Mathematical Formulas, which follow to the Koide-Formula, using the Input Values “Data of the Celestial Bodies of our Sun System and Physical Constants“, and the Connection of the Results

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1) Abstract:

In this report formulas are presented, which are based on the Koide-Formula and at which the input data are chosen from different fields of natural sciences. The Input data can be data of our celestial bodies, values of Physical Constants or just mathematical figures.
Besides the Koide-Formula with a result very close to the term “2/3”, also many other Formulas deliver unexpected results and connections, which also partly lead to the term “2/3”.
By that the assumption arises, that an unknown system might exist behind these astonishing results.

2) Modifications of the Koide-Formula with various Input Data and their Connection to each other and to Physical Constants:

The Koide-Formula[1] connects the masses of the three Leptons, namely Electron, Myon and Tauon and is readable as (verbatim taken from german wikipedia.de-entry “Yoshio Koide“[1]):

$$(m_e + m_\mu + m_\tau) / (\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 = 0.66666056 \qquad \text{(m0)}$$

The result is very close to the relation 2/3. This relation, namely 2/3 is valid for the **very Small**. Values for the masses m_e[2], m_μ[3], m_τ[4] are given at page 15.
Remark: The Elementar Particles Quark possess an electric charge either **+2/3e** or **-1/3e**!

The third Kepler Law[5] is given for a Sun System:

$$(RT_1 / T_2)^2 = (a_1 / a_2)^3 * (M + m_2) / (M + m_1) \qquad \text{with} \qquad \text{(KL)}$$

RT_1 and RT_2: Rotation Times of two considered Planets
a_1 and a_2: Distances (Big Half Axle) of these two considered Planets to the Sun
M: Mass of the Sun
m_1 and m_2: Masses of these two considered Planets

The relation "2 to 3" is also given for this law, in this case referring the exponents. This relation, namely 2/3 is now valid also for the **very Big**.

For further investigations the equatorial diameters of the eight planets of our sun system are taken in consideration and their values without the unit km are set in Formulas similar to the just presented Koide-Formula, but with 8 and 4 input values, for example at equations (Rel1) and (Rel2):

The equatorial diameters related to unit km of the eight planets are:
Mercury: D_1 = 4881 [km][6]; Venus: D_2 = 12103.6 [km][7]; Earth: D_3 = 12756.27 [km][8];
Mars: D_4 = 6792.4 [km][9]; Jupiter: D_5 = 142984 [km][10]; Saturn: D_6 = 120536 [km][11];
Uranus: D_7 = 51118 [km][12]; Neptune: D_8 = 49528 [km][13]

Input Values at Modifications of the Koide-Formula consist of the diameters of the eight planets:

$$F_{D1\text{-}8} = T_{N\_D1\text{-}8} / T_{D\_D1\text{-}8} = 0.166957 = 0.667829 : 4 \quad [\approx 2:3 * (1:4)] \qquad \text{(Rel1)}$$

with

$$T_{N\_D1\text{-}8} = (D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8) = 400699.3$$

$$T_{D\_D1\text{-}8} = (\sqrt{D_1} + \sqrt{D_2} + \sqrt{D_3} + \sqrt{D_4} + \sqrt{D_5} + \sqrt{D_6} + \sqrt{D_7} + \sqrt{D_8})^2 = \mathbf{2400011.9} \quad [\approx \mathbf{2.4 * 10^6}]$$

[At these formulas the downsized letter N stands for Numerator and the downsized letter D stands for Denominator. Term T_N is the Numerator and Term T_D is the Denominator of a formula F.]

Input Values consist of the diameters of the four mid planets (Number 3 to 6):

$F_{D3\text{-}6} = T_{N\_D3\text{-}6} / T_{D\_D3\text{-}6} = 0.333948 = 0.667897 : 2$ [≈ 2:3 * (1:2)] (Rel2)

with

$T_{N\_D3\text{-}6} = (D_3 + D_4 + D_5 + D_6) = 283068.7$

$T_{D\_D3\text{-}6} = (\sqrt{D_3} + \sqrt{D_4} + \sqrt{D_5} + \sqrt{D_6})^2 = 847642.2$

The relation of the just calculated results, which formulas are similar constructed to the Koide-Formula, is very close to the value 2:

$F_{D3\text{-}6} / F_{D1\text{-}8} = 0.333948 / 0.166957 =$ **2.000203 [≈ 2]** **(Rel3)**

Input Values are the diameters of the two first located and the two outer most planets (Numbers 1 and 2 as well as 7 and 8):

$F_{D12\text{-}78} = T_{N\_D12\text{-}78} / T_{D\_D12\text{-}78} = 0.297769$ [≈ 0.3] (Rel4)

with

$T_{N\_D12\text{-}78} = (D_1 + D_2 + D_7 + D_8) = 117630.6$

$T_{D\_D12\text{-}78} = (\sqrt{D_1} + \sqrt{D_2} + \sqrt{D_7} + \sqrt{D_8})^2 = 395040.3$

Why are the calculated values so close to full numbers or to full numbers at the fractions (for example 2/3, 1/3, 1/6 or 3/10)? Isn't it worth for further investigations to determine, if it is just random or if it is subject to a (superior) system?

Short Remark referring the Rotation Times (without unit day) of the Planets:

The sum of Rotation Times (in days) of the four most near Planets to the Sun is:

$RT_1 + RT_2 + RT_3 + RT_4 = 87.969 + 224.701 + 365.256 + 686.980 = 999.65 + 365.256$

[≈ 1000 + 365.256]

The Rotation Times day related to unit d (day) for the eight Planets are:

Mercury: $RT_1 = 87.969$ [d][6]; Venus: $RT_2 = 224.701$ [d][7];
Earth: $RT_3 = 365.256$ [d][8]; Mars: $RT_4 = 686.980$ [d][9];
Jupiter: $RT_5 = 4332.941$ [11 a, 315 d, 3 h][10]; Saturn: $RT_6 = 10758.424$ [29 a, 166 d][11];
Uranus: $RT_7 = 30685.504$ [84 a, 4d][12]; Neptune: $RT_8 = 60190.536$ [164.79 a][13]

Data of the equatorial Diameters D of Earth, Moon and Sun, of the Rotation Times RT and of the Distances (big Half Axles) Dist of Earth to Sun and to Moon, respectively:

$Ø_{Earth} = 12756.27$ [km][8]; $Ø_{Moon} = 3476$ [km][14]; $Ø_{Sun} = 1392684$ [km][15];
$RT_{Earth} = 365.256$ [days][8]; $RT_{Moon} = 27.3217$ [days][15];
$Dist_{E\text{-}S} = 149.6 * 10^6$ [km][8]; $Dist_{E\text{-}M} = 384400$ [km][15]

At the denominator term $T_{D\_D1\text{-}8}$ of equation (Rel1) the result is close to $2.4 * 10^6$ (= $2*1.2 * 10^6$). For the reference distance (big Half Axle in km) from Earth to Sun[5] one can give the following simple formula using the figures 4 and 12:

$(4*1.2)^{12} = 149.587 * 10^6$ [km] (A1)

Referring the Reference Distance Earth to Moon [without unit km] the following is remarkable:

$Dist_{E\text{-}M} = 384400 = 620^2$ (A2)

One is able to connect the figure 620 with the relation 2/3, which is won by the equations (m0) and (KL), and with help of 10-powers and the figure 70:

$(1000 * 2/3 - 620) / 70 = 2 / 3;$

Figure 70 can be closely described by the sun diameter (without unit km) and the figure 3.33 (see the use of this figure at the next pages) and by use of the logarithm operator:

$\ln(Ø_{Sun}) / \ln(70) = 3.329824$ [≈ 3.33]

By use of the circle figure π the reference distance (Big Half Axle) between Earth and Moon can be relatively simply described by the figures 2 and 3 at the exponents:

$$(20 * \pi^3)^2 - 5 * \pi^3 = 384400.65 \quad \text{(A3)}$$

Note: it is questionable, if this formula has any importance, but it delivers an astonishingly good result. The term "$20 * \pi^3$" (= 620.1255) amounts nearly to 620.

At the following formulas similar to the Koide-Formula are presented by use of diameter data (without unit km) of Moon and Sun and by use of the fraction 3/2:

$$F_{M\text{-}S} = T_{N\_M\text{-}S} / T_{D\_M\text{-}S} = 0.599235 \quad [\approx 0.6] \quad \text{(Rel5)}$$

with

$$T_{N\_M\text{-}S} = (Ø_{Moon}^{3/2} + Ø_{Sun}) = 1.597621 * 10^6 \quad [\approx 1.6 * 10^6]$$

$$T_{D\_M\text{-}S} = [(Ø_{Moon}^{3/2})^{0,5} + Ø_{Sun}^{0,5}]^2 = 2.666100 * 10^6 \quad [\approx 1.6 * 0.6^{-1} * 10^6]$$

The following formula possesses the input data of Earth instead of the one of Moon:

$$F_{E\text{-}S} = T_{N\_E\text{-}S} / T_{Z\_E\text{-}S} = 0.500036 \quad [\approx 0,5] \quad \text{(Rel6)}$$

mit

$$T_{N\_E\text{-}S} = (Ø_{Earth}^{3/2} + Ø_{Sun}) = 2.833423 * 10^6$$

$$T_{D\_E\text{-}S} = [(Ø_{Earth}^{3/2})^{0,5} + Ø_{Sun}^{0.5}]^2 = 5.666440 * 10^6$$

The values of formulas (Rel5) and (Rel6) are connected with each other:

$$F_{M\text{-}S} * F_{E\text{-}S} = 0.599235 * 0,500036 = 0,29964 \quad [\approx 0.3]$$

$$T_{D\_E\text{-}S} - T_{D\_M\text{-}S} = (5.66644 - 2.66610) * 10^6 = 3.00034 * 10^6 \quad [\approx 3 * 10^6]$$

Approximation for the Circle Figure π (Rotation Times are used without unit day):

$$Pi_{Appr1} = 2 * \sqrt{[\sqrt{Ø_{Earth}} / RT_{Earth} + \sqrt{Ø_{Moon}} / RT_{Moon}]} = 3.141415... \quad [\approx \pi = 3.14159\ldots]$$

Approximation for the root of Φ (Golden Ratio):

$$W\Phi_{Appr} = \sqrt{(Ø_{Earth} * RT_{Moon} + Ø_{Moon} * RT_{Earth})} / 1000 = 1.272066... \quad [\approx \sqrt{\Phi} = 1.27202...]$$

The last two formulas work with the same input data of earth and moon as used at Equations (Rel5) and (Rel6) and possess a form, which is harmonic and actually not too difficult to find (By that, probably someone might have found them before!).

With the figure 4.8 (see Equation A1 with the term 4.8^{12}) in combination with the data of our celestial bodies one is able to derive approximations, which closely correspond to full numbers multiplied by 10-powers. It is an open question, whether this has any importance, but nevertheless it is somehow astonishing, how simple they are constructed and how they lead two times very close to figure 1 in combination with 10-powers.

$Ø_{Sun} * Dist_{E\text{-}S} * 4.8$: $1392684 * 149.6*10^6 * 4.8 = 1.000059 * 10^{15}$

$Ø_{Moon} * Dist_{E\text{-}S} + 4.8*10^{11}$: $3476 * 149.6*10^6 + 4.8 * 10^{11} = 1.0000096 * 10^{12}$

$Dist_{E\text{-}M} * (10 - 4.8)$: $384400 * 5.2 = 1.99888 * 10^6$ [$\approx 2 * 10^6$]

Approximation with use of earth and sun diameters:

$(Ø_{Earth} * Ø_{Sun})^{0.5} = 1.33287 * 10^5$ [$\approx (1 + 0.333) * 10^5$; see repeated application of figure 666 (=2*333) at the next page]

A formula using the data of earth, moon and sun and the figure 20, which can be found also at Equation (A3) at page 3 and at Equation (Appr-7) at last page, is:

$(Ø_{Earth} * Ø_{Sun})^{0,5} - 20 * (Ø_{Earth} * Ø_{Moon})^{0,5} = 109.340$ [$\approx Ø_{Sun} / Ø_{Earth} = 109.176$]

Formulas based on the Koide-Formula with Masses of Electron, Proton, Neutron, Myon, Tauon:

Equation with Mass of Electron[2], Proton[16], Tauon[4]:

$$Rel_{m1} = (m_p + m_\tau) / \sqrt{(m_e * m_p)} = 123.9986 \quad [\approx 0.2 * 620]; \qquad (m1)$$

Figure 620 was firstly named at Equation (A2), where the distance Earth to Moon is expressed by the term ”620*620 (km)”.

Equation with Mass of Electron and Tauon:

$$Rel_{m2} = (\sqrt{m_e} + \sqrt{m_\tau}) / (60 * \sqrt{m_e}) = 0.999467 \quad [\approx 1]; \qquad (m2)$$

Equation with Mass of Electron, Proton and Myon[3]:

$$Rel_{m3} = (m_p^2 + m_\mu^2) / (10^6 * m_e^2) = 2^{1/2} + 1.9999962 \quad [\approx 2^{1/2} + 2]; \qquad (m3)$$

Very important Equation (m4) with Mass of Electron, Proton and Neutron[17]:

$$Rel_{m4} = (m_e + m_p + m_n) / \sqrt{(m_e^2 + m_p^2 + m_n^2)} = \mathbf{1.414598} \quad \mathbf{[\approx 0.1 * 14.146]} \qquad (m4)$$

$$Rel_{m5} = (m_e + m_p + m_n) / m_e = 3675.836 \quad [\approx (1.287 + 14.146)^3 = 3675.793] \qquad (m5)$$

Remarkable: $1.286*11 = 14.146$ and $(1286 + 1) * 0.777 = 1287 * 0.777 = 999.999$

Figure 14.146 had been derived earlier by the author (see description below) and has served as Input value for many approximations of Physical Constants.
By Modifications of the Koide-Formula it was possible to show that the figure 14.146 and the figure 1.4146, respectively not only serve as input values, but that the figure 1.4146 can be derived by a mathematical formula; that means: it is also an Output value. See also the Equations (S1) and (S2) at page 5 and page 6, at which the figure 1.286 and 12.86, respectively are derived.

Fine Structure Constant:

Performing a connection between the result of Equation (m4) and a Physical Constant, an approximation for the Fine Structure Constant α is presented, which is constructed of terms consisting of circle figure-powers. The author had found this very good approximation for the Inverse α^{-1} of the Fine Structure Constant more than 1 ½ years ago, by which the figure 14.146 is derived, which on the other hand serves as input value for other exact approximations for the Fine Structure Constant α and for other important Physical Constants.
This exact approximation using π-terms for the Inverse of the Fine Structure Constant is:

$$\alpha_{\#1}^{-1} = 1 * \pi^4 + 4 * \pi^2 + 1 * \pi^{-2} + 5 * \pi^{-4} - 4 * \pi^{-6} = 137.035999087\,382 \qquad (\alpha 1)$$

Deviation: $\alpha_{\#1}^{-1} - \alpha^{-1} = 3.7 * 10^{-9}$ [this is far within the tolerance $\pm 21*10^{-9}$]

with the setpoint value $\alpha^{-1} = 137.035999084(21)$[18]

The tolerance of the Inverse α^{-1} of the Fine Structure Constant ist given to $\pm 21*10^{-9}$, which delivers a tolerance width of $2*21*10^{-9} = 42*10^{-9}$. The smallest term π^{-6} ($=1.0402*10^{-3}$) of equation (α1) is bigger by the factor "$2.477*10^4$" than the tolerance width ($42*10^{-9}$) of the inverse α^{-1}.

The smallest π-term π^{-6} ($=1.0402*10^{-3}$) is bigger by the factor "$2.82*10^5$" than the difference "$3.7*10^{-9}$" between the calculated value $\alpha_{\#1}^{-1}$ and the value α^{-1} of the Fine Structure Constant given in literature[18] (see the value presented above).
[It is clear, that it leads to an exact value for the inverse α^{-1}, if one use the sum operation of terms π^{-M} with further increasing full even exponents M (up to M=18, that means exponent "-18" ; one consider the relation: $\pi^{-M} \approx 10^{-0.5*M}$) and at which only full numbers serve as multiplicators from minus 5 to plus 5 in front of the π-terms. But this application (up to M=18) doesn't happen here.]

Therefore the probability is small, that Equation (α1) with the applied π-terms (only even ±-numbers for the exponents) and the full numbers (from minus 5 to plus 5) for the multiplicators in front of the π-terms delivers such an exact result.

Figure 14.146 derived by the Multiplicators of Equation (α1):

There are the five multiplicators 1 - 4 - 1 - 5 - -4 in front of the five π-terms of Equation (α1). The five just presented figures are multiplied by 10-powers (with exponents from 1 to minus 3 in steps of 1, whereat the three terms with minus exponents and with basis 10 belong to the three multiplicators 1, 5 and -4, which belong to the π-terms with negative exponents) and then the terms are added together, which leads to the figure 14.146:

$$1*10^1 + 4*10^0 + 1*10^{-1} + 5*10^{-2} - 4*10^{-3} = \mathbf{14.146} \qquad \textbf{(S1)}$$

Is it too farfetched? Please look at the result of Equation (m4) at page 4!
Other formulas with the figures 14.146 and 1.286 (= 14.146/11) and connections to three-figures numbers with equal figures (for example 111) are:

$1286 * 777 = 999222$ [999 - 222 = 777]; $1286 - 666 = 620$; $44444 - 3*14146 - 3*666 = 2^3$

$$44.444 / (1.2 * \Phi^2) = 14.146748...; \qquad (\Phi 1)$$

$$\ln(\text{Ø}_{Sun}) = \ln(1392684) = 14.146743...$$

The last two results agree each other by five descendants behind the decimal point.

A formula with close approach to the figure 14.146 using the exponents 2 and 3:

$$44.444 / (1.2*\Phi^2) - 1/11^3 = 14.14599652 = 11*1.28599968 \quad [\approx 11*1.286] \qquad (\Phi 2)$$

Φ [= (5+5*√5) / 10 = 1.618034] is the coefficient of the Golden Ratio.

Other approximations for the inverse α^{-1} of the Fine Structure Constant by use of the figure 14.146 and the figures 144 and 666, which attributes will be described later, are Equations (α2) and (α3):

$$\alpha_{\#2}^{-1} = (1 - 1/144/666)^{1/8.88} * 14.146 * 1.44 * 6.66 / 0.99 = 137.035999088\,345 \qquad (\alpha 2)$$

Basis term at (α2): $137.03616 = 14.146 * 1.44 * 6.66 / 0.99 = 1.286 * 12 * 8.88$

Please look at the figure 8.88, which is used at the exponent and as multiplicator at the basis term!

$$\alpha_{\#3}^{-1} = 14.146 * 6.66 * [1 + 3/6.6 - 1/(2*0.66*666^2)] = 137.035999089\,089 \qquad (\alpha 3)$$

$$14.146 * 6.66 * (1 + 3/6.6) = 137.03616 \text{ (is the basis term)}$$

Within the angular brackets of Equation (α3) the figure 3 is located in the numerator of the mid term and the figure 2 is located in the denominator of the last term!

An approximation with the Golden Ratio is:

$$\alpha_{\#4}^{-1} = 0.999*144 - 6.66 - 0.4^2 - (1 / 20 / \Phi)^4 = 137.035999088\,137 \qquad (\alpha4)$$

basis term: $0.999*144 - 6.66 - 0.4^2 = 137.036$

With help of the figures Φ, π, 1.44 and 6.66 astonishingly many approximations with often extremely exact result values for the Physical Constants can be performed. The approximation examples for the Fine Structure Constant presented here are an extract from a report in german of the author, in which many approximations of Physical Constants are presented. The author will try to translate this report into english in the next future.

Please compare the result values of Equations (α1) to (α4), which all are very close to each other. Isn't that astonishing?

In this context the following simply constructed Equation (α5) fits:

$$\alpha_{\#5}^{-1} = 137.036 * (1 - 6.66 * 10^{-9}) = 137.035999087\,340 \qquad (\alpha5)$$

The deviaton of Equation (α1) to the one (α5) is about $4.2*10^{-11}$; and here not even the relative deviation is shown. This value would be smaller by a multiplication factor (1/137.036)!

A rough, but simple approximation of the inverse α^{-1} Fine Structure Constant with the figure 999 is:

$$\alpha_{\#6}^{-1} = 10^2 * 999 / 9^3 = 137.037037037... \qquad (\alpha6)$$

If the following applies to the readers: the author knows, that the plenty of the mathematical formulas can overtax one and by that one classifies the formulas as something normal. But that - the normality - isn't the case. If one comprises this, one can see the complexity, the harmony and elegance of the whole and one can guess of a system, from which only a fraction is recognizable for us.

The figure 666 takes some time to get used to, but with it and in combination with the figure 144 and 10-powers the Physical Constants, particularly the Fine Tuning-terms of many formulas are approximated with an incredible exactness. It is unknown to the author, why nearly always this exactness is reached in the described way. Investigations of this theme can surely contribute for clarification. Maybe the exactness - as given by the Equations (α1) to (α5) - is also possible with other full figures, but what is scarcely reachable in the opinion of the author. Just have again a look at the result of Equations (m4) and (S1) and the very simple formula (S2) and formula (S3) below, which leads to the figure 1.286 (=14.146/11), respectively by use of the figures 1.44 and 6.66.

[Please see the simple formulas with the data of earth and moon at the last page, by whom the author became aware of the figure 666. With further investigation it was observable, that the figure 666 harmonize well with the figure 144 and in combination with 10-powers.]

The next approximation (S2) for figure 1.286 underlies a different topic in comparison to Equation (m4). As input values of Equation (S2) are used 3 figures without - at the first glance - physical backround and the circle figure π. These four figures are: *Φ, π,1.44 and 6.66.*

<u>The sum of the 4 figures *Φ, π, 1.44 and 6.66* leads close to the figure 12.86:</u>

$$Sum_{4Z} = \Phi + \pi + 1.44 + 6.66 = \mathbf{12.8596} \qquad [\approx 12.86 = 10 * 14.146 / 11] \qquad \mathbf{(S2)}$$

A question referring the closeness of this sum value Sum_{4Z} to the figure 12.86 could be:
Which qualities do the four “versatiles“ figures *Φ, π, 1.44 and 6.66* possess in the figure range from 1 to 10 and apart from this?

In the following formula Rel_{11} the mass relation Rel_{m4} (Equation (m4) at page 4 above) is set in relation to the value Sum_{4Z}:

$$Rel_{11} = 100 * Rel_{m4} / (\Phi + \pi + 1.44 + 6.66) = 100 * 1.414598 / 12.8596 = 11.0003035$$

For illustration, that the figure 11 harmonizes with the four *versatile* figures, still another approximation for the inverse α^{-1} of the Fine Structure Constant is given. The formula (α7) is constructed by use of the figure 2, 11, 144, 666 and 10-powers:

$$\alpha_{\#7}^{-1} = [2*66.6 + 2*1.44 + (66.6)^{-1/2} + (1.44)^{-1/2}] / [1 - 100/(144*666)]^{1/(100*11)} =$$

$$= 137{,}03586910 / [1 - 100/(144*666)]^{1/(100*11)} = 137.035999070 \qquad (\alpha 7)$$

The result value of Equation (α7) lies within the tolerance range. Please see the Inverse of the multiplicator figure 2 at the exponents and the harmony at the basic term (= 137,03586910): two times the same multiplicator, the same basis and the same exponent.
Using values “1/1100.4“ and “1/1099.6“, respectively for the exponents of the Finetuning-term, the results of this approximation (α7) lie outside the tolerance!

Figure 11 is also used in the following approximation Rel_{m6} for the mass relation of Neutron to Proton by inclusion of the light velocity c[19]:

$$(m_n / m_p)^{0.5} * c_{wu} * 10^{-8} = 2.9999901 \qquad [\approx 3] \quad \text{with } c_{wu}\text{: light velocity c without units}$$

$$Rel_{m6} = [(3 * 10^8 / c_{wu}) * (1 - [1/144/666]^{11/10})]^2 = 1.0013784190 \qquad (m6)$$

Setpoint value is:

$$m_n / m_p = 1.67492749804 * 10^{-27}\ kg / 1.67262192369 * 10^{-27}\ kg = 1.0013784193$$

$$\text{Deviation:} \quad Rel_{m6} - m_n / m_p = \mathbf{-3.3 * 10^{-10}}$$

By use of the four *Versatile* Figures and of the Euler Figure e (=2.7182818) - to these five figures the wellknown term 2/3 is set as exponents“ - the followig formula astonishingly results to a value close to 9 plus the figure 1.286:

$$\Phi^{2/3} + e^{2/3} + \pi^{2/3} + 1.44^{2/3} + 6.66^{2/3} = \mathbf{1.286 + 9.000028} \qquad (S3)$$

The figure 9 harmonizes particularly with the figure 11, which is for example expressed at many formulas using the figure 99 (=9*11) in connection of 10-powers.
The fascinating serie formula for the circle figure π of the indian mathematician Srinivasa Ramanujan[20] contains the figures 9801 (= $9^2 * 11^2$) and 396 (= 4 * 9 * 11).

In this context an approximation for the mass relation Tauon to Electron is given:

$$Rel_{m7} = 0.999^{-2*0.99} * (2*\pi)^{0.999*4.44} = 3477.2429 \qquad (m7)$$

Exponent term 0.999*4.44 is equal the term 6.66*0.666.

An approximation for the mass relation Myon to Electron is:

$$Rel_{m8} = 0.999^{-2/1.14} * (2*\pi)^{2.9} = 0.999^{-2*0.99/(1+0.1*1.286)} * (2*\pi) * (2*\pi)^{1.9} = 206.7682821 \qquad (m8)$$

The results of Equation (m7) as well as Equation (m8) lie within the respective tolerance. The setpoint values are given at page 15.

Remarkable referring the exponents of basis 0.999 at Equations (m7) and (m8) are the connections, which lead to the figure 0.99 as well to figure 1.9 by use of the figure 1.286:

0.99 = (1 + 0.1*1.286) / 1.14 and 19 * 2 * 3 = 114;
100 * (10 + 1.286) / (2*3*99) = 1.9; $\Phi^{4/3}$ = 1.89955 [≈ 1,9; see use of term 4/3 at next pages]

As proof of these connections an approximation for the inverse of the Fine Structure Constant is given, at which also the just named figures 0.99, 1.286 and 1.9 come to use:

$$\alpha_{\#8}^{-1} = 0.99 * (\pi^4 + \pi^3 + \pi^2) / (1 - 1.286/10^4)^{4*1.9} = 137.0359990732 \qquad (\alpha 8)$$

If the term "4*1.9" at the exponent is tiny little changed to the term "4*(1.9 **±1*10^-6**)", the results of Equation (α8) lie outside the tolerance! Isn't that impressive?
At Equation (α8) the exponent term "4*1.9" is used, whereas at Equation (m8) the exponent term "0.6*1.9=1.14" is applied. Again the relation 2/3 is observable at the term "4*1.9 / (6*1.9/10)".

The following approximation ($\alpha_{\#9}^{-1}$) for the inverse of the Fine Structure Constant is a broadening of the Equation (α_{HR})[(21)] of R. Heyrovska, which is dependent on the Golden Ratio Φ:

$$\alpha_{HR}^{-1} = 360 / \Phi^2 - 2 / \Phi^3 = 137.035628095 \qquad (\alpha HR)$$

Equation (αHR) is widened by the term "1/(1666*Φ)", which results to Equation (α9):

$$\alpha_{\#9}^{-1} = 360 / \Phi^2 - 2 / \Phi^3 + 1 / (1666 * \Phi) = 137.035999064 \qquad (\alpha 9)$$

Figure 1666 can be described by the figures 1.286 and 0.19 by the following way:

$$1666 = 1000 * (1.286 + 2*0.19); \qquad \text{further } 1666 = 1000 + 666$$

Each approximation of the Fine Structure Constant given in this report is within the tolerance, most of them within a small result range. Only the result of Equation (α9) is located near to the tolerance limit, but still within the range.

Figure 1666 is also clearly visible in the following formula by use of elementary particle masses:

$$[(m_e + m_p + m_n) / m_e] / [(m_e^{1/3} + m_\mu^{1/3} + m_\tau^{1/3}) / m_e^{1/3}] = 166.605 = 0.1 * 1666.05 \qquad (m9)$$

An Adaptation of Equation (m9) is listed at Equation (m10), at which the figure 1.286 is used at the exponent instead of figure "1/3":

$$[(m_e + m_p + m_n) / m_e] / [(m_e^{1.286} + m_\mu^{1.286} + m_\tau^{1.286}) / m_e^{1.286}] = 0.099988 \quad [\approx 0.1] \qquad (m10)$$

In this section we have again a look at the Koide-Formula with the masses of the three Leptons Electron, Myon and Tauon.
The goal is to get information by using the result value 0.5, which is the exponent value of the terms in the denominator of the Koide-Formula (m0) presented at page 1.

$$Rel_{e\mu\tau\#} = T_{N\_e\mu\tau\#} / T_{D\_e\mu\tau\#} = (m_e + m_\mu + m_\tau) / (m_e^{Exp_\#} + m_\mu^{Exp_\#} + m_\tau^{Exp_\#})^{1/Exp_\#} = 0.5 \qquad (m11)$$

The exponent $Exp_\#$, which is necessary getting the result value 0.5, takes the value:

$$Exp_\# = 0.4050254 \quad [\approx (3/4)^{(1.2*\Phi*\Phi)} = 0.4050302]$$

Term $1.2*\Phi^2$ is already used at Equation (Φ2) at page 5!

The values for the numerator value $T_{N\_e\mu\tau\#}$ and denominator value $T_{D\_e\mu\tau\#}$ are:

$$T_{N\_e\mu\tau\#} = (m_e + m_\mu + m_\tau) / m_e = 3684.998 \quad [\approx 3685]$$

$$T_{D1\_e\mu\tau\#} = (m_e^{Exp_\#} + m_\mu^{Exp_\#} + m_\tau^{Exp_\#}) / m_e^{Exp_\#} = 36.848727 \quad [\approx 0.01 * T_{N\_e\mu\tau\#}]$$

$$T_{D\_e\mu\tau\#} = (m_e^{Exp_\#} + m_\mu^{Exp_\#} + m_\tau^{Exp_\#})^{1/Exp_\#} / m_e = 7369.997 \quad [\approx 200.007 * T_{D1\_e\mu\tau\#}]$$

One considers: with the - at the first look nondescript - exponent value $Exp_\#$ (= 0.4050254) one gets the result value 0.5 at Equation (m11) and furthermore a result value of the term $T_{D\_e\mu\tau\#}$, which is - very closely - 200 times bigger than the basis $T_{D1\_e\mu\tau\#}$.

Exponent value $Exp_{\#\Phi}$ used at upper formulas instead of exponent $Exp_{\#}$ (=0.4050254):

$Exp_{\#\Phi} = (3/4)^{(1.2*\Phi*\Phi)} = (4/3)^{(-1.2*\Phi*\Phi)} = 0.4050302$

The values for the numerator value $T_{N\_e\mu\tau\#\Phi}$ and denominator value $T_{D\_e\mu\tau\#\Phi}$ are:

$Rel_{e\mu\tau\#\Phi} = T_{N\_e\mu\tau\#\Phi} / T_{D\_e\mu\tau\#\Phi} =$

$= (m_e + m_\mu + m_\tau) / (m_e^{Exp_{\#\Phi}} + m_\mu^{Exp_{\#\Phi}} + m_\tau^{Exp_{\#\Phi}})^{1/Exp_{\#\Phi}} = 0.500010$ (m12)

$T_{N\_e\mu\tau\#\Phi} = (m_e + m_\mu + m_\tau) / m_e = 3684.998$ [$\approx$ 3685]

$T_{D1\_e\mu\tau\#\Phi} = (m_e^{Exp_{\#\Phi}} + m_\mu^{Exp_{\#\Phi}} + m_\tau^{Exp_{\#\Phi}}) / m_e^{Exp_{\#\Phi}} = 36.850004$ [$\approx 0.01 * T_{N\_e\mu\tau\#\Phi}$]

$T_{D\_e\mu\tau\#\Phi} = (m_e^{Exp_{\#\Phi}} + m_\mu^{Exp_{\#\Phi}} + m_\tau^{Exp_{\#\Phi}})^{1/Exp_{\#\Phi}} / m_e = 7369.855$ [$\approx 199.996 * T_{D1\_e\mu\tau\#\Phi}$]

Basis (4/3) of exponent $Exp_{\#\Phi}$ is close to result value of Equation (Rel8) given below.
Figures game: $Rel_{e\mu\tau\#\Phi} / Exp_{\#\Phi} = 0.500010 * (4/3)^{(1.2*\Phi*\Phi)} = 1.2344997$ [$\approx$ 1.2345 - figures serie]

Exponent value Exp_{m13} (=0.75*0.75) by use of upper Input Values:

$Exp_{m13} = (3/4)^2 = (0.75)^2 = 0.5625$

$Rel_{m13} = (m_e + m_\mu + m_\tau) / (m_e^{Exp_{m13}} + m_\mu^{Exp_{m13}} + m_\tau^{Exp_{m13}})^{1/Exp_{m13}} = 0.7500633$ (m13)

The result value of Equation (m13) is nearly the root of its exponent Exp_{m13}. See the results of Equations (Rel8) and (Rel8#) given further below, which are close to the inverse of value 3/4.

Adapted Koide-Formulas with Figures π, 4 and 6 (output values of the equations for circle, square, sphere and cube): circle diameter D is equal Length L of square/cube

Circumference of circle: $D * \pi$ Surface of circle: $D^2 * \pi / 4$
Circumference of square: $4 * L$ Surface of square: L^2

Surface of sphere: $D^2 * \pi$ Volume of sphere: $D^3 * \pi / 6$
Surface of cube: $6 * L^2$ Volume of cube: L^3

The exponent Exp_a for the following Equation (Rel7) is chosen in the way, that the result $Rel_{\pi46a}$ is very close to the value "2/3". The input value of this exponent is:

$Exp_a = 0.72559092$ [$\approx \Phi^{-2/3} = 0.72556263$; term $\Phi^{2/3}$ is used at Equation S3 at page 7!]

By use of this exponent value the following results are delivered:

$T_{N\_\pi46a} = \pi + 4 + 6 = 13.14159$

$T_{D1\_\pi46a} = (\pi^{Expa} + 4^{Expa} + 6^{Expa}) = 8.698634$

$T_{D\_\pi46a} = (\pi^{Expa} + 4^{Expa} + 6^{Expa})^{1/Expa} = 19.71239$

$Rel_{\pi46a} = T_{N\_\pi46a} / T_{D\_\pi46a} = 13.14159 / 19.71239 = 0.666666661$ [≈ 2/3] (Rel7)

We compare the just won results with the results of formulas, at which the exponent Exp_b is the inverse of exponent Exp_a:

$Exp_b = 1 / Exp_a = 1 / 0.72559092 = 1.37818704$ [$\approx \Phi^{2/3} = 1.3782408$]

By use of this exponent value the following results are delivered:

$T_{N\_\pi46b} = \pi + 4 + 6 = 13.14159$

$T_{D1\_\pi46b} = (\pi^{Expb} + 4^{Expb} + 6^{Expb}) = 23.41564$

$T_{D\_\pi46b} = (\pi^{Expb} + 4^{Expb} + 6^{Expb})^{1/Expb} = 9.856011$

$Rel_{\pi46b} = T_{N\_\pi46b} / T_{D\_\pi46b} = 13.14159 / 9.856011 = 1.333358$ [≈ 4/3] (Rel8)

The relation of the just presented values is very close to the figure 2:

$Rel_{\pi46b} / Rel_{\pi46a} = 1.333358 / 0.666666661 = 2.000037$ (Rel9)

The case, where a formula F_a adapted to the Koide-Formula with an exponent Exp_a possesses the result value 2/3 and the connected formula F_b with an exponent Exp_b (=1/Exp_b) possesses the double result value 4/3 compared to formula F_a, is the only case, at which the following is valid:

$F_a + F_b = 2 = F_b / F_a$

The sum of the results of the Equations (Rel7) and (Rel8) is:

$Rel_{\pi 46b} + Rel_{\pi 46a} = 1.333358 + 0.666666661 = 2.0000248$ (Rel10)

A question of the author is: why is the double value of Equation (Rel7) very close to the value of Equation (Rel8), at which the inverse of the exponent of the one of Equation (Rel7) is used? One consider in this context: $Exp_a \approx \Phi^{-2/3}$. See results using this term with factor 2/3 below.
Are expert mathematicians able to explain that?
Can these experts make the statement, that this is random?

With exponents $Exp_{a\#}$ ($= \Phi^{-2/3} = 0.72556263$) and $Exp_{b\#}$ ($= \Phi^{2/3}$) one gets the following results:

$Rel_{\pi 46a\#} = T_{N\_\pi 46a\#} / T_{D\_\pi 46a\#} = 13.14159 / 19.71353 = 0.666628$ [$\approx$ 2/3] (Rel7#)

$Rel_{\pi 46b\#} = T_{N\_\pi 46b\#} / T_{D\_\pi 46b\#} = 13.14159 / 9.855724 = 1.333397$ [$\approx$ 4/3] (Rel8#)

The relation of the just presented values is also close to the figure 2:

$Rel_{\pi 46b\#} / Rel_{\pi 46a\#} = 1.333397 / 0.666628 = 2.00021$ (Rel9#)

The sum of the results of the Equations (Rel7) and (Rel8) is:

$Rel_{\pi 46b\#} + Rel_{\pi 46a\#} = 1.333397 + 0.666628 = 2.000025$ (Rel10#)

<u>Radius of Electron and Proton:</u>

The relation of the radius of Electron(13) and Proton is:

$Rel_{re/rp} = r_e / r_p = 2.8179403262 * 10^{-15}\,m \;/\; 0.84087 * 10^{-15}\,m^{(22)} = 3.3512200$ (r1)

Hereby some noticeable relations are given:

$(r_e/r_p)^{1/0.666} = 6.146006 = 14.146 - 7.999994$ [$8 = 2^3$; Figures 2 and 3 connected with 8]

$(r_e/r_p)^{0.333} = 1.495866$ [$\approx (4*1.2)^{12} / 10^8 = 1.495873$; $Ø_{Sun} \approx (4*1.2)^{12}$]

$[1 + (r_e/r_p)^{0.5}]^{0.666} = 1.999653$ [$\approx$ 2; see exponent 0.666 before at the first relation]

$[1 + (r_e/r_p)^3]^{0.111} = 3.380642^{0.111} = 1.50023$ [$\approx 3 / 2 = 1.5$; see figure 0.111 below]

$Rel_{r1} = (1 + r_e/r_p) / [1 + (r_e/r_p)^3] = 0.112620 = 1 / 8.87955$ [$\approx 1 / 8.88 = 0.112613$]

$Rel_{r2} = (1 + r_e/r_p) / [1 + (r_e/r_p)^3]^{1/3} = 1.287099 = 1 / 0.776941$ [$\approx 1 / 0.777 = 1.287001$]

$Rel_{r3} = Rel_{r1}^{-1} / [1/ Rel_{r2} + 0.111] = 10.000050$

$[1 + (r_e/r_p)^3] + 1.286 = 38.6365 + 1.286 = 39.9225$ [$\approx 40 - 0.0777 = 39.9223$]

$[1 + (r_e/r_p)^3]^{1/3} + 1.286 = 3.380642 + 1.286 = 4.666642$ [$\approx 4 + 2/3$]

At the first four formulas of this section the figures 0.111, 0.333, 0.666 are input values at the exponents, whereat at the next four formulas the figures 0.777, 0.111, 8.88 and 0.0777 are Output values or part of them.
By multiplication of 10-powers the just mentioned figures are changed to full numbers, which all are divisible by the figure 111. The changed result values can be connected by simple plus-/minus operations using the figures 111 and 666: 777 = 666 + 111; 888 = 777 + 111

Equations (S1) to (S3) and the Equations (Rel1) to (Rel6) are approximations, therefore they are not exact. But the result values - particularly of the adapted Koide-Formulas - indicate to a system, which connects physical values of the very big with the ones of the very small and which often underly the factor 2/3.
A special observable aspect is, that formulas using figures of different areas lead nearly to the same results (once with mathematical input values - for example the Euler figure e, the Golden Ratio and the circle figure π -, once with physical input values, as for example mass values).

It is astonishing, how many items are connected with others:
The result 1.414598 (≈1.4146) of mass relation (m4), the result of Equation (α1), the derivation of figure 14.146 from Equation (α1) and the very exact approximations for Physical Constants by use of this figure and the proof of the versatility of the four figures "Φ, π, 1.44, 6.66".
These facts indicate to a system of huge probability, that connects the very Big with the very Small.

Therefore the following formulas and relations, respectively with the fraction 2/3 and figures 14.146 and 1.286, respectively are listed again:

$(m_e + m_\mu + m_\tau) / (\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 = 0.66666056$ [≈ 2 /3] Koide-Formula (m0)

$(T_1 / T_2)^2 = (a_1 / a_2)^3 * (M + m_2) / (M + m_1)$ Kepler-Law with exponents 2 and 3 (KL)

$44.444 / (1.2*\Phi^2) = 14.146748...;$

$\ln(\varnothing_{Sun}) = \ln(1392684) = 14.146743...;$

$44.444 / (1.2*\Phi^2) - 1/11^3 = 14.1459965;$ [see listed figure 11 below]

$44444 - 3*14146 - 3*666 = 2^3$ (Figures 2 and 3 are used at upper Equation!);

$\alpha_{\#1}^{-1} = 1*\pi^4 + 4*\pi^2 + 1*\pi^{-2} + 5*\pi^{-4} - 4*\pi^{-6} = 137.035999087\,382$ (α1)

$1*10^1 + 4*10^0 + 1*10^{-1} + 5*10^{-2} - 4*10^{-3} = \mathbf{14.146}$ **(S1)**

$Rel_{m4} = (m_e + m_p + m_n) / \sqrt{(m_e^2 + m_p^2 + m_n^2)} = \mathbf{1.414598}$ **[≈ 0.1 * 14.146]** **(m4)**

$Sum_{4Z} = \Phi + \pi + 1.44 + 6.66 = \mathbf{12.8596}$ **[≈ 12.86 = 10 * 14.146 / 11]** **(S2)**

$\Phi^{2/3} + e^{2/3} + \pi^{2/3} + 1.44^{2/3} + 6.66^{2/3} = 1.286 + 9.000028 = 1.286028 + 9$ (S3)

$Rel_{11} = 100 * Rel_{m4} / (\Phi + \pi + 1.44 + 6.66) = 100 * 1.414598 / 12.8596 = 11.0003035$ [≈ 11]

Formulas (S1) and (m4) have different input data and forms, but each leads exactly / closely to figure 14.146!
Formulas (S2) and (S3) have different exponents, but each leads closely to the figure 1.286!

3) Summary

The Koide-Formula with three lepton masses as input data delivers the result close to the fraction value 2/3. Modifications of this Formula are applied also to the masses of Electron, Proton and Neutron – hereby with variable exponent forms – and to other data with physical as well as mathematical backround and also to data of our eight planets.

A general Koide-Formula F could be written in the following way:

$F = (Inp_1 \pm Inp_2 \pm Inp_3 \pm Inp_4 \pm\ ...) / (Inp_1^{Exp} \pm Inp_2^{Exp} \pm Inp_3^{Exp} \pm Inp_4^{Exp} \pm\ ...)^{1/Exp}$

Inp_i: Input values with $i \geq 2$
Exp: Exponent at the terms in the denominator of F (z.B. 1/3, 0.5, 0.666; 2/3; 1.5; 1/0.666; 2; 3)

The Modifications of the Koide-Formula deliver also extraordinary results and visible connections - often to the fraction term "2/3" -, which can be interpreted as parts of an unknown system.
For example, the three Input Figures π, 4 and 6, which are the quantities at the equations for the circle surface and the sphere volume, show connections between a Formula F_a with its set result value 2/3 and a second Koide-Formula F_b, which exponent value is set to the inverse of the exponent value of Formula F_a and which result value 4/3 is (unforeseeably) close to the double of the result value of Formula F_a.
Further connections of very exact approximations for Physical Constants using the four versatile figures *Φ, π, 1.44, 6.66* are shown. Furthermore the use of figures *14.146* and *1.286* (=14.146/11), respectively, which can be astonishingly derived by certain Modifications of the Koide-Formula with use of input data of different fields of Physics and Mathematics, deliver exact approxima-

tions for several Physical Constants.

Possible qualities of the four versatile figures *Φ, π, 1.44, 6.66* are worth for further investigation and should be performed by professional mathematicians, who therefore might find many remarkable results in the opinion of the author.
A further investigation field could be the use of data of the Masses m_i, Rotation Times RT_i and Distances a_i (to the sun) of the planets i of our sun system at the general Koide-Formula F. According to the third Kepler Law one could use for example multiplication terms "a_i^x / Rt_i^y" with the ratio "x/y = 3/2". These investigations can be performed by any interesting mathematicians.

Literature and wikipedia.de- or other Internet-Entries:

The data of the physical Constants and the data of the celestial bodies of our sun system are taken in the mayority from the entries of Wikipedia Germany. The physical constants given in the corresponding entries refer mostly to the CODATA2018.

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[The Proton radius ist given at page 26 with subtitle *Muonic Conclusions*]

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Used Data for Physical Constants:

Fine Structure Constant α: $7.2973525693 * 10^{-2}$
Inverse of Fine Structure Constant α^{-1}: 137.035999084(21)[18]

Light velocity c: 299792458 m/s[19]

Mass of Electron m_e: $9.1093837015 * 10^{-31}$ kg[2]

Mass of Neutron m_n: $1.67492749804 * 10^{-27}$ kg[17]

Mass of Protons m_p: $1.67262192369 * 10^{-27}$ kg[16]

Mass of Myon m_μ: $1.883531627 * 10^{-28}$ kg[3]
Mass Relation Myon/Electron $MR_{\mu/e}$: 206.7682830(46)[3]

Mass of Tauon m_τ: $3.16754 * 10^{-27}$ kg[4]
Mass Relation of Tauon/Electron $MR_{\tau/e}$: 3477.23(23)[4]

Radius of Electron r_e : $2.8179403262 * 10^{-15}$ m[2]

Radius of Proton r_p: $0.84087 * 10^{-15}$ m[21]

The figures in the brackets behind the data descibe the uncertainty referring the last places of the given value. This uncertainty is given as estimated standard deviation of the given value to the real value of the Physical Constant.

Seven Approximations for the figure 666 dependent on data of earth and moon:

Diameters Ø are given without unit km and Rotation Times RT without unit day.

$Ø_{Earth} / \sqrt{RT_{Earth}} = 12756.27 / \sqrt{365.256} = \mathbf{667.460...}$ (Appr-1)

$Ø_{Moon} / \sqrt{RT_{Moon}} = 3476 / \sqrt{27.3217} = \mathbf{665.007}$ (Appr-2)

$Ø_{Earth} * \sqrt{RT_{Moon}} / 100 = \mathbf{666.772...}$ (Appr-3)

$Ø_{Moon} * \sqrt{RT_{Earth}} / 100 = \mathbf{664.322...}$ (Appr-4)

$(0{,}1 * Ø_{Earth})^{(1/1{,}1)} = \mathbf{665.863...}$ (Appr-5)

$(0{,}1 * Ø_{Moon})^{(1/0{,}9)} = \mathbf{665.920...}$ (Appr-6)

$20 * \sqrt{(RT_{Earth} + RT_{Moon}^2)} = 20 * \sqrt{(365.256 + 27.3217^2)} = \mathbf{666.853...}$ (Appr-7)

Mean Value of the seven 666-close result values:

$MV_{666} = (667.460 + 665.007 + 666.772 + 664.322 + 665.863 + 665.920 + 665.853) / 7 =$
$= 666.028...$

Noticable in this context:

$Distance_{Earth\text{-}Moon} = 384400$ km $= 620^2$ km;

$620^2 * (\sqrt{3} + 1/\sqrt{3}) / 2 - 620 / 2 = 620^2 * \sqrt{(2 * 2/3)} - 620 / 2 = \mathbf{666.0006659^2}$

See the use of figures 2 and 3 besides figure 620 in above formula!

Approximations of the Rotation Times of the Earth and Moon [in unit days]:

$10 * \sqrt{[2*666 + 6.66/\pi]} = 365.25607...$ (RT_{Earth})

$10 * \sqrt{[10000/(2*666) - \pi/66.6 + (666/10000)^2]} = 27.321735...$ (RT_{Moon})

What is firstly located in the Numerator/Denominator within the operator √, is secondly located in the Denominator/Numerator. Please look at the figures 2, 666 and 10000, which define the first and also the third term within the operator √ of the second formula (RT_{Moon}).