as they are carrying and transmitting momentums in a way-like manner [Figure 2,3]. As such, the photon is among a group of particles on a wave front transferring their momentums to other particles, and which can only be detected if falling within a range of values as described earlier. In reality then, what reaches the screen are two waves passing the slits, and which form the interference pattern. The probability of the detection as a photon (i.e., light particle) is higher near the constructive interference spots of the two waves that are coming from the slits.

The MM theory then resolves the paradox of wave-particle like duality, and ultimately verifies that the motions and transferring of momentums of an ensemble of M particles are in wave-like manner, while the momentum/energy of a single M particle can be detected by a detector in its range of detection.

Here is should be noted that it is impossible then to emit a single photon to any distance as an individual photon. Even assuming that such a singular photon would exist, it would transfer its momentum instantaneously to other M particles ahead.

Additionally, as can be observed in [Figure 2,3], the M particles which are closer to the crests, the light source, and the propagation axis of light are more likely to be detected as photons. In other words, the probability of detecting a particle as a photon is greater near the crests, the light source, and the axis of propagation. However, it is crucial to emphasize that this does not imply that the system behaves probabilistically, nor that there is any probabilistic nature of quantum systems.

2.3. Measurement and Detection

Measurement and detection are the cornerstone and building blocks of quantum mechanics, defining our understanding of the basic nature of reality at the smallest scales. In contemporary thought, the act of measurement defies basic intuition and pictures a world that is only explainable via probabilistic outcomes and uncertainty. In the double-slit experiment, the act of measurement and detection and their interpretations challenge the very foundations of our understanding of the universe. By realizing the complexities of measurement and detection in this experiment, the MM theory presents a unique and intuitive explanation of this seminal experiment and its determinist implications for the universe.

MM Theory addresses the act of detecting a photon or wave by underscoring that the detections cause disturbances that change the values and direction of the momentums of M particles. The disturbances can best be understood as disturbances in the wave patterns, rather than the wave functions. It is, then, such disturbances in the wave patterns that ultimately cause the collapse of the interference patterns on the screen. As shown in the instance illustrated in [Figure 4], as a result of the individual waves being disturbed by a detector, the interference pattern on the screen fails to form. This disturbance effects to what is seen on the screen as a collapse of the interference pattern.

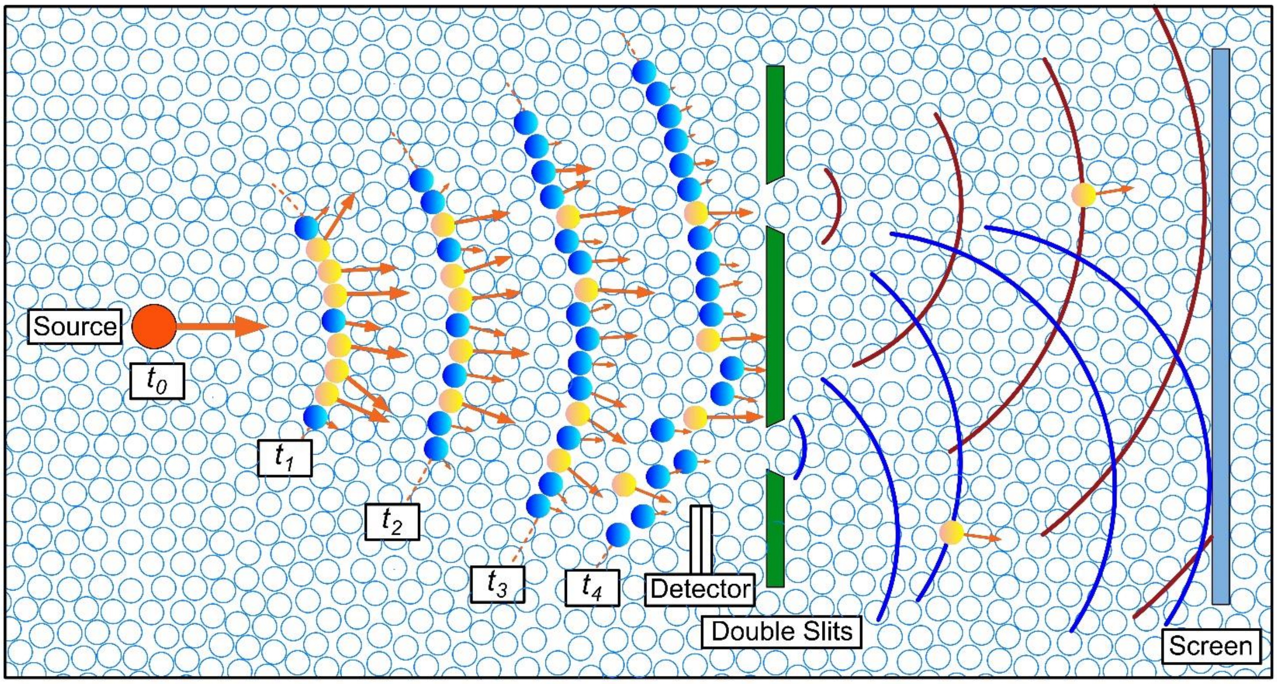

Fig. 4 Light emitting source forming a wave pattern prior to reaching the detector. The detector is placed prior to the slits, causing a collapse of the individual wave as well as the disturbance in the formation of an interference pattern on the screen; the same concept would apply if the detector is placed post the slits.

3. Fundamental Aspects of Quantum Mechanics (Interpretation)

3.1. Wave-Particle Duality of Light

Light is a WAVE of the motion of momentums that is conveyed via M PARTICLES.

3.2. Determinism and Indeterminism

The concepts of determinism and indeterminism are advocated by various schools of thought in contemporary physics. In classical physics, a system is always in a well-define state, where its properties and motion are determined via applicable classical laws. It is known that given a complete picture of a body's initial properties and positions, the final states of that body can be predicted with certainty. This is in contrast to quantum mechanics, which is foundationally based on indeterminism and uncertainty and where the exact properties of particles cannot be predicted until measured. Only a probabilistic picture of the particles can be presented. As explain earlier, MM Theory challenges quantum mechanics in this notion by explaining how it can be understood from a deterministic viewpoint. It is posited that a given system is always in a definite state with exact properties governed by classical laws of physics and deterministic theoretical foundations. As a result, if the properties and positions of particles are known, it is indeed theoretically possible to determine the final values of momentum and position of any particle.

3.3. Measurement and the Collapse of Wave

Quantum mechanics involves the concept commonly known as the “collapse of the wave-function,” whereby detection of a particle causes the disappearance of interference patterns on the screen in the seminal double-slit experiment. This is the foundation for quantum mechanics demonstrating that measurement plays a crucial role in determining the properties of a quantum system. This implies that making a measurement in this system fundamentally alters the quantum system, collapsing the wave function and forcing the system into a definite state.

MM Theory offers a distinct perspective by arguing that the act of measurement and its impact on wave patterns are logically interconnected and explainable. It explains the exact impact of measurement on the wave patterns, and centres on clearly describing how the disruption is causally responsible for the disappearance of the interference pattern. This challenges probabilistic interpretations of quantum mechanics, and proposes that the collapse of the wave pattern is a direct result of the impact of the act of measurement and not inherent in quantum systems themselves.

3.4. Observer Dependency

The conventional understanding of quantum mechanics highlights the key role that is played by the observer, suggesting that observations can influence measurement outcomes. This perspective suggests a critical connection tying the observer's actions with that of the quantum state. In contrast, MM Theory diverges from this perspective by underscoring the independence of any systems or states from the observer. It argues that measurement results are pre-determined via the underlying properties of any system rather than influenced by the presence of an observer. This theory again asserts a deterministic view within quantum mechanics, indicating that various observers interacting with the same quantum system would consistently obtain the same identical results. Consequently, MM Theory challenges the notion of observer dependence and promotes for the idea of an objective and deterministic nature in any system measurements.

3.5. Hidden Variables

There are no hidden variables in the MM approach to explaining any systems. It asserts that any system operates solely in a deterministic pathway using known variables and it does not depend on any hidden variables beyond those specifically defined. Among the various debates and arguments within the field, the MM theory consistently supports idea that quantum mechanics is intrinsically deterministic and it does not need to, in fact, require any hidden variables to explain any of the quantum phenomena.

4. Conclusion

This paper presents a substantial advancement in elucidating the wave-particle duality of light, a fundamental enigma in quantum mechanics, by employing the innovative MM Theory framework. MM Theory provides a cohesive and revolutionary explanation for observed phenomena, particularly in the context of the double-slit experiment via the depicting of light as a momentum-carrying wave via M particles. This approach not only presents a fresh perspective on a continuing debate but also reconciles the traditionally conflicting wave and particle notions of light.

The integration of MM Theory into quantum mechanics thus plays a vital role in resolving apparent contradictions within the discipline. Its rigorous analysis and theoretical novelty challenge established concepts regarding observer influence and the need for hidden variables. According to MM Theory, quantum measurements are truly objective processes that unveil the inherent characteristics of quantum systems, independent of the influence of observers' presence or actions. This viewpoint imparts a shift towards a deterministic interpretation of quantum mechanics and proposes that different observers interacting with the same quantum system would obtain the same results. Thus, MM Theory underscores the objective nature of quantum measurements and rejects the subjective interpretations that have historically prevailed.

Moreover, this paper elucidates fundamental aspects of quantum mechanics as envisioned by MM Theory, including the principles of quantum determinism and indeterminism. Within this framework, systems exist in defined states with specific properties and are governed by classical physics laws that dictate their future trajectories. The act of measurement has an impact on wave patterns and leads to an interference pattern collapse. MM theory proposes that measurement is an objective process that is independent on observer actions.

In addition, this theory asserts that no hidden variables exist within any system, which further reinforces its deterministic approach. It posits that systems operate solely on known variables, while eliminating the requirement for additional undefined parameters. This perspective contributes to more transparent and inherently deterministic understanding of quantum phenomena which aligns with MM Theory's emphasis on objective and consistent measurements.

The insights derived from this research not only lay the foundations for further theoretical exploration but also present new opportunities for technological advancements in the fields that are currently being promoted as quantum computing and information technology. Applying MM Theory to other aspects of quantum phenomena holds promise for enhancing our understanding of quantum mechanics and has the potential for revolutionizing and presenting novel reviews of quantum technologies, and their replacement with sub-branches in the field of MM theory. By bridging gaps between quantum mechanics and classical physics, MM Theory encourages deeper exploration into universal principles propelling us towards a more unified physics theory and the practical applications of these fields.

In conclusion, MM Theory's explication of light's wave-particle duality represents significant revolution in the current field of quantum mechanics. Its robust analytical foundation and theoretical concepts enhance our grasp of nature's intricate phenomena while paving the way for future discoveries and technological innovations. By setting groundwork for forthcoming theoretical and technological breakthroughs, MM Theory unlocks possibilities to unravel mysteries within the current realm of quantum physics and heralds us to an era marked by innovation and understanding of the universe's fundamental workings.

Acknowledgment

I extend my heartfelt appreciation to my son, Mohammad Mahvelati, for his invaluable assistance in editing this paper. His thorough focus on details, insightful recommendations, and constant support have significantly improved the precision and coherence of this work. His contributions have played a crucial role in the completion of this research.

Statements and Declarations:

Funding

The author declares that no funds, grants, or other support were received for this work.

Conflict of Interest

The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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