

Attempt at a Proof of Collatz Conjecture

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Abstract

This is an attempt of a proof of Collatz Conjecture.

A pair of natural numbers a and b that satisfy $[\alpha_N \cdot 3^a] = 2^b$ exist for all natural numbers N . However, α_N is a real number that satisfies $N \leq \alpha_N < N + 1$.

Now to modify the Collatz conjecture,

3 is multiplied a certain natural N , the exponentiating factor of maximum 2 of N is added. When repeated, this is perfectly equal to all natural numbers becoming 2^n .

Let's see an example Consider $N = 9$.

Naturally, it will progress in the following sequence 9 28 7 22 11 34 17 52 13 40 5 16 1. Here, instead of dividing by 2^k , adding 2^k leads to the following correspondence:

9 28 7 22 11 34 17 52 13 40 5 16 1

9 28 28 88 88 272 272 832 832 2560 2560 8192 8192

Naturally, 28 should be divided by 4, multiplied by 3, and added by 1; however, 88, which is the value obtained by multiplying 28 by 3 and then adding 4, is taken here. Here, 4 becomes the exponentiating factor of maximum 2 of 28.

Next, 22 should be divided by 2, multiplied by 3, and added by 1; however, it is substituted by 272, which is obtained by multiplying 88 by 3 and then adding 8 (the maximum factor of 88). Thus, the exponentiating factor of maximum 2 of 272 is 16.

Thus, $272 * 3 + 16 = 832$, $832 * 3 + 16 = 2560$, and $2560 * 3 + 16 = 8192$.

Further, since $8192 = 2^{13}$, the Collatz conjecture is satisfied.

Again,

A pair of natural numbers a and b that satisfy $[\alpha_N \cdot 3^a] = 2^b$ exist for all natural numbers. However, α_N is a real number that satisfies $N \leq \alpha_N < N + 1$. Let's assume that minimum a and b

that satisfy this for a certain N are a_0 and b_0 .

However, multiplying a certain N by 3 and adding 1 and then adding the exponential factor of maximum 2 of N is a monotonously increasing function; at the same time, each term obtains an increasingly larger exponentiate of 2 as a factor. Moreover, among the numbers up to that value, the largest exponentiate of 2 is obtained as the factor.

Therefore, after conducting this process a_0 times, 2^{b_0} is obtained as the factor and having a factor other than 2 in $N \leq a_N < N + 1$ constitutes a contradiction.

When stating the value of multiplying a certain natural number N by 3, adding 1, and then adding the exponential factor of maximum 2 of N as is $N_b, 3N_b < N_{b+1} < 5N_b$ established. Further, N_{b+1} needs to have an exponentiate of 2 that is larger than 2^b as a factor, which is satisfied only by

$$N_b = 2^b, N_{b+1} = 2^{b+2}.$$

Thus, when multiplying N by 3, adding 1, and then adding the exponential factor of maximum 2 of N a_0 times or less, the outcome becomes an exponential form of 2 that is 2^{b_0} or lower.

Let's put this into the form of the original conjecture: for every natural number, multiplication by 3 and addition of 1 a_0 times when N is odd and dividing it by 2 b_0 times when N is even results in 1.

Therefore, the Collatz conjecture is true.