# ELIMINATION OF ‘6’ AS THE SOLUTION TO HADWIGER-NELSON PROBLEM

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**Abstract:** Mathematicians are working day and night to solve problems like Riemann hypothesis but there are some handful amount of problems which are being ignored by the mass. Problems like this may hold low importance but still has some importance and who knows if this ‘some’ becomes a base for a huge discovery and in this paper, solution to such a problem has been addressed using a simple methodology.

**Introduction:** The **Hadwiger–Nelson problem**, named after Hugo Hadwiger and Edward Nelson, asks for the minimum number of required colors to color the points of any possible shape which lies at distance 1 unit from each other, such that no adjacent points have the same color[1]. In order to solve this problem, an extraordinary freedom has been given, through which the points can be colored in any possible sequential order. In layman’s term, in order to color the points, any possible ‘path’ can be maintained[1].

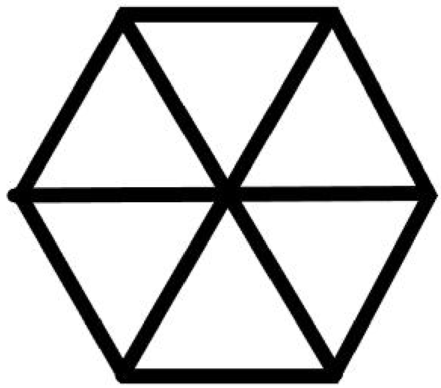
**Literature Review:** A lot of investigations has been done prior to this investigation and in those investigations, the investigators have shown that the possible minimum values of the solution to Hadwiger-Nelson problem are either 5, 6 or 7 because there are such tiles which are not possible to create in accordance with the conditions, with some certain number of colors ranging from 1 to 4. Again, it has been proven that all the possible shapes with points at one unit distance are possible to color with 7 and greater numbers of colors in any possible path maintained while coloring them[2]. In this paper, such a shape and ‘path’ has been discussed which can eliminate the solution which speaks of 6 to be the minimum value.

**Method :** The investigation follows a number of significantly easy steps-

**1)** At first 6 different colors were taken which will be used to color the points, the colors are red, orange, blue, pink, purple and green.

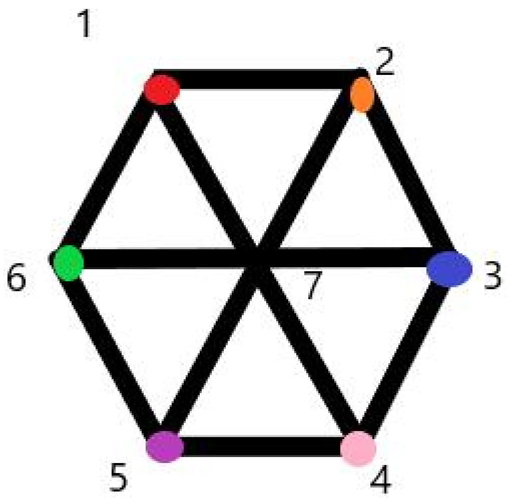
**2)** Then a regular hexagon was taken, here each of the 6 vertices lies at one unit distance from each other, since the sides are equal.

**3)** Then the opposite vertices were joined. The line segments bisect each other and intersect each other at the middle of the hexagon because by joining the opposite vertices, **"the hexagon has been turned into 6 congruent equilateral triangles."**



**4)** Since the intersection point lies at equal distance from each of the vertex, it can be also counted as a point. Thus, there are total 7 points in this hexagon, which satisfies the condition of 'one unit distance.'

**5)** Then, the point '1' was colored with red and the following 5 points were colored with the remaining 5 colors.



**6)** Now, it is found that there is no color left to color the 7th point. No matter with what color it is colored, a point with the same color will lie at one unit distance.

**Results:** After following the above described procedure, the coloration of the 7th point was not possible because no matter with what it was colored with, one of the adjacent point would have the same color and thus, it would be against the conditions. Hence, an conclusion can be reached which is the solution to this problem is not 6.

**Conclusion :** Thus, after following the mentioned procedures it can be said that, the solution to **Hadwiger–Nelson problem** can not be 6, either it is 5 or 7. This easy investigation might seem to have a a very small contribution but it was a great a leap to solve this problem. Now, the only question which remains is, whether the solution is 5 or not, after unraveling the truth, the mathematicians may finally have some rest. They are working day and night to solve the problem, many humungous tiles for it have been created too but still we have not been able to nullify it. But it can be hoped that the problem will be solved soon by someone else.

**Note:** This investigation used a very simple postulate in order to come to the conclusion, which is “A regular hexagon can be turned into 6 equilateral triangle by joining the opposite vertices.”

## Bibliography:


1. Coulson, D. (2004). On the chromatic number of plane tilings. *Journal of the Australian Mathematical Society*, *77*(2), 191–196. https://doi.org/10.1017/s1446788700013574

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