On Prime Cycles in Directed Graphs Built with Primality Test

Marcin Barylski

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Abstract

One of the most famous unsolved problems in mathematics is Collatz conjecture which is claiming that all positve numbers subjected to simple $3x + 1/2$ formula will eventually result in 1, with only one known cycle $(1, 4, 2, 1)$ present in the calculations. This work is devoted to finding cycles in other interesting sequences of integer numbers, constructed with the use of some aspect of primality test.

1 Introduction

Cycles in integer sequences can be beautiful and surprising about their quantity and lenght. Famous illustration of this statement is definitely is Collatz conjecture which is claiming that all positve numbers subjected to simple $3x+1$ (if x is odd) or $/2$ (if x is even) formula will eventually result in 1, with only one known cycle $(1, 4, 2, 1)$ present in the calculations. Collatz formula is taking advantage of divisibility by 2, the work presented in this paper is considering various sequences where primality of the term decides on argument or the next value in the sequence.

2 Plan for experiments

Experiments are planned against various sequences where integer primality plays a central role. In expedited cases the formula for sequence will take advantage of functions like: $GPF(n)$ - the greatest prime factor of n and $SOPF(n)$ - the smallest odd prime factor of n. P_i is the i-th prime. In presented results cycles of lenght n for sequence S_i are symbolized as a list $[a_1, a_2, \ldots, a_n]$, where $a_{j+1} = S(a_j)$ $(1 \leq j<n)$ and $a_1 = S(a_n)$. All terms of the sequences are primes. Experiments were run with framework [1].

For example, if sequence formula is $S_1(P_i) = GPF(P_i+P_{i+1}$, then we have:

$S_1(2) = GPF(2 + 3) = GPF(5) = 5$,
$S_1(3) = GPF(3 + 5) = GPF(8) = 2$,
$S_1(5) = GPF(5 + 7) = GPF(12) = 3$,
$S_1(7) = GPF(7 + 11) = GPF(18) = 3$,
$S_1(11) = GPF(11 + 13) = GPF(24) = 3$,
$S_1(13) = GPF(13 + 17) = GPF(30) = 5$.

These calculations are depiciting a cycle in this sequence: $S_1(S_1(S_1(2))) = 2$.

3 Results of experiments

Experiments were executed against first 50000 terms of each sequence. They allowed to find interesting cycles (Table 1). If graph (drawn as a graphical representation of S_i) is connected and has one cycle, then such case is similar to Collatz conjecture where all numbers subjected to the formula eventually land in one cycle.

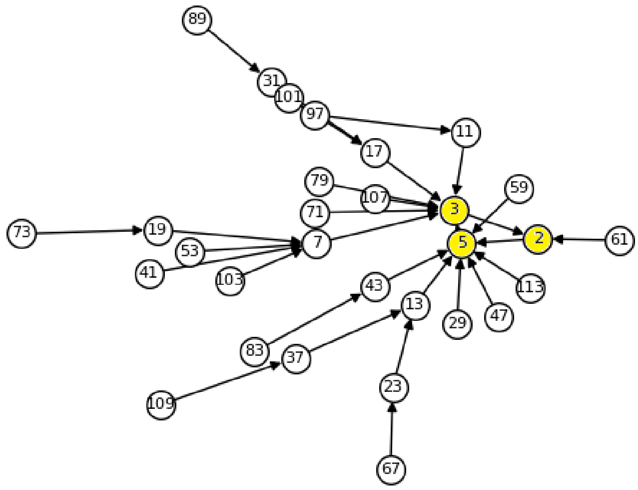

Figure 1: S_1 and its first 30 terms - example of connected graph (one cycle: [2, 5, 3] found so far)

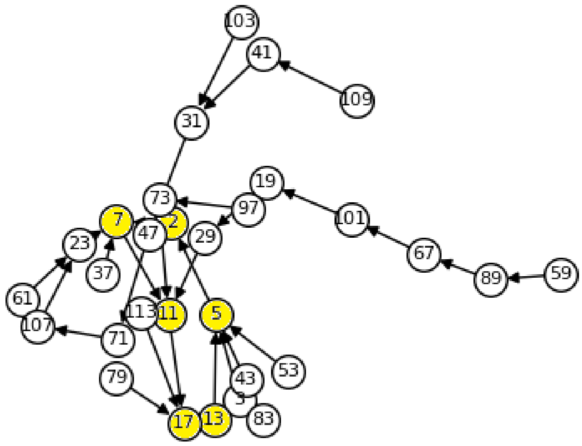

Figure 2: S_6 and its first 30 terms (one cycle [2, 7, 11, 17, 13, 5] found so far)

4 Summary and future work

Work allowed to find few interesting cycles. Directed graphs used to visualised sequences also revealed interesting shapes. Future work may be focused on checking further sequences. Three classes of sequences are interesting: class A - sequences producing one cycle; class B - sequences

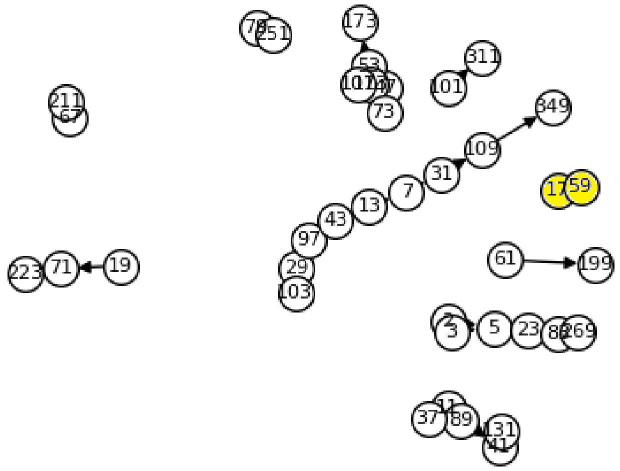

Figure 3: S_9 and its first 30 terms - example of disconnected graph (one cycle: [17, 59] found so far)

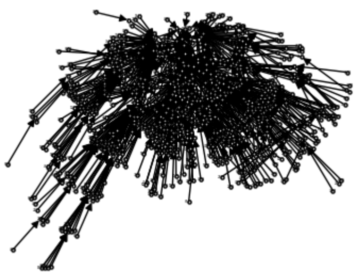

Figure 4: S_1 and its first 1000 terms

Figure 5: S_2 and its first 1000 terms

producing more than one cycle; class C - sequences without known cycle. Similarly to Collatz conjecture, the very interesting part of this analysis is actual proof, not empirical confirmation only.

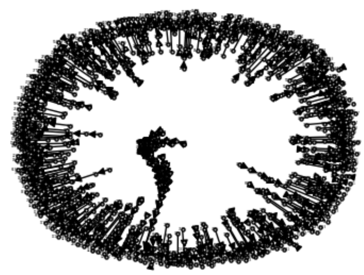

Figure 6: S_3 and its first 1000 terms

Figure 7: S_4 and its first 1000 terms

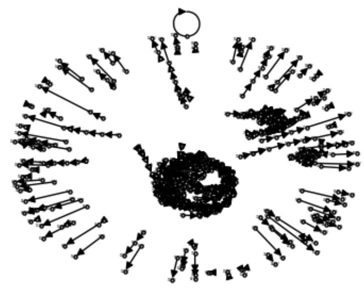

Figure 8: S_5 and its first 1000 terms

References

[1] *Library for various operations on primes.* https://github.com/mbarylsk/primes/

Figure 9: S_6 and its first 1000 terms

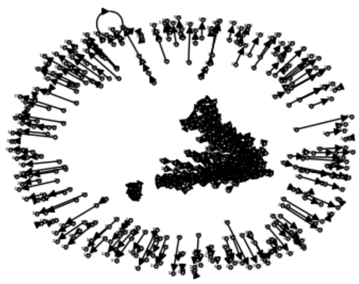

Figure 10: S_7 and its first 1000 terms

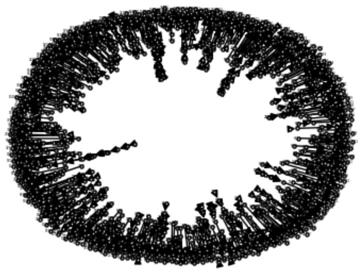

Figure 11: S_8 and its first 1000 terms

Figure 12: S_9 and its first 1000 terms

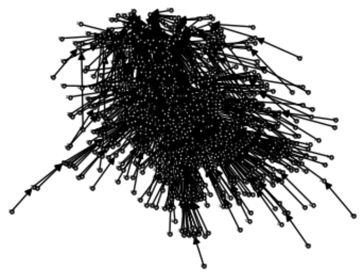

Figure 13: S_{10} and its first 1000 terms

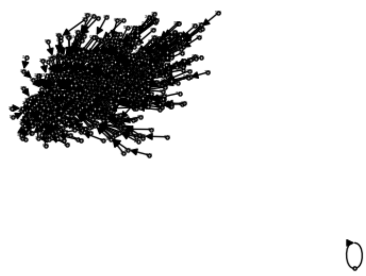

Figure 14: S_{11} and its first 1000 terms

Table 1: Sequences S_j subjected to experiments, results after 50000 iterations

j	Formula for $S_j(P_i)$	# of cycles found	cycles found
1	$GPF(P_i + P_{i+1})$	1	[2, 5, 3]
2	$GPF(2 \times P_i + 1))$	1	[3, 7, 5, 11, 23, 47, 19, 13]
3	$GPF(P_i \times P_{i+1} + 1)$	1	[2287, 479]
4	$GPF(2 \times P_i + 3)$	3	[3], [73, 149, 43, 89, 181], [61, 5, 13, 29]
5	$GPF(2 \times P_i + 5)$	4	[5], [739, 1483, 2971, 313, 631, 181, 367], [67, 139, 283, 571, 37, 79, 163, 331, 29, 7, 19, 43, 13, 31], [11, 3]
6	$GPF(3 \times P_i + 1)$	1	[2, 7, 11, 17, 13, 5]
7	$GPF(3 \times P_i + 2)$	3	[2], [389, 167, 503, 1511, 907], [131, 79, 239, 719, 127, 383, 1151, 691, 83, 251, 151, 13, 41, 5, 17, 53, 23, 71, 43]
8	$GPF(P_i^2 + 1)$	1	[89, 233]
9	$GPF(P_i + P_{i+1} + P_{i+2})$	3	[17, 59], [41, 131, 37, 11], [97, 43, 13, 7, 31, 109, 349, 1061, 103, 29]
10	$GPF(P_i + 1)$	1	[2, 3]
11	$GPF(P_i + 2)$	2	[2], [3, 5, 7]