N-dimensional representation of the set of prime numbers

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Abstract

This paper reveals/discusses the connection between the difference of prime numbers and the remainder of division by 6, the periodicity of the remainder.

Keywords: Prime numbers, intervals, periodicity.

Let's denote a prime number as (p_n). Difference between two p_n $i_k = p_n - p_{n-1}$; *n*, *k* = *1*, *2*, *3*, … . The difference between two odd numbers is even, except for the first two. We will represent this difference as an interval $i_k = \frac{p_n - p_{n-1}}{2}$. For each p_n we calculate the remainder of division by 6, $r = p_n \bmod 6$.

Each i_k is associated with a sequence p_n.

1-dimensional set $p_n(i_k)$.

2-dimensional set $p_n(i_{k_1}, i_{k_2})$, where $i_{k_2} = \frac{p_{n-1} - p_{n-2}}{2}$.

3-dimensional set $p_n(i_{k_1}, i_{k_2}, i_{k_3})$, where $i_{k_3} = \frac{p_{n-2} - p_{n-3}}{2}$.

Example 1: 1-dimensional set: $p_n(i_{k_1}) = 7, 11, 13, \dots, 31991$

	i_{k_1}	r	p						
n\n			1	2	3	4	5	6	7
1	0	0							
2	1	1	7	13	19	31			
3	2	5	11	17	23	41			
4	3	5	29						
5	4	1	97						
6	5	5	149						
7	6	1	211						
8	7	1	127						
9	8	5	1847						
10	9	1	541						
11	10	1	907						
12	11	5	1151						
13	12	1	1693						

Table 1

Table 1 is contained in its entirety in the file numbers1.txt. Program [A1] generates the file numbers1.txt.

In Table 1 there is no periodicity observed either in column r or in columns p. However, Theorem 1 holds.

Theorem 1. Numbers with equal intervals have the same remainder when divided by 6.
Example: $\mathrm{i}_{k_1} = 1, r = 1;\ 7 \bmod 6 = 1, 13 \bmod 6 = 1, 19 \bmod 6 = 1, \dots$ $\mathrm{i}_{k_1} = 2, r = 5;\ 11 \bmod 6 = 5, 17 \bmod 6 = 5, 23 \bmod 6 = 5, \dots$

Example 2. 2-dimensional set: $p_n(i_{k_1}, i_{k_2}) = 7, 11, 13, \dots, 99991$

	i_{k_1}, i_{k_2}	r	p						
n\n			1	2	3	4	5	6	7
1	0 0	0							
2	0 1	0							
3		0							
…	…	…							
37	1 0	0							
38	1 1	1	7						
39	1 2	1	13						
40	1 3	1	31						
41	1 4	0							
42	1 5	1	151						
43	1 6	1	523						
44	1 7	0							
45	1 8	1	1951						
46	1 9	1	1279						
47	1 10	0							
48	1 11	1	1153						
49	1 12	1	4549						
50	1 13	0							
51	1 14	1	3001						
52	1 15	1	6949						
53	1 16	0							
…	…	…							
74	2 1	5	11						
75	2 2	0							
76	2 3	5	41						
77	2 4	5	101						
78	2 5	0							
79	2 6	5	227						
80	2 7	5	131						
81	2 8	0							
82	2 9	5	1091						
83	2 10	5	911						
84	2 11	0							
85	2 12	5	1697						
86	2 13	5	3167						
87	2 14	0							
…	…	…							
110	3 1	1	37						
111	3 2	5	29						
112	3 3	5	59						
113	3 4	1	373						
114	3 5	5	257						
115	3 6	1	1549						
116	3 7	1	337						
117	3 8	5	3413						
118	3 9	1	547						
119	3 10	1	1663						
120	3 11	5	1979						
121	3 12	1	6427						
122	3 13	1	5563						
123	3 14	5	5153						
124	3 15	5							
…	…	…							
146	4 1	0							
147	4 2	1	409						

148	4 3	1	97						
149	4 4	0							
150	4 5	1	709						
151	4 6	1	487						
152	4 7	0							
…	…	…							

Table 2

Table 2 is contained in its entirety in the file numbers2.txt. Program [A2] generates the file numbers2.txt.

In tab. 2, periodicity appeared in columns *r* and *p*.

Let's imagine the set $p_n(i_{k_1}, i_{k_2})$ on the plane, Table 3. In each cell we place a number of prime numbers with identical pairs of intervals. If we imagine one number as one unit of measurement, then $p_n(i_{k_1}, i_{k_2})$ forms a pyramid. In this example, $k_1 * k_2 * h = 37 * 37 * 322$. Theorem 1 holds in the vertical columns of the pyramid.

$i_{k_1} \backslash i_{k_2}$	1	2	3	4	5	6	7	8	9	10	11	12	…
1	0	248	224	0	242	158	0	82	91	0	58	23	…
2	259	0	308	155	0	107	127	0	74	64	0	32	…
3	224	319	322	196	155	179	118	68	72	49	69	51	…
4	0	151	197	0	151	81	0	46	44	0	40	11	…
5	247	0	170	143	0	92	88	0	67	53	0	20	…
6	164	101	193	78	116	86	40	39	55	15	16	19	…
7	0	127	118	0	91	47	0	46	18	0	6	4	…
8	87	0	69	54	0	34	41	0	13	17	0	10	…
9	71	63	84	47	59	59	18	20	19	9	17	11	…
10	0	84	59	0	32	16	0	15	17	0	9	7	…
11	54	0	52	44	0	18	14	0	10	17	0	4	…
12	31	33	40	15	18	18	8	5	13	8	2	5	…
..	…	…	…	…	…	…	…	…	…	…	…	…	…

Table 3

Program [A2] generates table 3 into the file tab_2Interval.txt.

The periodicity of prime numbers over the intervals i_{k_1}, i_{k_2} is visible. Periodic pairs of intervals (i_{k_1}, i_{k_2}) are formed with the absence of prime numbers in them, yellow color.

Theorem 2. Prime numbers do not have a combination of intervals (i_(k_1),i_(k_2)), where a) $i_{k_1} = 3k_1 + 1, k_1 = 0, 1,2, \dots$, for each $i_{k_1}(i_{k_2} = 3k_2 + 1, k_2 = 0, 1,2,\dots)$; b) $i_{k_1} = 3k_1 + 2, k_1 = 0, 1,2, \dots$, for each $i_{k_1}(i_{k_2} = 3k_2 + 2, k_2 = 0, 1,2,\dots)$.

Let's show one more periodicity. Place the remainder of division by 6 into each cell: $r = p \bmod 6$.

0	1	2	3	4	5	6	7	8	9	10	11	12	…
1	0	1	1	0	1	1	0	1	1	0	1	1	
2	5	0	5	5	0	5	5	0	5	5	0	5	
3	5	1	5	5	1	5	5	1	1	5	1	5	
4	0	1	1	0	1	1	0	1	1	0	1	1	
5	5	0	5	5	0	5	5	0	5	5	0	5	
6	5	1	1	5	1	1	5	1	5	5	1	1	
7	0	1	1	0	1	1	0	1	1	0	1	1	
8	5	0	5	5	0	5	5	0	5	5	0	0	
9	5	1	1	5	1	5	5	1	5	5	1	1	
10	0	1	1	0	1	1	0	1	1	0	1	1	
11	5	0	5	5	0	5	5	0	5	0	0	0	
12	5	1	1	5	1	5	5	1	1	5	1	5	
…	0	1	1	0	1	1	0	1	1	0	1	1	

Table 4

Theorem 3. If the prime numbers (p_n) are sorted by two adjacent intervals, a table is built (i_{k_1}, i_{k_2}) and p_n is replaced by *r*, then we get period 3 in rows and columns. Every 1st row/column of a triple is filled with 1, every 2nd row/column of a triple is filled with 5, every 3rd row/column of a triple is filled chaotically with 1 or 5.

Example 3: 3-dimensional set: $p_n = 7, 11, 13, \dots, 99991$

	$i_{k_1}, i_{k_2}, i_{k_3}$	r	p						
n\n			1	2	3	4	5	6	7
1334	1 1 1	0	0						
1335	1 1 2	0	0						
1336	1 1 3	0	0						
…	…	0	0						
1367	1 1 34	0	0						
1368	1 1 35	0	0						
1369	1 2 0	0	0						
1370	1 2 1	1	13	…					

1371	1 2 2	0							
1372	1 2 3	1	43	…					
1373	1 2 4	1	103						
1374	1 2 5	0							
1375	1 2 6	1	229	…					
1376	1 2 7	1	313	…					
1377	1 2 8	0							
1378	1 2 9	1	1093	…					
1379	1 2 10	1	5743						
1380	1 2 11	0							
1381	1 2 12	1	1699						
1382	1 2 13	1	3169						
1383	1 2 14	0							
…	…								
2630	2 1 1	5	11						
2631	2 1 2	5	17						
2632	2 1 3	5	1613						
2633	2 1 4	0							
2634	2 1 5	5	197						
2635	2 1 6	5	2087						
2636	2 1 7	0							
2637	2 1 8	5	2243						
2638	2 1 9	5	1283						
2639	2 1 10	0							
2640	2 1 11	5	3257						
2641	2 1 12	5	6203						
2642	2 1 13	0							
…	…								
3926	3 1 1	0							
3927	3 1 2	1	79						
3928	3 1 3	1	37						
3929	3 1 4	0							
3930	3 1 5	1	157						
3931	3 1 6	1	1327						
3932	3 1 7	0							
3933	3 1 8	1	2137						
3934	3 1 9	1	4729						
3935	3 1 10	0							
…									
3962	3 2 1	5	29						
3963	3 2 2	0							
3964	3 2 3	5	89						
3965	3 2 4	5	509						
3966	3 2 5	0							
3967	3 2 6	5	683						
3968	3 2 7	5	137						
3969	3 2 8	0							
3970	3 2 9	5	3917						
3971	3 2 10	5	10253						
3972	3 2 11	0							
…									
3998	3 3 1	1	163						
3999	3 3 2	5	59						
4000	3 3 3	5	269						
4001	3 3 4	1	379						
4002	3 3 5	5	263						
4003	3 3 6	5	1913						
4004	3 3 7	1	4999						
4005	3 3 8	5	5309						

4006	3 3 9	1	34549						
4007	3 3 10	1	35323						
…									
4035	3 4 2	1	757						
4036	3 4 3	1	373						
4037	3 4 4	0							
4038	3 4 5	1	733						
4039	3 4 6	1	2473						
4040	3 4 7	0							
4041	3 4 8	1	4597						
4042	3 4 9	1	6043						
4043	3 4 10	0							
…									

Table 5

From tab. 5 we see that Theorems 1, 2, 3 are also true in the 3-dimensional representation of prime numbers. In addition, there is an additional periodicity.

Conclusion

Explain mathematically the dependence of the difference between prime numbers and the remainder of division by 6 $r\left(\frac{p_n-p_{n-1}}{2}\right)$. Prove theorems.

The idea of constructing prime numbers in the form of n-dimensional sets is described in the article [1].

Literature

1. Shtrom V. F., Scientific research of the SCO countries: synergy and integration 14.07.2023 г., THE MOVEMENT OF A LIQUID IN THE CASE WHEN THE VOLUME OF THE LIQUID... “PARADOX” TEA LEAF, DOI 10.34660/INF.2023.64.86.091

Applications

1. Shtrom V. F., PrimesL1_Interval_Console, https://drive.google.com/drive/folders/156E4v4V4KNZUGzMf9vGl1JSIg9ghNjfL?usp=sharing
2. Shtrom V. F., PrimesL2_Interva_Console, https://drive.google.com/drive/folders/1vp75riLO2mGFTtGMY1Vxhe3Hi6HojHT6?usp=sharing
3. Shtrom V. F., PrimesL3_Interva_Console, https://drive.google.com/drive/folders/1p8q5rO0AFTqwR8yWeQdc-cMdJQB3XBO8?usp=sharing