A Modified Born-Infeld Model of Electrons as Moving Clocks

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Abstract

A recently proposed modified Born-Infeld model of electrons includes an internal clock. De Broglie and other researchers have argued that such an internal clock may explain quantum-mechanical properties of electrons and other particles. Motivated by this hypothesis, this work proposes an interpretation of single-electron wave functions in terms of the mentioned model of electrons. Future numerical experiments that could test whether the model may be used to describe quantum-mechanical phenomena are also discussed.

1 Introduction

De Broglie hypothesized that electrons feature a periodic phenomenon or internal clock, which causes their wavelike, i.e., quantum-mechanical character [dB25, Bro60]. More recently, Müller claimed "that all of quantum mechanics can be derived from a picture of matter waves as clocks together with simple assumptions such as the principle of superposition" [Mül14]. One model for these internal clocks is a recently proposed modified Born-Infeld model of electrons [Kra23a], which describes electrons as rotating solutions of classical, nonlinear field equations that generalize Maxwell-Heaviside equations. While several properties of this model (e.g., mass, Compton frequency, electric charge, and magnetic moment) match properties of real electrons by design of the model, its intrinsic angular momentum also appears to match the spin of electrons [Kra24b], and it appears to show a Lorentz-type interaction with electromagnetic fields [Kra24a]. Thus, this model might be a suitable candidate for describing quantum-mechanical phenomena in a classical field theory.

The purpose of this work is to discuss qualitatively how the modified Born-Infeld model of electrons might be related to quantum-mechanical single-electron wave functions. Based on this discussion, numerical experiments are discussed that could quantitatively test the power of this classical model to describe quantum-mechanical phenomena. Actual numerical simulations are, unfortunately, beyond the scope of this work.

Section 2 discusses the modified Born-Infeld model of electrons [Kra23a] and Ballentine's ensemble interpretation of quantum mechanics [Bal70, Bal98], followed by a discussion of de Broglie's internal clock hypothesis [dB25, Bro60]. Section 3 describes the proposed interpretation of single-electron wave functions and how de Broglie's theory of the Double Solution [Bro60] might be related to it. Section 4 provides a discussion of the feasibility of future numerical simulations based on modified Born-Infeld field theory as a test of the proposed interpretation in various scenarios that are described in quantum mechanics by single-electron wave functions. Section 5 concludes this work.

2 Related Work

2.1 Modified Born-Infeld Field Theory

As in previous work [Kra24b], the Lagrangian density $\mathscr{L}$ of the modified Born-Infeld field theory is defined in SI units as

$$\mathscr{L} \stackrel{\text{def}}{=} \frac{b^2}{\mu_0}\left(1 - \sqrt{1 - \frac{1}{b^2}(\partial^\mu A^\nu)(\partial_\mu A_\nu)}\right), \tag{1}$$

with the Born-Infeld parameter b specifying the maximum magnetic field strength, the vacuum permeability μ_0, and the electromagnetic four-potential $(A^0, A^1, A^2, A^3) = (\phi/c, A_x, A_y, A_z)$. Basic Ricci

calculus is used as well as the Minkowski metric tensor η in the form $\mathrm{diag}(+1,-1,-1,-1)$. Electric field strength $\mathbf{E}$, magnetic field strength $\mathbf{B}$, and four-current density $(J^0, J^1, J^2, J^3) = (c\rho, J_x, J_y, J_z)$ are *defined* as follows:

$$\mathbf{E} \stackrel{\text{def}}{=} -\nabla\phi - \frac{\partial}{\partial t}\mathbf{A} \tag{2}$$

$$\mathbf{B} \stackrel{\text{def}}{=} \nabla \times \mathbf{A} \tag{3}$$

$$J^\nu \stackrel{\text{def}}{=} \frac{1}{\mu_0}\left(\partial_\mu \partial^\mu A^\nu - \partial_\mu \partial^\nu A^\mu\right) \qquad \text{for } \nu = 0, \ldots, 3 \tag{4}$$

More details about the notation are provided in previous work [Kra23b].

While these definitions guarantee that all four Maxwell-Heaviside equations are satisfied, the Lagrangian density $\mathscr{L}$ allows for particle-like field solutions that are not possible in standard electromagnetism. In order to find these solutions, the corresponding Euler-Lagrange equations have to be considered:

$$0 = \partial_\mu \left(\frac{\partial \mathscr{L}}{\partial\left(\partial_\mu A_\nu\right)}\right) - \frac{\partial \mathscr{L}}{\partial A_\nu} = \partial_\mu \left(\frac{1}{\mu_0} \frac{\partial^\mu A^\nu}{\sqrt{1 - \frac{1}{b^2}(\partial^\alpha A^\beta)(\partial_\alpha A_\beta)}}\right) - 0 \qquad \text{for } \nu = 0, \ldots, 3. \tag{5}$$

Thus:

$$0 = \partial_\mu \frac{\partial^\mu A^\nu}{\sqrt{1 - \frac{1}{b^2}(\partial^\alpha A^\beta)(\partial_\alpha A_\beta)}} \qquad \text{for } \nu = 0, \ldots, 3. \tag{6}$$

In the low-energy limit, i.e., for $(\partial^\alpha A^\beta)(\partial_\alpha A_\beta)/b^2 \approx 0$, the field equations become:

$$0 \approx \partial_\mu \partial^\mu A^\nu \qquad \text{for } \nu = 0, \ldots, 3. \tag{7}$$

As mentioned in previous work [Kra23b], these approximations have the same form as the inhomogeneous Maxwell-Heaviside equations for $J^\nu = 0$ in Lorenz gauge, i.e., with the gauge condition $\partial_\mu A^\mu = 0$. In the modified Born-Infeld model, however, the approximations $\partial_\mu \partial^\mu A^\nu \approx 0$ are valid without requiring a specific gauge condition, and they do not imply that $J^\nu = 0$. There is, however, a mechanism that might suppress low-energy solutions with $J^\nu \neq 0$ (e.g., longitudinal waves) at macroscopic scales as discussed in previous work [Kra23b]: Assuming that partial derivatives of the field equations are approximately 0, i.e.,

$$0 \approx \partial^\lambda \partial_\mu \partial^\mu A^\nu \qquad \text{for } \lambda = 0, \ldots, 3 \text{ and } \nu = 0, \ldots, 3, \tag{8}$$

it follows that

$$0 \approx \partial^\lambda J^\nu - \partial^\nu J^\lambda \qquad \text{for } \lambda = 0, \ldots, 3 \text{ and } \nu = 0, \ldots, 3. \tag{9}$$

In terms of ρ and $\mathbf{J}$, these approximations read:

$$\frac{\partial}{\partial t}\mathbf{J} \approx -c^2 \nabla \rho \qquad \text{and} \qquad \nabla \times \mathbf{J} \approx \mathbf{0}. \tag{10}$$

Considering the continuity equation $\frac{\partial}{\partial t}\rho = -\nabla \cdot \mathbf{J}$, a physical interpretation of $\frac{\partial}{\partial t}\mathbf{J} \approx -c^2 \nabla \rho$ might be that any gradient of charge density increases current density that diffuses this charge density. This might eliminate all charge densities at macroscopic scales for low field energies while stable, particle-like charges might tend to form at points with high field energies, where these approximations do not apply.

In order to model at least one kind of such particle-like charges, the nonlinear Euler-Lagrange equations in Eq. (6) were solved numerically in previous work [Kra23a] resulting in a rotating field solution with a peak moving at the speed of light on a circular orbit with a radius equal to an electron's reduced Compton wavelength. While most features of electrons (electric charge, magnetic moment, Compton frequency) were imposed on the solution, the Born-Infeld parameter was adjusted by matching the solution's rest mass energy to the invariant mass of electrons [Kra24b]. With these parameter values, the solution's intrinsic angular momentum matches the spin of electrons within the model's numerical accuracy [Kra24b]. Furthermore, sufficiently weak electromagnetic fields appear to result in a Lorentz-type force on this electron-like solution [Kra24a].

While these features suggest that this model shows many characteristics of a relativistic electron-like particle, it is less obvious whether this classical model shows quantum-mechanical features beyond its Compton frequency and intrinsic angular momentum. In order to prepare for the discussion of this question, the next section reviews Ballentine's ensemble interpretation of quantum mechanics.

2.2 Ensemble Interpretation of Quantum Mechanics

While most classical models of particles associate each particle with a specific trajectory of time-dependent positions and velocities, quantum mechanics is often interpreted to be in conflict with the notion of a specific trajectory of a particle. This supposed conflict may be overcome by a consistent interpretation of quantum mechanics that allows for the existence of specific trajectories of each particle. One such interpretation appears to be the ensemble interpretation of quantum mechanics by Ballentine [Bal70, Bal98], which is also known as the "Statistical Interpretation" of quantum mechanics. Ballentine explains the basic concepts using the example of a scattering experiment:

> "A particle is subjected to the *preparation* consisting of acceleration and collimation [...]. It scatters off the target through some angle θ, and is finally detected by one of the detectors [...]. A single *measurement* consists in the detection of the particle and hence the determination of the angle of scatter, θ. If the same preparation is repeated identically on a similar particle [...], the angle of scatter that results will, in general, be different. Individual events resulting from identical preparations are not reproducible." [Bal98, pages 43–44]

This is followed by an important footnote:

> "Whether this nonreproducibility is due to an indeterminism in nature, or merely to limitations (practical or fundamental) in the preparation procedure, is a question that we cannot, and need not, answer here. The statistical approach is applicable in any case." [Bal98, page 44]

Ballentine goes on to motivate the concept of a quantum-mechanical state:

> "A specific preparation determines not the outcome of the subsequent measurement, but the *probabilities* of the various possible outcomes. Since a preparation is independent of the specific measurement that may follow it, the preparation must determine probability distributions for all such possible measurements. This leads us to introduce the concept of a *state*, which is identified with the specification of a probability distribution for each observable. (An *observable* is a dynamical variable that can, in principle, be measured.) Any repeatable process that yields well-defined probabilities for all observables may be termed a *state preparation procedure*. [...] If two or more procedures generate the same set of probabilities, then these procedures are equivalent and are said to prepare the same *state*. The empirical content of a probability statement is revealed only in the relative frequencies in a sequence of events that result from the same (or an equivalent) state preparation procedure. Thus, although the primary definition of a *state* is the abstract set of probabilities for the various observables, it is also possible to associate a *state* with an *ensemble* of similarly prepared systems. However, it is important to remember that this ensemble is the conceptual infinite set of all such systems that may potentially result from the same state preparation procedure, and not a concrete set of systems that coexist in space." [Bal98, pages 45–46]

Hopefully, these lengthy quotes illustrate that Ballentine's ensemble interpretation of quantum mechanics allows for the possibility of specific particle trajectories. The cited textbook on quantum mechanics [Bal98] does not only describe his ensemble interpretation, but applies it to many concepts in quantum mechanics. For example, quantum-mechanical wave functions in scattering experiments are interpreted in this way:

> "The quantum state does not describe the position of the incident particle, but rather it gives the probability density, $|\psi(\mathbf{r})|^2$, for it to be a distance $\mathbf{r}$ from the target. Similarly the state does not describe the actual flux of particles, but rather the *probability flux*, which is the net probability per unit time that a particle crosses a unit area. [...] [W]e can write the probability flux [...] as $\mathbf{J} = \frac{\hbar}{\mu}\mathrm{Im}(\psi^*\nabla\psi)$." [Bal98, pages 425–426]

This ensemble interpretation of wave functions is the foundation of the interpretation proposed in this work.

2.3 Quantum Mechanics as a Theory of Moving Clocks

One of the advantages of Ballentine's ensemble interpretation of quantum mechanics in the context of this work is that it avoids many non-classical concepts, for example, wave function collapse. Thus, it allows us to discuss many quantum-mechanical phenomena in terms of classical field theories. One of the earliest attempts in this direction was de Broglie's concept of *phase waves*, which "establish a link between motion of a material body and propagation of a wave, and thereby permit envisioning the possibility of a synthesis of these antagonistic theories on the nature of radiation" [dB25, page 10]. De Broglie based the concept of phase waves on hypothetical internal clocks, which he described in this way:

> "One may imagine that, by cause of a meta law of Nature, to each portion of energy with a proper mass m_0, one may associate a periodic phenomenon of [Compton] frequency ν_0, such that one finds: $h\nu_0 = m_0c^2$. The [Compton] frequency ν_0 is to be measured, of course, in the rest frame of the energy packet." [dB25, page 8]

Based on special relativity, he concluded:

> "A periodic phenomenon is seen by a stationary observer to exhibit the frequency $\nu_1 = h^{-1}m_0c^2\sqrt{1-\beta^2}$ that appears constantly in phase with a wave having frequency $\nu = h^{-1}m_0c^2/\sqrt{1-\beta^2}$ propagating in the same direction with [phase] velocity $V = c/\beta$." [dB25, page 9]

Notably, he observed that this wave "can not represent transport of energy" [dB25, pages 9–10]; instead "this wave represents a spacial distribution of *phase*, that is to say, it is a '*phase wave*' " [dB25, page 10]. Furthermore, he found that "[t]he group velocity of phase waves equals the velocity of its associated body" [dB25, page 11].

De Broglie's doctoral thesis successfully explained many quantum-mechanical phenomena based on his concept of phase waves and, therefore, based on his hypothesis of particles with internal clocks. More recently, Müller discussed many works related to this idea and claimed "that all of quantum mechanics can be derived from a picture of matter waves as clocks together with simple assumptions such as the principle of superposition" [Mül14]. However, de Broglie's ambition went beyond quantum mechanics of point particles. Many years later, he described his early ideas in this way:

> "I was imbued with classical ideas of the possibility of representing phenomena in an objective and deterministic way within the framework of space-time. It therefore seemed to me that the wave-particle association must take the following form: the particle would be some sort of singularity at the center of an extended wave phenomenon—a phenomenon of which it would be an integral part, and the motion of this singularity, although almost certainly behaving according to the new dynamical laws [of quantum physics], must involve, in conformity with the classical picture, a trajectory in space and a velocity determined for every point of this trajectory. So it occurred to me that the plane monochromatic Ψ wave, associated in my earlier reasoning with the uniform and rectilinear motion of a free particle, did not actually describe reality, but that it could give in a precise way only the *phase* of the wave phenomenon surrounding the particle, since the constant amplitude a_0 could not represent the true amplitude of this phenomenon. Indeed, the true amplitude seemed to me to involve a singularity—the particle; and this amplitude had to diminish with the distance from that singularity." [Bro60, pages 7–8]

And in the same book:

> "So I boldly laid down a hypothesis—that of the Double Solution—according to which the linear equations of [quantum-physical] Wave Mechanics admitted two kinds of solution: the continuous Ψ solutions one normally thinks of—the statistical nature of which was beginning to become clearly apparent at that time, thanks to the work of Born, and 'singularity' [u] solutions that would have a concrete meaning and be the true physical representation

> of the wave-particle dualism. Particles would then be clearly localized in space, as in the classical picture, but they would be *incorporated* in an extended wave phenomenon. [...] [T]he motion of the singularity was to be dependent on all the obstacles that hindered the free propagation of the wave phenomenon surrounding it and there would result from this a reaction of the wave phenomenon on the particle [...]. And in this way the appearance of interference and diffraction phenomena would be explained." [Bro60, pages 89–90]

During his academic career, de Broglie's concept of the singularity u solutions evolved substantially:

> "A very important point to be elucidated is the question of the nature of u solutions having a singularity of the kind postulated by the theory of the Double Solution. A certain number of reasons [...] have led me to modify quite considerably the ideas I originally expressed in 1927. At that time I considered the u wave as a solution of the *linear* equation of propagation of [quantum-physical] Wave Mechanics, which would involve a singularity in the mathematical sense of the word. It seems to me at the present time absolutely certain that the idea of singularity must be replaced by that of a very small singular region—in general, mobile—where the u function would take on very large values and obey a *non-linear* equation. Only outside this very small singular region would the u function approximately obey the linear equation of propagation of current [quantum-physical] Wave Mechanics. This new way of defining the u wave is in conformity with Vigier's ideas. Vigier thinks that in this way it would be possible to reconcile the theory of the Double Solution with the ideas of Einstein—who always sought to represent particles by singular regions in the field—and perhaps also with the non-linear electromagnetism of Born." [Bro60, pages 95–96]

Since the most prominent "non-linear electromagnetism of Born" is Born-Infeld theory [BIF34], the last sentence raises the question whether the modified Born-Infeld model of electrons described in Section 2.1 may be considered a successor of de Broglie's theory of the Double Solution. This question is addressed in the next section.

3 Statistical Description of Moving Clocks by Wave Functions

This section introduces an interpretation of wave functions as statistical descriptions of ensembles of moving clocks; specifically, the kind of moving clocks considered in de Broglie's internal clock hypothesis; for example, the rotating wave solution described in Section 2.1. This interpretation is based on Ballentine's ensemble interpretation of quantum mechanics cited in Section 2.2. However, while Ballentine argues that a statistical interpretation does not need to explain the nonreproducibility of experiments, the interpretation discussed here explains this nonreproducibility by a lack of control over the phase of the employed moving clock, or—more specifically—lack of control over the angle of rotation of a rotating wave solution.

3.1 Wave Function of a Single, Freely Moving Clock

In its own rest frame, the rotating wave solution mentioned in Section 2.1 is rotating at the Compton frequency of electrons m_0c^2/h, which means that the electric potential ϕ and the magnetic vector potential $\mathbf{A}$ at most points of the solution is oscillating with this frequency—the exceptions being the points on the axis of rotation. This feature qualifies the rotating wave solution as a model of de Broglie's hypothetical internal clock of a particle, which might be observed as a phase wave (and phase of a wave function) with its group velocity equal to the particle's velocity v and its phase velocity equal to c^2/v as mentioned in Section 2.3.

To describe wave functions mathematically, this section follows de Broglie's notation [Bro60] except for minor changes. Based on a particle's velocity $\mathbf{v}$, speed $v \stackrel{\text{def}}{=} |\mathbf{v}|$, and rest mass m_0, de Broglie defined its speed β relative to the speed of light c, energy W, and momentum $\mathbf{p}$ [Bro60, page 3]:

$$\beta \stackrel{\text{def}}{=} \frac{v}{c}, \qquad W \stackrel{\text{def}}{=} \frac{m_0c^2}{\sqrt{1-\beta^2}}, \qquad \mathbf{p} \stackrel{\text{def}}{=} \frac{m_0\mathbf{v}}{\sqrt{1-\beta^2}}. \tag{11}$$

With these values, the de Broglie wavelength λ of the corresponding wave function on a particle's trajectory may be computed using the Planck constant h [Bro60, page 6]:

$$\lambda \stackrel{\text{def}}{=} \frac{c^2}{v}\frac{h}{W} = \frac{h}{p}. \tag{12}$$

The complex wave function $\Psi(\mathbf{r}(t), t)$ on a trajectory $\mathbf{r}(t)$ of a free particle moving with constant velocity $\mathbf{v}$ may then be expressed as follows using the angular frequency $W/\hbar$, the angular wavenumber $\mathbf{p}/\hbar$, and an amplitude a_0 [Bro60, page 7]:

$$\Psi(\mathbf{r}(t), t) \stackrel{\text{def}}{=} a_0 e^{\frac{i}{\hbar}(Wt - \mathbf{p}\cdot\mathbf{r}(t))}. \tag{13}$$

Note that this definition of $\Psi(\mathbf{r}(t), t)$ is limited to the spacetime points of a specific trajectory $\mathbf{r}(t)$.

Considering the rotating wave solution mentioned above, the trajectory $\mathbf{r}(t)$ specifies the movement of its rotation center, and the wave function $\Psi(\mathbf{r}(t), t)$ describes the overall oscillation of the electromagnetic potential of the whole rotating wave solution. (Interestingly, the electromagnetic potential at the rotation center is not oscillating since it is part of the rotation axis, but all other points join the oscillation described by the phase of the wave function.) It might be worth repeating that the wave function describes only the overall oscillation of the whole rotating wave solution. The actual oscillation of the electric potential ϕ and magnetic vector potential $\mathbf{A}$ at specific points in spacetime is described only indirectly. Thus, Ψ is linked to a physical wave, but only in a rather complicated way. As mentioned by Ballentine, it would be a misinterpretation of Ψ to think of it as a physical wave in ordinary space [Bal98, page 99].

3.2 Wave Function of an Ensemble of Freely Moving Clocks

As mentioned above, the de Broglie wavelength λ and, therefore, the wave function $\Psi(\mathbf{r}(t), t)$ in Eq. (13) is only defined on a particle's trajectory $\mathbf{r}(t)$ for a constant velocity $\mathbf{v}$, which raises the question of how to define $\Psi(\mathbf{r}, t)$ for more points in spacetime.

One answer is to fill spacetime with many non-crossing trajectories for the same constant velocity $\mathbf{v}$. This is straightforward for a constant velocity because the trajectories are just straight lines, thus, spacetime may be filled with translated (in space and time) copies of one trajectory. One may choose an arbitrary additive offset to the complex phase of $\Psi(\mathbf{r}, t)$ on each trajectory, for example, 0, which results in this new definition:

$$\Psi(\mathbf{r}, t) \stackrel{\text{def}}{=} a_0 e^{\frac{i}{\hbar}(Wt - \mathbf{p}\cdot\mathbf{r})}. \tag{14}$$

Compared to the previous definition of the wave function in Eq. (13), the only change is the replacement $\mathbf{r}(t) \to \mathbf{r}$. However, the interpretation of this new definition is very different: Instead of describing a phase wave on a single trajectory, this new definition describes an ensemble of phase waves on infinitely many non-crossing trajectories such that every point $(\mathbf{r}, t)$ in spacetime is associated with exactly one trajectory, which then allows us to define a wave function as in Eq. (13) on that trajectory through the point $(\mathbf{r}, t)$.

While it is not an issue for freely moving particles, it is worth noting that a wave function like $\Psi(\mathbf{r}, t)$ cannot represent an ensemble of crossing or self-crossing trajectories. It is also remarkable how overcoming the limitation to a single trajectory in Eq. (13) led naturally to an ensemble interpretation of the wave function in Eq. (14).

One of the advantages of the extended wave function $\Psi(\mathbf{r}, t)$ is that it is a solution to certain partial differential equations. As discussed by de Broglie [Bro60, pages 25–27], the wave function $\Psi(\mathbf{r}, t)$ solves a relativistic equation that may be derived from the relativistic energy-momentum relation

$$\frac{W^2}{c^2} = m_0^2 c^2 + \mathbf{p}\cdot\mathbf{p} \tag{15}$$

by applying the substitutions

$$W \to -i\hbar\frac{\partial}{\partial t} \qquad \text{and} \qquad \mathbf{p} \to i\hbar\,\nabla, \tag{16}$$

resulting in the Klein-Gordon equation for a spinless particle

$$\Box\Psi(\mathbf{r}, t) + \frac{m_0^2 c^2}{\hbar^2}\Psi(\mathbf{r}, t) = 0, \tag{17}$$

where $\Box$ is the d'Alembert operator:

$$\Box \stackrel{\text{def}}{=} \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}. \tag{18}$$

As discussed by de Broglie [Bro60, pages 17–21], for very small velocities compared to the speed of light, i.e., $v \ll c$, the wave function $\Psi(\mathbf{r}, t)$ satisfies the non-relativistic Schrödinger equation for a spinless particle

$$-\frac{\hbar^2}{2m}\triangle\Psi(\mathbf{r}, t) + V(\mathbf{r})\Psi(\mathbf{r}, t) = -i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r}, t) \tag{19}$$

where $\triangle$ is the Laplace operator

$$\triangle \stackrel{\text{def}}{=} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{20}$$

and $V(\mathbf{r})$ is the particle's potential energy, which is part of its non-relativistic total energy E:

$$E = \frac{1}{2}mv^2 + V(\mathbf{r}). \tag{21}$$

The wave function $\Psi(\mathbf{r}, t)$ in Eq. (14) satisfies the Schrödinger equation if we set $W = E$, $p = mv = \sqrt{2m(E - V(\mathbf{r}))}$, and $V(\mathbf{r}) = \text{const}$ for a freely moving particle.

While $\Psi(\mathbf{r}, t)$ in Eq. (14) was constructed based on properties of phase waves, one may also compute $\Psi(\mathbf{r}, t)$ as a solution to one of the mentioned single-particle equations. This does not affect the proposed ensemble interpretation; i.e., finding a wave function $\Psi(\mathbf{r}, t)$ as a solution to a single-particle equation implies finding a space-filling ensemble of non-crossing particle trajectories. In fact, de Broglie (and later Bohm) showed how to compute non-crossing trajectories from $\Psi(\mathbf{r}, t)$ as mentioned in the next section.

3.3 Amplitude of Wave Function and Its Interpretation

In the non-relativistic case, i.e., for $v \ll c$, and, therefore, $\mathbf{p} = m\mathbf{v}$, it is possible to compute $\mathbf{v}$ for any point in spacetime from $\Psi(\mathbf{r}, t) = a_0 e^{\frac{i}{\hbar}(Wt - \mathbf{p}\cdot\mathbf{r})}$ as shown by de Broglie [Bro60, page 33]:

$$\mathbf{v} = \frac{1}{|\Psi(\mathbf{r}, t)|^2}\frac{\hbar}{2im}\left(\Psi(\mathbf{r}, t)\nabla\Psi^*(\mathbf{r}, t) - \Psi^*(\mathbf{r}, t)\nabla\Psi(\mathbf{r}, t)\right) = -\frac{1}{m}\nabla\varphi(\mathbf{r}, t). \tag{22}$$

where $\varphi(\mathbf{r}, t)$ is the phase of $\Psi(\mathbf{r}, t)$ scaled by factor $\hbar$, i.e., $\Psi(\mathbf{r}, t) = a(\mathbf{r}, t)e^{\frac{i}{\hbar}\varphi(\mathbf{r},t)}$, and $\Psi^*(\mathbf{r}, t) = a(\mathbf{r}, t)e^{-\frac{i}{\hbar}\varphi(\mathbf{r},t)}$. This is an example of de Broglie's "guidance formula", which is part of his "pilot-wave theory" [Bro60, page 90], where it may be used to compute non-crossing trajectories of particles from solutions of the Schrödinger equation. In standard quantum mechanics, on the other hand, $\mathbf{v}$ would be considered the velocity of a probability flux (see Section 2.2).

As discussed by de Broglie [Bro60, pages 32–33], $a^2(\mathbf{r}, t) = \Psi^*(\mathbf{r}, t)\Psi(\mathbf{r}, t) = |\Psi(\mathbf{r}, t)|^2$ is interpreted as a probability density $\rho(\mathbf{r}, t) \stackrel{\text{def}}{=} \sqrt{a(\mathbf{r}, t)}$ (for a suitably normalized wave function $\Psi(\mathbf{r}, t)$), which is supported by the observation that the Schrödinger equation implies a continuity equation for $\rho(\mathbf{r}, t)$:

$$\frac{\partial}{\partial t}\rho(\mathbf{r}, t) + \nabla\cdot(\rho(\mathbf{r}, t)\mathbf{v}(\mathbf{r}, t)) = 0. \tag{23}$$

(De Broglie also discussed the corresponding equations for the case of the relativistic Klein-Gordon equation [Bro60, pages 38–39].)

The interpretation proposed here is that the Schrödinger equation in Eq. (19) for a single, spinless particle combines the required conservation of probability density (as expressed by the continuity equation in Eq. (23)) with the apparent oscillation of a moving clock (as described by de Broglie's phase waves and expressed by Eq. (13) for the case of a free particle) along one of many trajectories that form an ensemble of space-filling, non-crossing trajectories of a non-relativistic, spinless particle in a potential $V(\mathbf{r})$.

This interpretation appears to be compatible with Ballentine's ensemble interpretation (see Section 2.2) if particles' initial trajectories in a statistical experiment are determined by the preparation of particles, which usually results in some knowledge of the experimentalist about the initial trajectories,

e.g., the distribution of particles' velocities after the preparation. Furthermore, the initial phases of the internal clocks of the prepared particles are assumed to be unknown to the experimentalist. Solutions to the Schrödinger equation may then provide probabilities of possible outcomes of a measurement of the particles depending on the knowledge of the experimentalist about the preparation of the particles. The experimentalist's knowledge is always limited because the phases of the prepared particles are always assumed to be unknown.

The main advantage of the interpretation proposed here is the possibility of using rotating wave solutions of the modified Born-Infeld model of electrons (see Section 2.1) as the moving clocks that are described by the wave function $\Psi(\mathbf{r}, t)$. Thus, $\Psi(\mathbf{r}, t)$ might provide an approximate, statistical description (at the scale of picometers) of much more complicated processes on the level of the nonlinear modified Born-Infeld field theory (at the scale of femtometers). Section 4 discusses future numerical experiments to numerically test this hypothesis.

3.4 Relation to de Broglie's Theory of the Double Solution

Since de Broglie's theory of the Double Solution has been introduced in Section 2.3, it might be worthwhile to compare it to the interpretation of quantum-mechanical wave functions presented in the previous section.

De Broglie's u solution, which includes a nonlinear "singular region" [Bro60, page 95], resembles the rotating wave solution of the modified Born-Infeld model of electrons discussed in Section 2.1. However, the details are quite different as illustrated by the following description of the u solution by de Broglie: "[O]utside this very small singular region would the u function approximately obey the linear equation of propagation of current [quantum-physical] Wave Mechanics" [Bro60, page 95]. In contrast to this, the modified Born-Infeld field theory for low energies does not approximate quantum-mechanical field equations but electromagnetic wave equations in Lorenz gauge as mentioned in Section 2.1. As discussed at the end of Section 3.1, the relation between rotating wave solutions of the modified Born-Infeld model and quantum-mechanical wave functions is rather complicated—supposedly more complicated than the relation between quantum-mechanical wave functions and u solutions as imagined by de Broglie.

In the interpretation proposed here, quantum mechanics of single electrons is considered an effective theory of the statistical behaviour of rotating wave solutions of the modified Born-Infeld field theory at much larger length scales than the size of their nonlinear peaks. Since de Broglie never completed the theory of the Double Solution, it is impossible to know exactly how it relates to this interpretation. However, there are good reasons to assume that de Broglie would have appreciated the attempt to "reconcile the theory of the Double Solution [...] with the non-linear electromagnetism of Born." [Bro60, page 95]

4 Discussion of Future Numerical Experiments

As mentioned at the end of Section 3.3, numerical simulations might be able to reject or confirm the proposed interpretation of quantum-mechanical wave functions as statistical descriptions of ensembles of moving clocks and, more specifically, ensembles of rotating wave solutions of the modified Born-Infeld model of electrons. In order to plan such simulations, this section discusses potential scenarios and their practical feasibility. To structure the discussion, scenarios involving unbound wave functions are discussed first, followed by scenarios involving bound wave functions.

4.1 Unbound Wave Functions

A wave function of a freely moving electron is a basic example of an unbound wave function. Unfortunately, it is unclear how the numerical simulation of a freely moving rotating wave solution could provide any further insights beyond what simulations of resting rotating wave solutions can provide.

The simulation of a moving electron that interacts with a short-ranged, external electric potential, i.e., a basic scattering experiment, is likely to provide more insights. Such a simulation might show that the nonlinear peak of the moving rotating wave solution is scattered by an external potential similarly to how de Broglie described such scenarios for a "singularity" in the theory of the Double Solution:

> "[T]he motion of the singularity was to be dependent on all the obstacles that hindered the free propagation of the wave phenomenon surrounding it and there would result from this a reaction of the wave phenomenon on the particle [...]. And in this way the appearance of interference and diffraction phenomena would be explained." [Bro60, page 90]

A very basic setup for the simulation of a scattering experiment could include a fixed initial velocity and starting position of the rotating wave solution, while the simulation would have to be repeated for many different values of the initial phase of the electron, i.e., different values of the initial angle of rotation of the rotating wave solution. The simulation would then let the rotating wave solution move toward the external potential by simulating the field equations of the modified Born-Infeld field theory. Depending on the initial phase, the moving rotating wave solution would interact differently with the static potential, and be scattered in various directions. In this way, these numerical simulations could map values of the initial phase to resulting scattering directions. If this mapping matches probability distributions predicted by quantum mechanics, one could conclude that at least some aspects of quantum-mechanical scattering may be described by the modified Born-Infeld model of electrons.

The challenge with this kind of experiment is that it appears to require a full four-dimensional numerical simulation down to the (femtometer) scale of the nonlinear peak of the rotating wave solution because only the nonlinear peak interacts with the external potential. (A linear superposition principle applies to the other, low-energy parts of the rotating wave solution.) In principle, this kind of simulation appears to be possible; however, it would require considerably more computational resources than previous simulations of the rotating wave solutions, where the time dependency was eliminated by assuming a rotation of the solution with constant angular velocity [Kra23a]. Without reduction to a three-dimensional problem, the required simulations are far more demanding. Therefore, the next section is about scenarios involving bound wave functions while avoiding unbound wave functions.

4.2 Bound Wave Functions

Common examples of bound electron wave functions include hydrogen-like atoms, e.g., a single electron bound to an external electric Coulomb potential. Since the interpretation of wave functions proposed here is based on electrons moving on specific trajectories, it shares some features with the Bohr model of hydrogen atoms, which assumes circular orbits of electrons, and the Bohr-Sommerfeld model, which predicts elliptical orbits of electrons, and thereby successfully explains the spectral fine structure of hydrogen.

A stationary wave function for a bound electron of energy E may be written in the form

$$\Psi(\mathbf{r},t) = a(\mathbf{r})e^{\frac{i}{\hbar}Et+\frac{i}{\hbar}\varphi(\mathbf{r})}. \tag{24}$$

The interpretation proposed here assumes that this wave function describes an ensemble of closed orbits, which may be computed from its (continuous) phase $\varphi(\mathbf{r})$ (see Section 3.3). If $\mathbf{r}(\xi)$ specifies one of those closed orbits, then $\varphi(\mathbf{r}(\xi))$ describes a snapshot of a phase wave of de Broglie wavelength λ (see Section 3.1). Therefore, the length of any closed orbit $\mathbf{r}(\xi)$ must be a whole multiple of the de Broglie wavelength λ, which is equivalent to the quantization condition in the Bohr model. In the words of de Broglie:

> "[A] trajectory of a moving particle is identical to a ray of a phase wave, along which frequency $[E/h]$ is constant (because total energy $[E]$ is constant) and with variable velocity, whose value we shall not attempt to calculate. Propagation is, therefore, analogue to a liquid wave in a channel closed on itself but of variable depth. It is physically obvious, that to have a stable regime, the length of the channel must be resonant with the wave; in other words, the points of a wave located at whole multiples of the wave length l, must be in phase. The resonance condition is $l = n\lambda$ if the wave length is constant, and $\oint(\nu/V)\mathrm{d}l = n$ (integer) in the general case." [dB25, page 28]

It might be worth repeating that if an electron is on orbit $\mathbf{r}(\xi)$ then $\varphi(\mathbf{r})$ is only relevant for points $\mathbf{r}$ on this orbit $\mathbf{r}(\xi)$. Values of $\varphi(\mathbf{r})$ for other points $\mathbf{r}$ are irrelevant for an electron on orbit $\mathbf{r}(\xi)$.

With this important caveat in mind, $\varphi(\mathbf{r}(\xi))$ may be interpreted as phase offsets of an oscillating system that is driven by an external, oscillating force (with frequency E/h) applied to points on orbit

$\mathbf{r}(\xi)$ with the full time-dependency specified by $\Psi(\mathbf{r}(\xi), t)$. The oscillating system is not restricted to orbit $\mathbf{r}(\xi)$, but the information about the oscillation provided by the wave function is limited to points on orbit $\mathbf{r}(\xi)$. The forced oscillation is comparable to the forced vibration of many mechanical systems, e.g., sound boards of string instruments, air columns in wind instruments, or metal plates showing Chladni figures.

This interpretation of $\Psi(\mathbf{r}(\xi), t)$ as description of a driven oscillation on orbit $\mathbf{r}(\xi)$ raises the question of a description of the resulting forced oscillation at points outside this orbit. It is unclear whether quantum mechanics can provide an answer to this question. If, however, a rotating wave solution (see Section 2.1) is employed as model of an orbiting electron, the full solutions $\phi(\mathbf{r}, t)$ and $\mathbf{A}(\mathbf{r}, t)$ for the system (including the "external" electric Coulomb potential) may well provide an answer.

At the time of writing, this full solution has not been computed. However, it might be possible to speculate about its features by considering an analogy with a two-dimensional bent waveguide, where the inner "reflecting boundary" corresponds to the external Coulomb potential with a nonlinear peak at its center, which (because of the non-linearity) reflects incoming electromagnetic waves (including longitudinal electromagnetic waves, see Section 2.1). The outer "reflecting boundary" of this waveguide corresponds to the orbiting rotating wave solution, which might appear to send out electromagnetic waves in phase with the incoming waves that where reflected at the external Coulomb potential. As in the case of bent waveguides, the space between these two "reflecting boundaries" might then be filled with a wave that moves along the orbit with a phase velocity higher than the speed of light, which might be the phase velocity of the electron's phase wave (see Section 2.3).

In contrast to the case of the previous section, it might be possible to simplify the simulation of this system, for example, by limiting the simulation to longitudinal waves of the electric potential $\phi(\mathbf{r}, t)$ and considering the whole orbit as an emitter of such waves with phases specified by $\Psi(\mathbf{r}(\xi), t)$ instead of simulating a moving rotating wave solution. Thus, it might be possible to remove the movement of the electron from the simulation while still simulating essential features of the system. Therefore, scenarios involving bound wave functions appear to be more suitable for numerical simulations based on the modified Born-Infeld electron model.

5 Conclusion

This work presents an interpretation of single-electron wave functions as statistical descriptions of ensembles of trajectories of a single electron. Based on this interpretation, a numerical simulation of hydrogen-like atoms in terms of modified Born-Infeld field theory is proposed to further investigate whether quantum-mechanical phenomena may be understood as emerging features of this field theory.

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A Revisions

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