Space (multidimensional model)

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Abstract

The article constructs a visual model of a multidimensional space that displays the properties of intersections of multidimensional spaces. The model reveals some unusual aspects of multidimensional spaces.[1][2]

1 Philosophy of multidimensional space

Any measurement process is, in fact, an external relationship of some measured bodies or processes with other material bodies or processes acting as measuring instruments (watches, rulers, any instruments, etc.).

The external nature of spatial measurements left its mark on the formation of the corresponding natural and mathematical concepts. In particular, this was expressed in the idea of three-dimensionality of space. Real things, bodies, processes that a person encounters in practical activity are voluminous. Essentially, volume (or capacity) represents real spatial extent. Space cannot be anything other than a collection of cubic meters. However, the expression of real volume in cubic meters (centimeters, kilometers, etc.) was the result of a long development, primarily of economic, but also scientific practice. The need to measure sown areas, distances over which herds were driven, migrations were made or hunters left, in fact, led to the fact that the initial basis of spatial measurements was length and its abstract expression - line.

Volume in Euclidean geometry is three-dimensional because it is based on a line taken one-dimensionally; lines form a two-dimensional plane, and from planes a three-dimensional volume is built. Although this path is optimal and best satisfies the needs of practice, it is still not the only possible one. Archaeological data confirm that units of volume (capacity) are historically as ancient as natural units of time and length (day, month, foot, etc.). It can be assumed that if the practical needs of primitive people had brought to the fore not the measurement of areas and distances, but the measurement of volumes, then the development of geometric science could have followed a path different from that laid out by Euclid. They say, for example: such and such a room (cave, temple, house, hall, etc.) is larger than another; the new device (machine) is more compact and takes up less space (less space) than the previous model. Despite the approximate nature of the above comparisons, the real spatial volume is expressed here in one dimension: in the relation "more - less". If, on the basis of similar or analogous comparisons, we develop units of measurement of one-dimensional volumes and put them as the basis of some imaginary geometry, then the concept of a line in it could be completely different: for example, expressed in three dimensions, say, as the third root of a unit of one-dimensional volume. Although such a performance seems pretentious at first glance, in reality there is nothing unusual about it. When measuring the surface of a table with a ruler, a one-dimensional line is not obtained using operations with two volumes (since both the ruler and the table are volumetric, the surface of which, as a side of the real volume, is being measured)? The resulting line and measured length, as well as their numerical values, are the result of a certain comparison of real volumetric objects.

Neither two-, nor three-, nor four-dimensionality, nor any other multidimensionality are identical to real extension, but reflect certain aspects of the objective relationships in which it can be located. The material world is the world of Euclid, and the world of Lobachevsky, and the world of Riemann, and the world of Minkowski, because in the concepts of any of the geometries associated with the names of these outstanding scientists, it is possible to describe and reflect real spatial extent as a universal attribute of material reality.[3]

2 Model of multidimensional space

In three-dimensional space the Pythagorean theorem is valid

$$r^2 = x^2 + y^2 + z^2 \tag{2.1}$$

where r is the distance between any two points in space. It is known that the entire content of Euclidean geometry can be derived from the relation (2.1). Indeed, for example, in Descartes' geometry, the Pythagorean theorem is an axiom.

Let's consider a set consisting of three points (Fig.1). Here the points are symbols, elements of the set. Instead of three dots, you can draw, for example, three crocodiles.

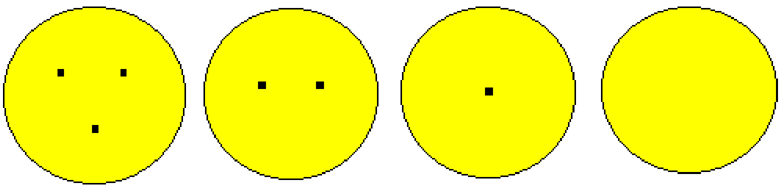

Figure 1: Set of points

Let us assign a set of points to the set of dimensions of the 3-dimensional space. Then a 3-dimensional space corresponds to a set of three points, a 2-dimensional space corresponds to a set of two points, a 1-dimensional space corresponds to a set of one point, 0-dimensional - the empty set of points.

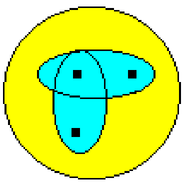

Figure 2: Intersection of subsets of points in a set of three points

Let's consider the intersections of subsets of points in a set of three points (Fig.2) Recall that an intersection is a subset that belongs to both intersecting subsets. In Fig.2 subsets intersect, each of which consists of two points. As you can see, subsets of two points can intersect at one point. In 3-dimensional space, this corresponds to the intersection of two 2-dimensional planes intersecting along a 1-dimensional line.

Let's look at Fig.3. Here the intersection of two subsets of two points and one point occurs along the empty set of points.

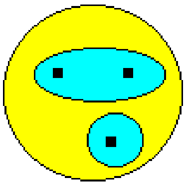

Figure 3: Intersection of subsets of two points and one point

In 3-dimensional space, this corresponds to the intersection of a 1-dimensional line and a 2-dimensional plane at a 0-dimensional point .

Similarly, we can consider intersections in 2-dimensional space and 1-dimensional space. The correspondence between the set of points and the set of dimensions will be complete.

Let us now consider a set of four points, which corresponds to 4-dimensional space (Fig. 4)

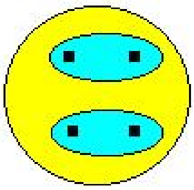

Figure 4: Set of four points

As you can see, in 4-dimensional space two 2-dimensional planes can intersect at a 0-dimensional point, which is impossible to do in 3-dimensional space. This can be visualized by projecting a 4-faceted angle onto a plane in the same way as projecting a 3-facet angle onto a plane and imagining that all angles at a vertex are 4-faces are straight. In such a 4-hedron, any two opposite faces (coordinate planes) intersect at one point. Nobody is surprised that in 3-dimensional space 3 coordinate planes intersect at one 0-dimensional point.

From Fig.2 and Fig.3 it becomes clear why it is psychologically impossible to imagine the intersection of two planes at one point in 3-dimensional space. We live in the space of 3 dimensions and cannot go into the 4 dimension. Any combination of intersections of subspaces in the 3th space will not lead to the intersection of two planes at one point (see Fig. 2). After all, they must intersect along an empty set of points, which is impossible.

In general, if we consider a set of n points, which corresponds to a n-dimensional space, then it is easy to find that the following relation holds

$$l \geq m + k - n \tag{2.2}$$

where l is a subset of points at the intersection of the subsets m and k; n is the entire set of points.

In the theory of finite-dimensional vector spaces there is a similar relation

$$\dim l \geq \dim m + \dim k - \dim n \tag{2.3}$$

where "dim" is the "dimension"; $\dim l$ is the dimension of the subspace resulting from the intersection of the subspaces m and k; $\dim n$ is the dimension of the ambient space.[4]

Let us have infinite-dimensional spaces$M$ and K. Then in our model their intersection will be displayed as a subset L of an infinite number of points (Fig. 5)

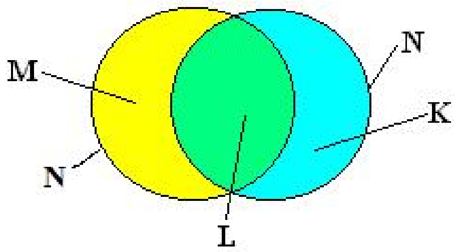

Figure 5: Subsets of an infinite number of points

that is, a continuous continuous region. The equations (2.2) and (2.3) here will look like

$$L \geq M + K - N \tag{2.4}$$

Let us now consider a set of 9 points, which corresponds to a 9-dimensional space (Fig.6)

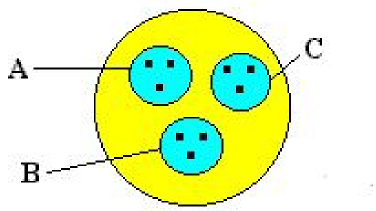

Figure 6: Set of nine points

If this set is divided into subsets of three points - A, B, C, then it is easy to see that the intersections of the subsets A, B, C are similar to the intersections of subsets of three points. In a 9-dimensional space, this means that its three 3-dimensional subspaces can intersect at one point and be mutually orthogonal. Thus, the 3-dimensional subspace in this case can play the role of a coordinate "axis". The concept of the angle between such "axes" is also generalized here. Then what corresponds to 2-dimensional planes in 3-dimensional space will be a 6-dimensional subspace here.

We took three points each in A, B, C just as an example. Let there be n points in A, B, C. Then we get an analogue of a $3\,n$-dimensional space. A "cube," for example, in such a space may look like this (Fig.7)

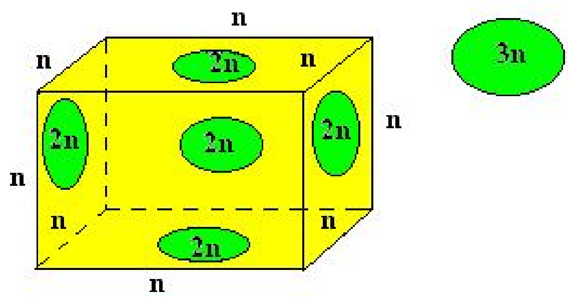

Figure 7: "Cube" in 3n-dimensional space

Here, each edge is n-dimensional, each face is $2\,n$-dimensional, the cube itself is $3\,n$-dimensional. If we take its n-dimensional subspace as a "line" in a $3\,n$-dimensional space, then with this definition we obtain the usual 3 -dimensional geometry, where each point can be characterized by three numbers in relation to n-dimensional coordinate "axes". The only difference from 3-dimensional space will be that the "length" of this "line" will be measured in meters to the power of n (cm, km, etc.), that is μ^n. The Pythagorean theorem in this case will have the form

$$r^2_{\mu^{2n}} = x^2_{\mu^{2n}} + y^2_{\mu^{2n}} + z^2_{\mu^{2n}} \tag{2.5}$$

With this definition, such a "3-dimensional" geometry will formally be no different from the 3-dimensional geometry of Euclidean with all its content.

In principle, n can be directed to infinity and we get "3-dimensional" geometry with an infinite number of internal degrees of freedom. "Points" in such a space (that is, very small regions) will be infinite-dimensional.

We come to the conclusion that even if observers use the formalism of 3-dimensional geometry, space itself can be multi- and even infinite-dimensional in the sense described above, but additional dimensions of space are unobservable (for example, curled into rings with Planck radius). At what level this multidimensionality manifests itself is a matter of physics.

References

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